GPS Data processing: Code and Phase

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http://www.gage.es

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This course is based on the book edited by gAGE:

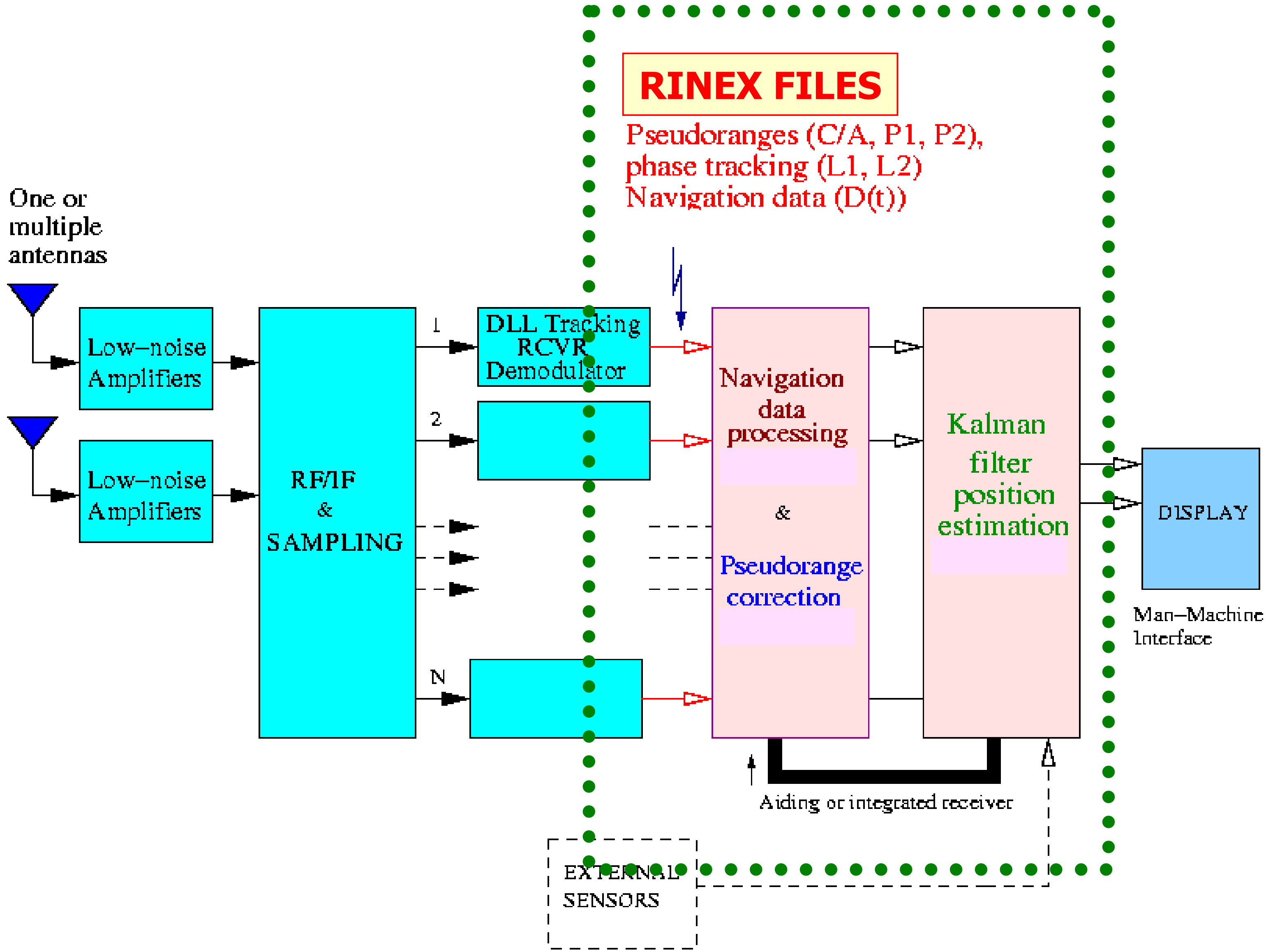
GPS data processing: code and phase.
Algorithms, techniques and recipes.

(available at http://www.gage.es)

These slides show some exercises and examples whose data files (with actual GPS data) and corresponding software (with source code) are provided in the book.

Summary

- **Introduction.**
- **GPS measurements and their combinations.**
- **Satellite coordinates.**
- **The model.**
- **Navigation equations.**
- **Code and phase differential positioning.**
  - Floating versus fixing ambiguities.


RINEX FILES
Pseudoranges (C/A, P1, P2),
phase tracking (L1, L2)
Navigation data (D(t))
One or multiple antennas
Low–noise Amplifiers
Low–noise Amplifiers
RF/IF & SAMPLING
1
2
N
DLL Tracking RCVR Demodulator
Navigation data processing
&
Pseudorange correction
Kalman filter position estimation
DISPLAY
Man–Machine Interface
Aiding or integrated receiver
EXTERNAL SENSORS

Specific Objectives:

- To learn about GPS observables (code and phase), their characteristics, properties, combinations and applications.
- To learn how to calculate satellites orbits and clocks from navigation message. To know the achievable precision.
- To learn how to model pseudodistance for code and phase measurements. This includes calculation of: 1) Coordinates at emission epoch, 2) Ionospheric delay (Klobuchar model), 3) Tropospheric delay, 4) relativistic correction, 5) clocks offsets and satellite instrumental delays, 6) phase wind-up, etc.
- To learn how to set and solve the navigation equation system using least-squares or Kalman filter (algorithm level).
- To know how to use phase differential positioning: Floating and fixing ambiguities.

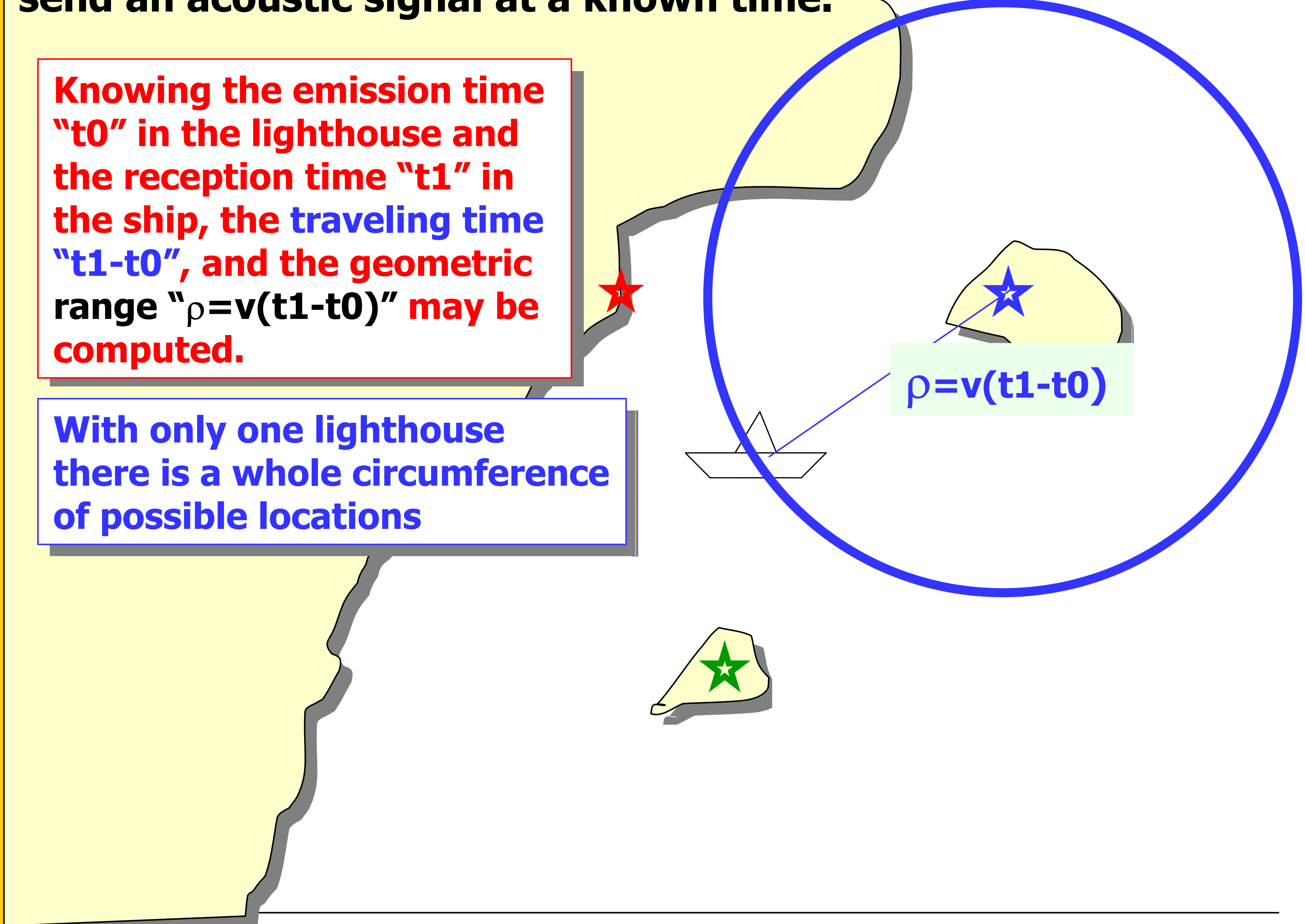
To get tools and skills to process and analyze GPS data.
To implement algorithms for satellite navigation.

Introduction

A ship determines its location from a set on lighthouses that send an acoustic signal at a known time.

Knowing the emission time "t0" in the lighthouse and the reception time "t1" in the ship, the traveling time "t1-t0", and the geometric range "ρ=v(t1-t0)" may be computed.

With only one lighthouse there is a whole circumference of possible locations



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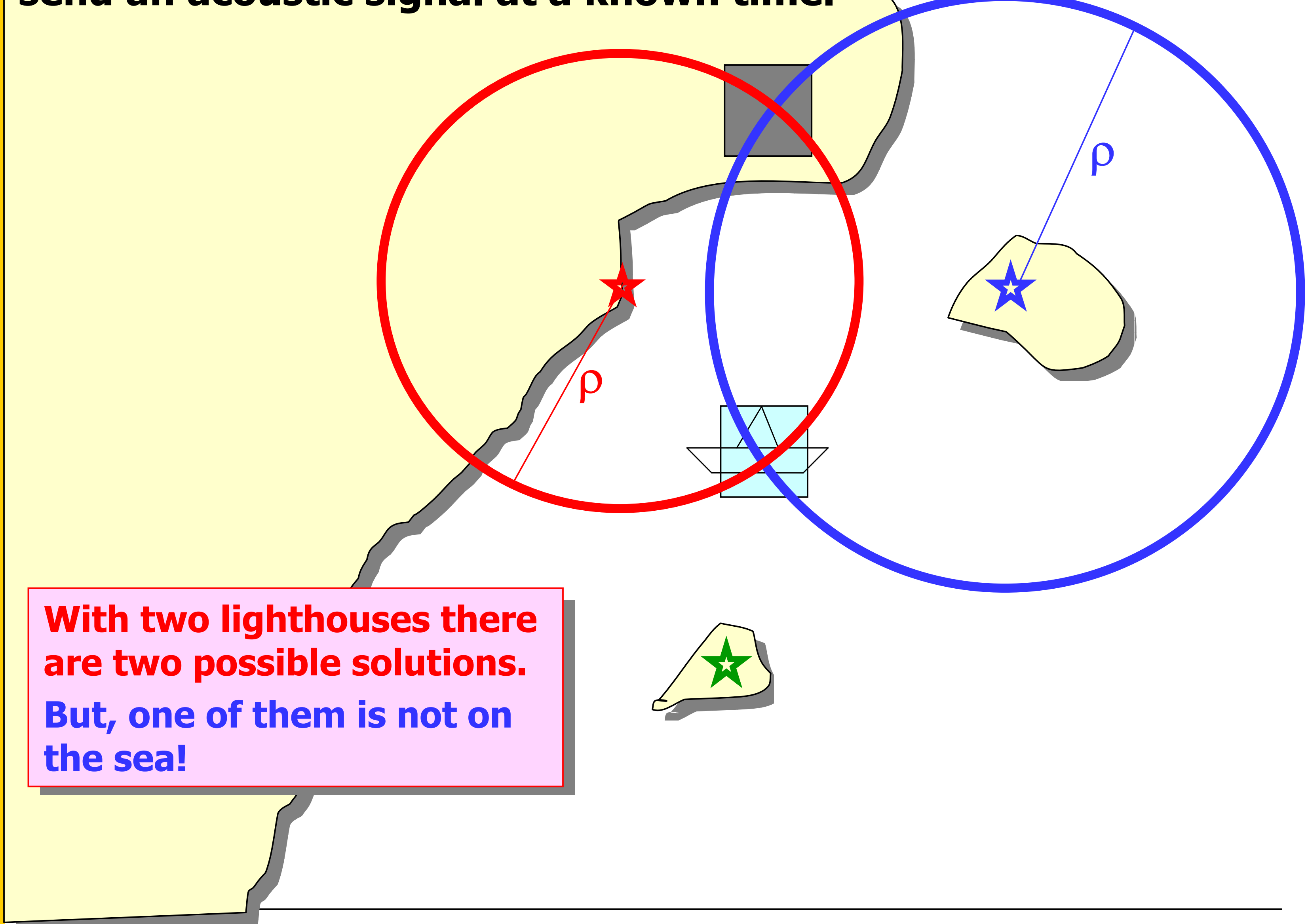
A ship determines its location from a set on lighthouses that send an acoustic signal at a known time.



With two lighthouses there are two possible solutions.

But, one of them is not on the sea!

