Galileo Masterclass Brazil (GMB) 2022

Lab 1 - Code and Correlation

Felix Antreich

GALILEO INFORMATION CENTRE Brazil

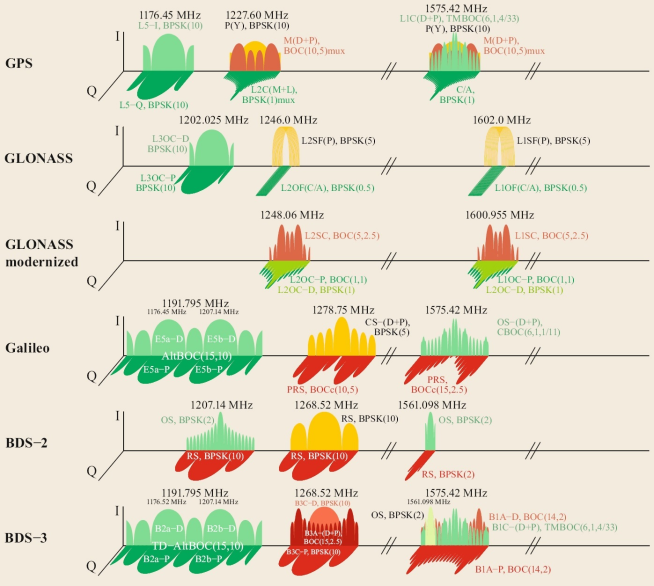
GALILEO EGNOS

Outline

GNSS Frequency Bands

GPS L1 C/A Code Signal

Current GNSS Signals



Source: Springer Handbook of Global Navigation Satellite Systems, P. J. Teunissen and O. Montenbruck, Eds. Springer International Publishing, 2017.

Outline

GNSS Frequency Bands

GPS L1 C/A Code Signal

Overview

- L1 C/A code signal used to be only the acquisition signal for the GPS military service
- After 2000 the encription on the signal was removed and thus the signal became available for commercial and private users
- Carrier frequency: $f_c = 1575.42$ MHz (so-called L1 carrier)
- PR sequence: Gold codes with $N_d = 1023$, $T_d = 1\ ms$, and $T_c = \frac{0.001s}{1023} \approx 977.52\ ns$
- Navigation data rate is $50Hz$
- Maximum available bandwidth: $B_{max} \approx 30\ MHz$

Chip Pulse Shape (1)

The rectangular chip pulse shape can be described by

$$p_{\sqcap}(t) = \frac{1}{\sqrt{T_c}} \left(U(t + \frac{T_c}{2}) - U(t - \frac{T_c}{2}) \right),$$

where $U(t)$ denotes the unit step or Heaviside's unit step function

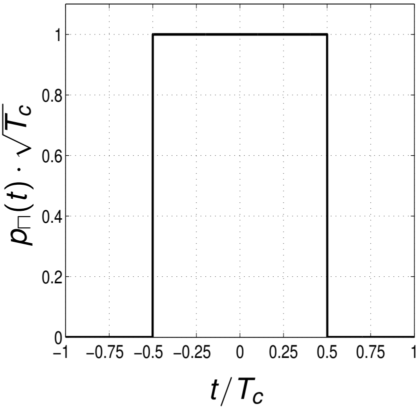
$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases},$$

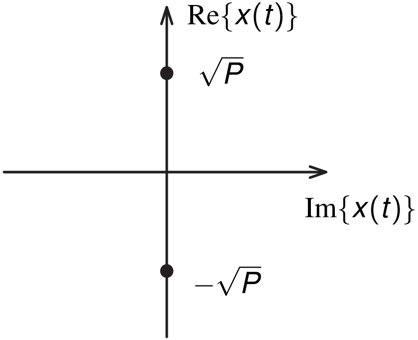
and

$$\int_{-\infty}^{\infty} |p_{\sqcap}(t)|^2 \, dt = 1$$

- The rectangular chip pulse shape can be considered as the *classical* chip pulse shape, which originally was used for early spread spectrum signals
- We get a binary phase shift keying (BPSK) signal

Chip Pulse Shape (2)





Fourier Transform and Autocorrelation

The Fourier transform of $p_\sqcap(t)$ reads

$$P_\sqcap(f) = \frac{\sqrt{T_c}\,\sin(\pi f T_c)}{\pi f T_c} = \sqrt{T_c}\,\mathrm{sinc}(f T_c)$$

