Galileo Masterclass Brazil (GMB) 2022

Lecture 5 - Propagation Aspects

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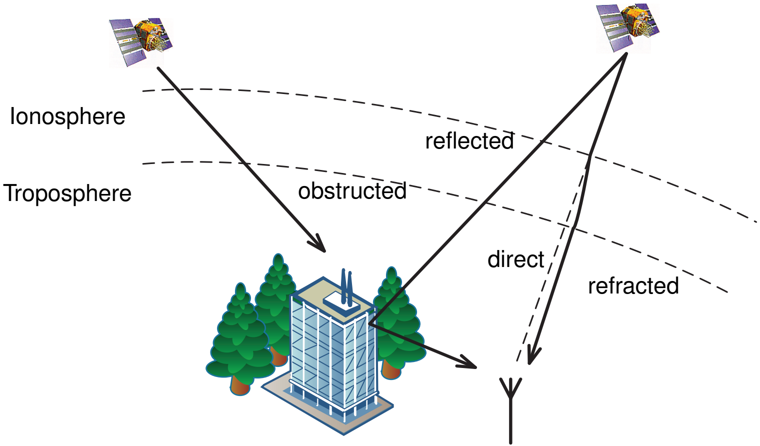
Outline

Propagation Effects



Passband Signal

The passband signal of one satellite can be given as

$$\begin{aligned}\tilde{s}(t) &= \sqrt{2}\mathrm{Re}\left\{s(t)\mathrm{e}^{\mathrm{j}2\pi f_c t}\right\} \\ &= \sqrt{2}s_I(t)\cos(2\pi f_c t) - \sqrt{2}s_Q(t)\sin(2\pi f_c t)\end{aligned}$$

with carrier frequency f_c and the equivalent baseband signal

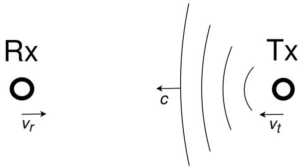
$$s(t) = s_I(t) + \mathrm{j}s_Q(t).$$

- In the following we do not consider free space loss (the link budget was discussed in lecture 6)
- The different effects are discussed separately, but in reality are to be considered in a cascaded form

Outline

Frequency Shift (1)

The Doppler effect (or the Doppler shift) is the change in frequency or wavelength of a wave (or other periodic event) for an observer moving relative to its source.



In case a receiver and a transmitter are moving towards each other the shift between emitted frequency f and observed frequency $\tilde{f}$ for an electromagnetic wave in vacuum can be given as

$$\tilde{f} = f \frac{\sqrt{1 - \frac{(v_r + v_t)^2}{c^2}}}{1 - \frac{(v_r + v_t)}{c}} = f \frac{\sqrt{1 - \frac{\Delta v^2}{c^2}}}{1 - \frac{\Delta v}{c}}$$

Frequency Shift (2)

In case $\Delta v \ll c$ and $\frac{\Delta v}{c} < 1$ we can write

$$\tilde{f} = f\frac{\sqrt{1 - \frac{\Delta v^2}{c^2}}}{1 - \frac{\Delta v}{c}} \approx f\frac{1}{1 - \frac{\Delta v}{c}} = f\sum_{n=0}^{\infty}\left(\frac{\Delta v}{c}\right)^n$$

using

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \,, \; \forall_x |x| < 1$$

As $\Delta v \ll c$ we get

$$\tilde{f} = f\sum_{n=0}^{\infty}\left(\frac{\Delta v}{c}\right)^n \approx f(1 + \frac{\Delta v}{c})$$

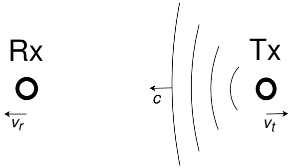
Thus, the Doppler shift can be given as

$$f_D = f\,\frac{\Delta v}{c}$$

Frequency Shift (3)

In case a receiver and a transmitter are moving away from each other the shift between emitted frequency f and observed frequency $\tilde{f}$ of a electromagnetic wave in vacuum can be given as

$$\tilde{f} = f\frac{\sqrt{1-\frac{(v_r+v_t)^2}{c^2}}}{1+\frac{(v_r+v_t)}{c}} = f\frac{\sqrt{1-\frac{\Delta v^2}{c^2}}}{1+\frac{\Delta v}{c}}$$



Frequency Shift (4)

In case $\Delta v \ll c$ and $\frac{\Delta v}{c} < 1$ we can write

$$\tilde{f} = f \frac{\sqrt{1 - \frac{\Delta v^2}{c^2}}}{1 + \frac{\Delta v}{c}} \approx f \frac{1}{1 + \frac{\Delta v}{c}} = f \sum_{n=0}^{\infty} (-1)^n \left(\frac{\Delta v}{c} \right)^n$$

using

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n \, x^n \, , \; \forall_x |x| < 1$$

As $\Delta v \ll c$ we get

$$\tilde{f} = f \sum_{n=0}^{\infty} (-1)^n \left(\frac{\Delta v}{c} \right)^n \approx f(1 - \frac{\Delta v}{c})$$