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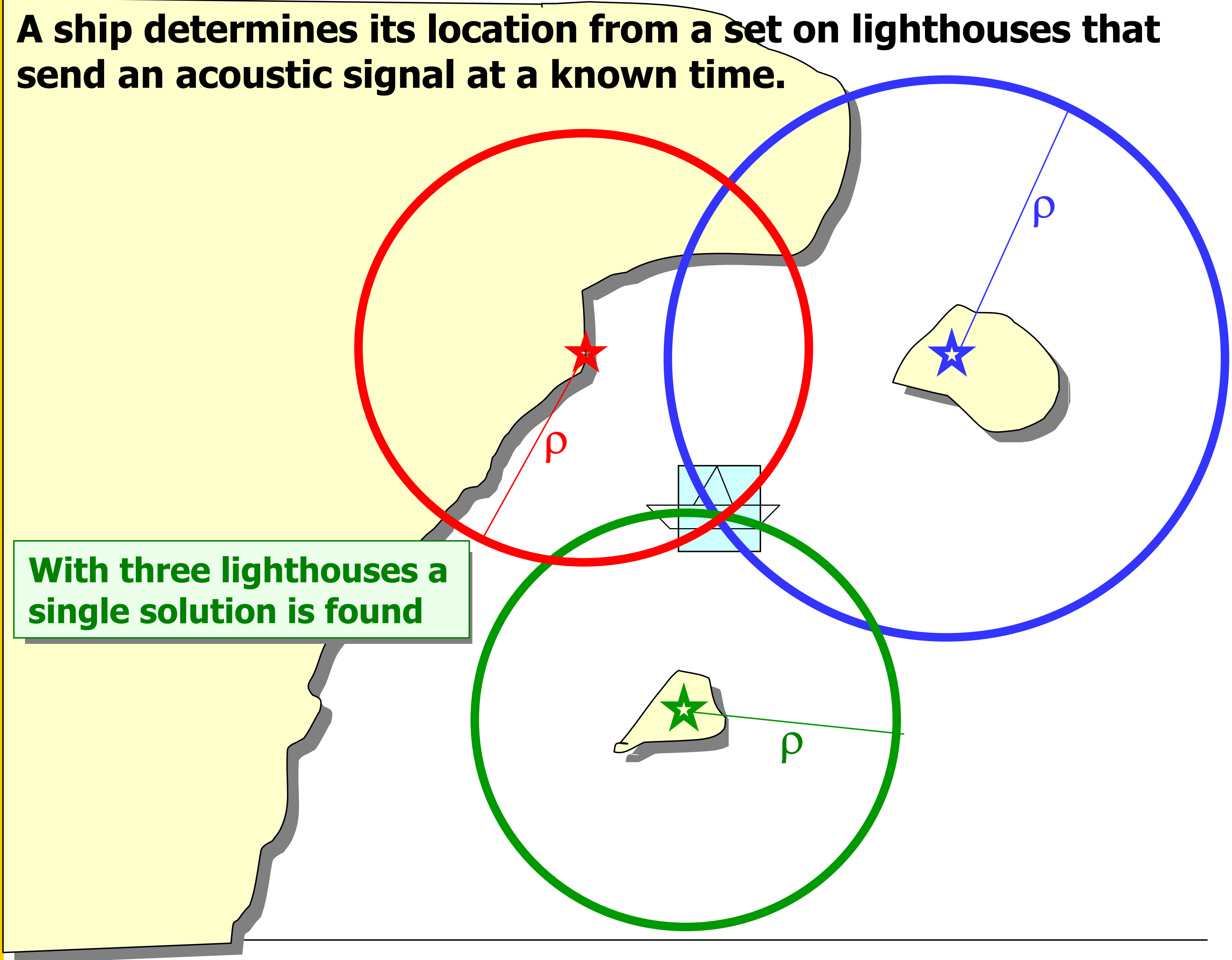
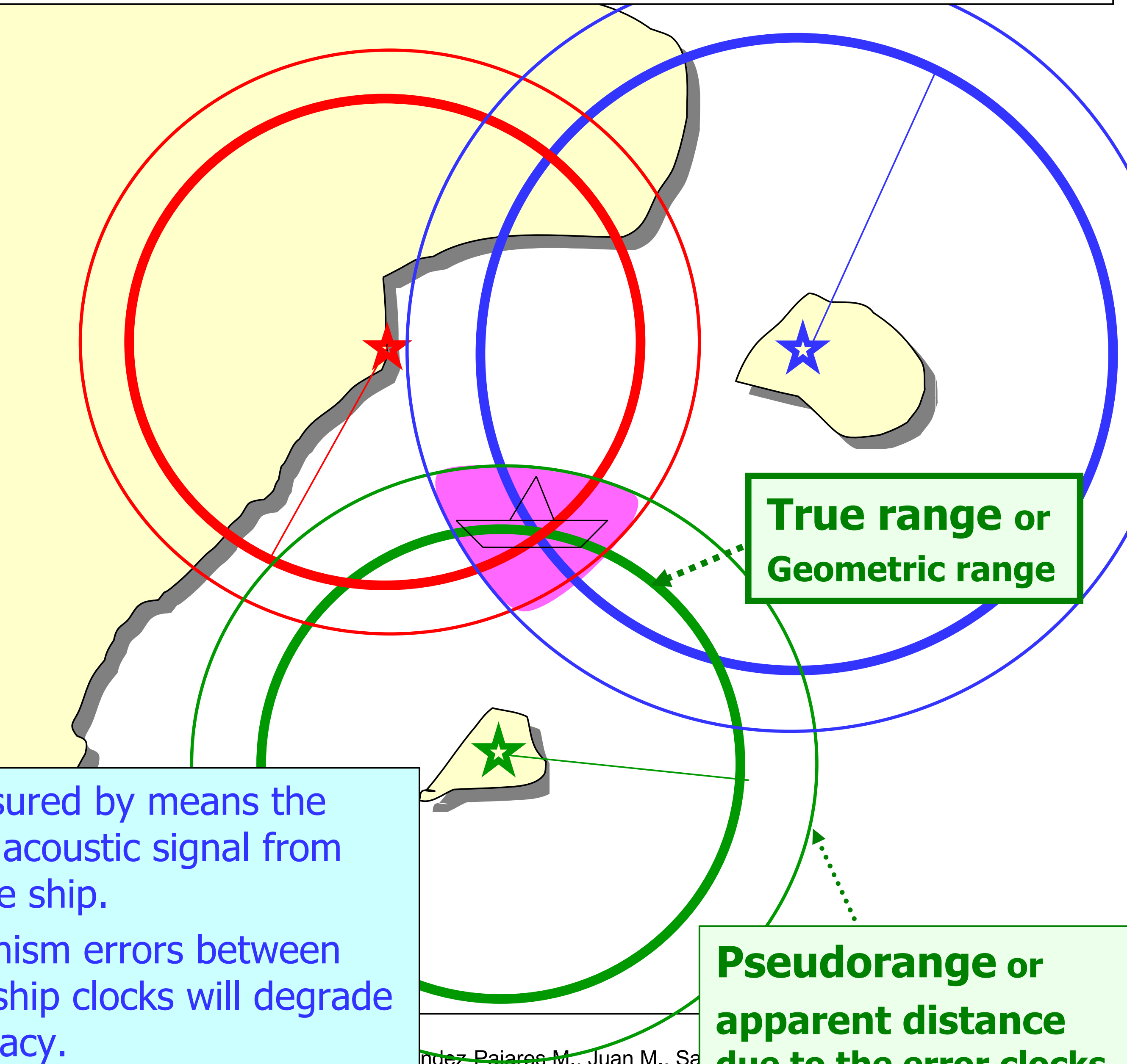
A ship determines its location from a set on lighthouses that send an acoustic signal at a known time.



Errors in the clocks (lighthouses and ship) synchronism affects the accuracy

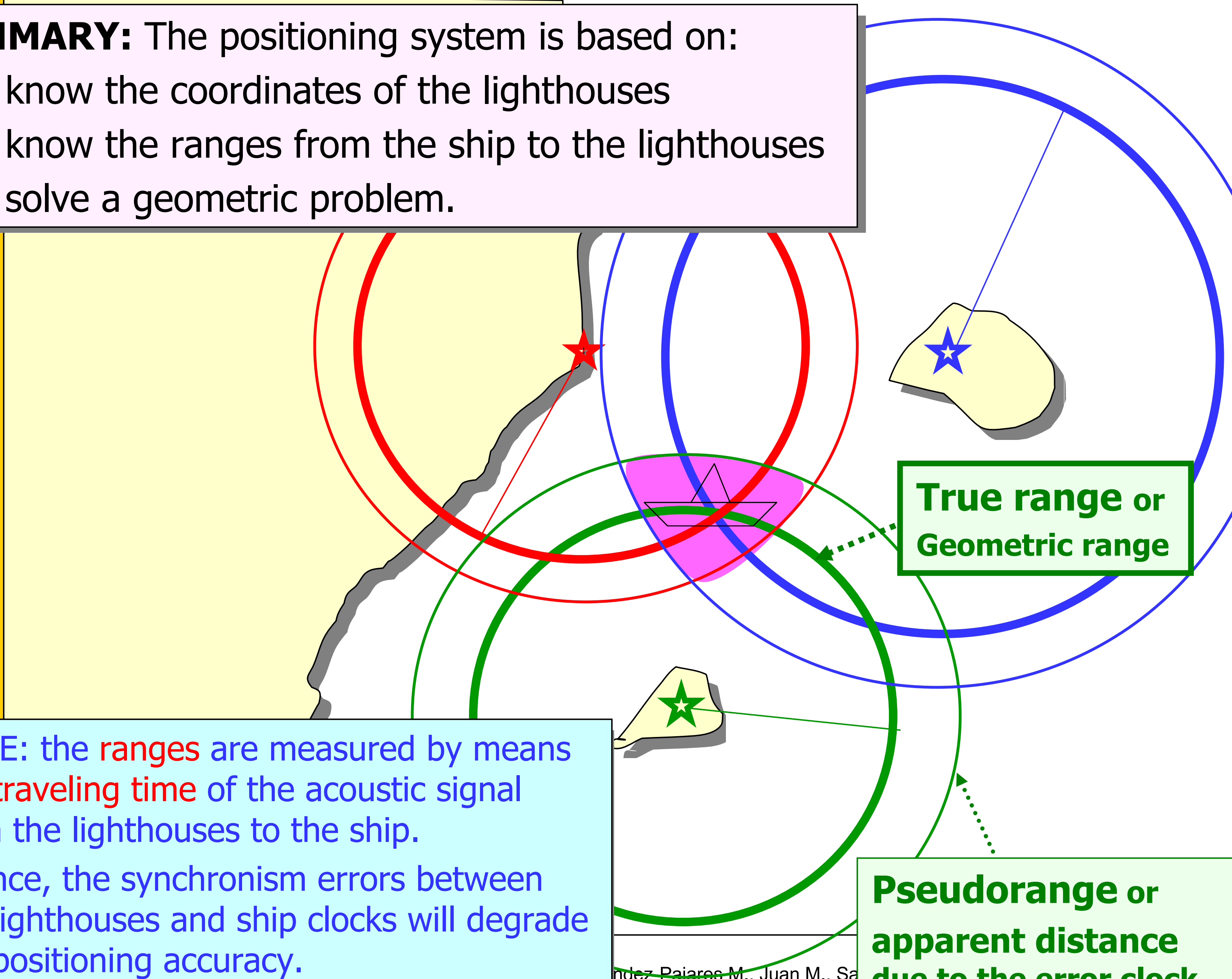


The ranges are measured by means the traveling time of the acoustic signal from the lighthouses to the ship.

Thence, the synchronism errors between the lighthouses and ship clocks will degrade the positioning accuracy.

SUMMARY: The positioning system is based on:

- To know the coordinates of the lighthouses
- To know the ranges from the ship to the lighthouses
- To solve a geometric problem.



NOTE: the ranges are measured by means the traveling time of the acoustic signal from the lighthouses to the ship.

Thence, the synchronism errors between the lighthouses and ship clocks will degrade the positioning accuracy.

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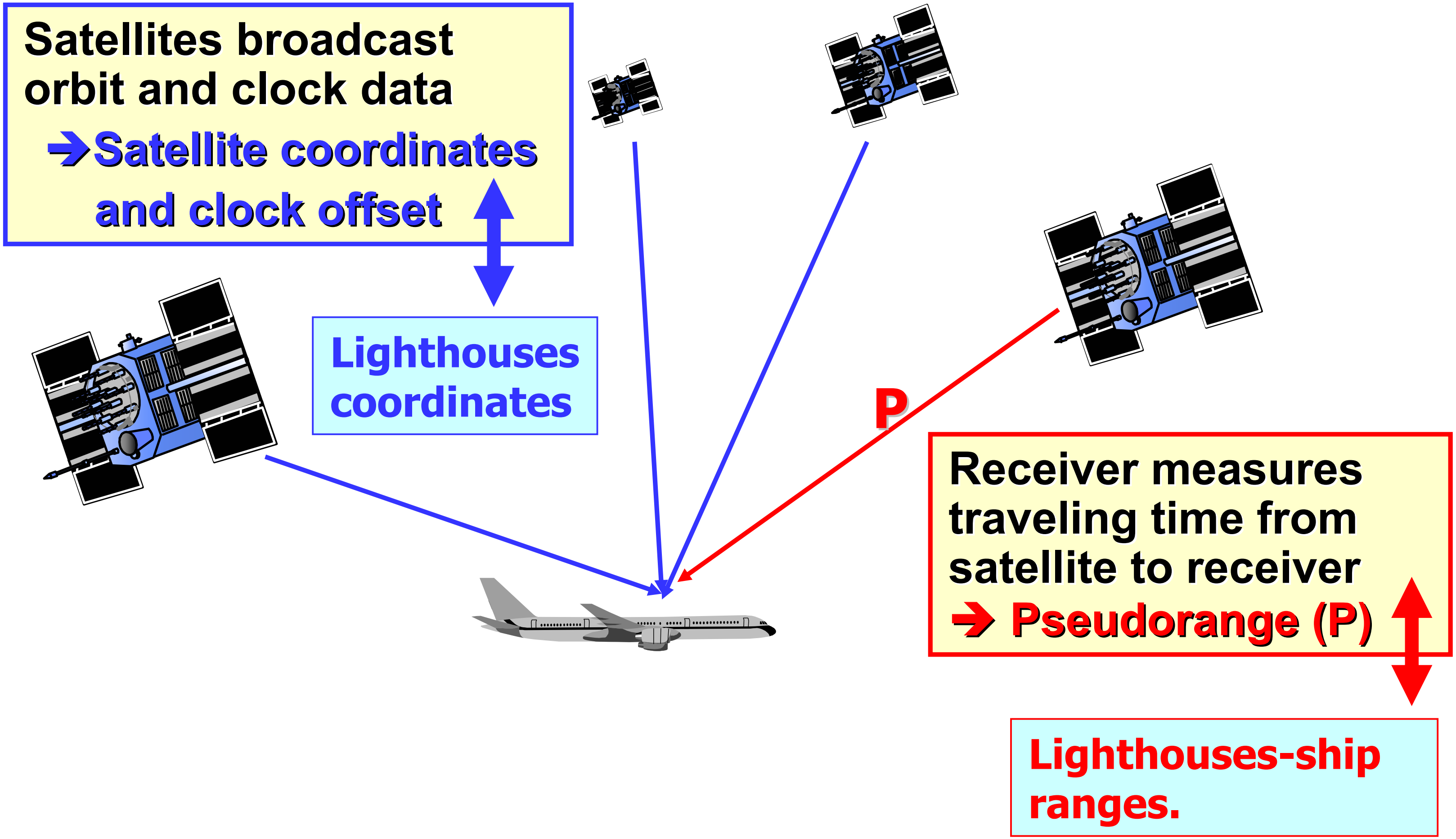
How GPS Works



Thence, the receiver coordinates are found **solving a geometrical problem**: from sat. coordinates and ranges

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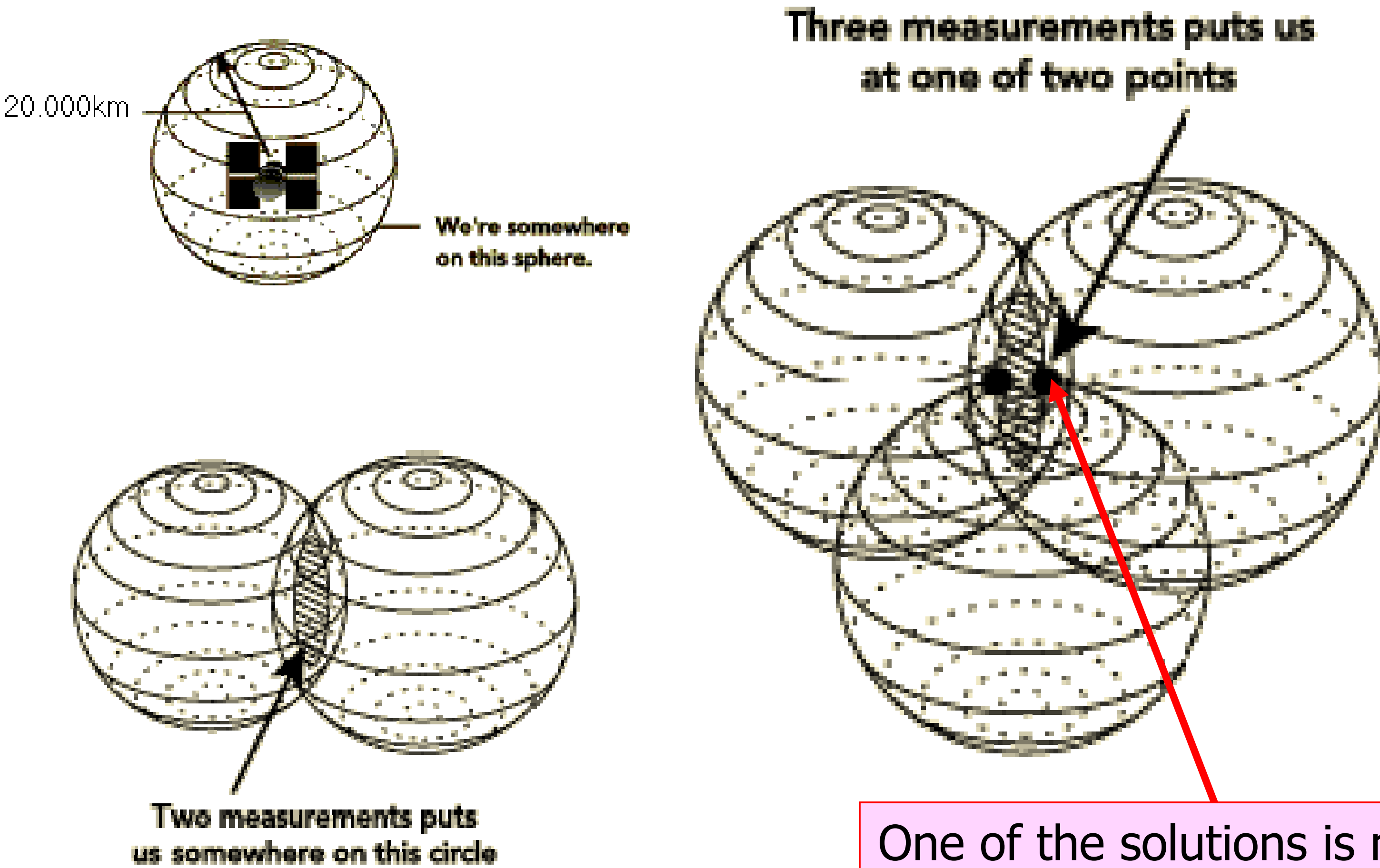
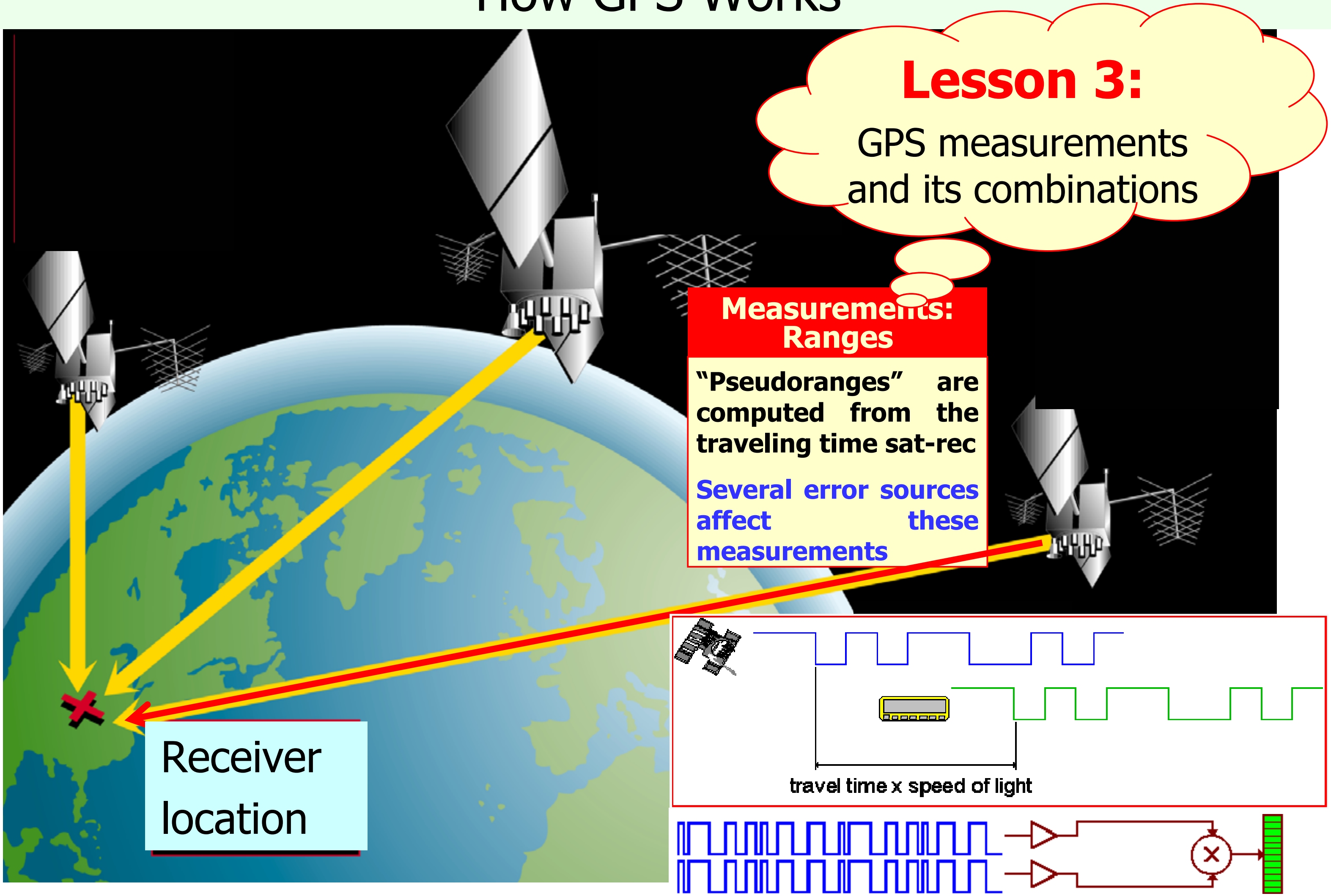
How GPS Works