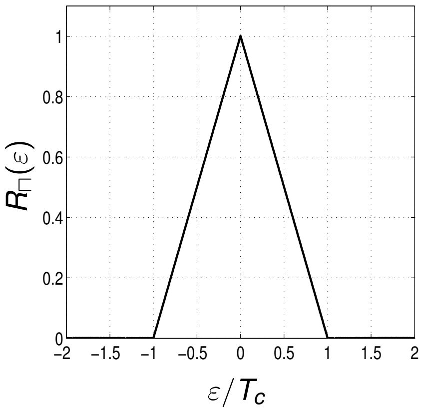
Here, the sinc function is defined as

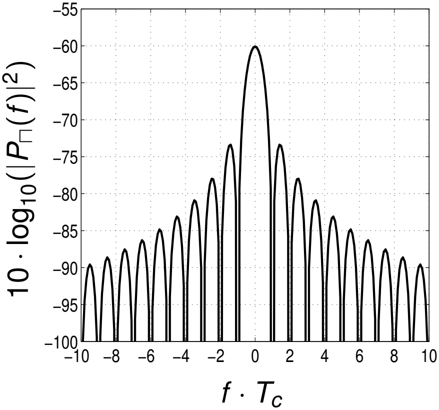
$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

The autocorrelation function can be given as

$$\begin{aligned} R_\sqcap(\varepsilon) &= \int_{-\infty}^{\infty} |P_\sqcap(f)|^2\, \mathrm{e}^{\mathrm{j}2\pi f \varepsilon}\, df = \int_{-\infty}^{\infty} T_c\, \mathrm{sinc}^2(f T_c)\, \mathrm{e}^{\mathrm{j}2\pi f \varepsilon}\, df \\ &= \int_{-\infty}^{\infty} p_\sqcap(t) p_\sqcap(t+\varepsilon)\, dt = \begin{cases} 1 - \frac{|\varepsilon|}{T_c}, & |\varepsilon| < T_c \\ 0, & \mathrm{else} \end{cases} . \end{aligned}$$

Autocorrelation and Power Spectral Density (PSD)



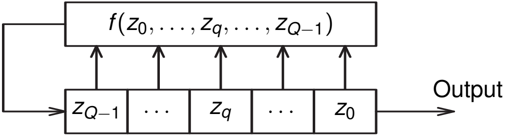


$$B \to \infty$$

PR Sequence Generator (1)

- The PR sequences (Gold codes) can be generated with two linear feedback shift registers (LFSR)
- A LFSR is a shift register whose input binary state (0 or 1) is a linear function of its previous states
- A Q-stage shift register consists of Q consecutive two-state stages (flip-flops) driven by a clock
- At each pulse of the clock the state of each stage is shifted to the next stage in line to the right of the register
- In order to convert the Q-stage shift register into a sequence generator a feedback loop is incorporated, which calculates a new term for the left-most stage, based on the states of the Q previous states
- At the right-most stage of the register the generated sequence is outputted

PR Sequence Generator (2)



The boolean feedback function can be expressed as a modulo 2 sum of the Q-stages $z_q \in \{0, 1\}$ element-wise multiplied by feedback coefficients $c_q \in \{0, 1\}$

$$f(z_0, \ldots, z_q, \ldots z_{Q-1}) = c_0 z_0 \oplus \ldots \oplus c_q z_q \oplus \cdots \oplus c_{Q-1} z_{Q-1}$$

where

$\oplus$	0	1
0	0	1
1	1	0

$\cdot$	0	1
0	0	0
1	0	1

- The content of the Q stages is called a state of the shift register
- $(\tilde{a}_0, \ldots, \tilde{a}_q, \ldots, \tilde{a}_{Q-1})$ is called the initial state of the shift register which generates the sequence $\{\tilde{a}_m\}$

PR Sequence Generator (3)

- Gold sequences are produced by modulo 2 sum of two sequences each of length $M = 2^n - 1$ in their various phases
- For the GPS C/A L1 signal the PR sequences are Gold codes with $M = N_d = 1023$ and $n = 10$
- The PR sequences are generated with two LFSRs with $Q = 10$ stages
- the initial states of the LFSRs is $(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$

The feedback functions for the two LFSRs are

$$f_1(z_{1,0}, \dots, z_{1,9}) = z_{1,0} \oplus z_{1,7}$$
$$f_2(z_{2,0}, \dots, z_{2,9}) = z_{2,0} \oplus z_{2,1} \oplus z_{2,2} \oplus z_{2,4} \oplus z_{2,7} \oplus z_{2,8}.$$

PR Sequence Generator (4)

To generate the PR sequence the states of the LFSRs are connected as

$$\tilde{d}_m = z_{1,0} \oplus z_{2,a} \oplus z_{2,b}$$

with $\tilde{d}_m \in \{0, 1\}$ and

$$d_m = -2\tilde{d}_m + 1.$$

The phases $z_{2,a}$ and $z_{2,b}$ are selected to produce the respective PR sequence for each satellite.

PR Sequence Generator (5)

PR sequence	a	b
1	8	4
2	7	3
3	6	2
4	5	1
5	9	1
6	8	0
7	9	2
8	8	1
9	7	0
10	8	7
11	7	6
12	5	4
13	4	3
14	3	2
15	2	1
16	1	0

PR sequence	a	b
17	9	6
18	8	5
19	7	4
20	6	3
21	5	2
22	4	1
23	9	7
24	6	4
25	5	3
26	4	2
27	3	1
28	2	0
29	9	4
30	8	3
31	7	2
32	6	1