Thus, the Doppler shift can be given as

$$f_D = -f \, \frac{\Delta v}{c}$$

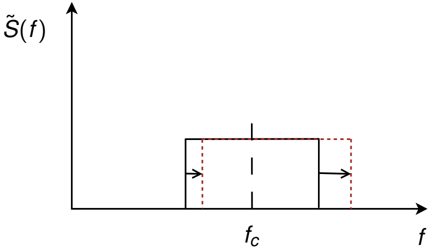
Doppler Effect on a Passband Signal

Doppler shift of complete frequency support of signal $\tilde{s}(t)$

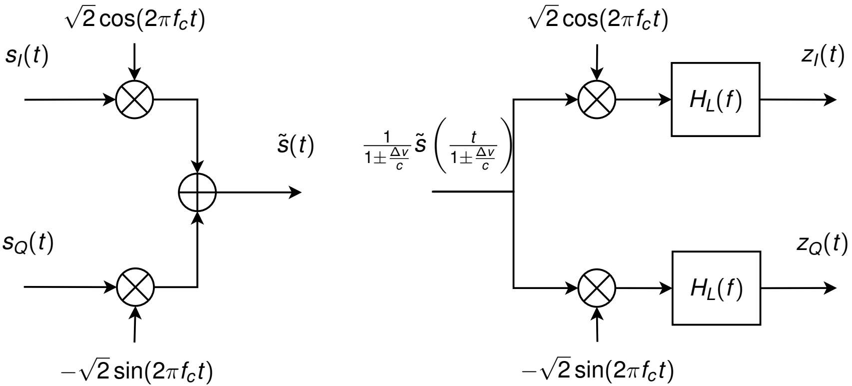
$$\left|\frac{1}{1 \pm \frac{\Delta v}{c}}\right| s\left(\frac{t}{1 \pm \frac{\Delta v}{c}}\right) \circ\!\!-\!\!\bullet \; \tilde{S}\left(f\left(1 \pm \frac{\Delta v}{c}\right)\right)$$

$$|a|\, \tilde{s}(a\,t) \circ\!\!-\!\!\bullet \; \tilde{S}\left(\frac{f}{a}\right)$$

Thus, higher frequencies are affected more than lower frequencies



Transmitter and Baseband Receiver Model



$$H_L(f) = \begin{cases} 1 & , |f| \leq \tilde{B} \\ 0 & , |f| > \tilde{B} \end{cases}$$

The bandwidth $\tilde{B}$ needs to be sufficiently large to cover the signal bandwidth B plus the Doppler effect

Transmitter and Baseband Receiver Model, cont'd

We can derive the inphase and quadrature component of the baseband received signal as

$$z_I(t) = |a|\, s_I(a\,t) \cos(2\pi f_c t(a-1)) - |a|\, s_Q(a\,t) \sin(2\pi f_c t(a-1))$$
$$z_Q(t) = |a|\, s_I(a\,t) \sin(2\pi f_c t(a-1)) + |a|\, s_Q(a\,t) \cos(2\pi f_c t(a-1))$$

with

$$a = \frac{1}{1 \pm \frac{\Delta v}{c}}.$$

Thus, we can write

$$\begin{aligned} z(t) = z_I(t) + \mathrm{j}\, z_Q(t) &= |a|\, s_I(a\,t)\, \mathrm{e}^{\mathrm{j}2\pi f_c t(a-1)} + \mathrm{j}|a|\, s_Q(a\,t)\, \mathrm{e}^{\mathrm{j}2\pi f_c t(a-1)} \\ &= |a|\, s_I(a\,t)\, \mathrm{e}^{\mathrm{j}2\pi\left(\pm f_c \frac{\Delta v}{c}\right)at} + \mathrm{j}|a|\, s_Q(a\,t)\, \mathrm{e}^{\mathrm{j}2\pi\left(\pm f_c \frac{\Delta v}{c}\right)at} \\ &\approx s_I(t)\, \mathrm{e}^{\mathrm{j}2\pi\left(\pm f_c \frac{\Delta v}{c}\right)t} + \mathrm{j}\, s_Q(t)\, \mathrm{e}^{\mathrm{j}2\pi\left(\pm f_c \frac{\Delta v}{c}\right)t} \end{aligned}$$

However, this approximation is only valid iff $1 \pm \frac{\Delta v}{c} \approx 1$ and the observation interval of the signal is short enough

Outline

Structure of the Ionosphere (1)

- The ionosphere is a shell of electrons, electrically charged atoms, and molecules that surrounds the earth, stretching from a height of about 50 km to more than 1000 km
- Ultraviolet radiation of the sun breaks the bonds relating the electrons to the atoms when traversing the upper atmosphere
- Thus, there is a large number of free electrons and ions in the ionosphere
- The free electrons in the ionosphere affect propagation of radio waves
- An (partially) ionized gas is called plasma