One of the solutions is not on the Earth surface.

How GPS Works

How GPS Works



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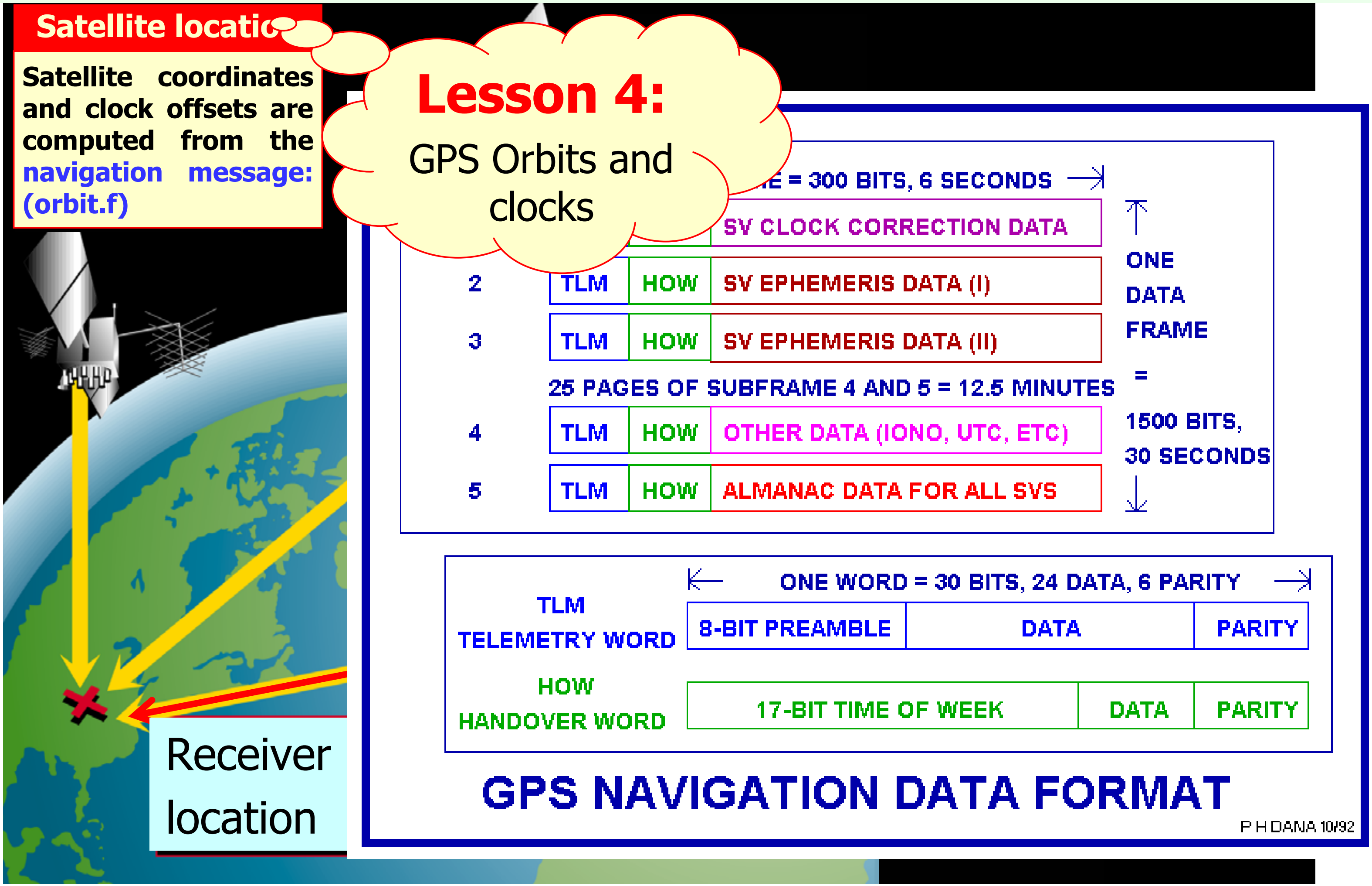
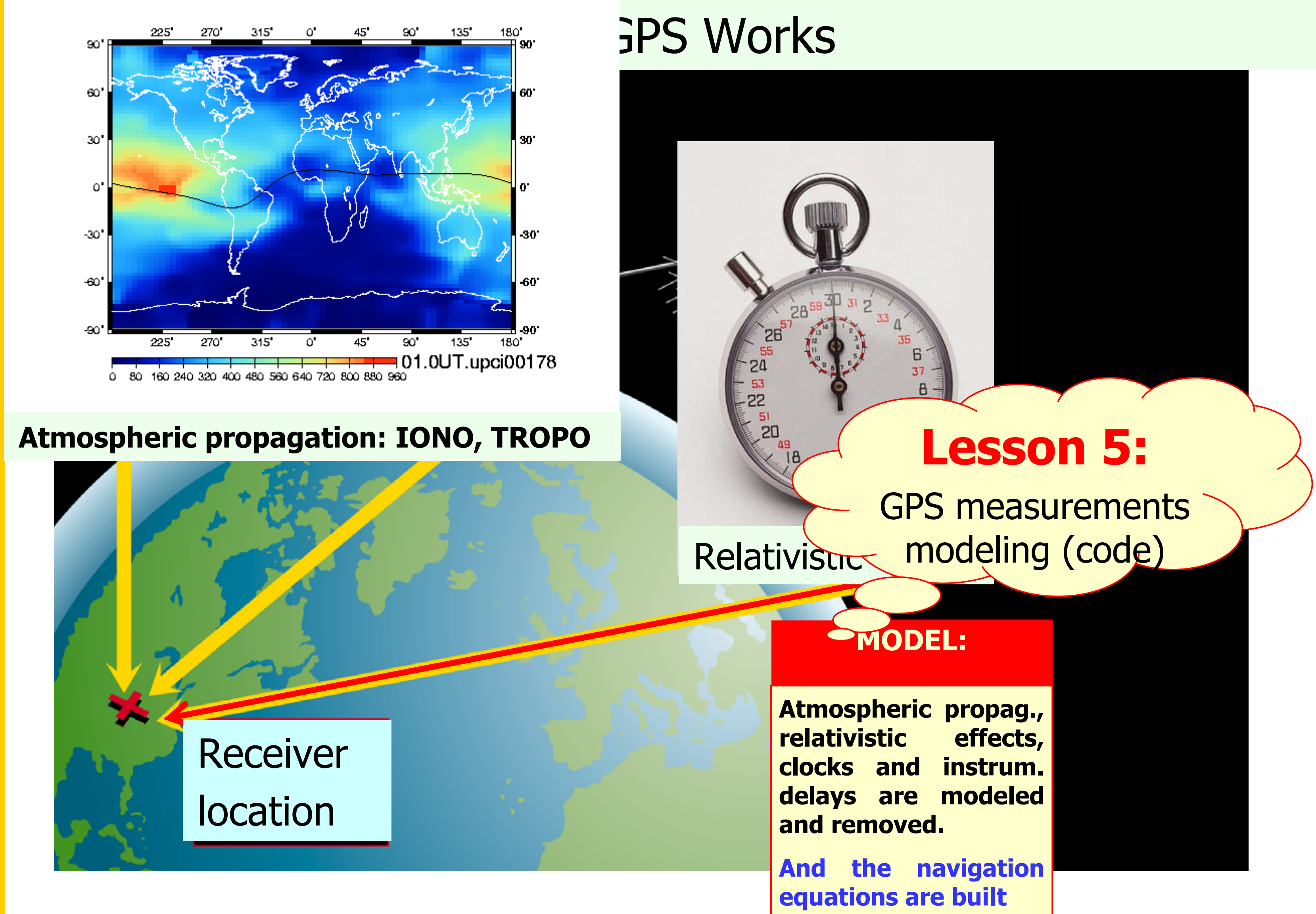
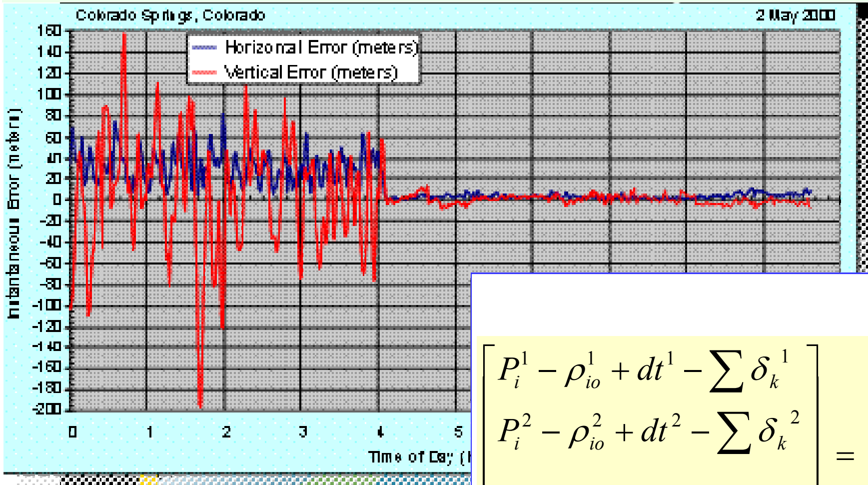
How GPS Works



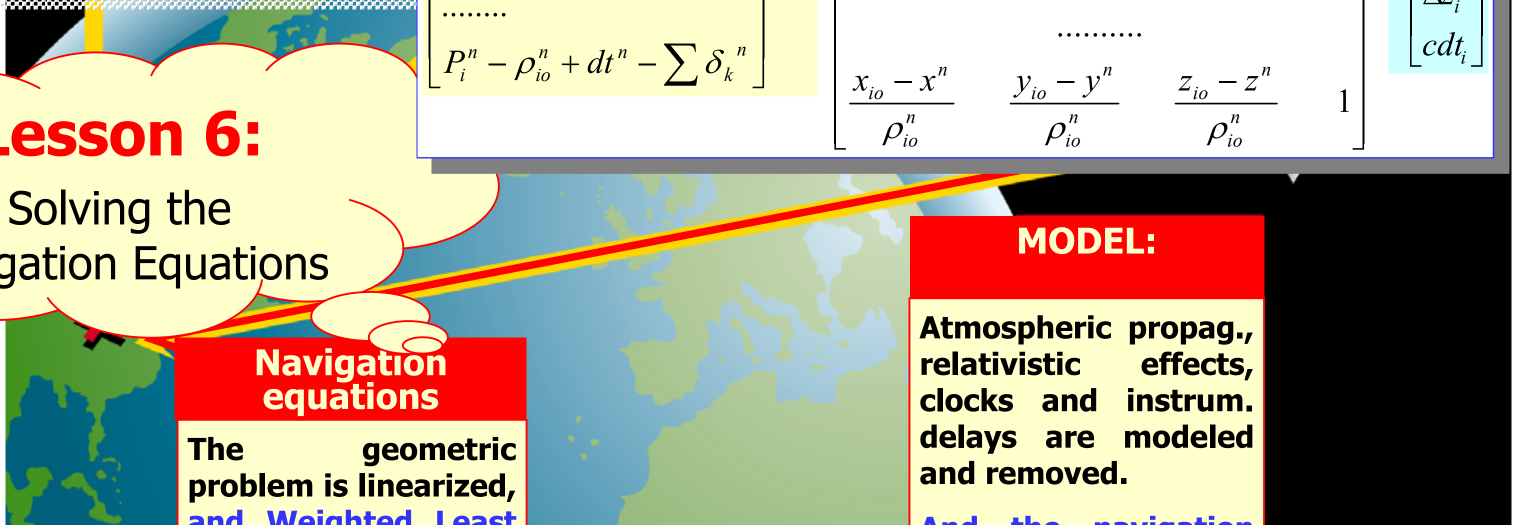
GPS Works


225° 270° 315° 0° 45° 90° 135° 180°
90° 60° 30° 0° -30° -60° -90°
0 80 160 240 320 400 480 560 640 720 800 880 960
01.0UT.upci00178
Atmospheric propagation: IONO, TROPO
Relativistic
Lesson 5:
GPS measurements modeling (code)
Receiver location
MODEL:
Atmospheric propag., relativistic effects, clocks and instrum. delays are modeled and removed.
And the navigation equations are built

How GPS Works



$$\begin{bmatrix} P_i^1-\rho_{io}^1+dt^1-\sum\delta_k^{\,1}\\ P_i^2-\rho_{io}^2+dt^2-\sum\delta_k^{\,2}\\ \cdots\cdots\\ P_i^n-\rho_{io}^n+dt^n-\sum\delta_k^{\,n} \end{bmatrix} = \begin{bmatrix} \frac{x_{io}-x^1}{\rho_{io}^1} & \frac{y_{io}-y^1}{\rho_{io}^1} & \frac{z_{io}-z^1}{\rho_{io}^1} & 1\\ \frac{x_{io}-x^2}{\rho_{io}^2} & \frac{y_{io}-y^2}{\rho_{io}^2} & \frac{z_{io}-z^2}{\rho_{io}^2} & 1\\ & \cdots\cdots\cdots & & \\ \frac{x_{io}-x^n}{\rho_{io}^n} & \frac{y_{io}-y^n}{\rho_{io}^n} & \frac{z_{io}-z^n}{\rho_{io}^n} & 1 \end{bmatrix} \begin{bmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\\ cdt_i\end{bmatrix}$$



Lesson 6:

