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BarcelonaTECH, Spain

Lecture 10

Satellite orbits and clocks computation and accuracy

Professors: Dr. J. Sanz Subirana, Dr. J.M. Juan Zornoza
and Dr. Adrià Rovira García

Authorship statement

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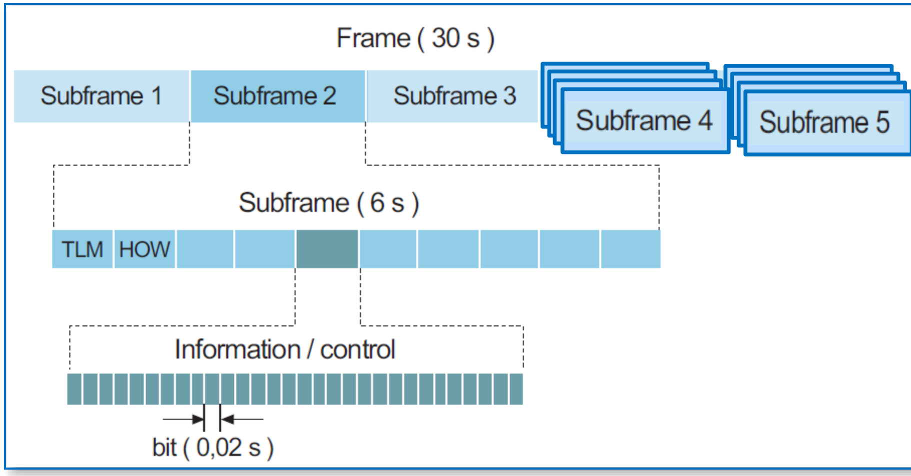
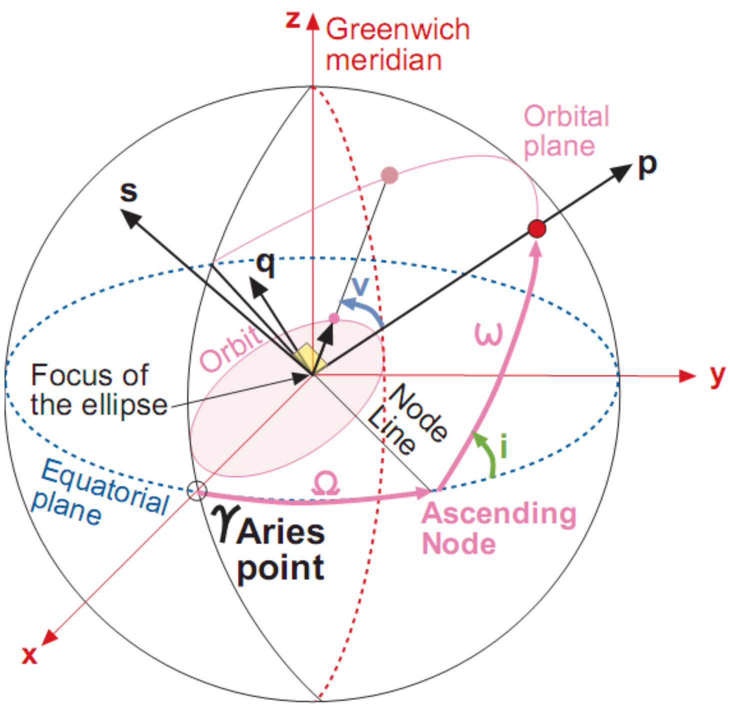
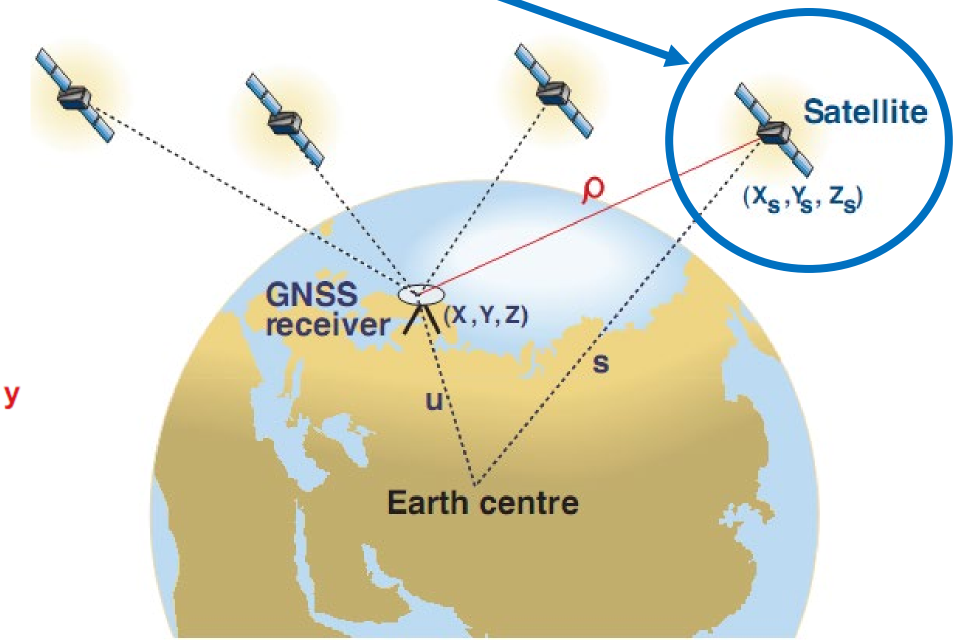


The GPS navigation message provides pseudo-Keplerian elements to compute satellite coordinates





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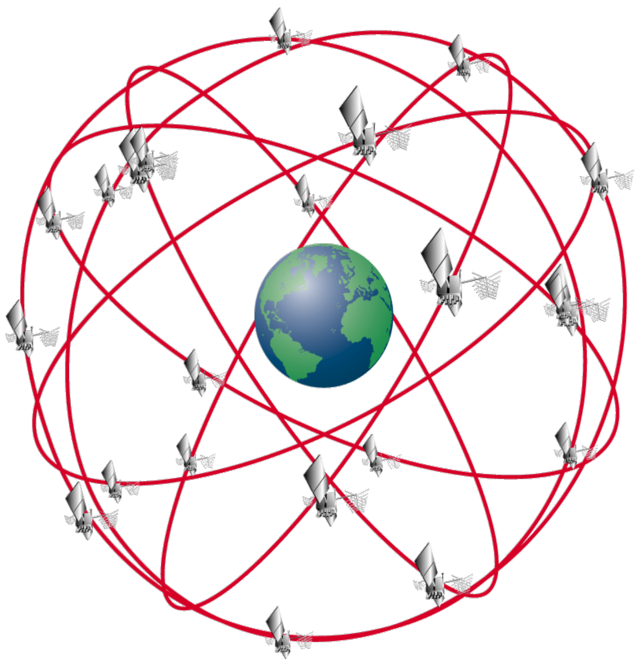
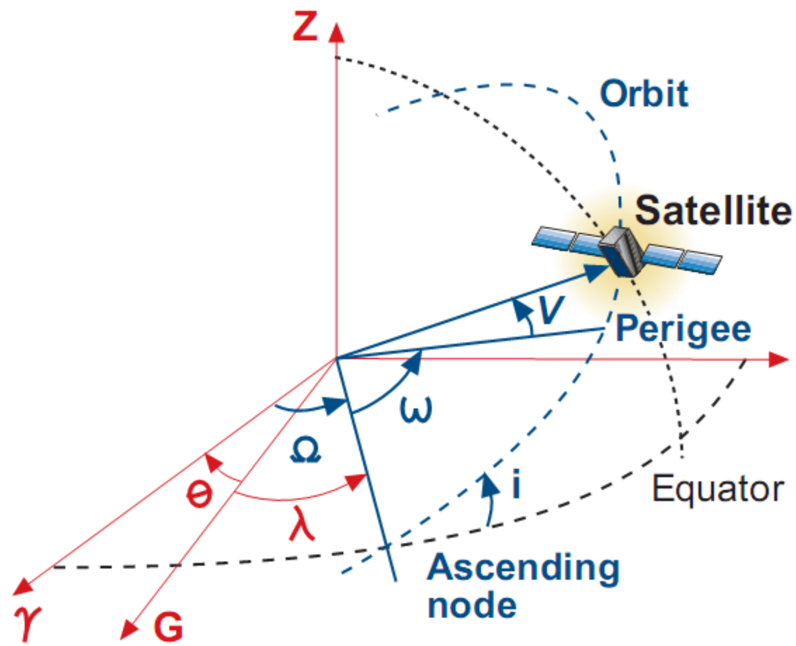
(X, Y, Z, Vx, Vy, Vz) → (a, e, i, Ω, ω, V)

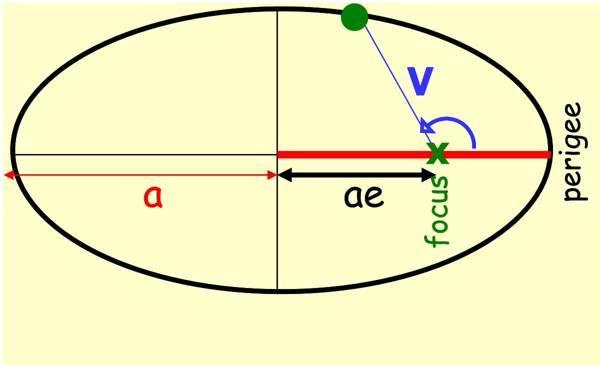
6 values are needed (x,y,z,vx,vy,vz) to provide the position and velocity of a body. They can be map into the **six Keplerian elements** (a, e, i, Ω, ω, V), which provides the "natural" representation of the orbit!

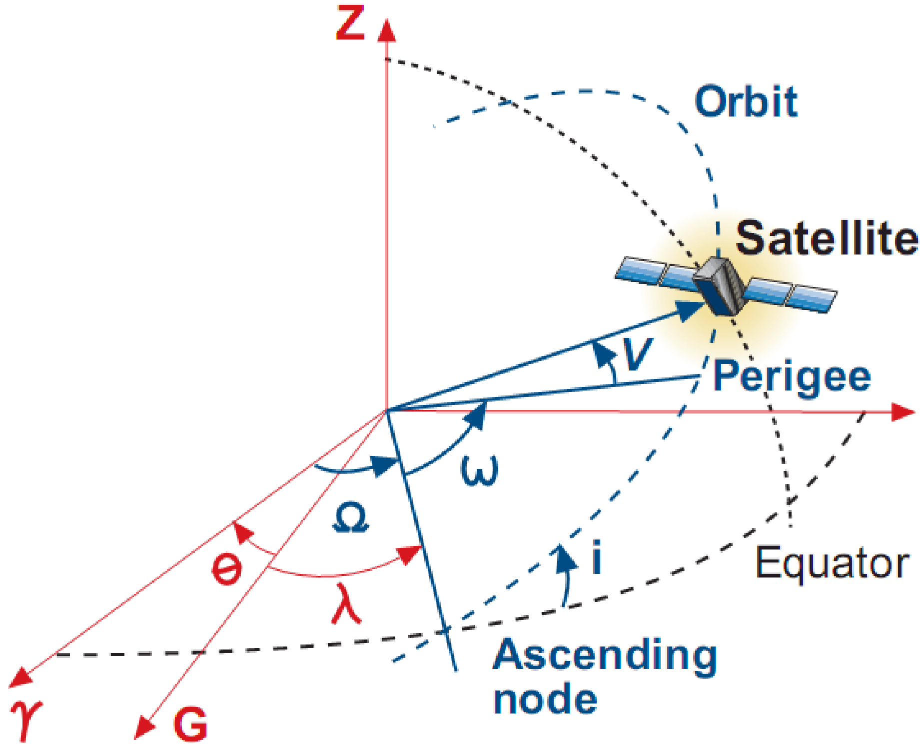




$(a, e, \; i, \Omega, \omega, \; V)$

- a, e: orbit shape
- i, Ω, ω: orbit orientation
- V: position in the orbit





- i inclination
- ω argument of perigee
- Ω arg. ascending node (Aries)
- λ arg. ascending node (Greenwich)
- V true anomaly
- θ sidereal time
- γ vernal equinox
- G Greenwich meridian

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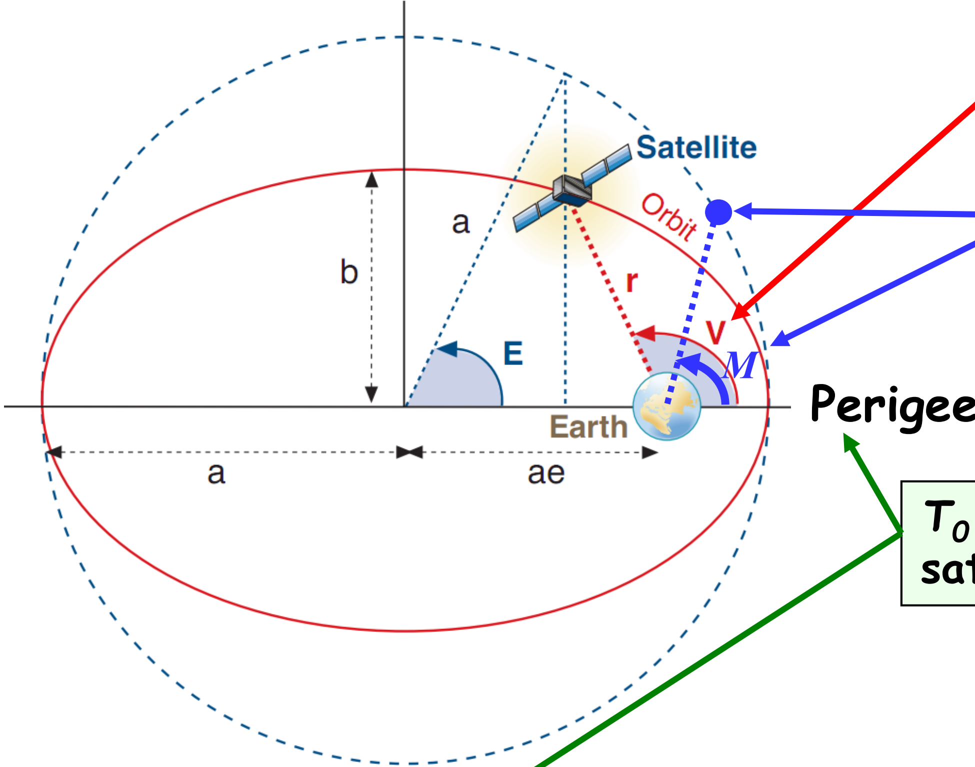