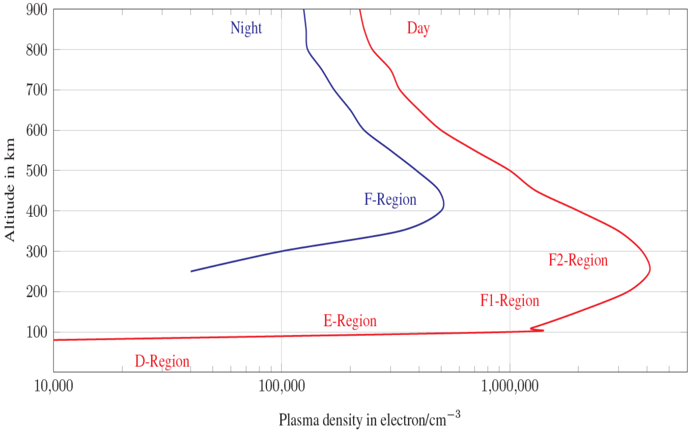
Structure of the Ionosphere (2)

- At frequencies below about 30 MHz the ionosphere acts like a mirror reflecting radio waves back to earth $\rightarrow$ long distance communication
- The speed of a radio wave in the ionosphere is determined by the density of the electrons
- Electron density is quantified by counting the number of electrons in a vertical column with a cross-sectional area of 1 m^2 $\rightarrow$ this number is called total electron content (TEC)
- TEC is a function of the amount of incident solar radiation
- On the night side of earth free electrons have a tendency to recombine with ions $\rightarrow$ reducing TEC
- TEC above a particular location of the earth has strong diurnal variation

Structure of the Ionosphere (3)

- Changes in the TEC can also occur on much shorter time scales
- One of the phenomena responsible for such changes is the traveling ionospheric disturbance (TID)
- TIDs are manifestations of waves in the upper atmosphere believed to be caused in part by severe weather fronts and volcanic eruptions
- There are also seasonal variations in TEC and variations that follow the sun´s 27-day rotational period and the 11-year cycle of the solar activity

Layers of the Ionosphere



- D-Region: 50-90 km, very low electron density
- E-Region: 90-125 km, vanishes rapidly after dark
- F-Region: 125-1000 km, peak electron density at 200 -400 km

Propagation Through the Ionosphere

Propagation of the signal through the ionosphere can be approximated by the transfer function

$$H_i(f) = A_i\, \mathrm{e}^{-\mathrm{j}2\pi f(\tau_i(f)+\tau_v)} = A_i\, \mathrm{e}^{\mathrm{j}\phi_i(f)}$$

- A_i: amplitude response (attenuation of the signal is negligible for frequencies way above 30 MHz)
- $\phi_i(f)$: phase response
- τ_v: delay of the signal in vacuum
- $\tau_i(f)$: delay introduced by the ionosphere

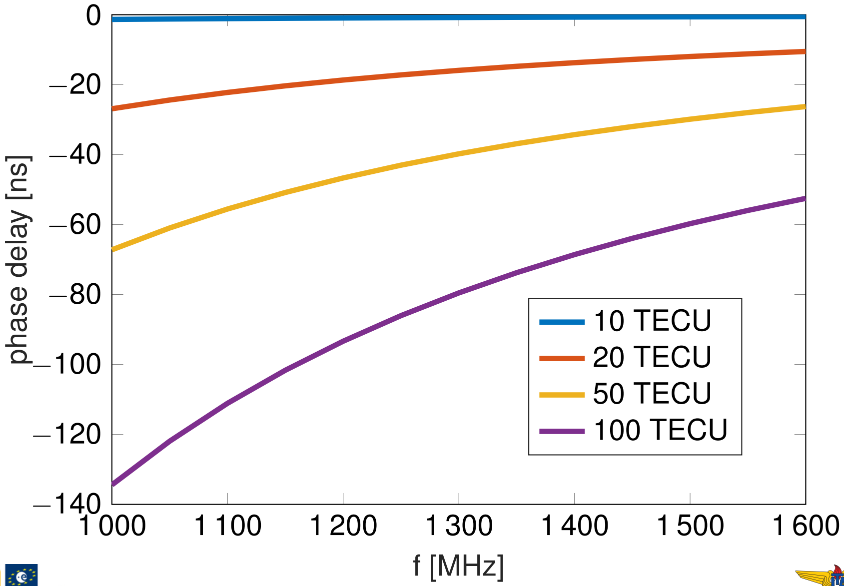
In a first order approximation we can write

$$\tau_i(f) \approx -\frac{\frac{r_e c^2}{\pi} \int N_e dl}{c\, f^2} \approx -\frac{40.3\,\mathrm{m}^3\mathrm{s}^{-2} \int N_e dl}{c\, f^2}$$

- c: speed of light
- r_e: electron radius
- N_e: electron density (electrons m^{-3})

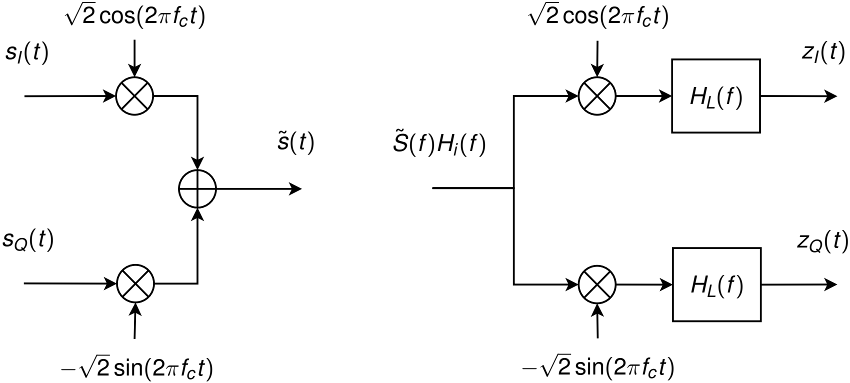
The integral of N_e along the raypath l defines slant total electron content (STEC) in units of TEC (UTEC, 10^{16} electrons m^{-2})