Solving the navigation Equations

Navigation equations

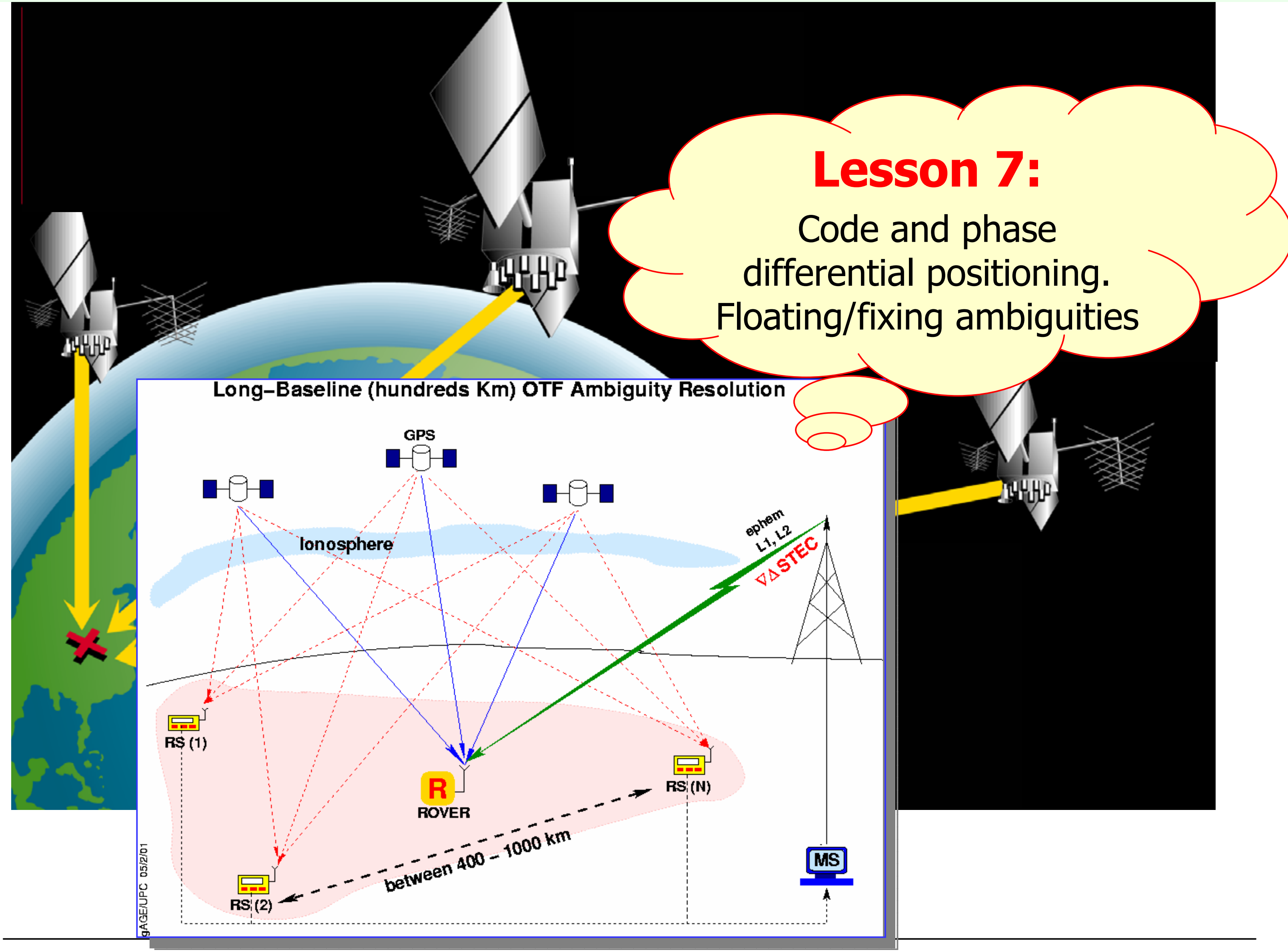
The geometric problem is linearized, and Weighted Least Mean Squares or Kalman filter are used to compute the solution.

MODEL:

Atmospheric propag., relativistic effects, clocks and instrum. delays are modeled and removed.

And the navigation equations are built

How GPS Works



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Lesson 3

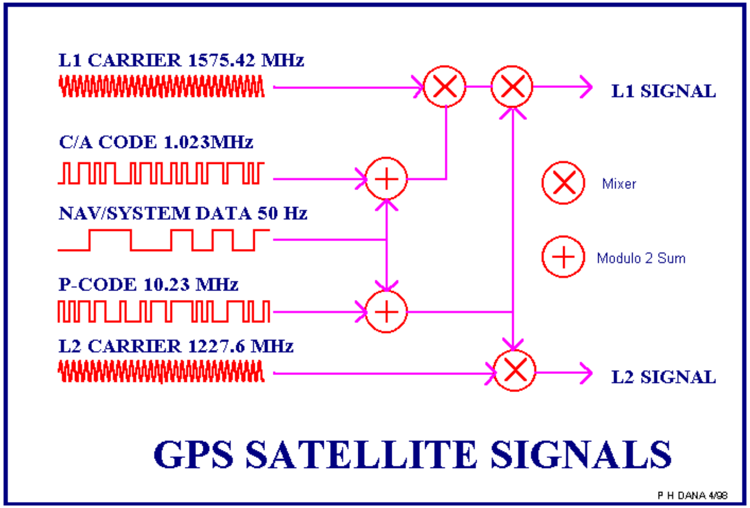
GPS measurements and their combinations: The RINEX files

GPS SIGNAL STRUCTURE

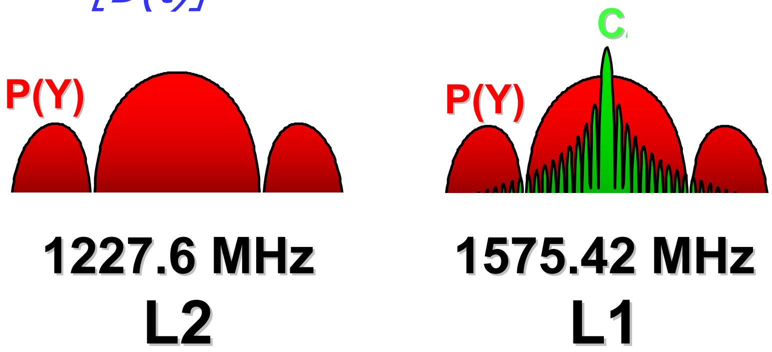
Two carriers in L-band:

- L1=154 fo=1575.42 Mhz
- L2=120 fo=1227.60 Mhz

where fo=10.23 Mhz



- C/A-code for civilian users *[C(t)]*
- P-code only for military and authorized users *[P(t)]*
- Navigation message with satellite ephemeris and clock corrections *[D(t)]*



$$L_1(t) = a_1 P(t) D(t) \sin(f_1 t) + a_1 C(t) D(t) \cos(f_1 t)$$

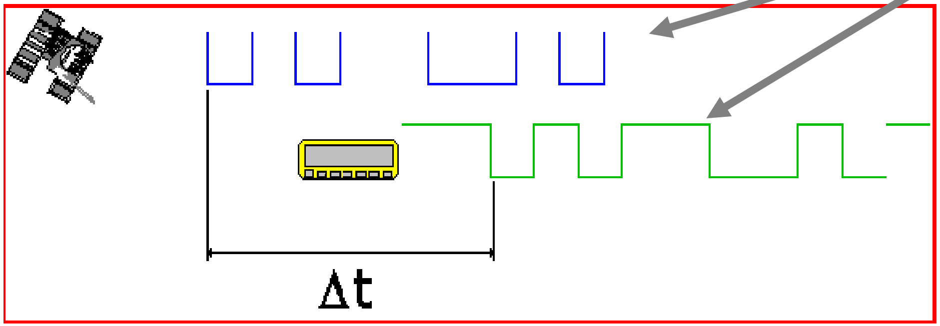
$$L_2(t) = a_2 P(t) D(t) \sin(f_2 t)$$

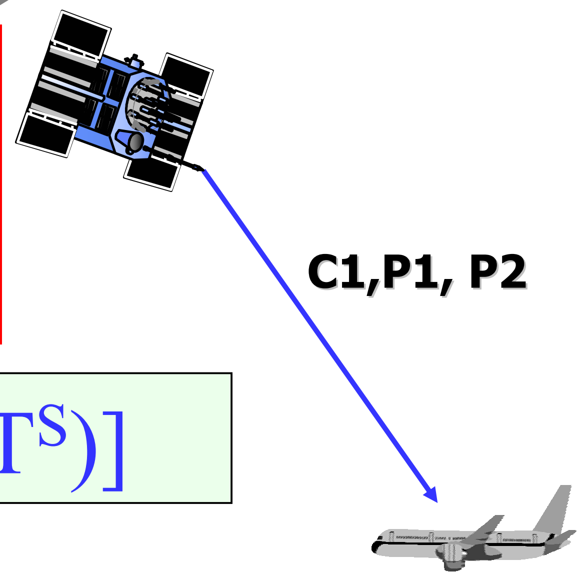
GPS Pseudorange Measurements

$$L_1(t) = a_1 \boxed{P(t)} D(t)\sin(f_1 t) + a_1 C(t) D(t)\cos(f_1 t)$$

$$L_2(t) = a_2 P(t) D(t)\sin(f_2 t)$$

binary code P





$$P1 = c\,\Delta t = c\,[t_{rec}(T_R) - t^{sat}(T^S)]$$

From hereafter we will call:

- C1 pseudorange computed from C binary code (on frequency 1)
- P1 pseudorange computed from P binary code (on frequency 1)
- P2 pseudorange computed from P binary code (on frequency 2)

GPS Phase Measuremen

$$L_1(t) = a_1 P(t) D(t) \sin(f_1 t) + a_1 C(t) D(t) \cos(f_1 t$$

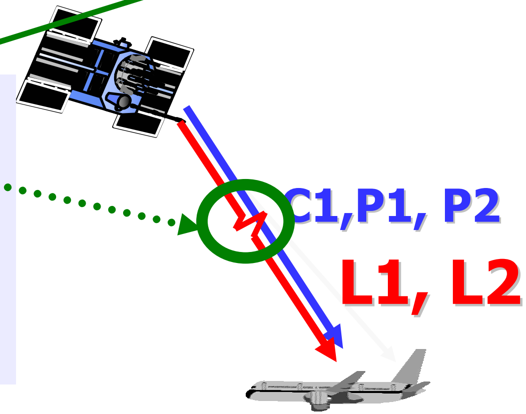
$$L_2(t) = a_2 P(t) D(t) \sin(f_2 t)$$

$$\Delta f = f_r - f_e = -\frac{\dot{\rho}}{c} f_e$$

$$\rho = \int \dot{\rho} = -c \int \frac{\Delta f}{f_e} + ctt$$

Carrier phase L

"pseudoranges" **L1, L2** (containing unknown bias) can be also measured from the carrier phases *$L_1(t)$, $L_2(t)$* (*integrated Doppler*)

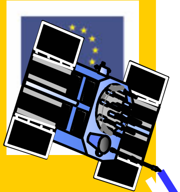


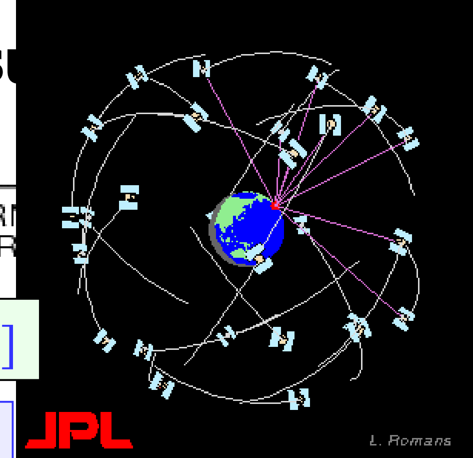
From hereafter we will call:

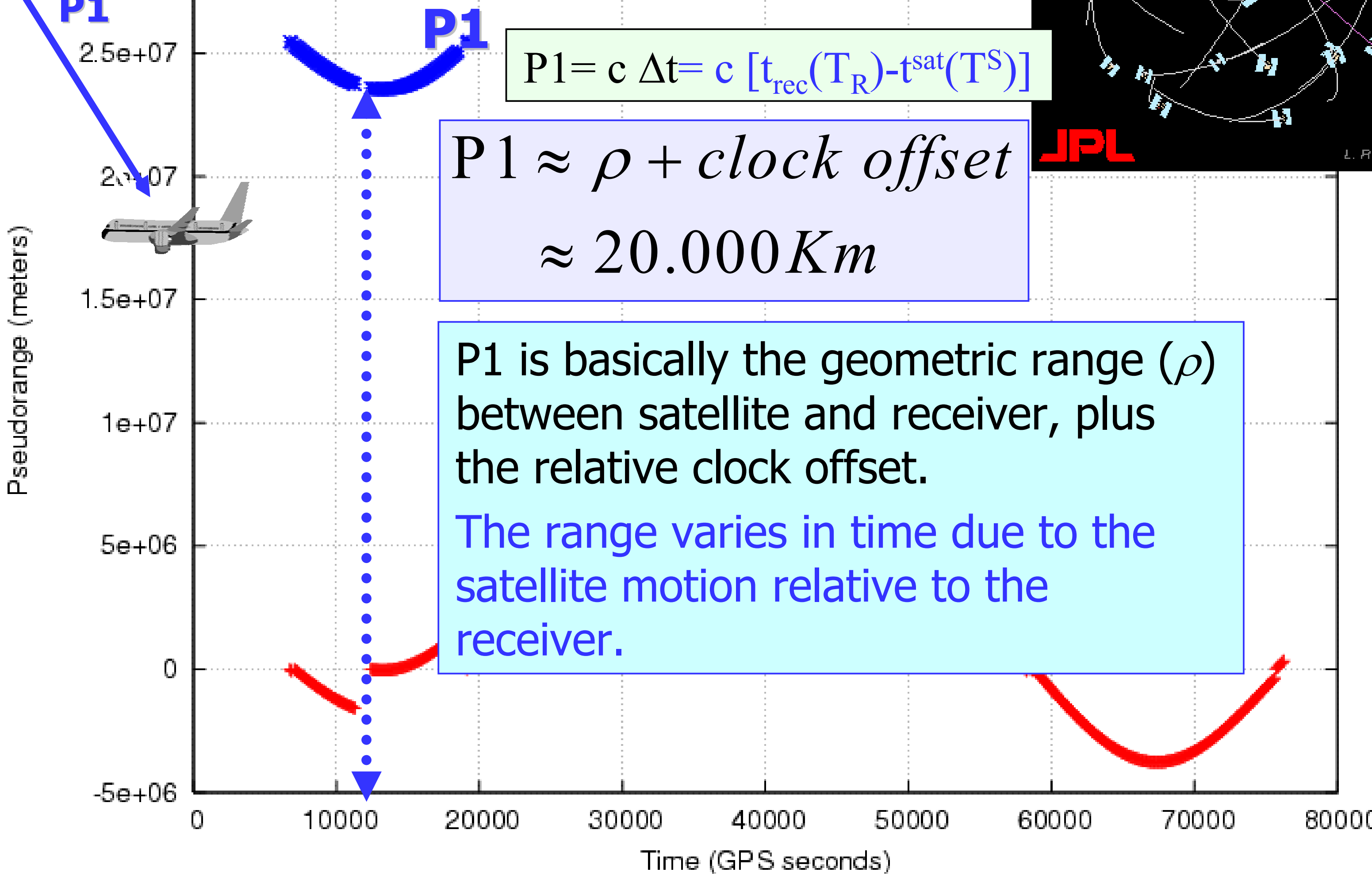
- L1 phase measur. computed from the carrier phase on frequency 1
- L2 phase measur. computed from the carrier phase on frequency 2

- C1 pseudorange computed from C binary code (on frequency 1)
- P1 pseudorange computed from P binary code (on frequency 1)
- P2 pseudorange computed from P binary code (on frequency 2)

Phase and Code pseudorange meas







$P1 = c\,\Delta t = c\,[t_{rec}(T_R) - t^{sat}(T^S)]$

$$P1 \approx \rho + clock\ offset$$
$$\approx 20.000\,Km$$

P1 is basically the geometric range (ρ) between satellite and receiver, plus the relative clock offset.

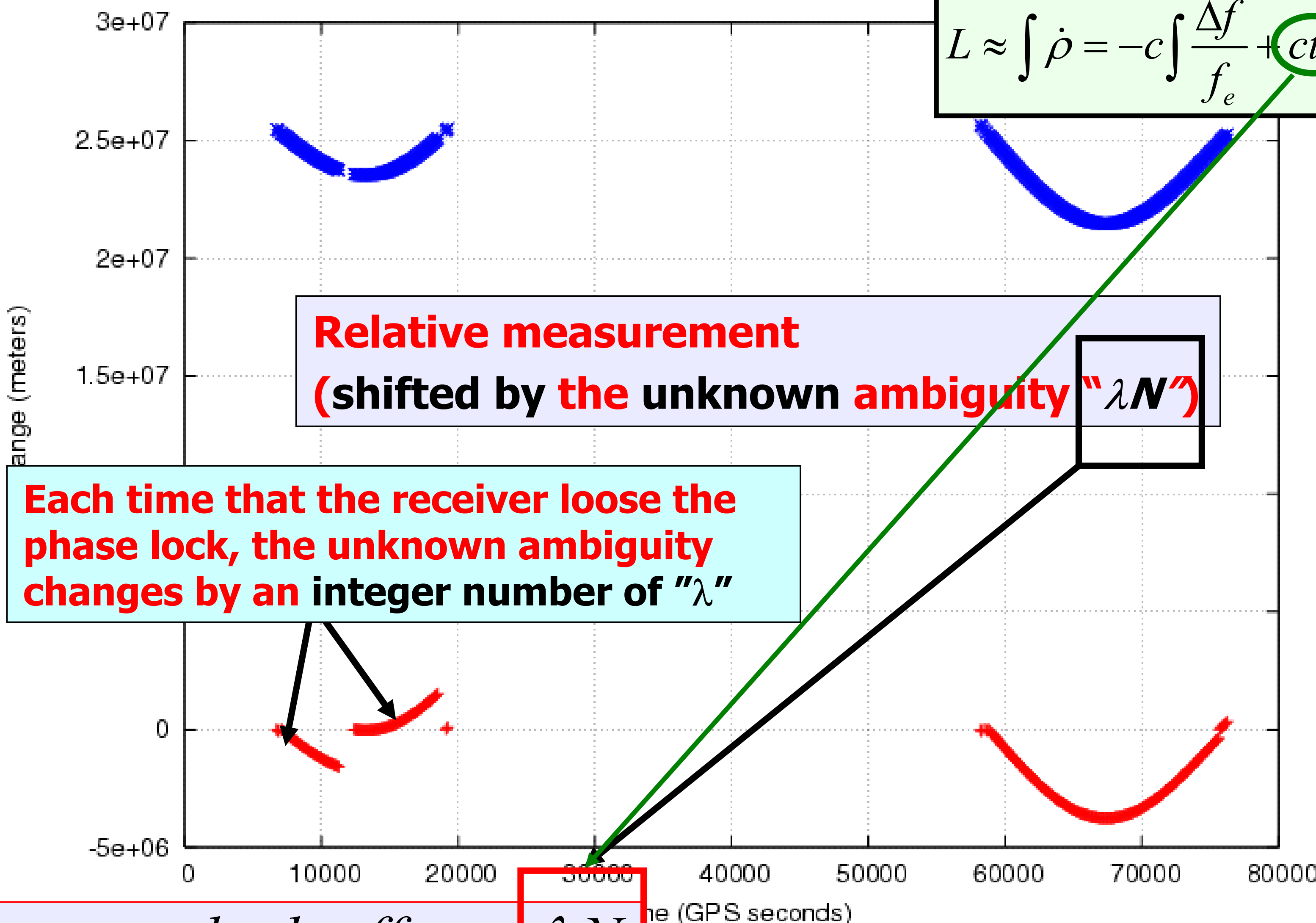
The range varies in time due to the satellite motion relative to the receiver.