True anomaly $V(t)$

Fictitious body moving at velocity $n=2\pi/P=constant$

➔ Mean anomaly $M(t)$

T_0 : time of passage by satellite's perigee

$$t \rightarrow \left[T_0 ,\quad n=\frac{2\pi}{P} ,\quad \mu, a, e \right] \rightarrow V(t)$$

$\longleftrightarrow$

$$M(t) = n(t - T_0) \quad ; \quad n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

$$E(t) = M(t) + e \sin E(t)$$

$$V(t) = 2 \arctan \left[\sqrt{\frac{1+e}{1-e}} \tan \frac{E(t)}{2} \right]$$

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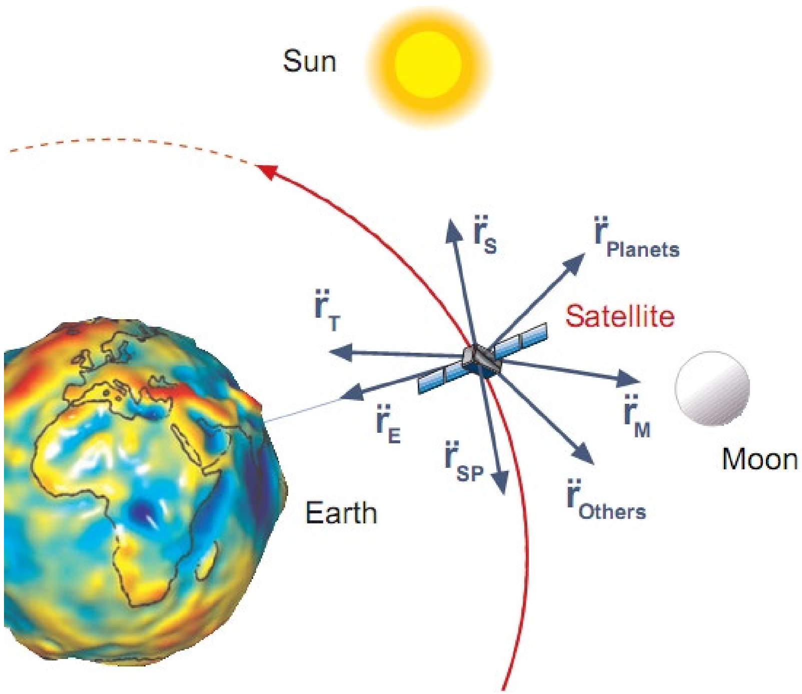
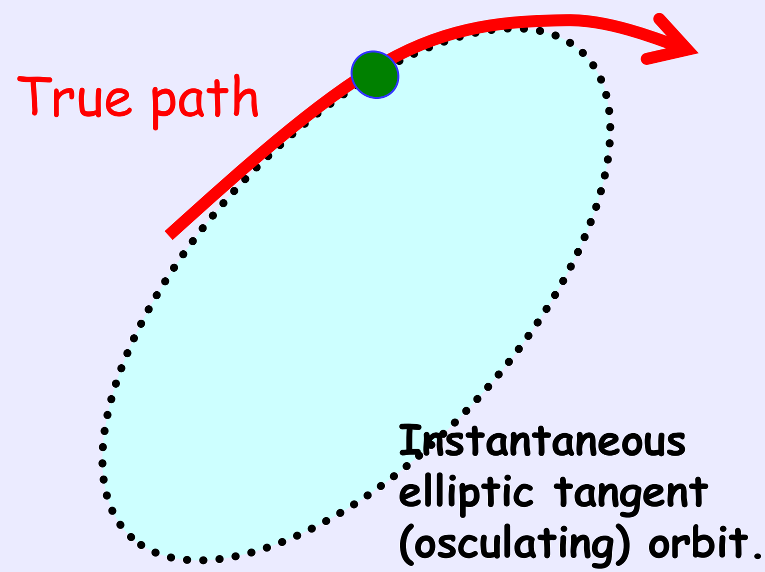
Due to the non-spherical nature of gravitational potential, the attraction of the Sun and Moon, the solar radiation pressure, etc., **the true satellite path deviates from the elliptic orbit**.



At any time an elliptical orbit tangent to the true path can be defined. This is the "osculating orbit", whose Keplerian elements vary with time "t":

$$a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$$



Different magnitudes of perturbation and their effects on GPS orbits

Perturbation	Acceleration (m/s^2)	Orbital effect in 3 hours	Orbital effect in 3 days
Central force (as a reference)	0.56		
J_2	$5 \cdot 10^{-5}$	2 km	14 km
Rest of the harmonics	$3 \cdot 10^{-7}$	50–80 m	100–1500 m
Solar + Moon grav.	$5 \cdot 10^{-6}$	5–150 m	1000–3000 m
Tidal effects	$1 \cdot 10^{-9}$	–	0.5–1.0 m
Solar rad. pressure	$1 \cdot 10^{-7}$	5–10 m	100–800 m

GLONASS Broadcast orbit integration terms

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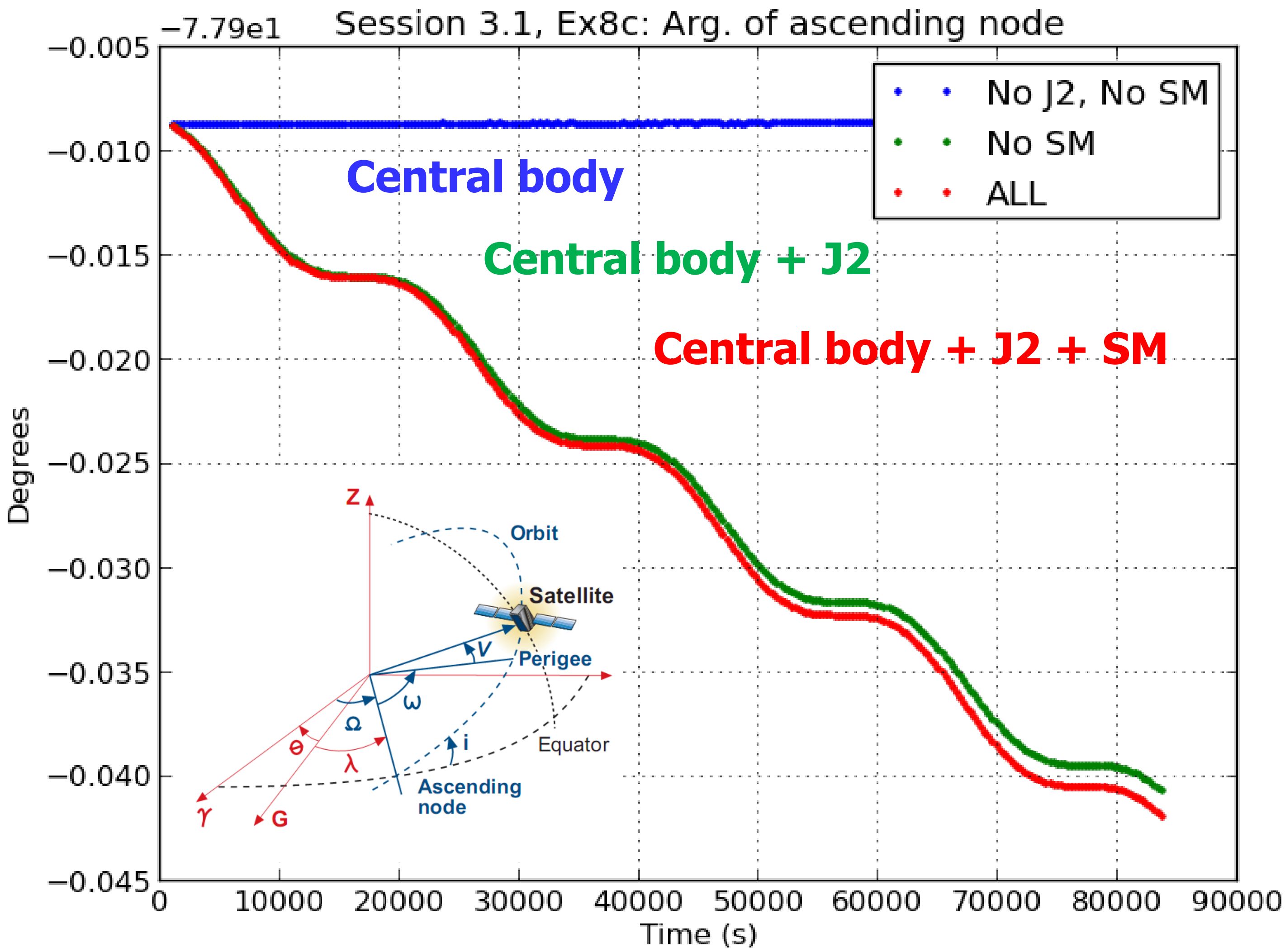
Session 3.1, Ex8c: Arg. of ascending node
−7.79e1
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030
−0.035
−0.040
−0.045
Degrees
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
Time (s)
No J2, No SM
No SM
ALL
Central body
Central body + J2
Central body + J2 + SM
Z
Orbit
Satellite
V
Perigee
ω
Ω
θ
λ
i
Equator
Ascending node
γ
G

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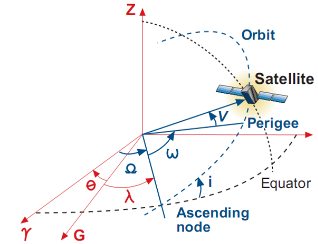

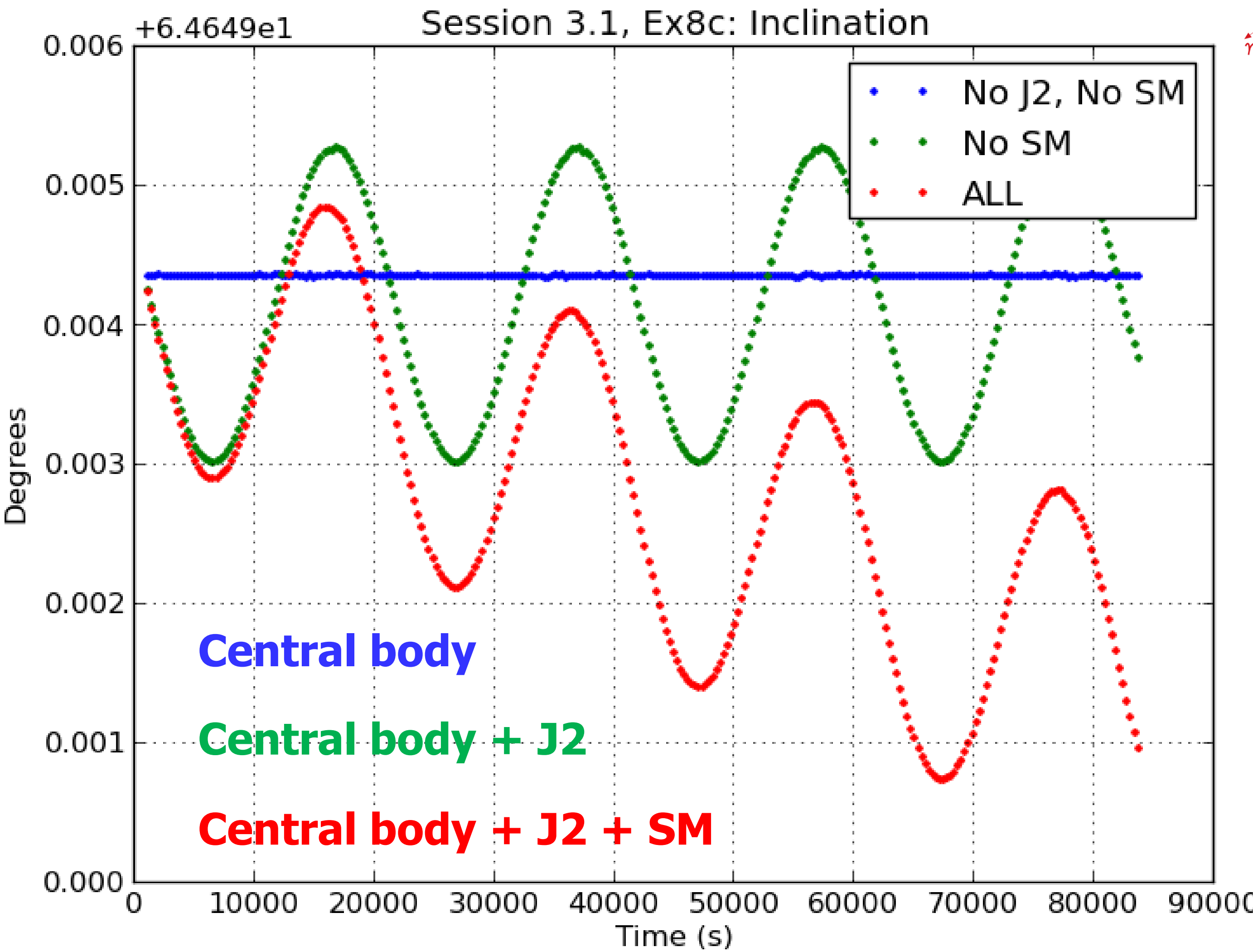

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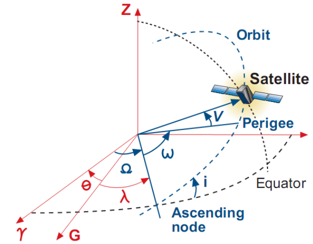