Phase Delay



0
−20
−40
−60
−80
−100
−120
−140
phase delay [ns]
1 000
1 100
1 200
1 300
1 400
1 500
1 600
f [MHz]
10 TECU
20 TECU
50 TECU
100 TECU

Transmitter and Baseband Receiver Model (1)



$$H_L(f) = \begin{cases} 1 & , |f| \leq B \\ 0 & , |f| > B \end{cases}$$

The baseband equivalent signal of $\tilde{s}(t)$ is assumed to be strictly bandlimited to B.

Transmitter and Baseband Receiver Model (2)

We can derive the inphase and quadrature component of the baseband received signal as

$$z_I(t) \circ\!\!-\!\!\bullet \frac{1}{2}(S_I(f) - \mathrm{j}S_Q(f))H_i(f - f_c) + \frac{1}{2}(S_I(f) + \mathrm{j}S_Q(f))H_i(f + f_c)$$
$$z_Q(t) \circ\!\!-\!\!\bullet \frac{\mathrm{j}}{2}(S_I(f) - \mathrm{j}S_Q(f))H_i(f - f_c) - \frac{\mathrm{j}}{2}(S_I(f) + \mathrm{j}S_Q(f))H_i(f + f_c)$$

Thus, the complex baseband equivalent signal can be written

$$z(t) = z_I(t) + \mathrm{j}z_Q(t) \circ\!\!-\!\!\bullet (S_I(f) + \mathrm{j}S_Q(f))H_i(f + f_c)$$

Narrowband Approximation (1)

Suppose a quasi-sinusoidal signal $\tilde{s}(t) = a(t)\cos(2\pi f_c t + \varphi)$[1] is propagating through the ionosphere. At the output of $H_i(f)$ we get

$$\tilde{S}(f)H_i(f) \bullet\!\!-\!\!\circ\, z(t) \approx A_i\, a(t-\tau_g)\cos(2\pi f_c(t-\tau_p)+\varphi)$$

where the group delay for $\tau_v = 0$ is given as

$$\tau_g = -\frac{\partial\phi_i(f)}{2\pi\;\partial f}\bigg|_{f=f_c} = -\frac{\partial}{\partial f}\frac{40.3\,\mathrm{m}^3\mathrm{s}^{-2}\;\int N_e dl}{c\,f}\bigg|_{f=f_c} = -\tau_i(f_c)$$

and the phase delay is given as

$$\tau_p = -\frac{\phi_i(f_c)}{2\pi\;f_c} = -\frac{40.3\,\mathrm{m}^3\mathrm{s}^{-2}\;\int N_e dl}{c\,f_c^2} = \tau_i(f_c)$$

[1] its amplitude envelop a(t) is changing slowly in relation to the frequency f_c of the sinusoid, $|\partial \log(a(t))/\partial t| << 2\pi f_c$

Narrowband Approximation (2)

- The carrier of the signal undergoes an apparent phase advance while the complex envelope of the signal undergoes an apparent delay with respect to propagation in vacuum
- The carrier appears to be propagating faster while the complex envelope appears to propagate slower than it would through vacuum
- Since TEC varies over time and the signal eventually passes through different paths in the ionosphere the signal experiences variations of phase delay and group delay $\rightarrow$ this phenomenon in satellite navigation is called code-carrier divergence

Estimation of STEC

The difference in phase delay for two signals at two different frequencies (f_1 and f_2) passing through the same path (channel) $h_i(t)$ is

$$\begin{aligned}\Delta_{f_1-f_2} &= \frac{40.3\,\mathrm{m}^3\mathrm{s}^{-2} \int N_e dl}{c}\left(\frac{1}{f_2^2}-\frac{1}{f_1^2}\right)\\ &= \frac{40.3\,\mathrm{m}^3\mathrm{s}^{-2} \int N_e dl}{c}\left(\frac{f_1^2-f_2^2}{f_1^2 f_2^2}\right)\end{aligned}$$

Knowing the two frequencies f_1 and f_2 and measuring the difference in phase delay $\Delta_{f_1-f_2}$ we can estimate $\int N_e dl$ or STEC $\rightarrow$ correction of ionosphere effects (phase and group delay)