P1 is an **absolute** measurement **(unambiguous)**

Phase and Code pseudorange mea

$$\Delta f = f_r - f_e = -\frac{\dot{\rho}}{c} f_e$$

$$L \approx \int \dot{\rho} = -c \int \frac{\Delta f}{f_e} + ctt$$



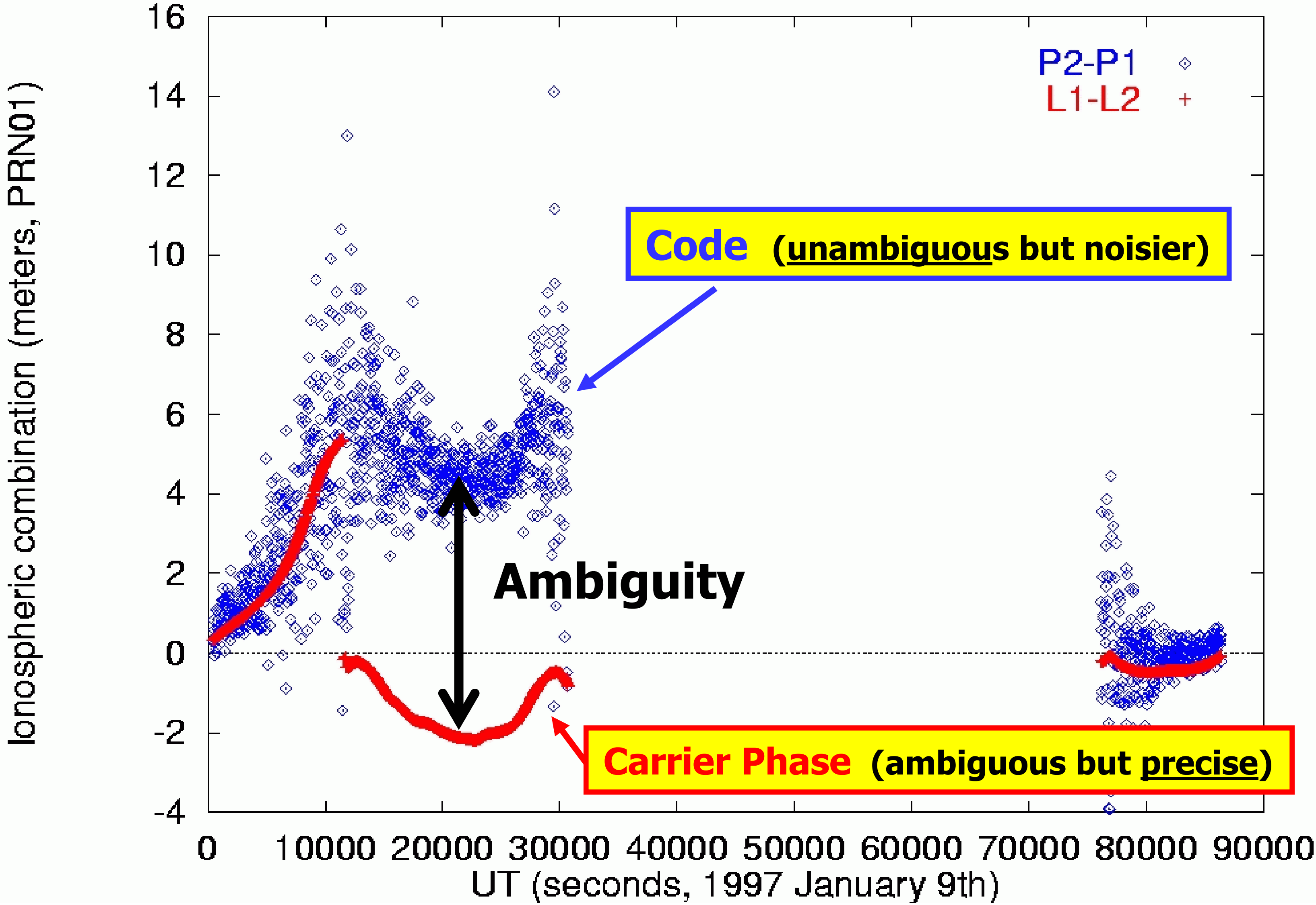
Relative measurement

(shifted by the unknown ambiguity "λN")

Each time that the receiver loose the phase lock, the unknown ambiguity changes by an integer number of "λ"

$$L1 \approx \rho + clock\ offset + \boxed{\lambda N}$$

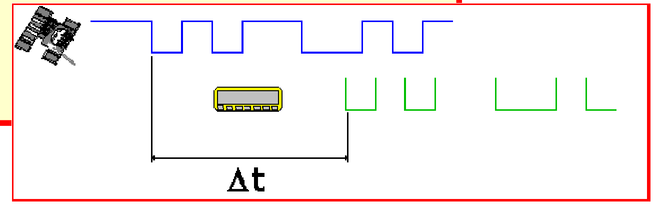
Code and phase measurements



GPS measurements: Code and Phase pseudoranges

$$L_1(t) = a_1 P(t) D(t) \sin(f_1 t) + a_1 C(t) D(t) \cos(f_1 t)$$

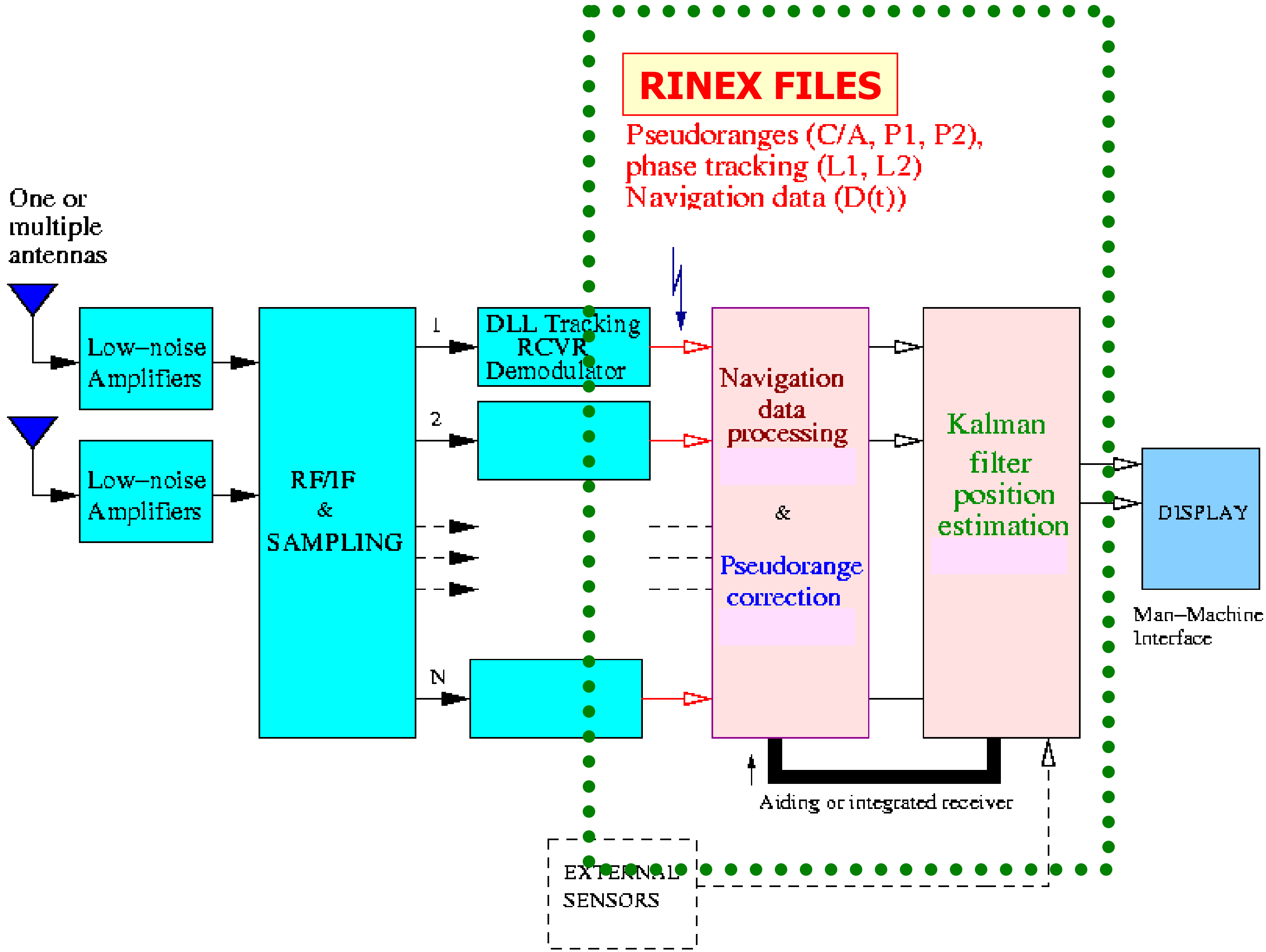
$$L_2(t) = a_2 P(t) D(t) \sin(f_2 t)$$



Antispoofing (A/S):
The code P is encrypted to Y.
→ Only the code C at frequency L1 is available.

	Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
Code measurements			
C1	300 m	3 m	**Unambiguous** but noisier
P1 (Y1): encrypted	30 m	30 cm	
P2 (Y2): encrypted	30 m	30 cm	
Phase measurements			
L1	19.05 cm	2 mm	**Precise** but ambiguous
L2	24.45 cm	2 mm	

[*] the codes can be smoothed with the phases in order to reduce noise (i.e, C1 smoothed with L1 → 50 cm noise)



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RINEX measurement file

HEADER

```
     2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX  AUSLIG              10-JAN-97 10:19     PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN)  - COCOS ISLAND      COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
     -0.000000000103          HARDWARE CALIBRATION (S)      COMMENT
     -0.000000054663          CLOCK OFFSET (S)              COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 uslig                                   OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:     >500  >100   >50   >10    >5    >0   bad   n/a    COMMENT
 sig:        9     8     7     6     5     4     1     0    COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
```

MEASUREMENTS

```
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127                               1118481.28445  22127685.4014  <=====  1
  22672                               8969469.30045  22672158.5184  <=====  25
  22594902.367    -12949753.825 7  -10090708.53945  22594903.7394  <=====  9
  22731128.796    -11621184.951 7   -9055464.16945  22731130.0094  <=====  5
  24610920.702      -924108.174 6    -720085.67045  24610920.0404  <=====  23
  20718775.074    -18605935.474 9  -14498133.97346  20718775.6074  <=====  17
  20842713.610    -19083282.892 9  -14870090.55546  20842713.4814  <=====  6
```

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        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
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 SNR:     >500  >100   >50   >10    >5    >0   bad   n/a    COMMENT
 sig:        9     8     7     6     5     4     1     0    COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014   <=====  1
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184   <===== 25
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394   <=====  9
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094   <=====  5
  24610920.702     -924108.174 6   -720085.67045  24610920.0404   <===== 23
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  20842713.610   -19083282.892 9 -14870090.55546  20842713.4814   <=====  6
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Australian Regional GPS Network (ARGN)  - COCOS ISLAND      COMMENT
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COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
                    MARGOLIN T                              ANT # / TYPE
                    624 -1337769.9813                       APPROX POSITION XYZ
                    000         0.0000                      ANTENNA: DELTA H/E/N
     1      1                                               WAVELENGTH FACT L1/2
     5     C1     L1     L2     P2     P1                   # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:    >500   >100                                        COMMENT
 sig:       9      8                                        COMMENT
    30                                                      INTERVAL
  1997      1      9      0      7   30.0000000             TIME OF FIRST OBS
  1997      1      9     23     59   30.0000000             TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014   <=====  1
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184   <=====  25
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394   <=====  9
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094   <=====  5
  24610920.702     -924108.174 6    -720085.                      <=====  23
  20718                          9 -14498133.                     <=====  17
  20842                          9 -14870090.                     <=====  6
```

Measurement time (receive time tags)

Number of tracked satellites

Epoch flag 0: OK

One satellite per row

RINEX measurement file