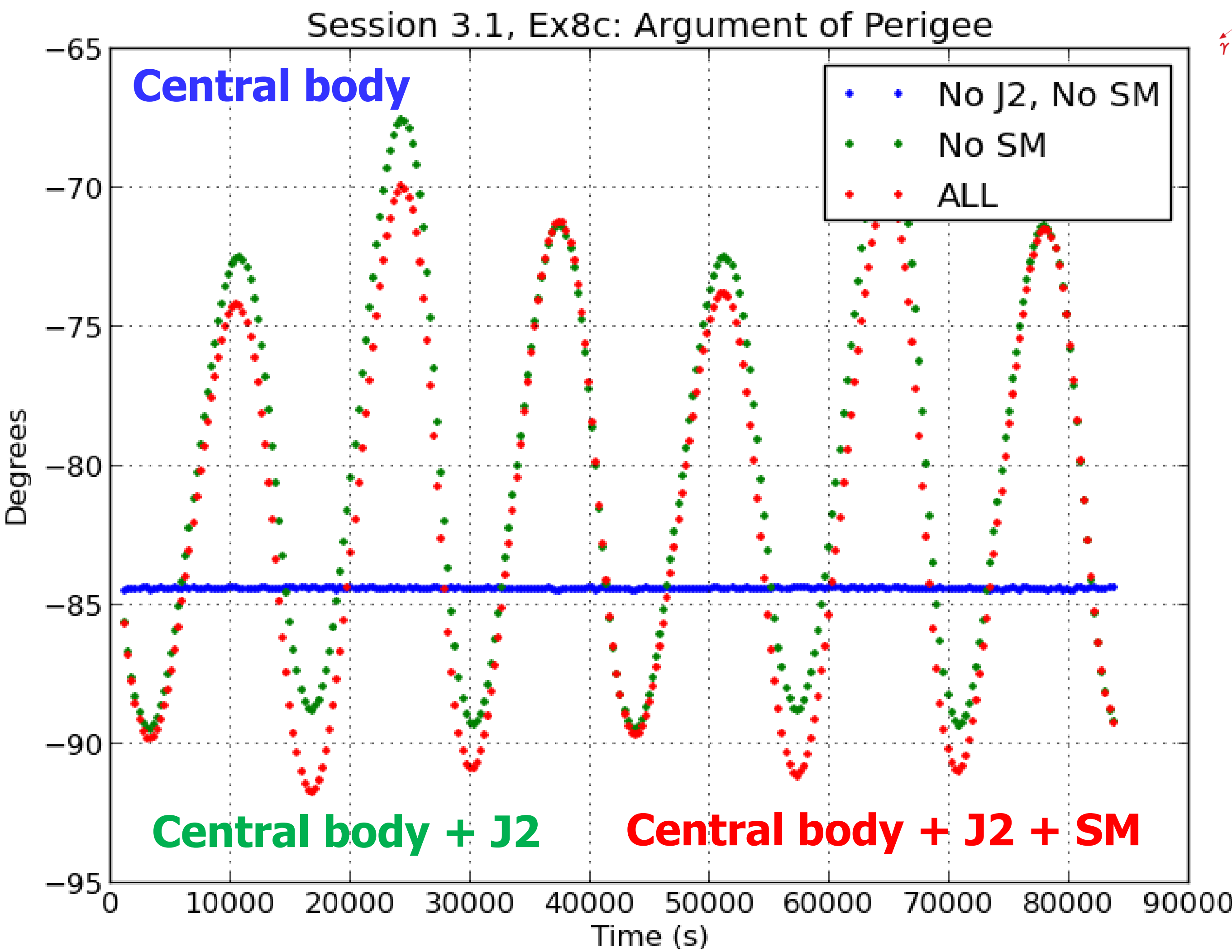

Calculation of osculating orbital elements from position and velocity (rv2osc.f)

$$(x, y, z, v_x, v_z, v_z) \Rightarrow (a, e, i, \Omega, \omega, M)$$

$$\vec{c} = \vec{r} \times \vec{v} \Longrightarrow p = \frac{c^2}{\mu} \Longrightarrow p$$
$$\mathrm{v}^2 = \mu(2/r - 1/a) \Longrightarrow a$$
$$\mathrm{p} = \mathrm{a}\ (1 - \mathrm{e}^2) \Longrightarrow e$$

$$\vec{c} = c\vec{S} \Longrightarrow \Omega = \arctan(-c_x/c_y); \quad i = \arccos(c_z/c) \Longrightarrow \Omega, i$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cos(\omega+V) - \sin\Omega\sin(\omega+V)\cos i \\ \sin\Omega\cos(\omega+V) + \cos\Omega\sin(\omega+V)\cos i \\ \sin(\omega+V)\sin i \end{pmatrix} \Longrightarrow \omega + V$$

$$r = \frac{p}{1 + e\cos(V)} \Longrightarrow \omega, V$$

$$\tan(E/2) = \left(\frac{1-e}{1+e}\right)^{1/2} \tan(V/2) \quad ; \quad M = E - e\sin E \Longrightarrow M$$

Calculation of position and velocity from osculating orbital elements (osc2rv.f)

$$(a, e, i, \Omega, \omega, \underbrace{T; t}_{V}) \Rightarrow (x, y, z, v_x, v_z, v_z)$$

$$t \Longrightarrow M \Longrightarrow E \Longrightarrow (r, V)$$

$$M = n(t - T) \qquad M = E - e \sin E \qquad r = a(1 - e\cos E)$$

$$\tan(V/2) = \left(\frac{1+e}{1-e}\right)^{1/2} \tan(E/2)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} ; \quad \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{na^2}{r}\{\vec{Q}(1-e^2)^{1/2}\cos E - \vec{P}\sin E\}$$

Where:

$$\begin{aligned} \mathrm{R} &= \mathrm{R}_3(-\Omega) R_1(-i) R_3(-\omega) = \\ &= \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \mathrm{P}_x & \mathrm{Q}_x & \mathrm{S}_x \\ \mathrm{P}_y & \mathrm{Q}_y & \mathrm{S}_y \\ \mathrm{P}_z & \mathrm{Q}_z & \mathrm{S}_z \end{pmatrix} = [\vec{P}\ \vec{Q}\ \vec{S}] \end{aligned}$$

Exercise: Orbital elements variation:

File 1995-10-18.eci contains the precise position and velocities of GPS satellites every 5 minutes for October 18th, 1995 in a Earth-Centred Inertial system (ECI) **[from JPL/NASA server:**
ftp://sideshow.jpl.nasa.gov/pub/gipsy_products]

a) Use program "**rv2osc**" to compute the instantaneous orbital eleme[nts]

(X, Y, Z, Vx, Vy, Vz) → $(a, e, i, \Omega, \omega, V)$

b) Plot the orbital elements in function of time to show their variation: $a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$

c) Compare with the broadcast orbital elements

Solution:

a) cat 1995-10-18.eci|rv2osc> orb.dat

b) See the following plots

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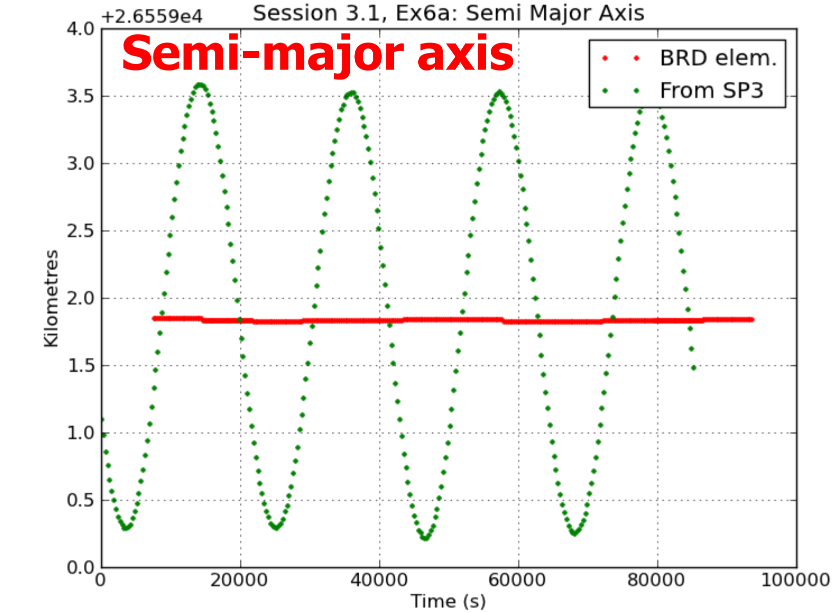
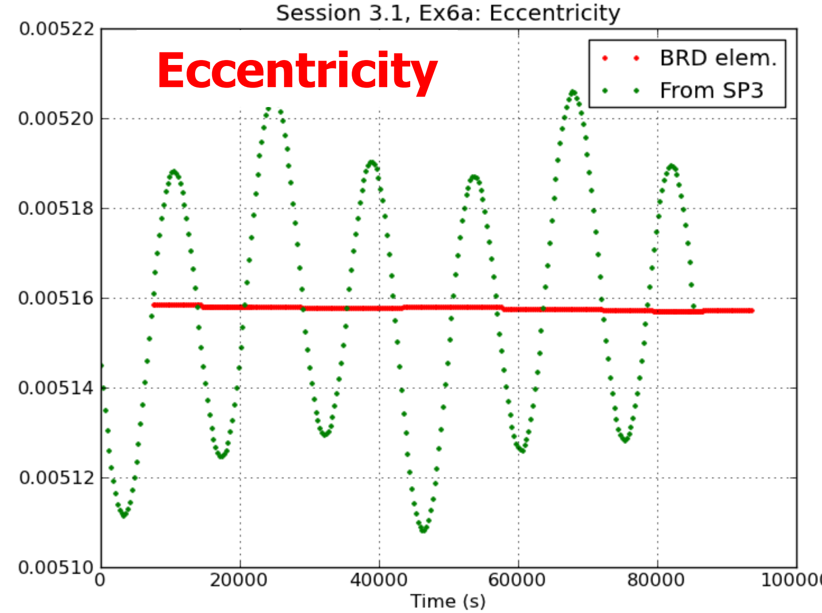
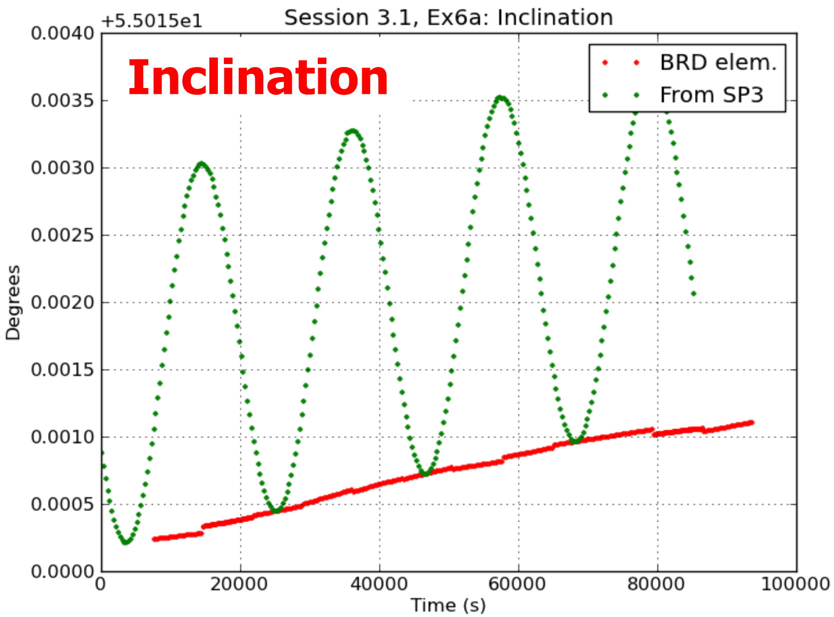
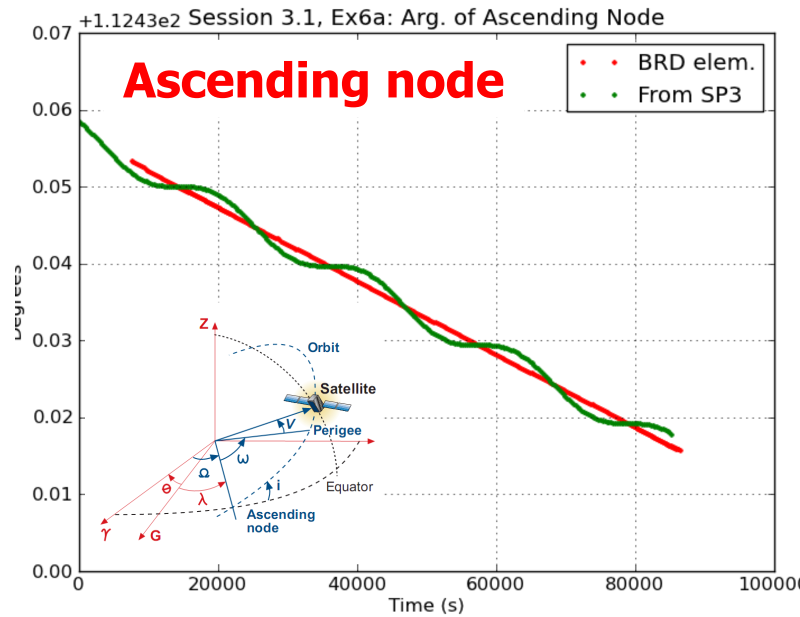
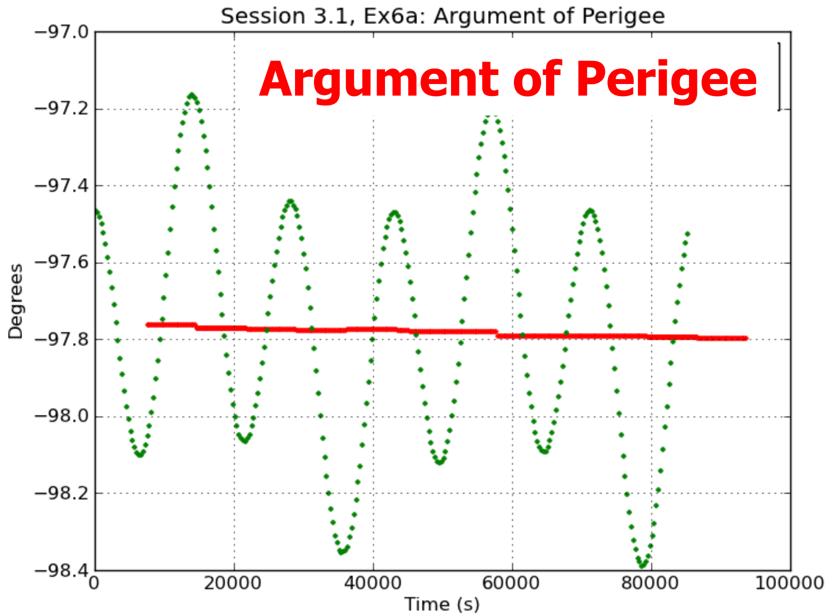
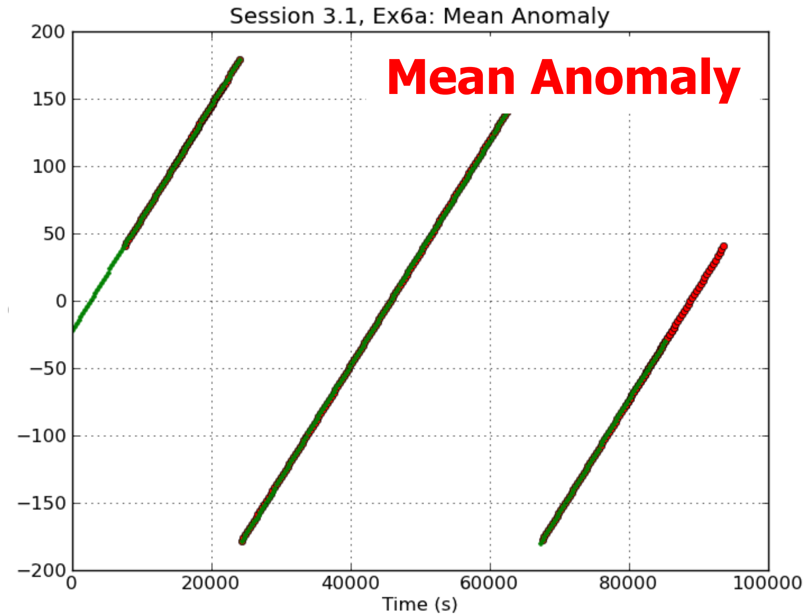