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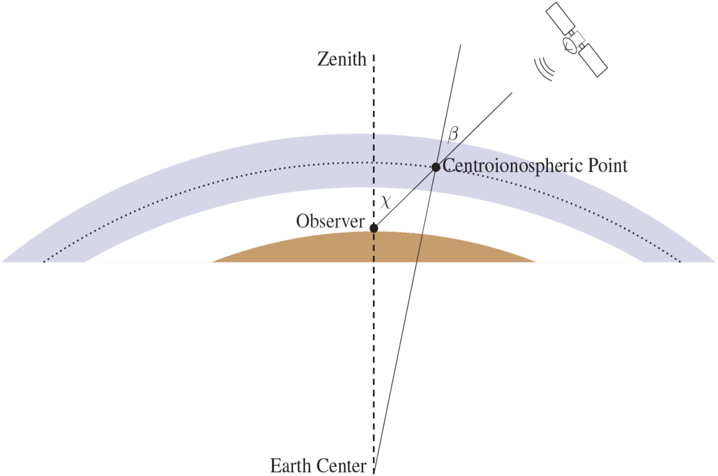
Ionospheric Mapping Function



courtesy of Friederike Fohlmeister, German Aerospace Center (DLR)

Ionospheric Mapping Function, cont 'd

Based on the measured STEC and assuming that the ionosphere is a thin layer at 400 km height, one can derive vertical TEC by

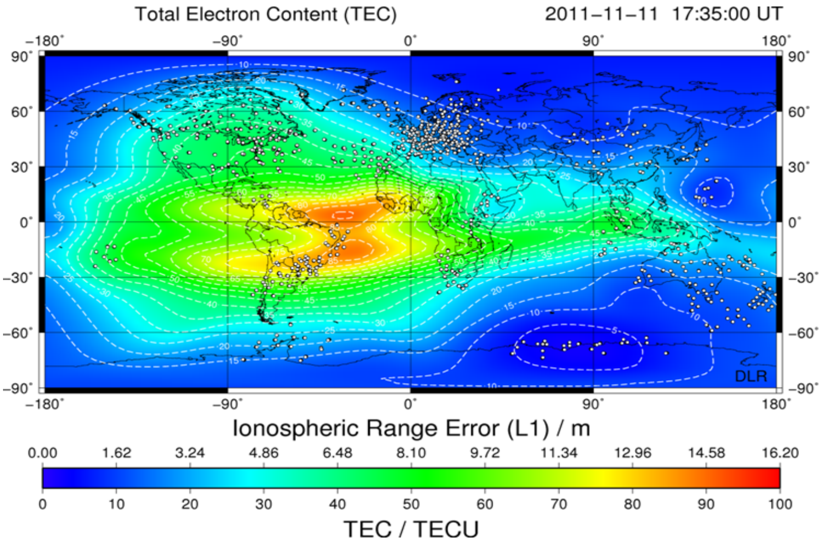
$$N_e^v = N_e \sqrt{1 - \frac{\sin^2(\chi)}{(1 + h/R)^2}}$$

with

$$\sin(\beta) = \frac{\sin(\chi)}{(1 + h/R)}$$

- h: height above Earth (400 km)
- R: Earth radius (average earth radius: 6371 km)

TEC Maps

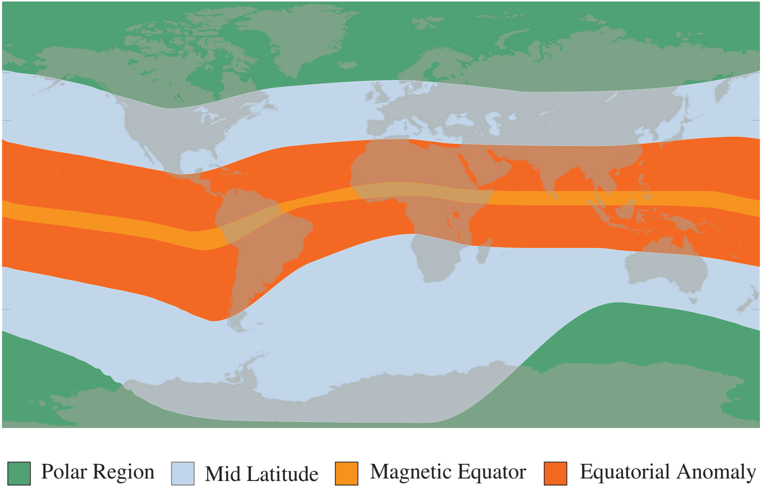


courtesy of German Aerospace Center (DLR), http://swaciweb.dlr.de/

Ionospheric Scintillation Effects

- Equatorial region: Flow inversion of the equatorial plasma during evening hours (dusk) leads to Rayleigh-Taylor instabilities (RTI) and plasma bubbles cause amplitude and phase scintillations
- Strong amplitude fading with deep fades of up to 15 - 20dB are possible (amplitude scintillations)
- Polar region: geomagnetic storms cause phase scintillation
- Both amplitude and phase scintillations can cause outage or errors in positioning (loss of lock signal tracking) or data transmission

Intensity of Ionospheric Scintillations



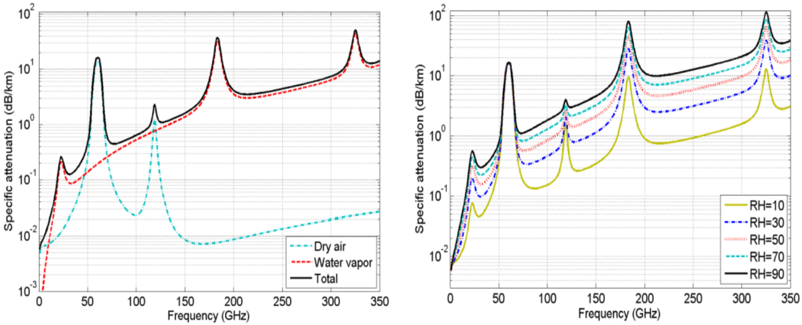
courtesy of Friederike Fohlmeister, German Aerospace Center (DLR)