```
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327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:     >500  >100   >50   >10    >5    >0   bad   n/a    COMMENT
 sig:        9     8     7     6     5     4     1     0    COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.000000                  TIME OF FIRST OBS
  1997     1     9    23    59   30.000000                  TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105  -14268715.899 8 -11118481.28445  22127685.4014   <====== 1
  22672158.746  -11510817.892 7  -8969469.30045  22672158.5184   <====== 25
  22594902.367  -12949753.825 7 -10090708.53945  22594903.7394   <====== 9
  22731128.796  -11621184.951 7  -9055464.16945  22731130.0094   <====== 5
  24610920.702    -924108.174 6   -720085.67045  24610920.0404   <====== 23
  20718775.074  -18605935.474 9 -14498133.97346  20718775.6074   <====== 17
  20842713.610  -19083282.892 9 -14870090.55546  20842713.4814   <====== 6
```

Synthetic P2 (A/S=on)

S/N indicator

Loss of lock indicator

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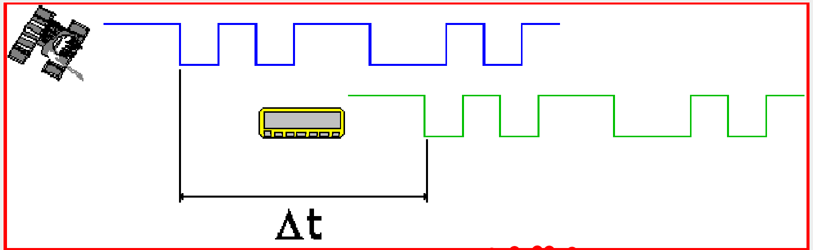
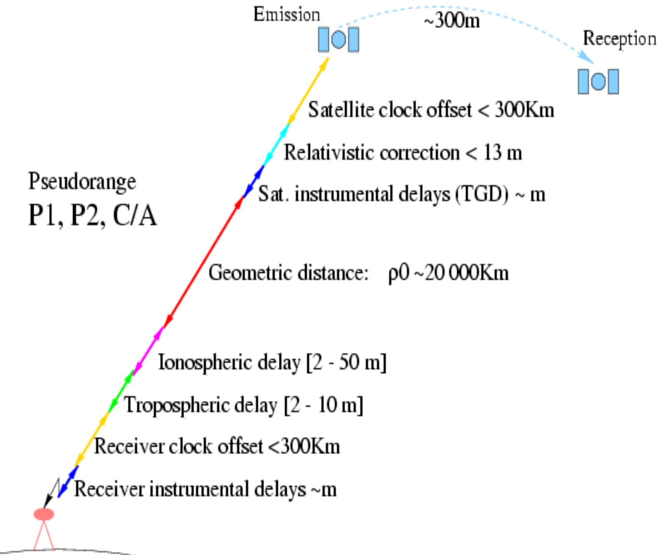


$$P = c\,\Delta t = c\,[t_{rec}(T_R) - t^{sat}(T^S)]$$



$$P_{rec}^{sat} = \rho_{rec}^{sat} + c\cdot\left(dt_{rec} - dt^{sat}\right) + \sum \delta$$

Geometric range

Clock offsets

$$\sum \delta = rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{rec}^{sat} + K_{rec} + K^{sat} + \varepsilon$$

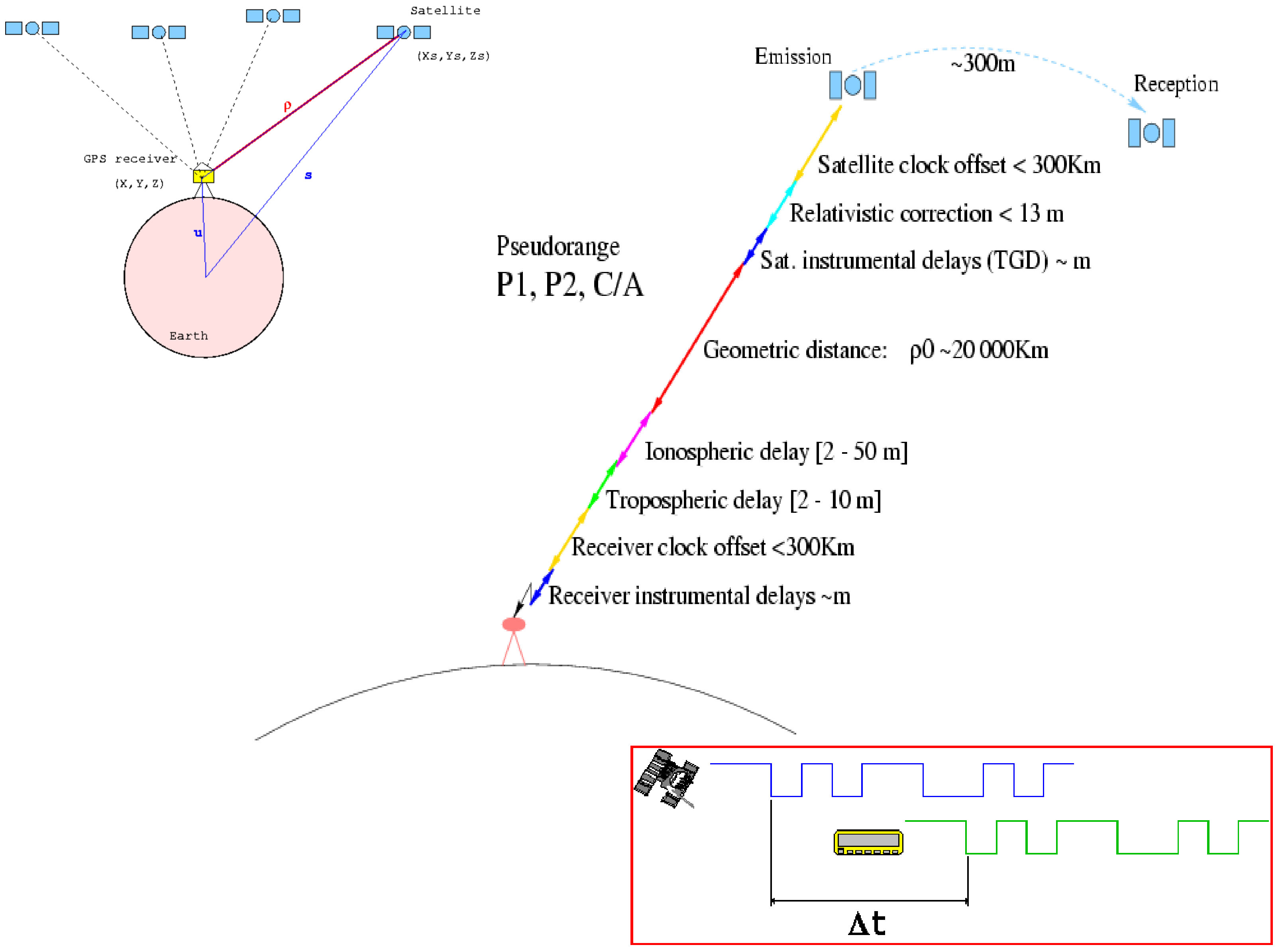
Relativistic effects

Tropospheric delay

Ionospheric delay

Instrumental delays

noise


Satellite
(Xs,Ys,Zs)
ρ
s
u
GPS receiver
(X,Y,Z)
Earth
Emission
~300m
Reception
Satellite clock offset < 300Km
Relativistic correction < 13 m
Sat. instrumental delays (TGD) ~ m
Pseudorange
P1, P2, C/A
Geometric distance: ρ0 ~20 000Km
Ionospheric delay [2 - 50 m]
Tropospheric delay [2 - 10 m]
Receiver clock offset <300Km
Receiver instrumental delays ~m
Δt

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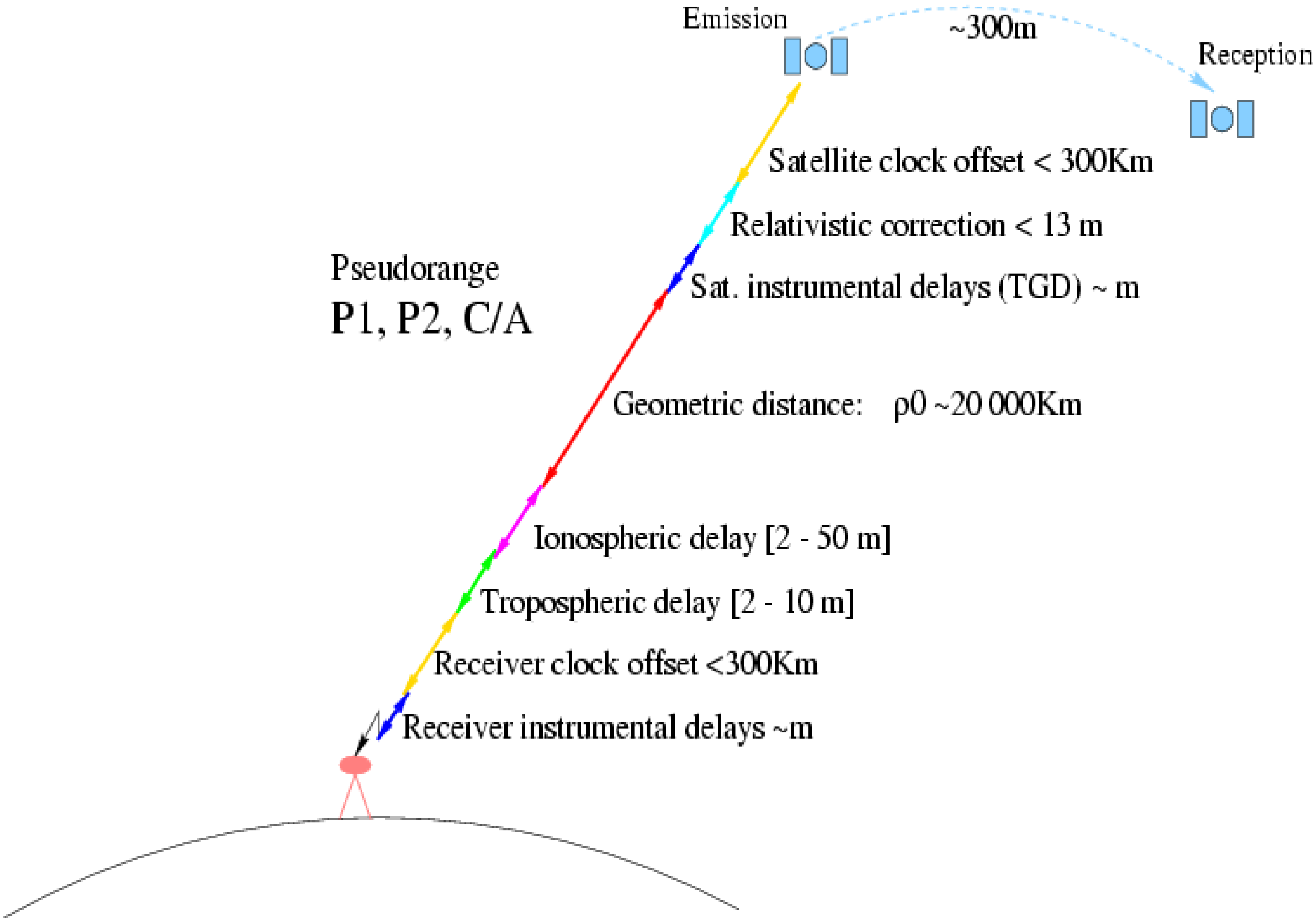

Exercise 1:

a) Using the file 95oct18casa___r0.rnx, generate the "txt" file 95oct18casa.a (with data ordered in columns).

b) Plot code and phase measurements for satellite PRN28 and discuss the results.

Resolution:

a)

```
cat 95oct18casa___r0.rnx| rnx2txt > 95oct18casa.a
```

b) See next plots:

An example of program to read the RINEX: rnx2txt

RINEX file → rnx2txt → txt file

Ficheros RINEX: observables