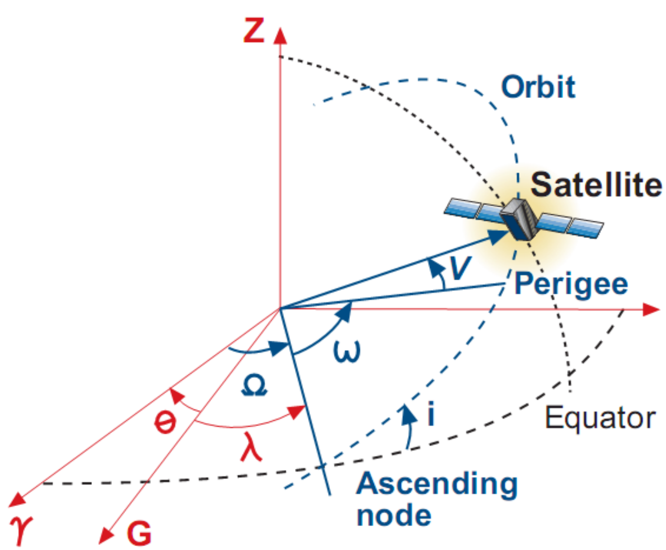


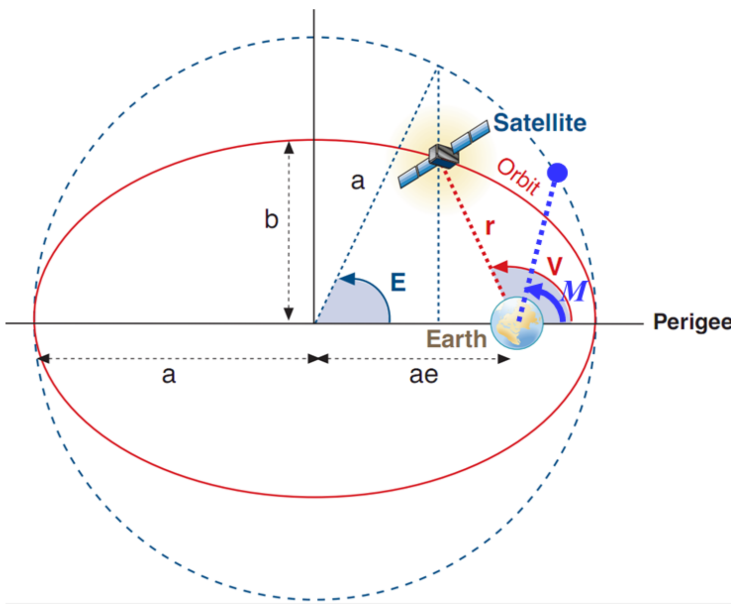





Session 3.1, Ex6a: Argument of Perigee
Argument of Perigee
Degrees
Time (s)


Session 3.1, Ex6a: Mean Anomaly
Mean Anomaly
Time (s)


Z
Orbit
Satellite
V
Perigee
ω
Ω
θ
λ
i
Equator
Ascending
node
γ
G


Satellite
Orbit
a
b
r
V
E
M
Perigee
Earth
a
ae

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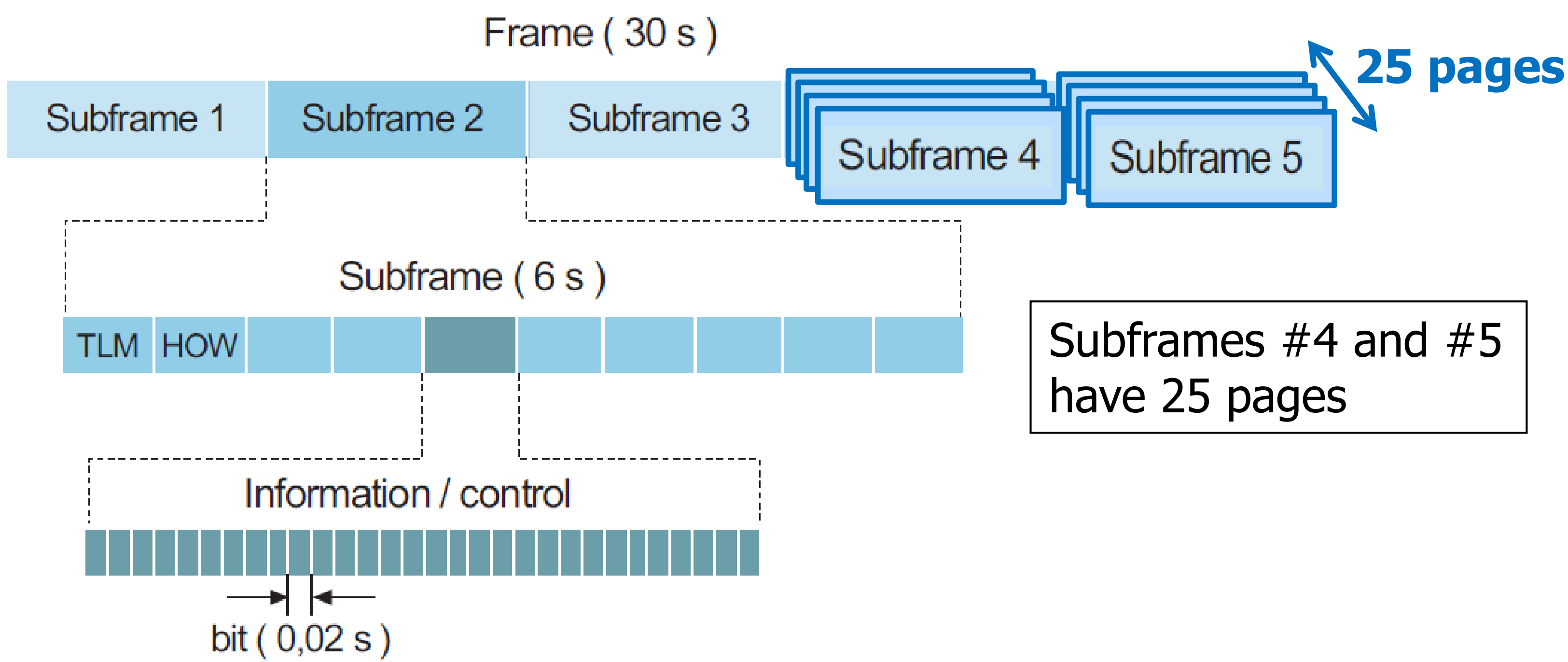
GPS navigation message



Subframes #4 and #5 have 25 pages

One Master Frame includes All 25 pages of Subframes #4 and #5 → 25 x 30s = **12.5 min**

Subframe 1 contains information about the parameters to be applied to **satellite clock** status for its correction. These values are polynomial coefficients that allow time onboard to be converted to GPS time. The subframe also contains information on satellite health condition.

Subframes 2 and 3 contain **satellite ephemerides**.

Subframe 4 provides **ionospheric model** parameters (in order to adjust for ionospheric refraction), UTC information, part of the **almanac**, and indications whether the A/S is activated or not (which transforms the P code into encrypted Y code).

Subframe 5 contains data from the **almanac** and on constellation status. It allows rapid identification of the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.

Ephemeris in navigation message

Parameter	Explanation
$IODE$	Series number of ephemerides data
t_{oe}	Ephemerides reference epoch
$\sqrt{a}$	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω	Ascending node's right ascension
Δn	Mean motion difference
$\dot{i}$	rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
c_{uc}, c_{us}	Latitude argument correction
c_{rc}, c_{rs}	Orbital radius correction
c_{ic}, c_{is}	Inclination correction

In order to calculate WGS84 satellite coordinates, you should apply de following algorithm [GPS/SPS-SS, table 2-15] (see in the book FORTRAN subroutine orbit.f)

RINEX ephemeris file

```
     2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
XPRINT v1.1         gAGE                00/08/17 09:31:37   PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE                               COMMENT
   +1.7695E-08 +2.2352E-08 -1.1921E-07 -1.1921E-07          ION ALPHA
   +1.1878E+05 +1.4746E+05 -1.3107E+05 -3.2768E+05          ION BETA
   +1.955777406693E-08+1.598721155460E-14   405504     1064 DELTA_UTC: A0,A1,T,W
    13                                                      LEAP SECONDS
                                                            END OF HEADER

03 00  5 30 10  0 40.0+7.855705916882E-06+3.524291969370E-12+0.000000000000E+00
   +1.010000000000E+02+6.500000000000E+01+5.456298524109E-09+5.530285585107E-01   Mo
   +3.475695848465E-06+1.308503560722E-03+2.641230821609E-06+5.153678266525E+03   e, √a
   +2.088000000000E+05+1.117587089539E-08+7.472176136643E-01-1.862645149231E-09   TOE, Ω
   +9.412719852649E-01+3.163750000000E+02+1.125448382894E+00-8.826796182859E-09   io, ω
   +1.239337382719E-10+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00
   +4.000000000000E+00+0.000000000000E+00-4.190951585770E-09+6.130000000000E+02   TGD
   +2.044980000000E+05+0.000000000000E+00+0.000000000000E+00+0.000000000000E+00

06 00  5 30 10  0  0.0+1.636799424887E-06+0.000000000000E+00+0.000000000000E+00
   +6.000000000000E+01+5.100000000000E+01+5.198073527168E-09-5.601816471398E-01
   +2.635642886162E-06+6.763593177311E-03+2.468004822731E-06+5.153726325989E+03
   +2.088000000000E+05+1.862645149231E-08+7.894129138508E-01+8.195638656616E-08
   +9.487675576456E-01+3.229687500000E+02-2.409256713064E+00-8.734292400447E-09
   +4.714481929846E-11+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00
```

3.1. Computation of satellite coordinates from navigation message (orbit.f)

- **Computation of t_k time since ephemerids reference epoch t_{oe} (t and t_{oe} are given in GPS seconds of week):**

$$t_k = t - t_{oe}$$

- **Computation of mean anomaly M_k for t_k,**

$$M_k = M_0 + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n \right) t_k$$

- **Iterative resolution of Kepler's equation in order to compute eccentric anomaly E_k :**

$$M_k = E_k - e \sin E_k$$

- **Calculation of true anomaly v_k :**

$$v_k = \arctan \left(\frac{\sqrt{1-e^2} \sin E_k}{\cos E_k - e} \right)$$

- **Computation of latitude argument u_k from perigee argument W, true anomaly v_k and corrections c_{uc} and c_{us}:**

$$u_k = \omega + v_k + c_{uc} \cos 2(\omega + v_k) + c_{us} \sin 2(\omega + v_k)$$

- **Computation of radial distance r_k, taking into consideration corrections c_{rc} and c_{rs}:**

$$r_k = a\left(1 - 2\cos E_k\right) + c_{rc}\cos 2\left(\omega + v_k\right) + c_{rs}\sin 2\left(\omega + v_k\right)$$

- **Calculation of orbital plane inclination i_k from inclination i_o at reference epoch t_{oe} and corrections c_{ic} and c_{is} :**

$$i_k = i_0 + it_k + c_{ic}\cos 2\left(\omega + v_k\right) + c_{is}\sin 2\left(\omega + v_k\right)$$