Outline

Structure of the Troposphere

- Propagation in the troposphere for L-band signals (ca. 1000 - 1800 MHz) is mainly characterized by the tropospheric delay
- Attenuation of L-band signals is rather small
- Delay is caused by natural gases in the atmosphere
- Gases extend beyond the boundary of the troposphere at 9-16 km above see level
- Tropospheric propagation is non-dispersive for L-band signals, i.e. phase and group delay of the signal stay the same when propagation through the troposphere
- The propagation speed of the complex envelop and the carrier of the signal are lower than that in vacuum
- Tropospheric delay is dependent on the dry gases and water vapor

Attenuation



(a) (b)

RH: relative humidity

Taken from A. Mohammed Al-Saegh, A. Sali, J. S. Mandeep, A. Ismail, A. H.J. Al-Jumaily and C. Gomes,"Atmospheric Propagation Model for Satellite Communications", in "MATLAB Applications for the Practical Engineer", K. Bennett (Ed.), InTech, 2014.

Propagation Through the Troposphere (1)

Propagation of the signal through the troposphere can be approximated by the transfer function

$$H_t(f) = A_t \, \mathrm{e}^{-\mathrm{j}2\pi f(\tau_t+\tau_v)} = A_t \, \mathrm{e}^{\mathrm{j}\phi_t}$$

- A_t: amplitude response
- ϕ_t: phase response
- τ_v: delay of the signal in vacuum
- τ_t: delay introduced by the troposphere

For the zenith (elevation $\vartheta = 90^\circ$ wrt horizon) delay τ_t^z we can write

$$\tau_t^z = \tau_h^z + \tau_w^z$$

- τ_h^z: hydrostatic zenith delay
- τ_w^z: wet zenith delay

Propagation Through the Troposphere (2)

Where following the derivation of Leick[2] we get

$$\tau_h^z = 10^{-6} \int N_d(h)dh$$

$$\tau_w^z = 10^{-6} \int N_w(h)dh$$

with

$$N_d(h) \approx k_1 \frac{p(h)}{T(h)}$$

$$N_w(h) \approx k_2 \frac{p_w(h)}{T(h)} + k_3 \frac{p_w(h)}{T^2(h)}$$

- $p(h)$: total atmospheric pressure at height h [mbar]
- $T(h)$: absolute temperature in Kelvin [K]
- $p_w(h)$: partial pressure of water vapor [mbar]
- k_1, k_2, k_3: physical constants based on theory and on experiments[3]

[2] Leick, A., 1994. GPS Satellite Surveying. Wiley-Interscience Publication, USA.

[3] $k_1 = 77.60$ K/mbar, $k_2 = 22.10$ K/mbar, $k_3 = 370100$ K^2/mbar

Model Zenith Delay

Saastamoine's model[4] for τ_h^z can be given as

$$\tau_h^z = \frac{1}{c}\frac{2.2768 \cdot 10^{-3}\,\mathrm{m\ mbar}^{-1}\ p_0}{1 - 2.66 \cdot 10^{-3}\cos(2\theta) - 2.8 \cdot 10^{-7}\,\mathrm{m}^{-1}\ h}$$

- p_0: total pressure at orthometric height h [mbar]
- θ: latitude

Following the model of Mendes and Langley[5] for τ_w^z we get

$$\tau_w^z = \frac{1}{c}1.22 \cdot 10^{-2}\,\mathrm{m} + 9.43 \cdot 10^{-3}\mathrm{m\ mbar}^{-1} p_{w,0}$$

- $p_{w,0}$: surface partial water vapor pressure [mbar]

[4] J. Saastamoinen, "Atmospheric Correction for the Troposphere and Stratosphere in Radio Ranging of Satellites," in The use of Artificial Satellites for Geodesy, Geophys. Monogr., AGU vol. 15, pp. 247-251, 1972.

[5] V. B. Mendes and R. B. Langley,"Tropospheric Zenith Delay Prediction Accuracy for High-Precision GPS Positioning and Navigation", Navigation, 46: 25–34, 1999.

Tropospheric Mapping Function (1)

Using a mapping $m_h(\vartheta)$ for the hydrostatic delay and a mapping function $m_w(\vartheta)$ we can write for the wet delay

$$\tau_t = \tau_h + \tau_w = m_h(\vartheta)\tau_h^z + m_w(\vartheta)\tau_w^z$$

Applying Niell's [6] mapping function we get

$$m_h(\vartheta) = \frac{1 + \frac{a}{1+\frac{b}{(1+c)}}}{\cos(\vartheta) + \frac{a}{\cos(\vartheta)+\frac{b}{\cos(\vartheta)+c}}} + \frac{h}{1000}\left(\frac{1}{\cos(\vartheta)} - \frac{1 + \frac{a_h}{1+\frac{b_h}{(1+c_h)}}}{\cos(\vartheta) + \frac{a_h}{\cos(\vartheta)+\frac{b_h}{\cos(\vartheta)+c_h}}}\right)$$