```
     2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX  AUSLIG              10-JAN-97 10:19     PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN)  - COCOS ISLAND      COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
     -0.000000000103          HARDWARE CALIBRATION (S)      COMMENT
     -0.000000054663          CLOCK OFFSET (S)              COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:   >500  >100   >50   >10    >5    >0   bad   n/a      COMMENT
 sig:      9     8     7     6     5     4     1     0      COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105  -14268715.899 8 -11118481.28445  22127685.4014  <=====  1
  22672158.746  -11510817.892 7  -8969469.30045  22672158.5184  <===== 25
  22594902.367  -12949753.825 7 -10090708.53945  22594903.7394  <=====  9
  22731128.796  -11621184.951 7  -9055464.16945  22731130.0094  <=====  5
  24610920.702    -924108.174 6   -720085.67045  24610920.0404  <===== 23
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  20842713.610  -19083282.892 9 -14870090.55546  20842713.4814  <=====  6
```

sta	Doy	sec	PRN	**L1**	**L2**	**C1/P1**	**P2**	
casa	291	0.30	14	3832855.061	3832852.989	20764791.163	20764791.889	0
casa	291	0.30	15	-1932753.473	-1932752.152	23025605.133	23025608.420	0
casa	291	0.30	18	-191430.529	-191430.252	24656587.151	24656589.163	0
casa	291	0.30	22	-2444624.551	-2444621.901	22508513.354	22508514.937	0
casa	291	0.30	25	-2949812.363	-2949808.662	22258999.265	22258999.632	0
casa	291	0.30	29	-1670473.859	-1670472.308	22409115.477	22409113.535	0
casa	291	30.30	14	-3840286.848	-3840284.776	20757359.266	20757360.162	0
casa	291	30.30	15	-1914283.549	-1914282.239	23044075.112	23044078.373	0
casa	291	30.30	18	-207329.153	-207328.868	24640688.394	24640690.635	0
casa	291	30.30	22	-2458225.787	-2458223.122	22494912.015	22494913.648	0
casa	291	30.30	25	2935690.693	2935686.992	22273121.015	22273121.208	0
casa	291	30.30	29	1681115.594	1681114.037	22398473.854	22398474.354	0
casa	291	60.30	14	-3847635.893	-3847633.821	20756013.062	20756016.988	0
casa	291	60.30	15	-1895770.976	-1895769.678	23062588.653	23062591.023	0
casa	291	60.30	18	-223219.005	-223218.710	24624798.015	24624800.800	0
casa	291	60.30	22	-2471750.391	-2471747.715	22481387.436	22481388.934	0
casa	291	60.30	25	-2921510.699	-2921507.003	22287301.001	22287301.347	0
casa	291	60.30	29	-1691735.199	-1691733.640	22387854.082	22387854.429	0
casa	291	90.30	14	-3854900.513	-3854898.440	20742745.403	20742746.301	0
casa	291	90.30	15	-1877216.337	-1877215.051	23081143.251	23081145.556	0
casa	291	90.30	18	-239097.495	-239097.188	24608919.738	24608922.072	0
casa	291	90.30	22	2485199.565	2485196.877	22467938.317	22467939.902	0
casa	291	90.30	25	2907272.793	2907269.105	22301538.849	22301539.273	0
casa	291	90.30	29	-1702332.295	-1702330.732	22377257.010	22377257.225	0
casa	291	120.30	14	-3862079.748	-3862077.674	20735565.158	20735566.939	0
casa	291	120.30	15	-1858620.266	-1858618.992	23099739.479	23099741.640	0
casa	291	120.30	18	-254966.378	-254966.052	24593053.725	24593053.168	0

The RINEX file is convert to a "columnar format" to easily plot its contents and to analyze the measurements (the public domain free tool **"gnuplot"** is used in the book to make the plots).

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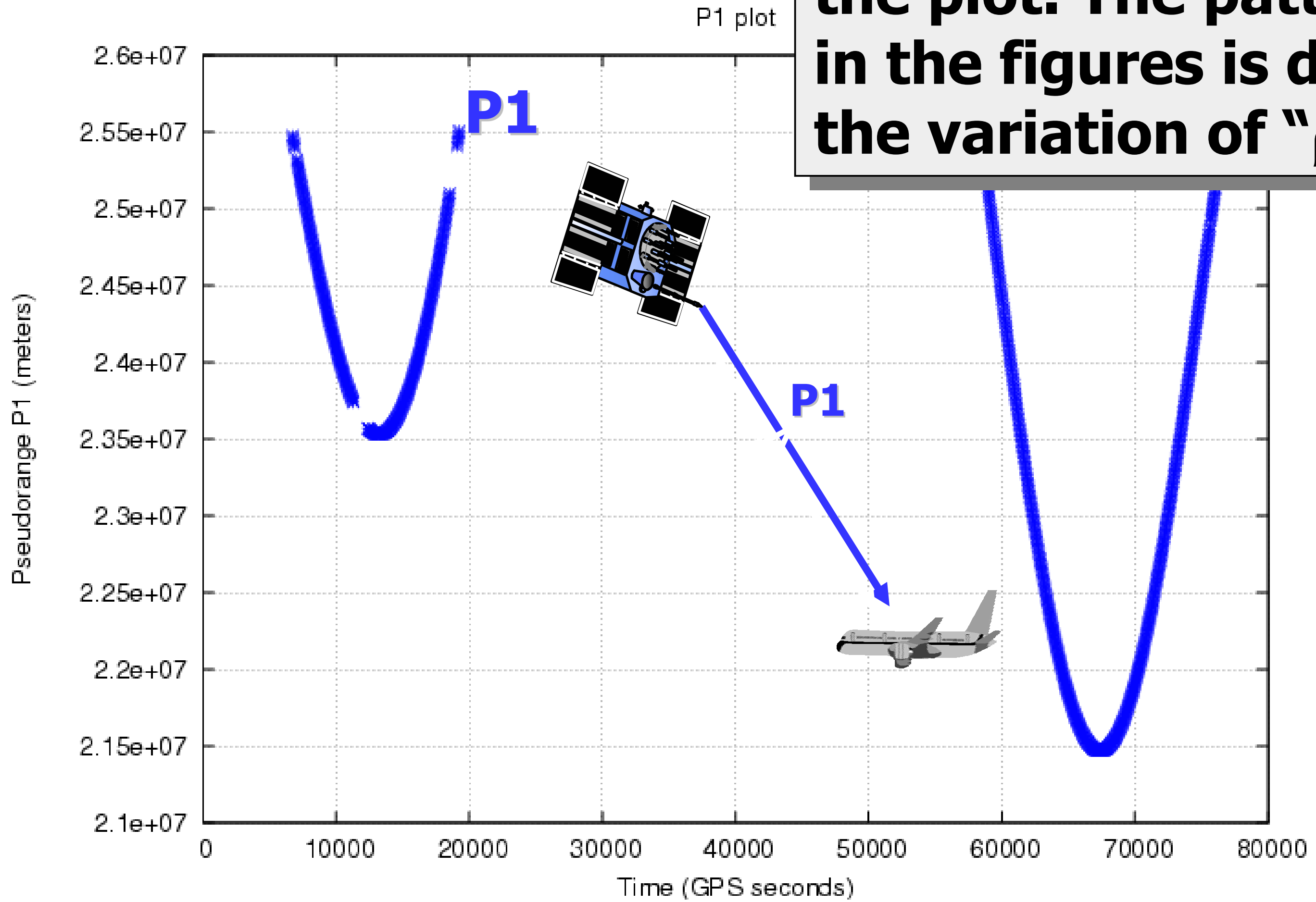
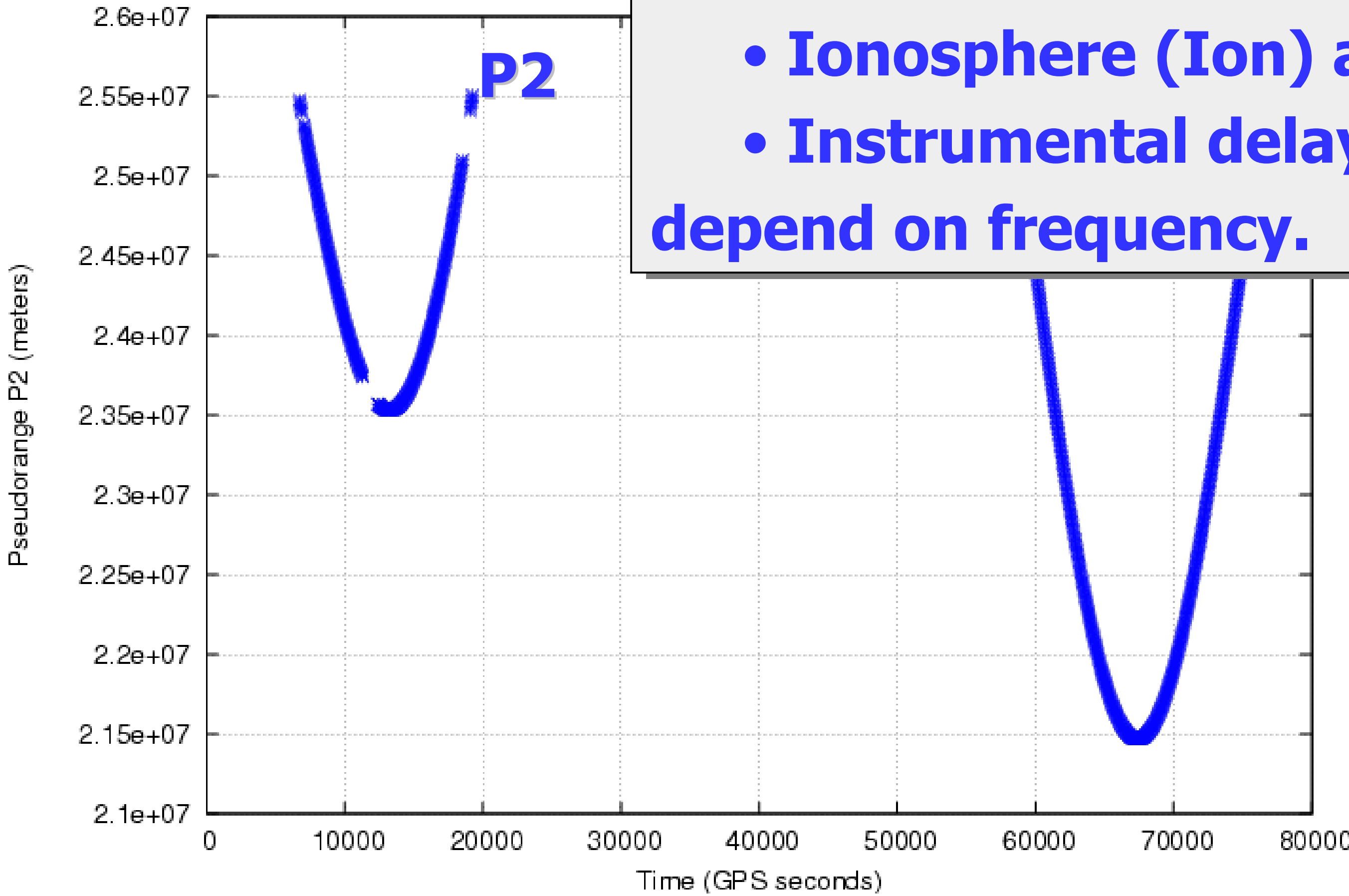
Code measurements

The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ"



$$P_{1sta}^{sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon$$

Code measurements



Similar plot for code measurements at f2.

Notice that

- **Ionosphere (Ion) and**
- **Instrumental delays (K)**

depend on frequency.

$$P_{2sta}^{sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_{2}^{sat} + \varepsilon$$

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...ase measurements

Ionosphere delays code and advances phase measurements

Code measurements: C1,P1,P2

$$C_{1sta}^{sat};$$

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_{1}^{sat} + \varepsilon$$

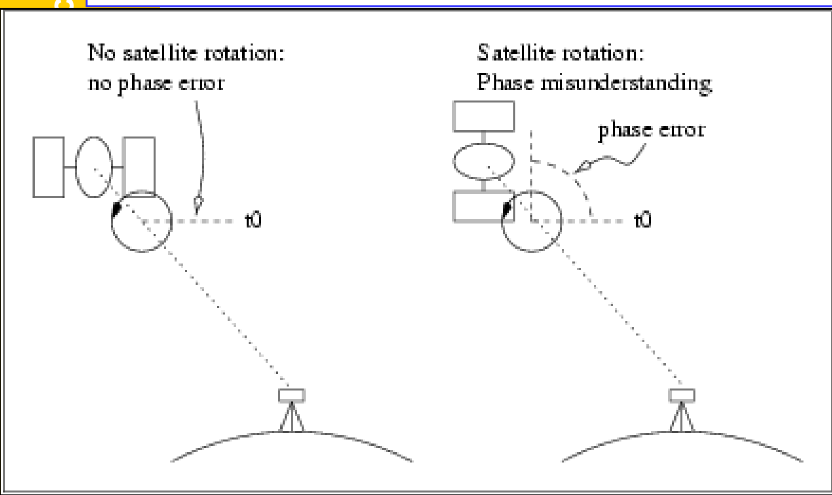
$$P_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_{2}^{sat} + \varepsilon$$

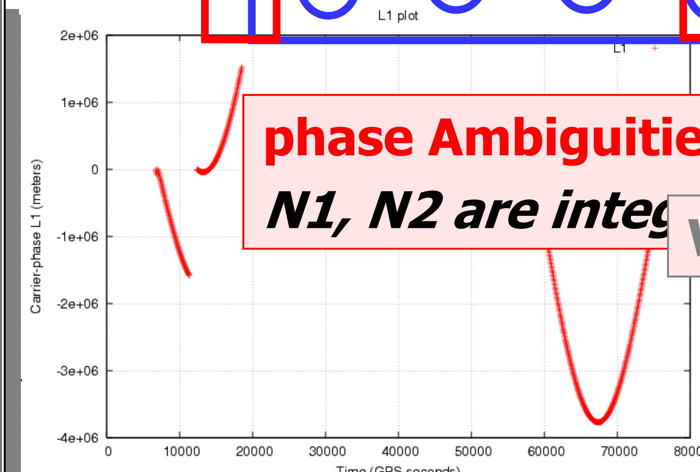
Frequency dependent

Phase measurements: L1,L2

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + k_{1sta} + k_{1}^{sat} + \lambda_1 N_1 + w_1 + \varepsilon$$

$$L_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + k_{2sta} + k_{2}^{sat} + \lambda_2 N_2 + w_2 + \varepsilon$$





phase Ambiguities

N1, N2 are integ...

Wind Up

Phase measurements

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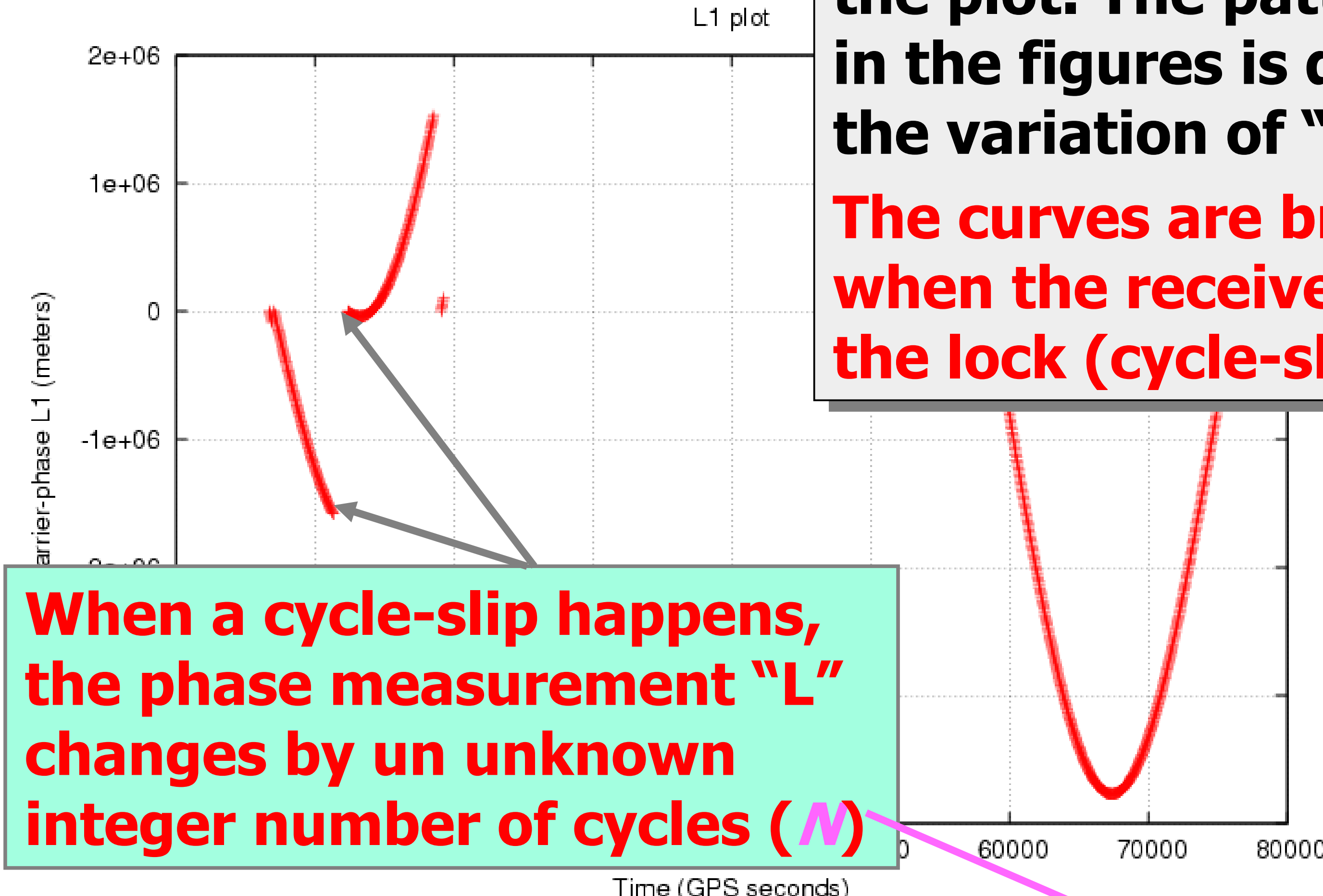