- **Computation of ascending node longitude Ω_k (Greenwich), from longitude Ω_0 at start of GPS week, corrected from apparent variation of sidereal time at Greenwich between start of week and and reference time $t_k=t-t_{oe}$, and also corrected from change of ascending node longitude since reference epoch t_{oe}.**

$$\Omega_k = \Omega_0 + \left(\Omega - \omega_E\right)t_k - \omega_E t_{oe}$$

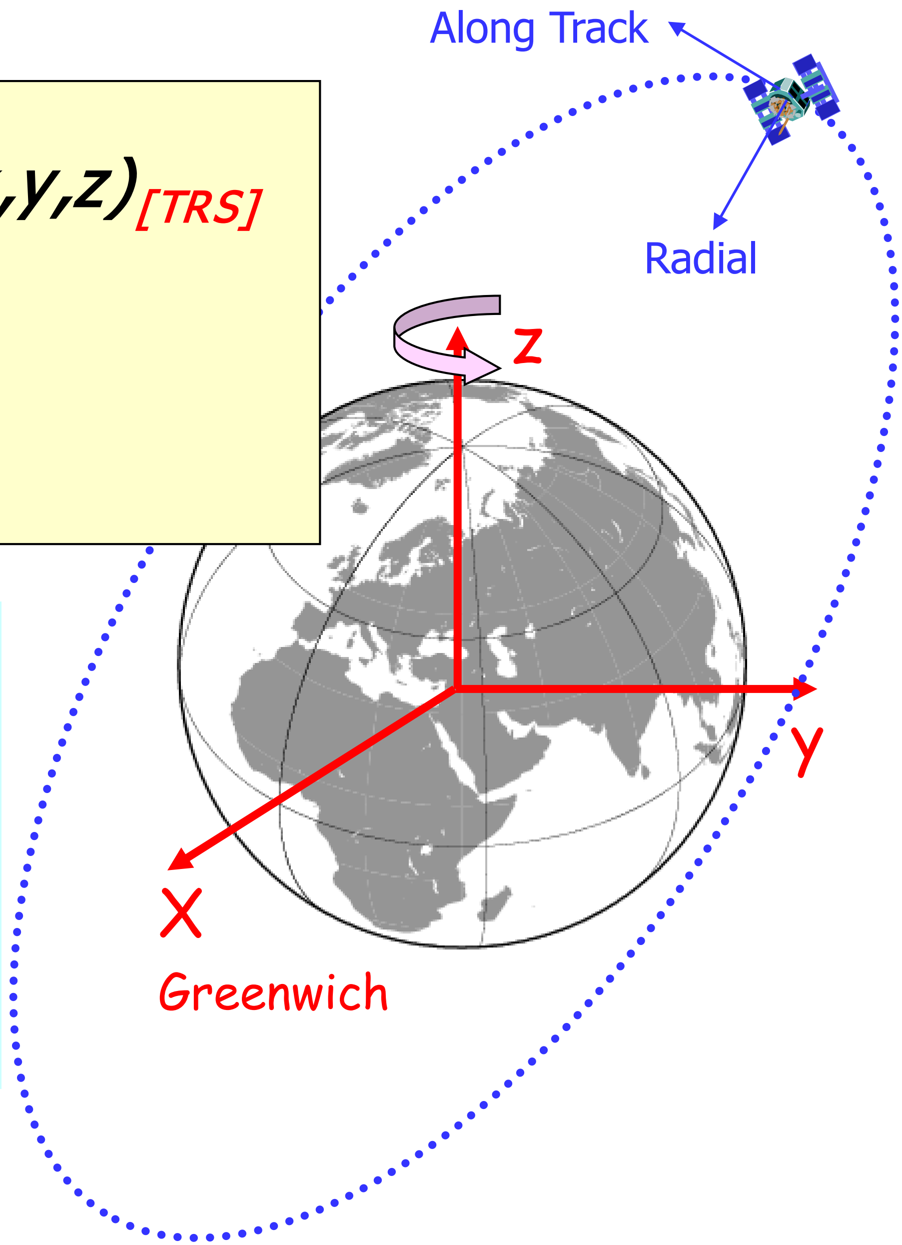
- **Calculation of coordinates in CTS system, applying three rotations (around u_k, i_k, Ω_k) :**

$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = \mathbf{R}_3(-\Omega_k)\mathbf{R}_1(-i_k)\mathbf{R}_3(-u_k)\begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$

$t \rightarrow$ **Orbit.f** $\rightarrow (x,y,z)_{[TRS]}$

↑

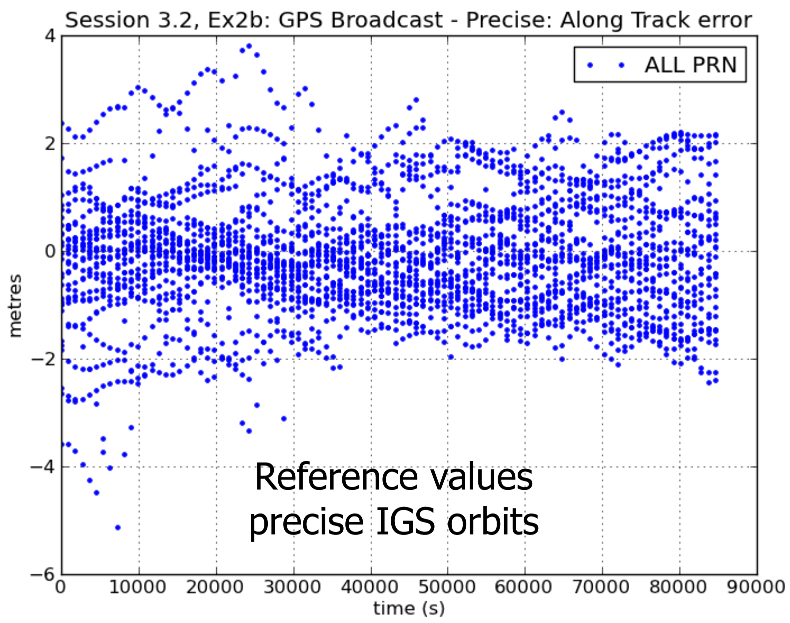
Nav. message (ephemeris)

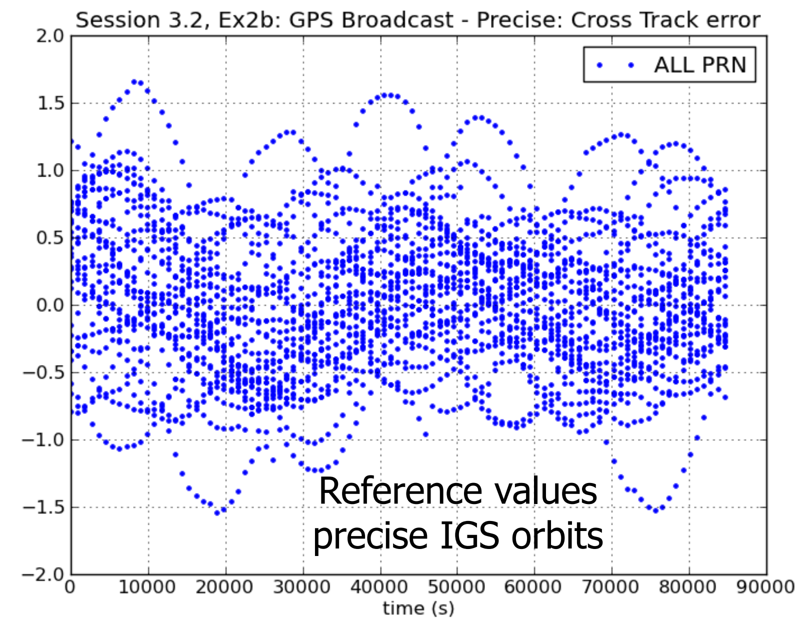


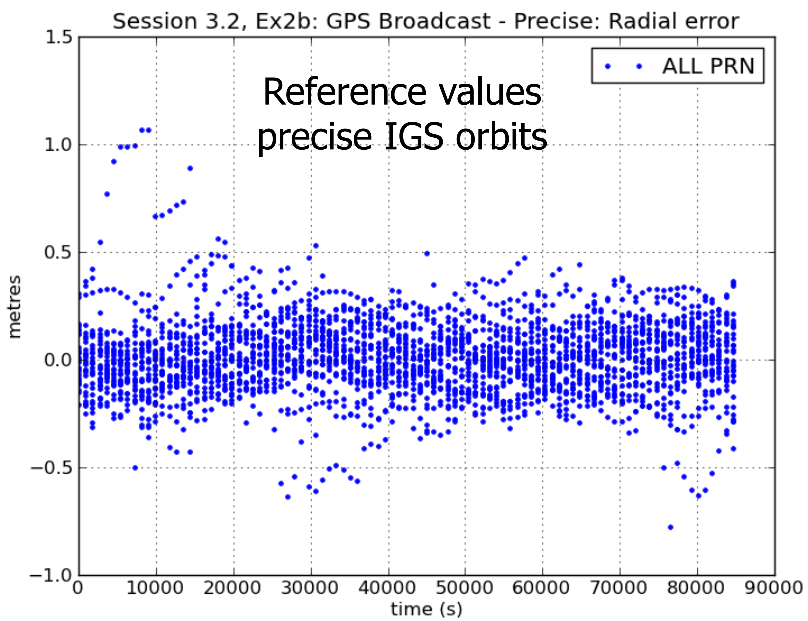
Conventional Terrestrial Reference System **(TRS):**

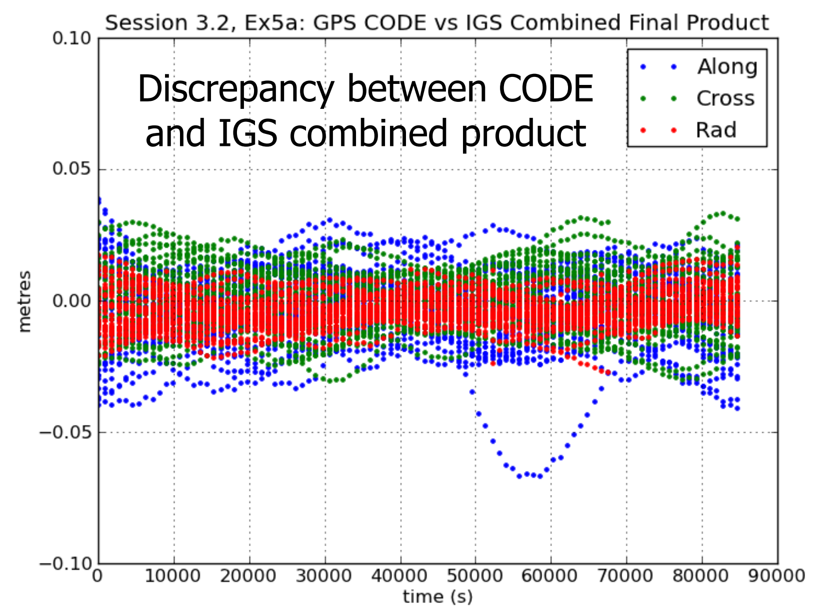
Earth Centered, Earth-Fixed (ECEF) ➔

the reference system rotates with Earth.

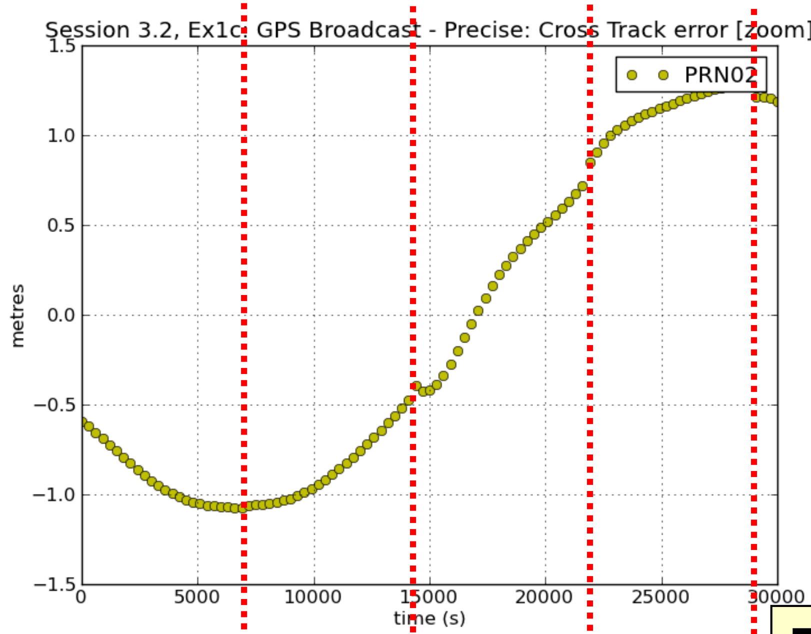


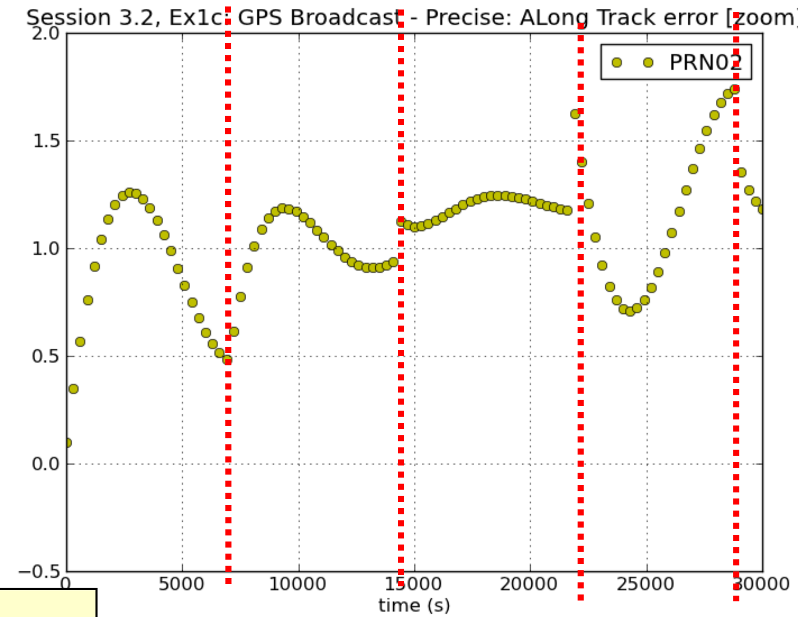




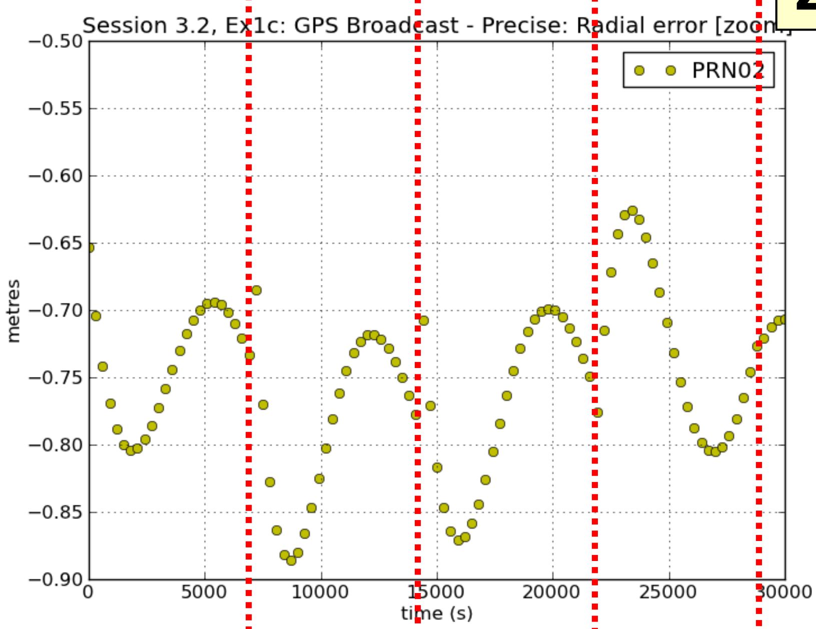


Antenna Phase Centre Correction has been applied





Zoom



Broadcast Orbit Updates

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3.2 Computation of satellite coordinates from precise products.