[6] A. E. Niell,"Global Mapping Functions for the Atmospheric Delay at Radio Wavelengths", J. Geophys. Res., 101(B2):3227–3246, 1996

Tropospheric Mapping Function (2)

and

$$m_w(\vartheta) = \frac{1 + \frac{a}{1+\frac{b}{(1+c)}}}{\cos(\vartheta) + \frac{a}{\cos(\vartheta)+\frac{b}{\cos(\vartheta)+c}}}$$

where before substitution the coefficients a, b, and c in $m_h(\vartheta)$ they must be corrected for periodic terms following the general formula

$$a(\vartheta, D) = \tilde{a} - a_p \cos\left(2\pi \frac{D - D_0}{365.25}\right)$$

- D: day of the year
- D_0: 28 or 211 for stations/users in the Southern or Northern Hemisphere

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Tropospheric Mapping Function (3)

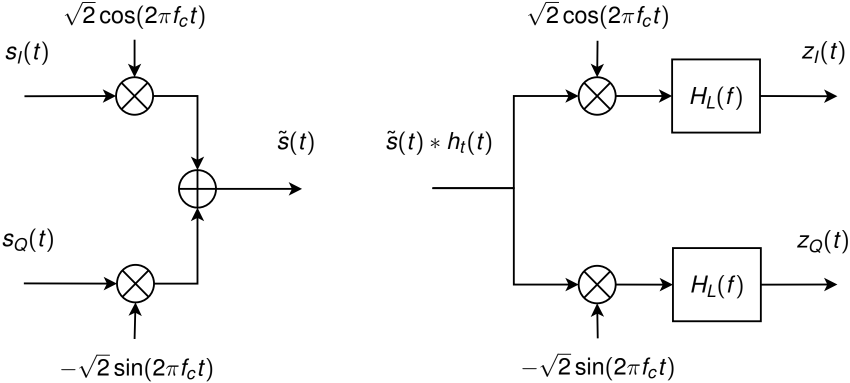
The coefficients for $m_h(\vartheta)$ are

ϑ	$\tilde{a} \cdot 10^3$	$\tilde{b} \cdot 10^3$	$\tilde{c} \cdot 10^3$	$a_p \cdot 10^5$	$b_p \cdot 10^5$	$c_p \cdot 10^5$
15°	1.2769934	2.9153695	62.610505	0	0	0
30°	1.2683230	209152299	62.837393	1.2709626	2.1414979	9.0128400
45°	102465397	209288445	63.721774	2.6523662	3.0160779	4.3497037
60°	102196049	209022565	63.824265	3.4000452	7.2562722	84.795348
75°	102045996	2.9024912	64.258455	4.1202191	11.723375	170.37206
	$a_h \cdot 10^5$	$b_h \cdot 10^5$	$c_h \cdot 10^5$			
	2.53	5.49	1.14			

and for $m_w(\vartheta)$ are

ϑ	$a \cdot 10^4$	$b \cdot 10^3$	$c \cdot 10^2$
15°	5.8021897	1.4275268	4.3472961
30°	5.6794847	1.5138625	4.6729510
45°	5.8118019	1.4572752	4.3908931
60°	5.9727542	1.5007428	4.4626982
75°	6.1641693	1.7599082	5.4736038

Transmitter and Baseband Receiver Model (1)



$$H_L(f) = \begin{cases} 1 & , |f| \leq B \\ 0 & , |f| > B \end{cases}$$

The baseband equivalent signal of $\tilde{s}(t)$ is assumed to be strictly bandlimited to B.

Transmitter and Baseband Receiver Model (2)

We can derive the inphase and quadrature component of the baseband received signal as

$$
\begin{aligned}
z_I(t) &= [(s(t) * h_t(t))\cos(2\pi f_c t)] * h_L(t) \\
&\circ\!\!-\!\!\!-\!\!\bullet \frac{1}{2}(S_I(f) - \mathrm{j}S_Q(f))H_t(f - f_c) + \frac{1}{2}(S_I(f) + \mathrm{j}S_Q(f))H_t(f + f_c) \\
z_Q(t) &= -[(s(t) * h_t(t))\sin(2\pi f_c t)] * h_L(t) \\
&\circ\!\!-\!\!\!-\!\!\bullet \frac{\mathrm{j}}{2}(S_I(f) - \mathrm{j}S_Q(f))H_t(f - f_c) - \frac{\mathrm{j}}{2}(S_I(f) + \mathrm{j}S_Q(f))H_t(f + f_c)
\end{aligned}
$$

Thus, the complex baseband equivalent signal can be written as

$$
\begin{aligned}
z(t) &= z_I(t) + \mathrm{j}z_Q(t) \\
&= (s_I(t) + \mathrm{j}s_Q(t)) * h_t(t)\, \mathrm{e}^{-\mathrm{j}2\pi f_c t} \\
&= (s_I(t) + \mathrm{j}s_Q(t)) * A_t\, \delta(t - (\tau_t + \tau_v))\, \mathrm{e}^{-\mathrm{j}2\pi f_c(\tau_t + \tau_v)} \\
&\circ\!\!-\!\!\!-\!\!\bullet (S_I(f) + \mathrm{j}S_Q(f))H_t(f + f_c)
\end{aligned}
$$