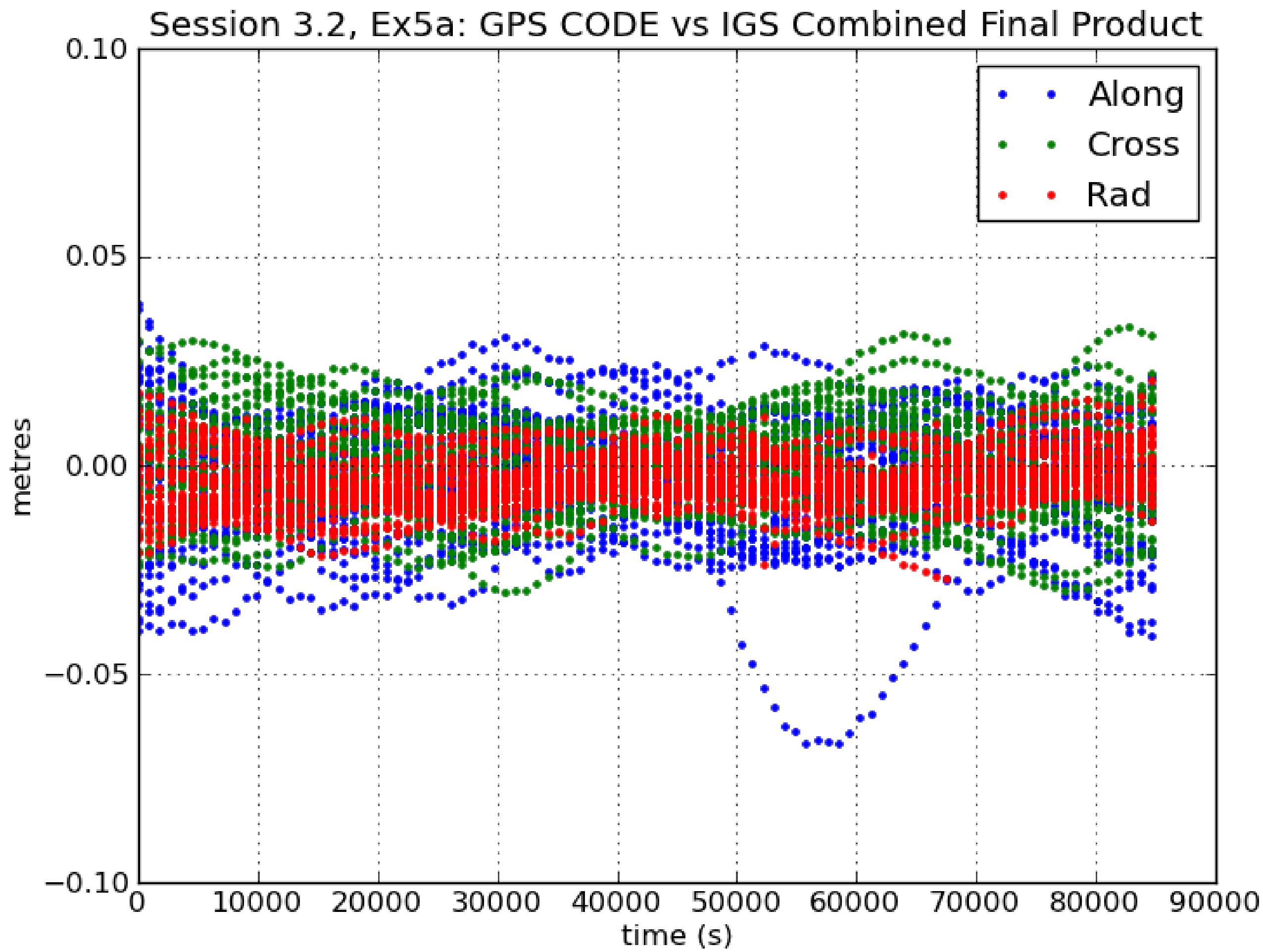
Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server **`http://igscb.jpl.nasa.gov`**

Orbits are given by (x,y,z) coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

$$
\begin{aligned}
P_n(x) &= \sum_{i=1}^{n} y_i \frac{\prod_{j\neq i}(x-x_j)}{\prod_{j\neq i}(x_i-x_j)} \\
&= y_1 \frac{x-x_2}{x_1-x_2}\cdots\frac{x-x_n}{x_1-x_n}+\cdots \\
&\quad + y_i \frac{x-x_1}{x_i-x_1}\cdots\frac{x-x_{i-1}}{x_i-x_{i-1}}\frac{x-x_{i+1}}{x_i-x_{i+1}}\cdots\frac{x-x_n}{x_i-x_n}+\cdots \\
&\quad + y_n \frac{x-x_1}{x_n-x_1}\cdots\frac{x-x_{n-1}}{x_n-x_{n-1}}
\end{aligned}
$$

IGS orbit and clock products (for PPP):

Discrepancy between CODE and IGS combined product.



Contents

GPS Satellite Clock computation: Broadcast message

$$dt^{sat} = a_0 + a_1 (t - t_0) + a_2 (t - t_0)^2$$

```
 2              NAVIGATION DATA     GPS                 RINEX VERSION / TYPE
srx/v1.8.1.4        BAI                 95/10/19 03:18:35   PGM / RUN BY / DATE
CASA                                                        COMMENT
 -2444431.2031 -4428688.6270  3875750.1442                  COMMENT
                                                            END OF HEADER
14 95 10 18 00 51 44.0 1.129414886236D-05 1.136868377216D-13 0.000000000000D+00
    1.730000000000D+02-5.175000000000D+01 4.375182243902D-09-5.836427291652D-01
   -2.712011337280D-06 2.427505562082D-03 8.568167686462D-06 5.153718931198D+03
    2.623040000000D+05 4.470348358154D-08 1.698435481558D+00 1.676380634308D-08
    9.636381916043D-01 2.153437500000D+02 3.056960010495D+00-8.030691653399D-09
   -5.178787145843D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.730000000000D+02
    2.592180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

PRN: 14

t0 (YY MM DD H M S): 95 10 18 00 51 44.0

a0: 1.129414886236D-05

a1: 1.136868377216D-13

a2: 0.000000000000D+00

Computation of satellite clocks from precise products

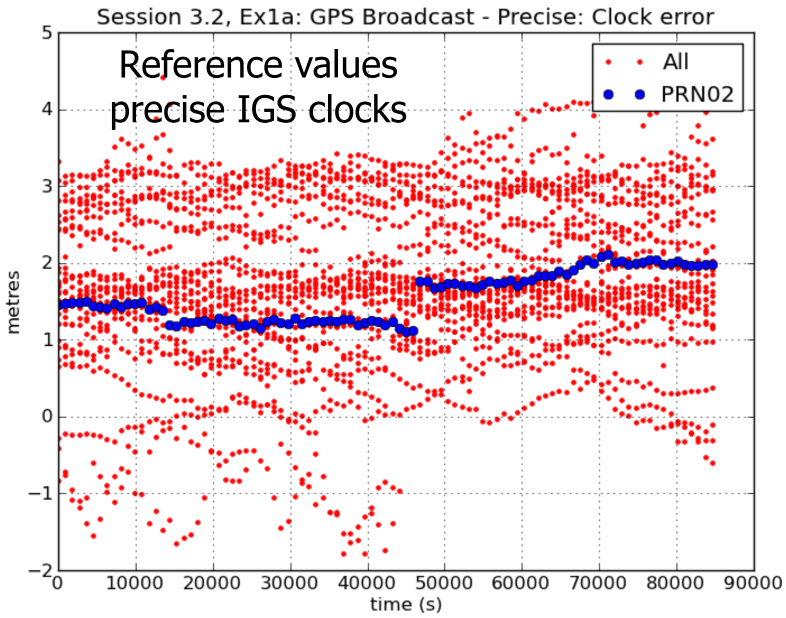
Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**

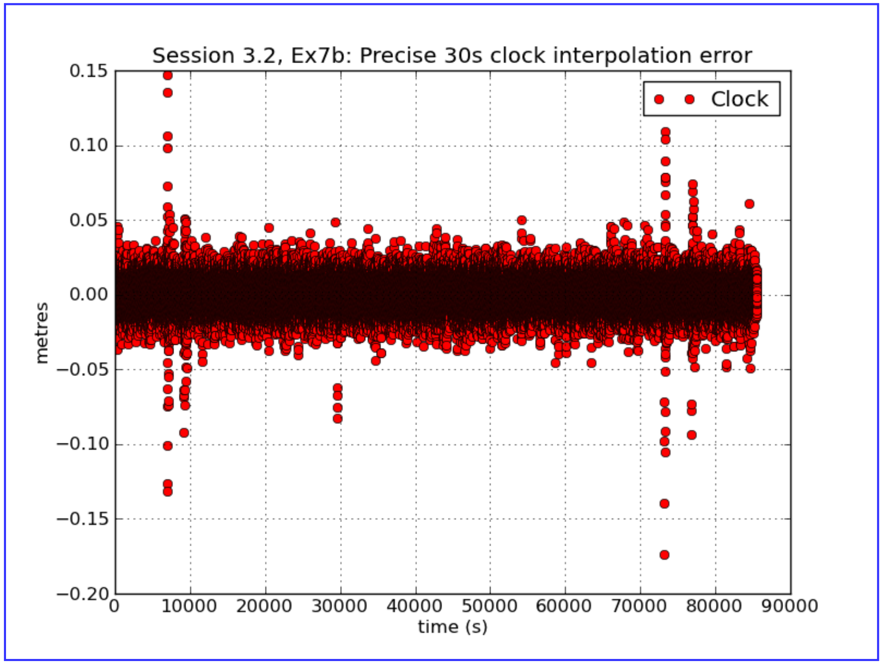
They are providing precise orbits and clock files with a sampling rate of 15 min (SP3 files), as well as precise clock files with a sample rate of 5 min and 30 s (CLK files).

Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.

Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.

SA=off


Session 3.2, Ex1a: GPS Broadcast - Precise: Clock error
Reference values
precise IGS clocks
All
PRN02
metres
time (s)


Session 3.2, Ex7b: Precise 30s clock interpolation error
Clock
metres
time (s)

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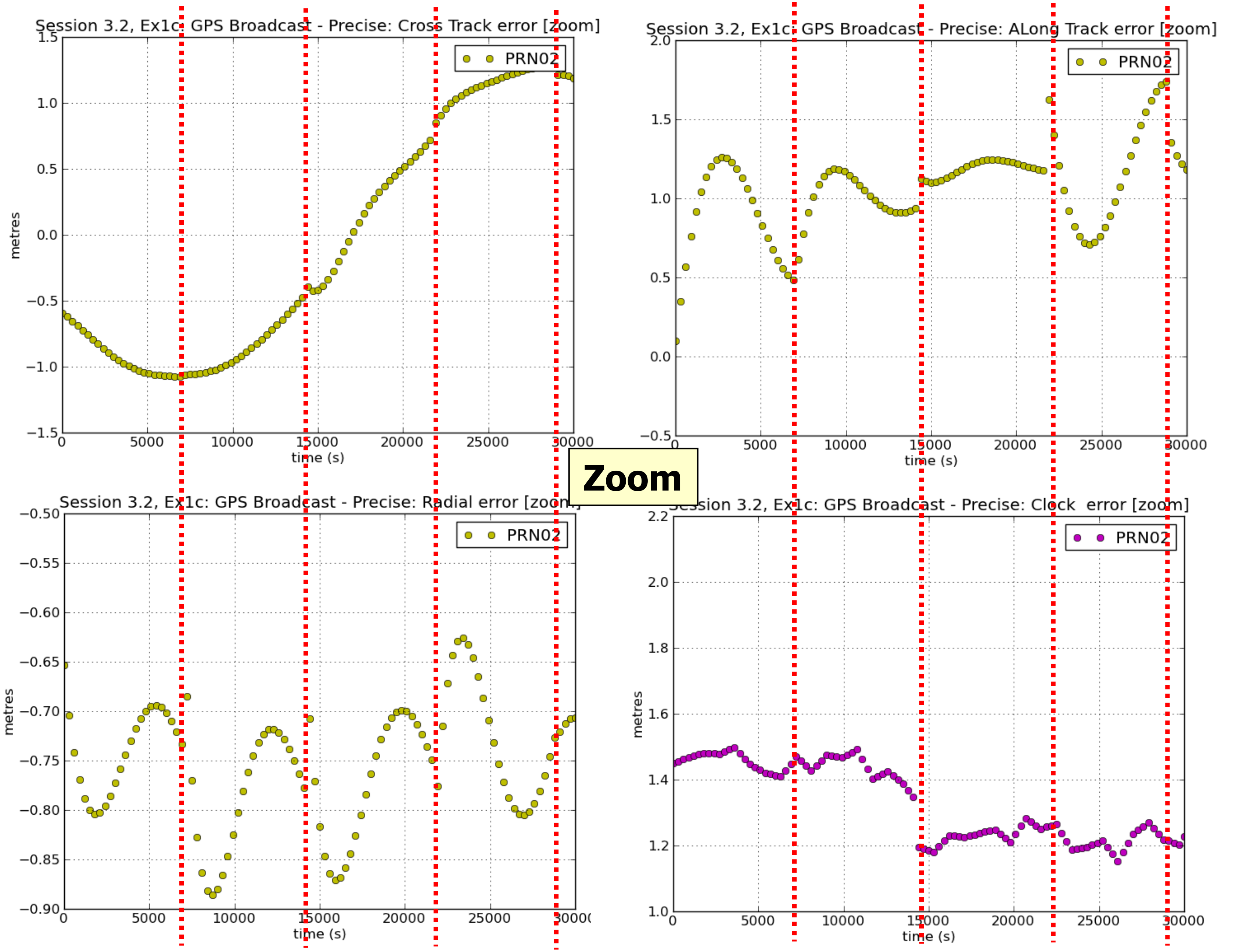