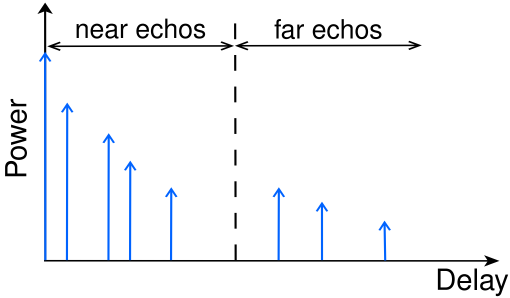
Outline

Multipath Characteristics (1)

- Reflected or diffracted replicas of the desired signal
- The path traveled by a reflection is always longer than the direct path (line-of-sight, LOS)
- Multipath arrivals are delayed relative to the LOS signal and they usually have less power than the LOS signal
- In case the multipath delay is large (e.g. greater than twice the spreading code symbol period), a GNSS receiver can readily resolve the multipath
- The direct path can be strongly attenuated (shadowing), e.g. when the direct path propagates through foliage or a structure
- In such cases multipath power could be even stronger than LOS signal power (indoor or outdoor)

Multipath Characteristics (2)

- Multipath signals with shorter relative delay with respect to the LOS signal will be influencing the ranging performance
- Multipath signals with longer delay will not influence the ranging performance



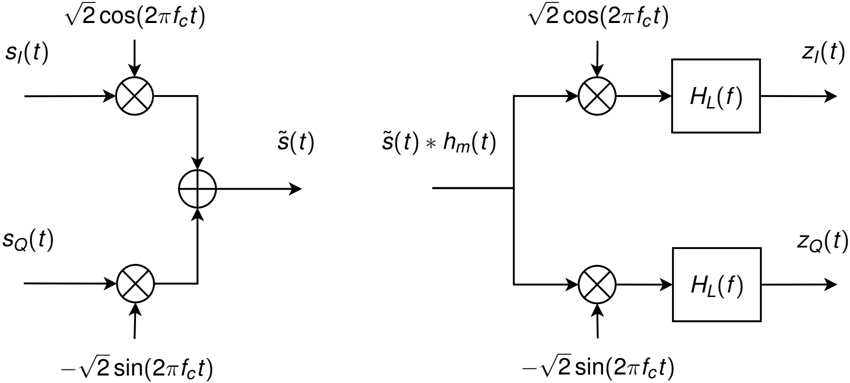
Multipath Propagation

Multipath propagation can be characterized by the transfer function

$$H_m(f) = 1 * \delta(f - \tilde{f}_0) + \sum_{m=1}^{M} \alpha_m \, \mathrm{e}^{-\mathrm{j}2\pi f \tau_m} * \delta(f - \tilde{f}_m)$$

- $\tilde{f}_0$, $\tilde{f}_m$: Doppler shift for LOS and multipath signals
- α_m: relative amplitude of multipath with respect to LOS signal
- τ_m: relative delay of multipath with respect to LOS signal

Transmitter and Baseband Receiver Model (1)



$$H_L(f) = \begin{cases} 1 & , |f| \leq B \\ 0 & , |f| > B \end{cases}$$

The baseband equivalent signal of $\tilde{s}(t)$ is assumed to be strictly bandlimited to B.

Transmitter and Baseband Receiver Model (2)

We can derive the inphase and quadrature component of the baseband received signal as

$$z_I(t) = [(s(t) * h_m(t))\cos(2\pi f_c t)] * h_L(t)$$

$$\circ\!\!-\!\!\bullet\ \frac{1}{2}(S_I(f) - \mathrm{j}S_Q(f))H_m(f - f_c) + \frac{1}{2}(S_I(f) + \mathrm{j}S_Q(f))H_m(f + f_c)$$

$$z_Q(t) = -[(s(t) * h_m(t))\sin(2\pi f_c t)] * h_L(t)$$

$$\circ\!\!-\!\!\bullet\ \frac{\mathrm{j}}{2}(S_I(f) - \mathrm{j}S_Q(f))H_m(f - f_c) - \frac{\mathrm{j}}{2}(S_I(f) + \mathrm{j}S_Q(f))H_m(f + f_c)$$

Thus, the complex baseband equivalent signal can be written

$$\begin{aligned} z(t) &= z_I(t) + \mathrm{j}z_Q(t) \\ &= (s_I(t) + \mathrm{j}s_Q(t)) * h_m(t)\,\mathrm{e}^{-\mathrm{j}2\pi f_c t} \\ &= (s_I(t) + \mathrm{j}s_Q(t)) * \left(\delta(t)\mathrm{e}^{-\mathrm{j}2\pi \tilde{f}_0 t} + \sum_{m=1}^{M} \alpha_m \delta(t - \tau_m)\mathrm{e}^{-\mathrm{j}2\pi(\tilde{f}_m t + f_c \tau_m)}\right) \end{aligned}$$

$$\circ\!\!-\!\!\bullet\ (S_I(f) + \mathrm{j}S_Q(f))H_m(f + f_c)$$