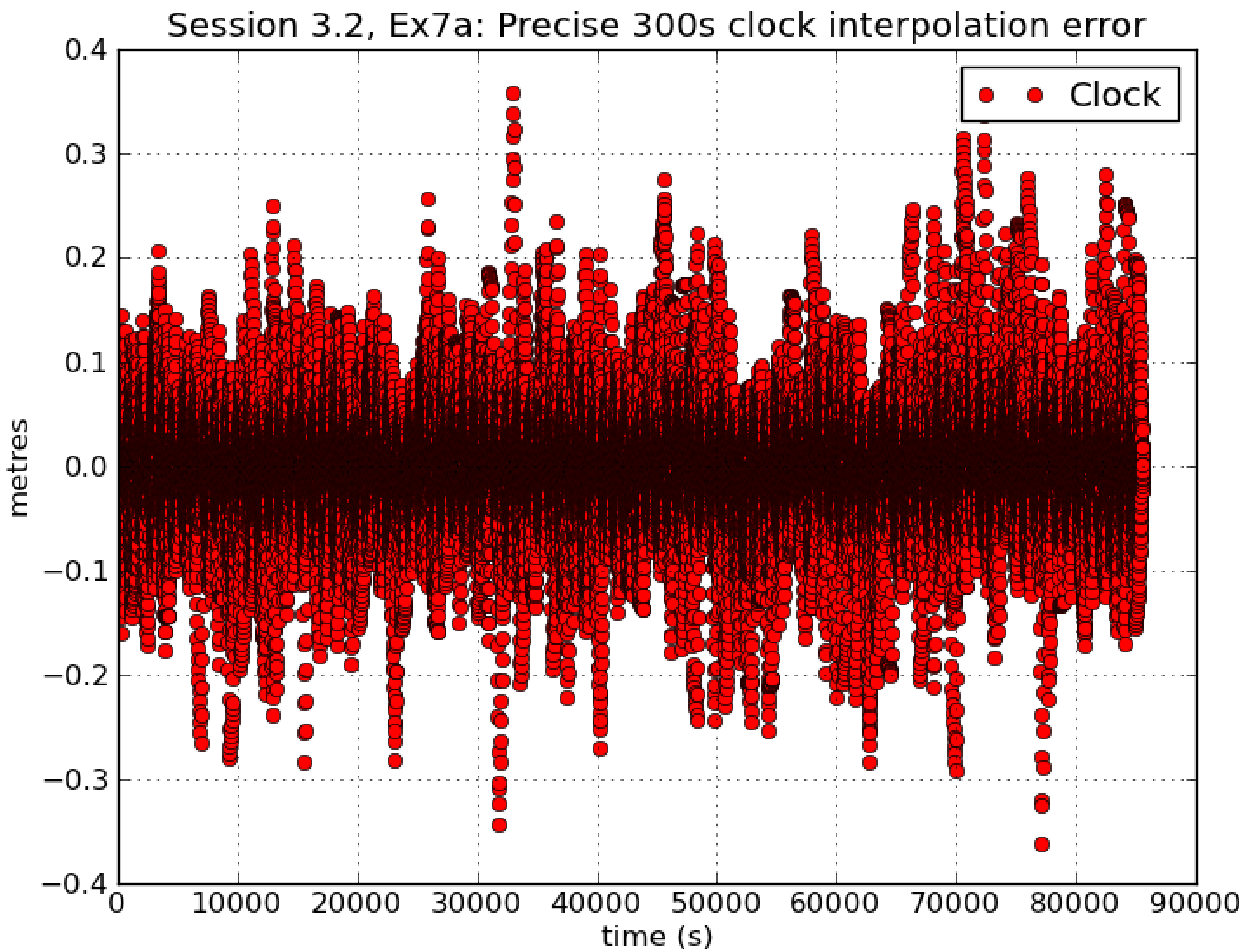
BarcelonaTECH, Spain

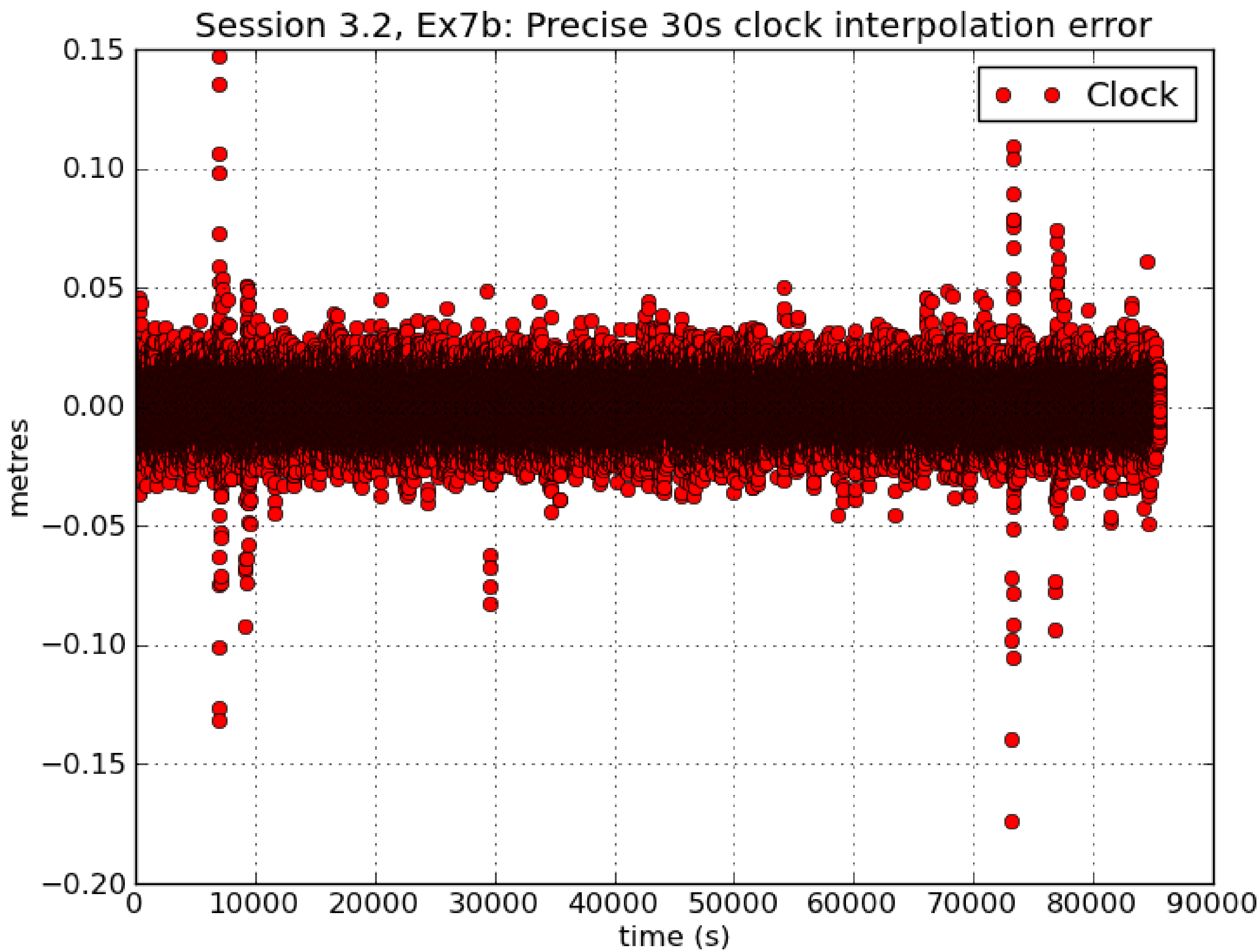


Broadcast Orbit & Clock Updates

Precise Clock Interpolation: 300s samples

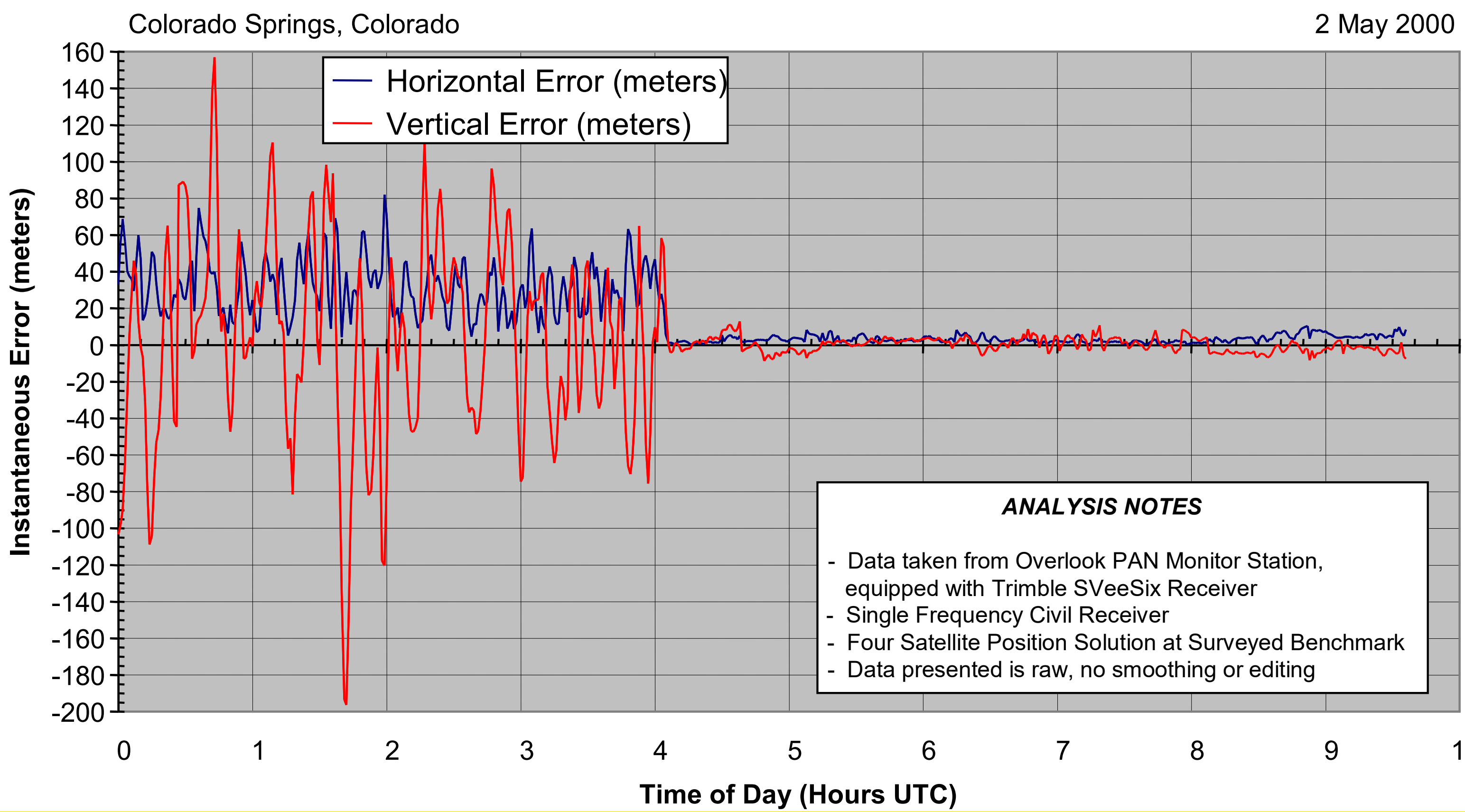


Precise Clock Interpolation: 30s samples

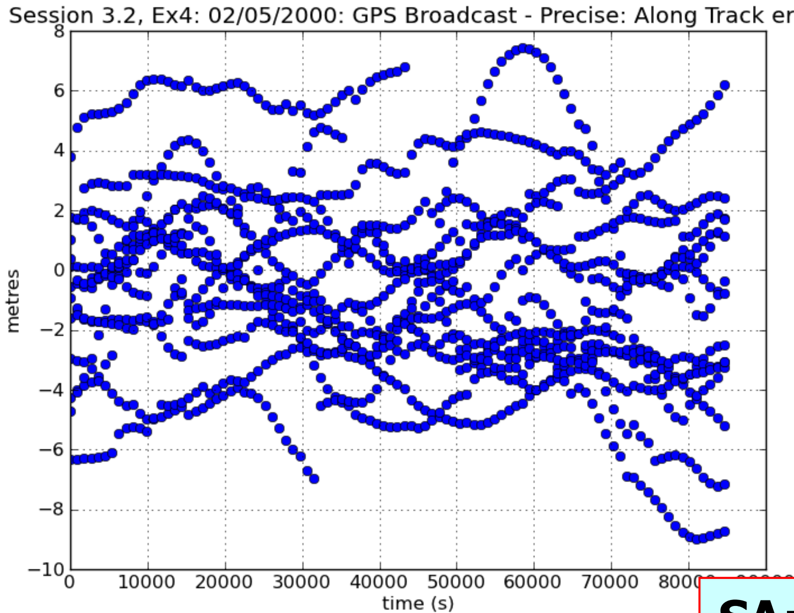


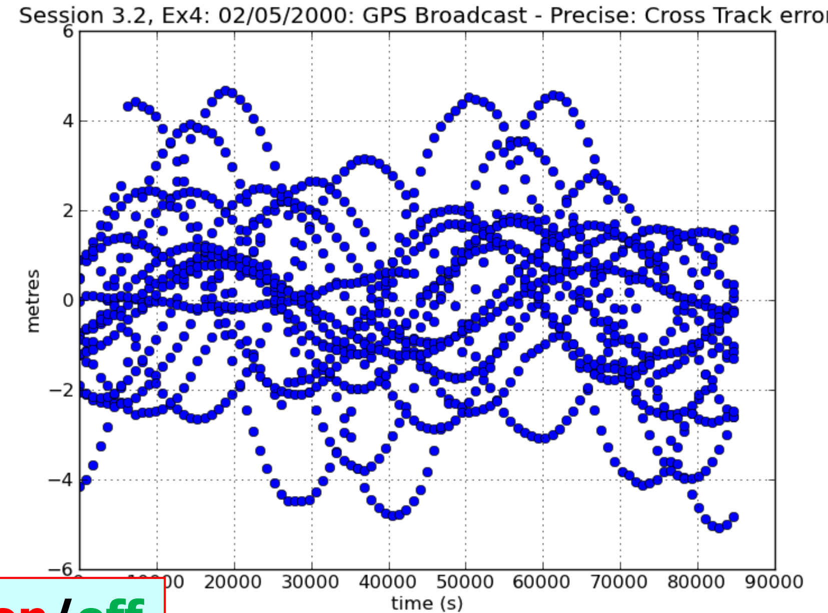
Selective Availability (S/A): Intentional degradation of satellite clocks and broadcast ephemeris. (from 25 March, 1990)

GPS Before and After S/A was switched off

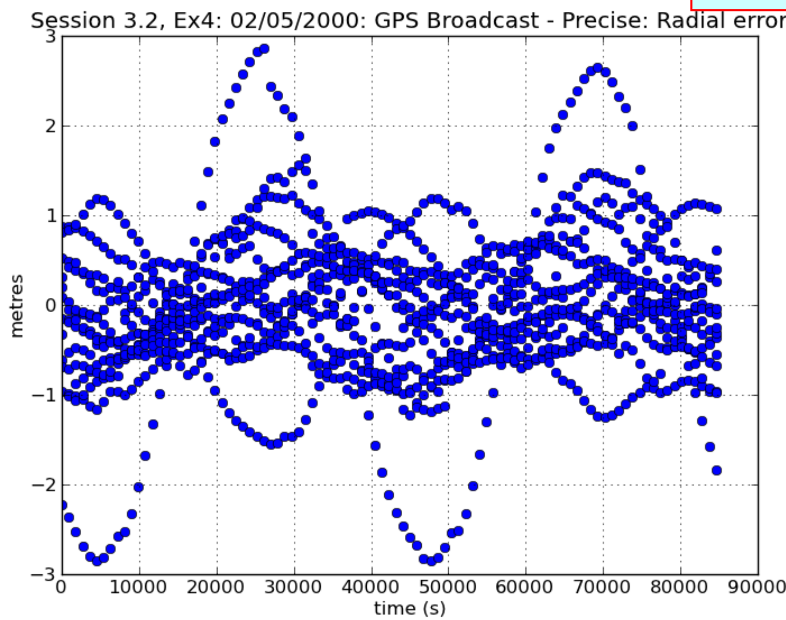


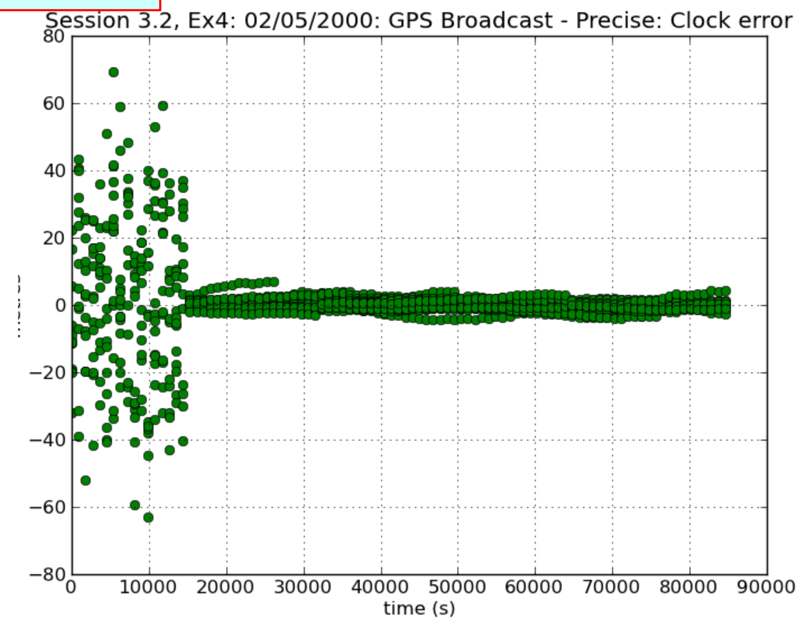
S/A was switched off at 2nd May 2000 and **Permanently removed in 2008**

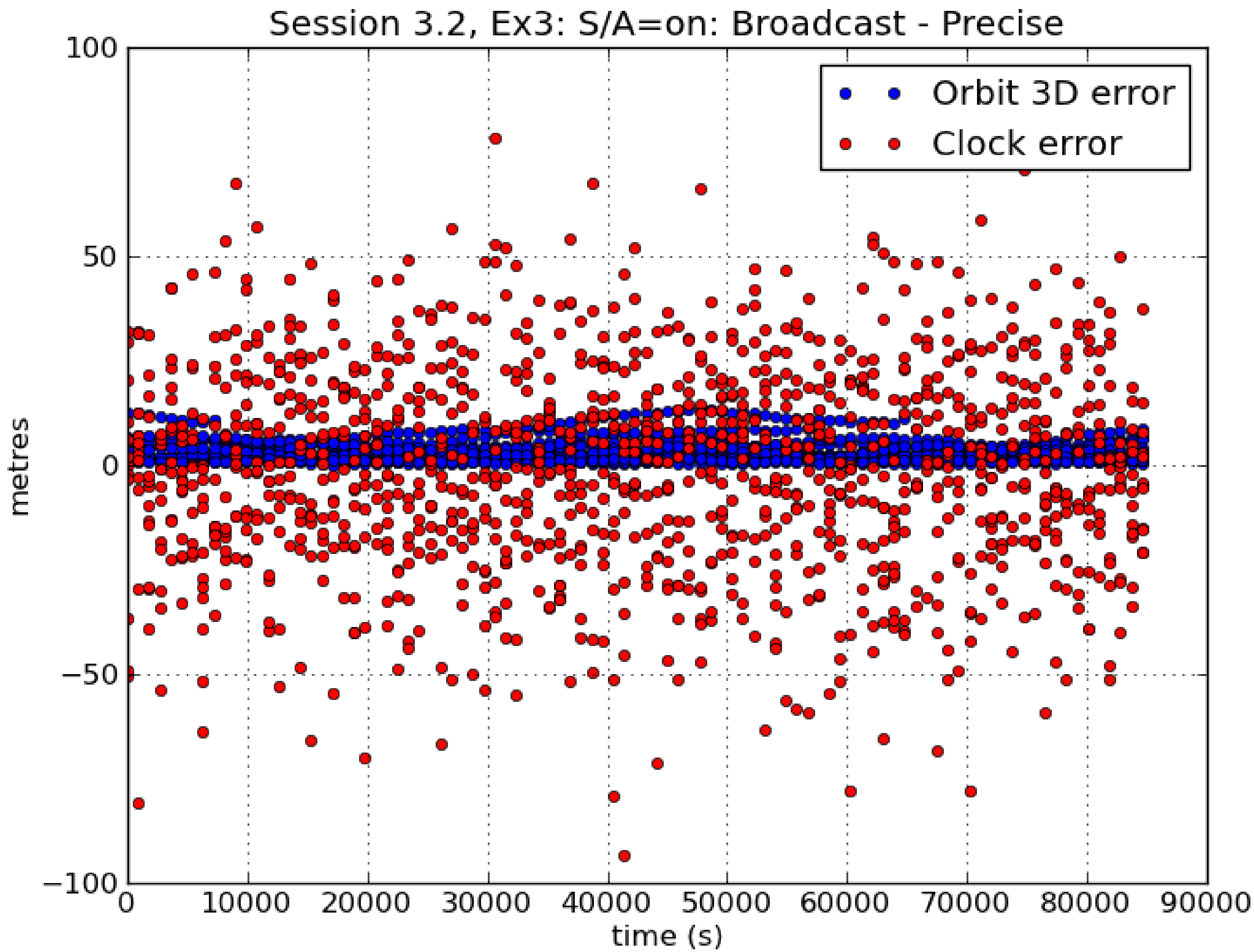

Session 3.2, Ex4: 02/05/2000: GPS Broadcast - Precise: Along Track err
metres
time (s)

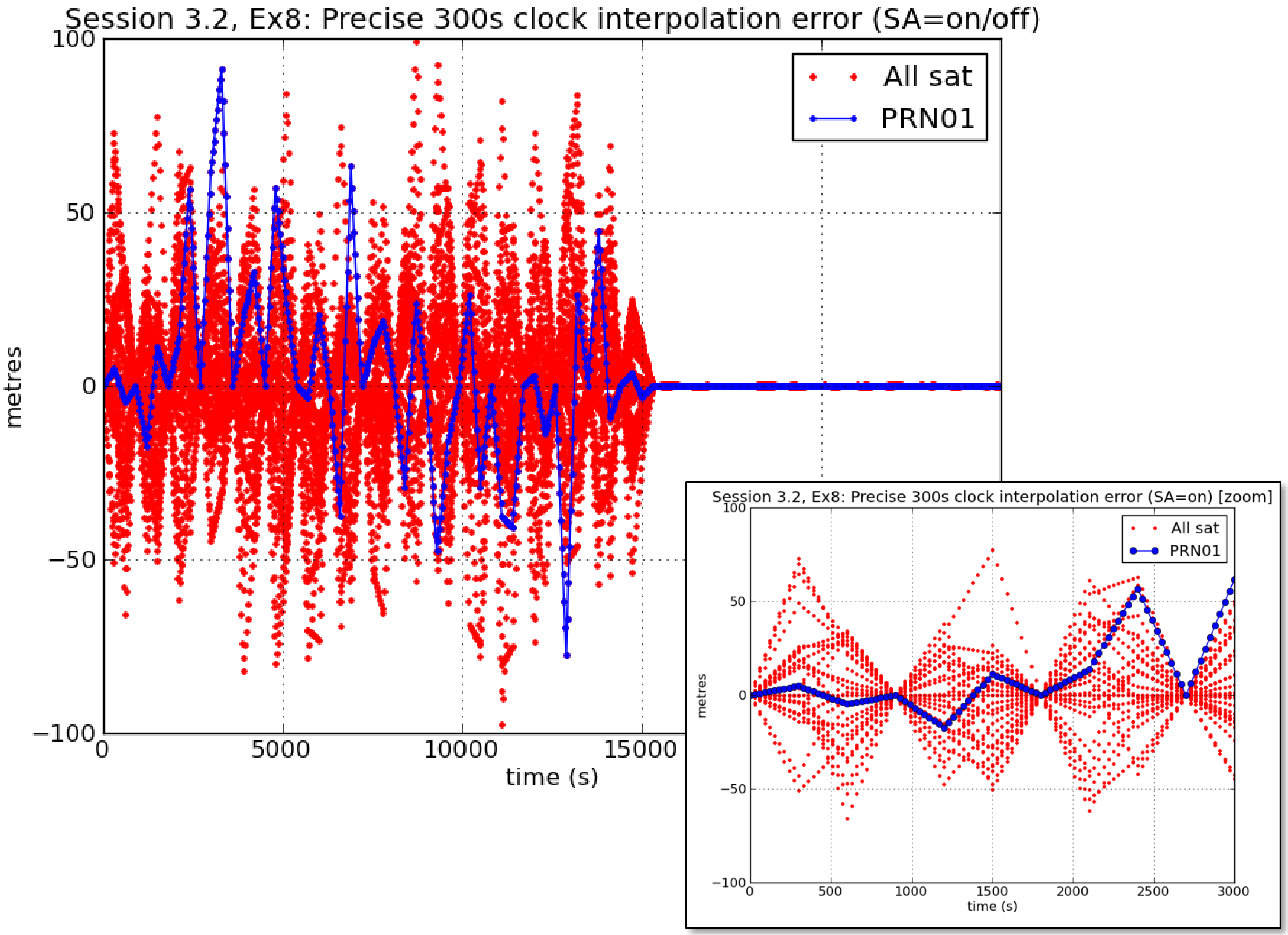

Session 3.2, Ex4: 02/05/2000: GPS Broadcast - Precise: Cross Track error
metres
time (s)

SA=on/off


Session 3.2, Ex4: 02/05/2000: GPS Broadcast - Precise: Radial error
metres
time (s)


Session 3.2, Ex4: 02/05/2000: GPS Broadcast - Precise: Clock error
time (s)


Session 3.2, Ex3: S/A=on: Broadcast - Precise
Orbit 3D error
Clock error
metres
time (s)
100
50
0
−50
−100
0
10000
20000
30000
40000
50000
60000
70000
80000
90000


Session 3.2, Ex8: Precise 300s clock interpolation error (SA=on/off)
All sat
PRN01
metres
time (s)
100
50
0
−50
−100
0
5000
10000
15000
Session 3.2, Ex8: Precise 300s clock interpolation error (SA=on) [zoom]
All sat
PRN01
metres
time (s)
500
1000
1500
2000
2500
3000

References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

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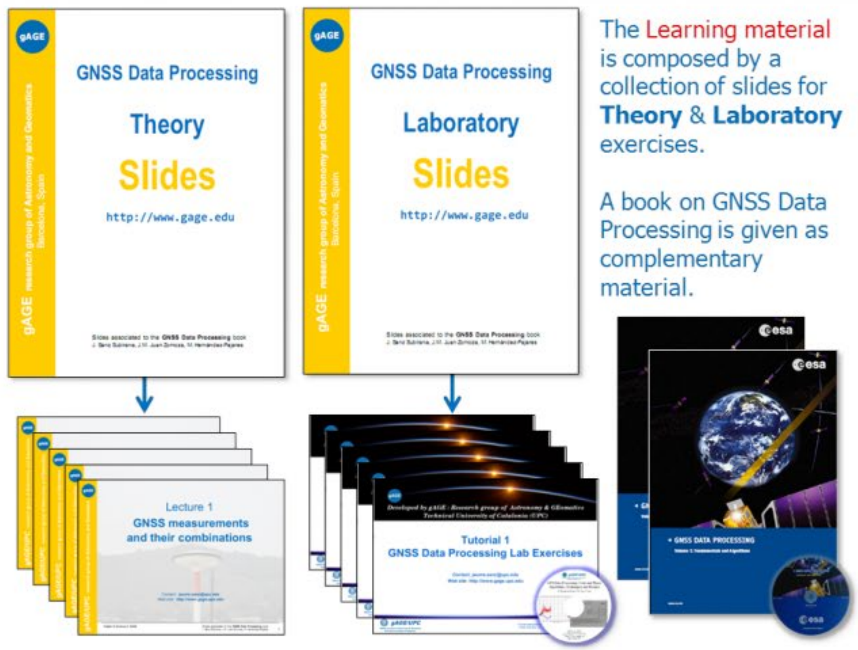
Tue, 09/03/2013 - 15:39 — jaume.sanz



The Learning material is composed by a collection of slides for **Theory** & **Laboratory** exercises.

A book on GNSS Data Processing is given as complementary material.

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