The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ".

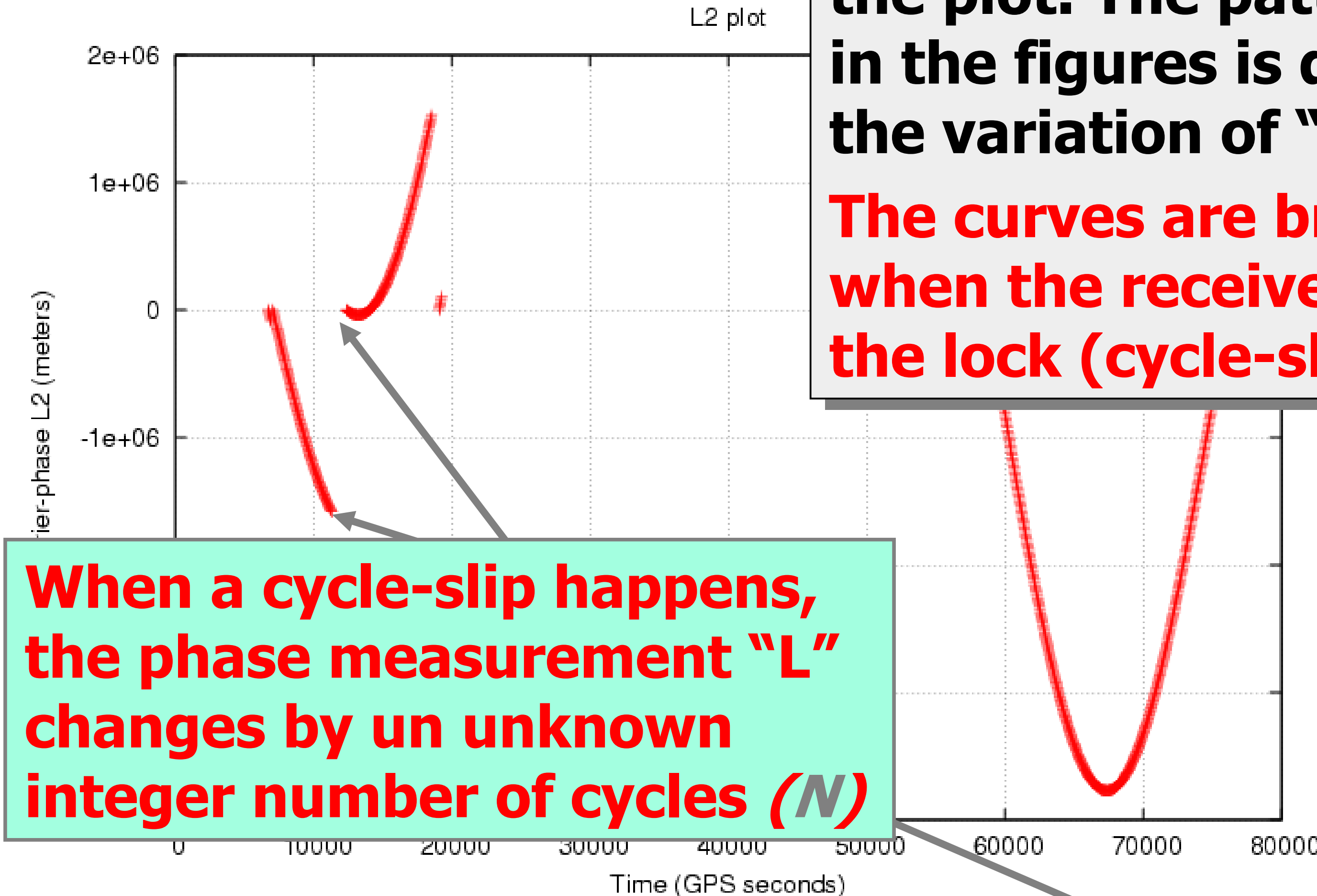
The curves are broken when the receiver loss the lock (cycle-slip).



When a cycle-slip happens, the phase measurement "L" changes by un unknown integer number of cycles (*N*)

$$L_{1sta}^{sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + rel_{sta}^{sat} + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + k_{1sta} + k_1^{sat} + \boxed{\lambda_1 N_1} + w_1 + \varepsilon$$

Phase measurements



The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ".

The curves are broken when the receiver loss the lock (cycle-slip).

When a cycle-slip happens, the phase measurement "L" changes by un unknown integer number of cycles *(N)*

$$L_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + k_{2sta} + k_2^{sat} + \boxed{\lambda_2 N_2} + w_2 + \varepsilon$$

Combination of measurements:

- Ionospheric combination
- Ionosphere-Free combination
- Wide-lane and Narrow-lane combinations

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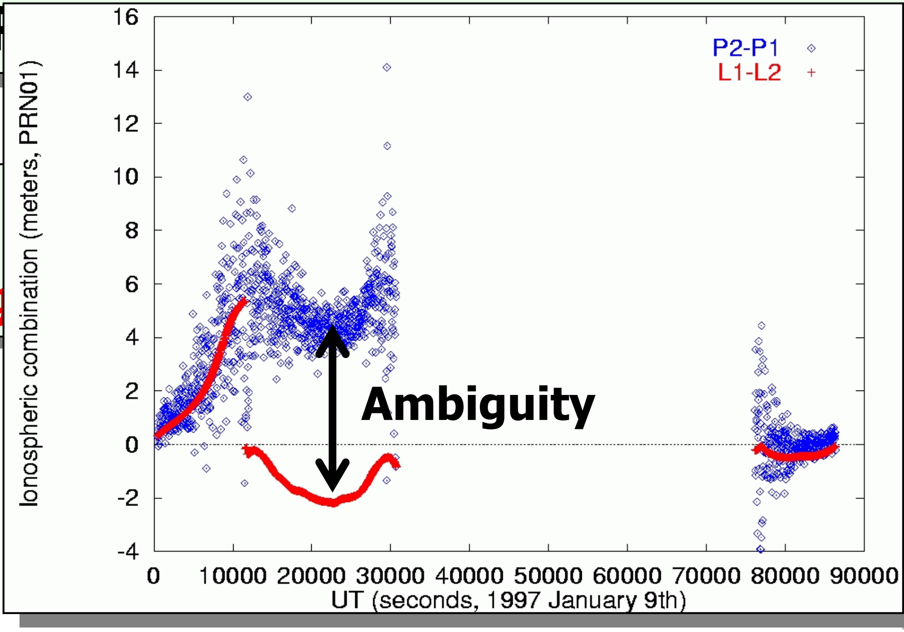
1. IONOSPHER

PI= P2-P1=Iono+ctt

LI= L1-L2= Iono+ctt+Ambig



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na, Spain

Code measurements: C1,P1,P2

P_2-P_1

$$P_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} + \boxed{Ion_{1sta}^{sat} + K_{1sta} + K_{1}^{sat}} + \varepsilon$$

$$P_{2sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} + \boxed{Ion_{2sta}^{sat} + K_{2sta} + K_{2}^{sat}} + \varepsilon$$

Phase measurements: L1,L2

L_1-L_2

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} \boxed{- Ion_{1sta}^{sat} + k_{1sta} + k_{1}^{sat}} \boxed{+ \lambda_1 N_1} + w_1 + \varepsilon$$

$$L_{2sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} \boxed{- Ion_{2sta}^{sat} + k_{2sta} + k_{2}^{sat}} \boxed{+ \lambda_2 N_2} + w_2 + \varepsilon$$

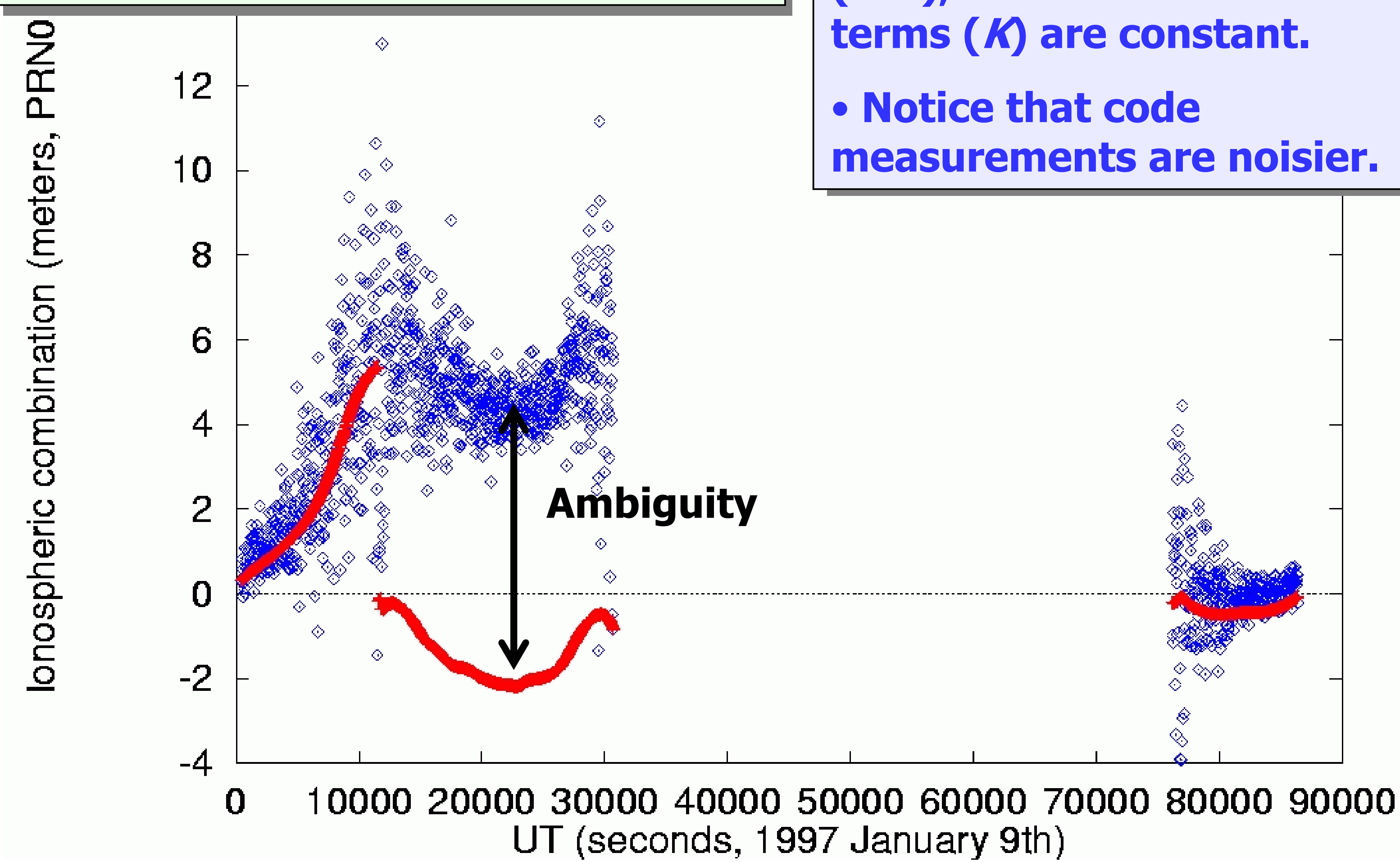
phase Ambiguities

1. IONOSPHERIC COMBINATION

PI= P2-P1=Iono+ctt

LI= L1-L2= Iono+ctt+Ambig

- The pattern corresponds to the ionospheric refraction (*Ion*), because the other terms (*K*) are constant.
- Notice that code measurements are noisier.

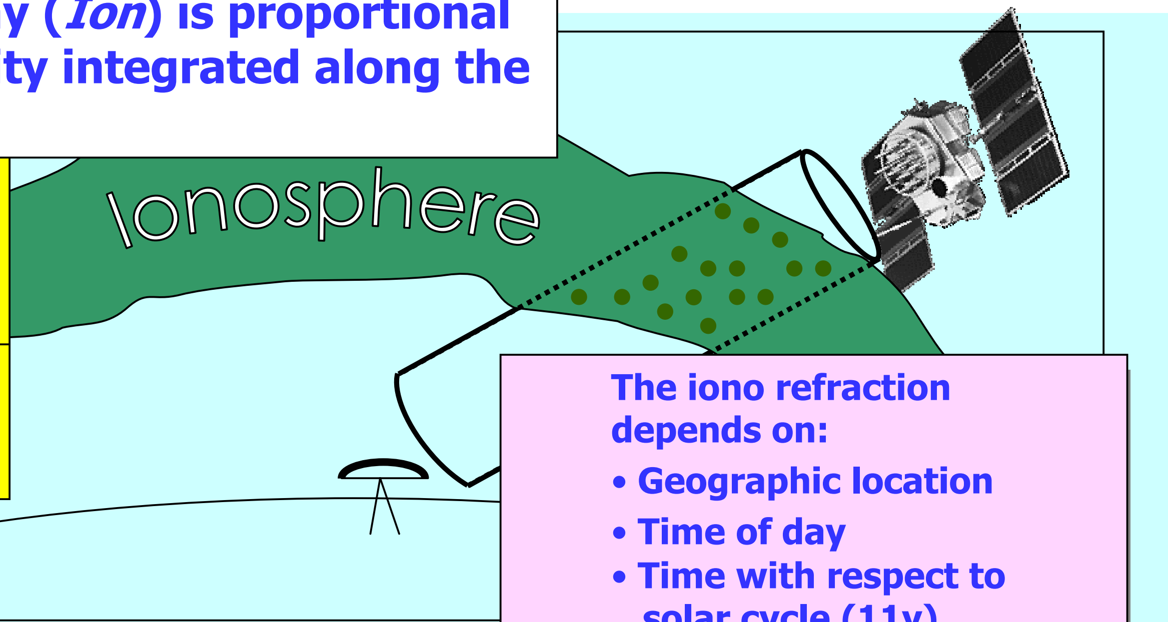


1. IONOSPHERIC COMBINATION

The ionospheric delay (*Ion*) is proportional to the electron density integrated along the ray path (*STEC*).

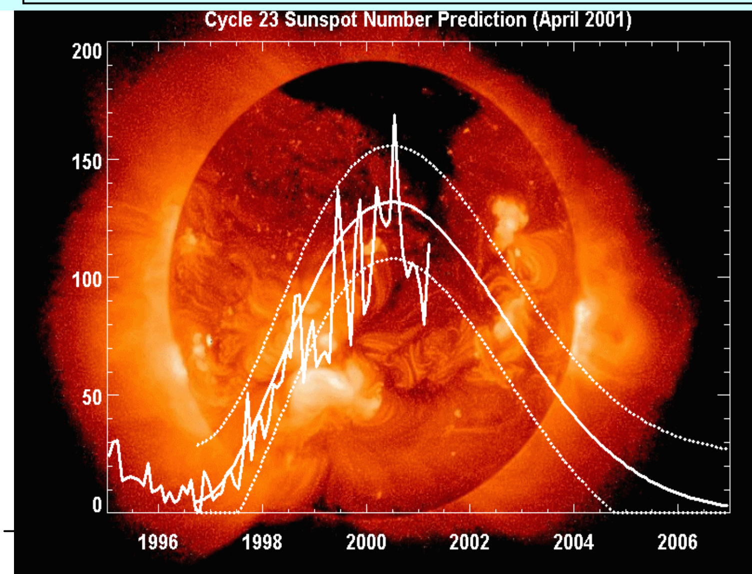
$$Ion = \frac{40.3}{f^2} STEC$$

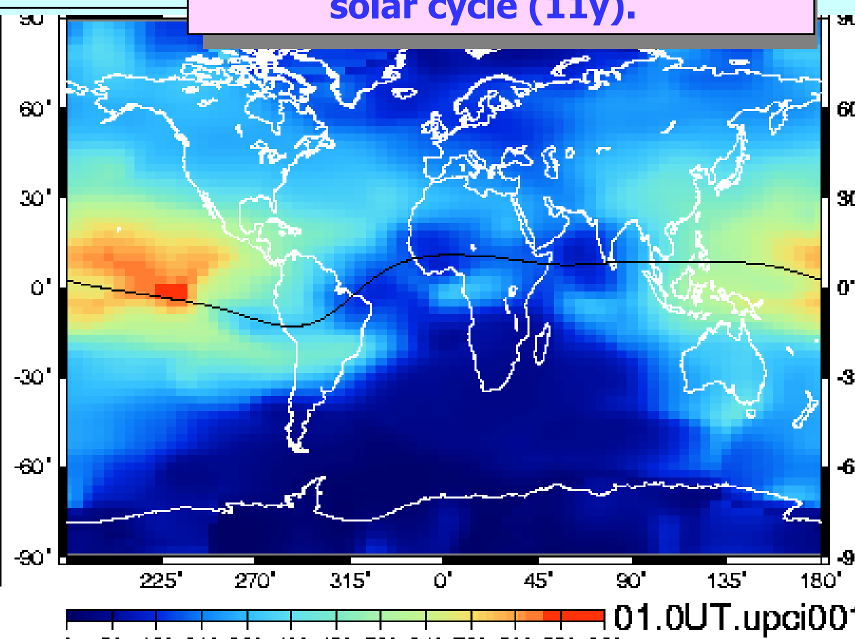
$$STEC = \int_{\vec{r}[GPStransmitter]}^{\vec{r}[GPSreceiver]} N_e(\vec{r},t)\,dr$$



The iono refraction depends on:

- **Geographic location**
- **Time of day**
- **Time with respect to solar cycle (11y).**





JEAGAL, 2004-2005

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1. IONOSPHERIC COMBINATION

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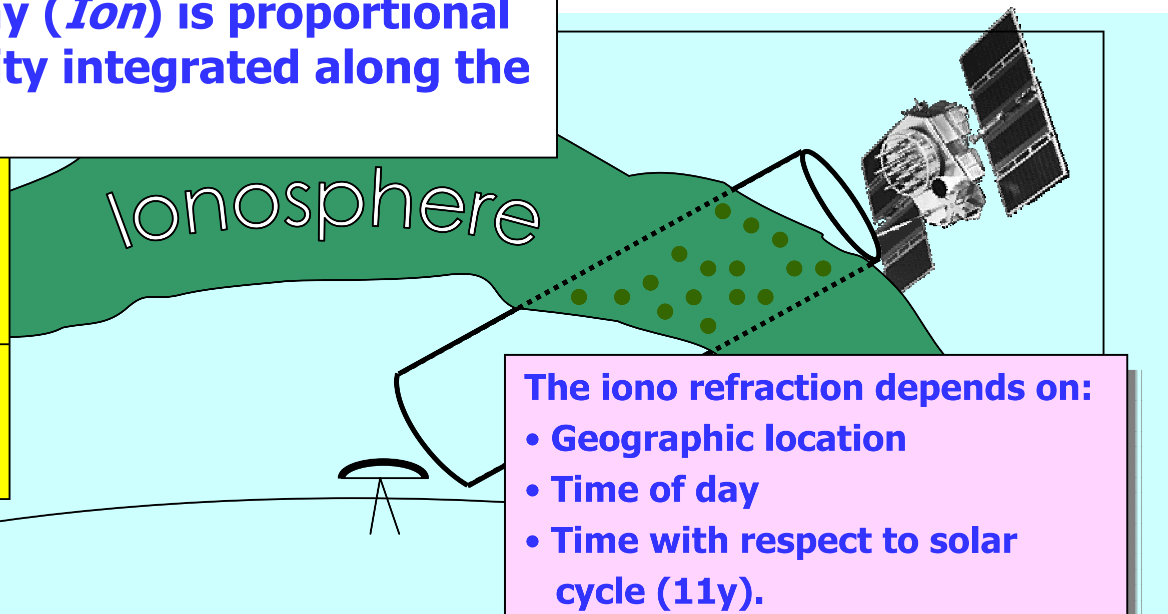

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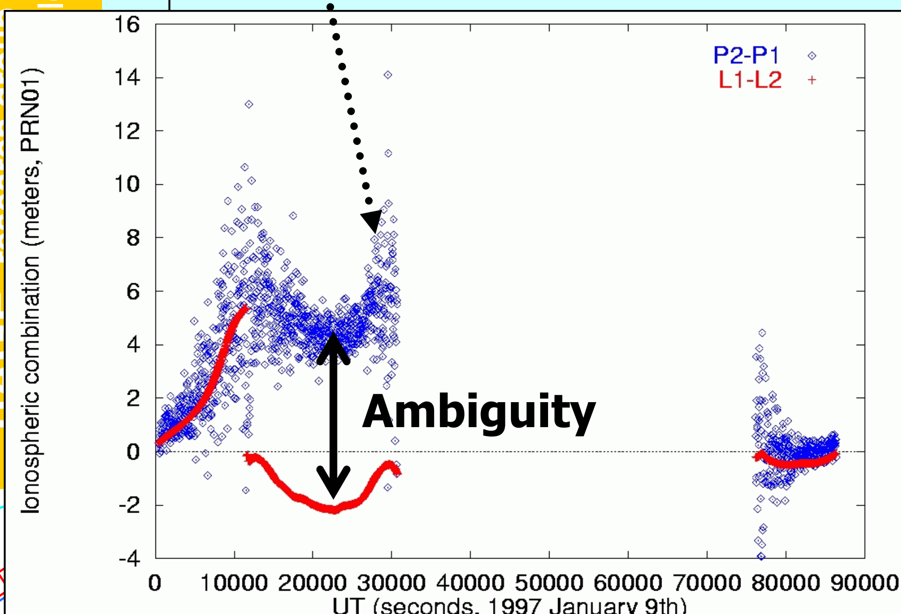

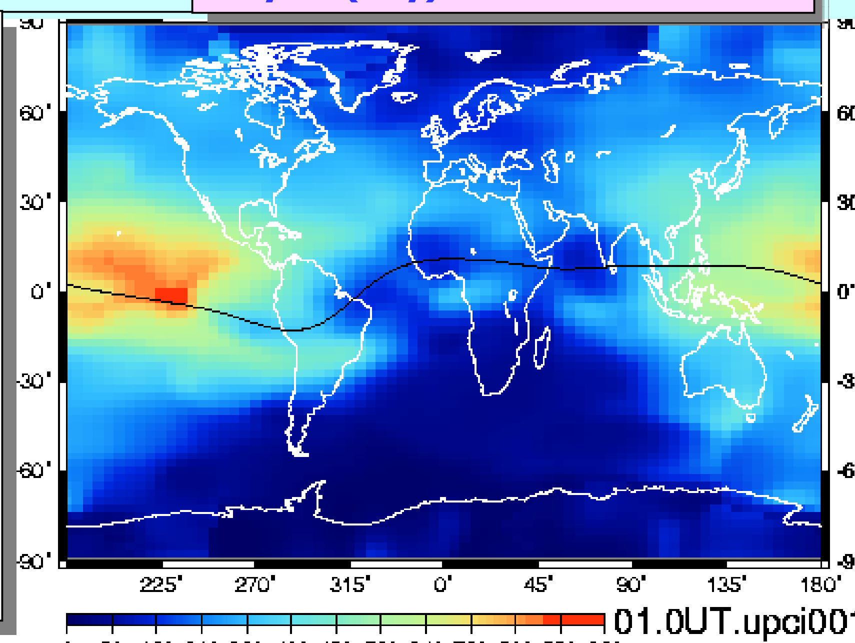

2. IONOSPHERE-FREE COMBINATION *(Pc,Lc)*

The ionospheric refraction depends on the inverse of the squared frequency and can be removed up to 99.9% combining *f1* and *f2* signals:

$$Ion = \frac{40.3}{f^2} STEC$$

$$Pc = \frac{f_1^{\,2} P_1 - f_2^{\,2} P_2}{f_1^{\,2} - f_2^{\,2}} \qquad Lc = \frac{f_1^{\,2} L_1 - f_2^{\,2} L_2}{f_1^{\,2} - f_2^{\,2}}$$

$$Pc_{sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + K_{c\,sta} + \varepsilon$$

$$Lc_{sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + k_{c\,sta} + k_c^{\,sat} + \lambda_c Rc + w_c + \varepsilon$$

- **The ionospheric refraction has been removed in *Lc* and *Pc***
- **λ_c = *10.7 cm***

The *Rc* ambiguities are NOT integers!!

$$Rc = \frac{\lambda_W}{\lambda_1} N1 - \frac{\lambda_W}{\lambda_2} N2$$

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Comments:

Two-frequency receivers are needed to apply the ionosphere-free combination.

If a one-frequency receiver is used, a ionospheric model must be applied to remove the ionospheric refraction. The GPS navigation message provides the parameters of the Klobuchar model which accounts for more than 60% of the ionospheric delay.

Narrow-lane (Pw) and Wide-lane Combination Lw

The wide-lane combination Lw provides a signal with a large wave-length (**λ=86.2cm~4*λ1**). This makes it very useful for detecting cycle-slips through the Melbourne-Wübbena combination: **Lw-Pw**

$$Pw = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2}$$

$$Lw = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$$

The same sign

$$Pw_{sta}^{sat} = \rho_{sta}^{sat} + c \cdot (dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{w\,sta}^{\;sat} + K_{w\,sta} + K_w^{\;sat} + \varepsilon$$

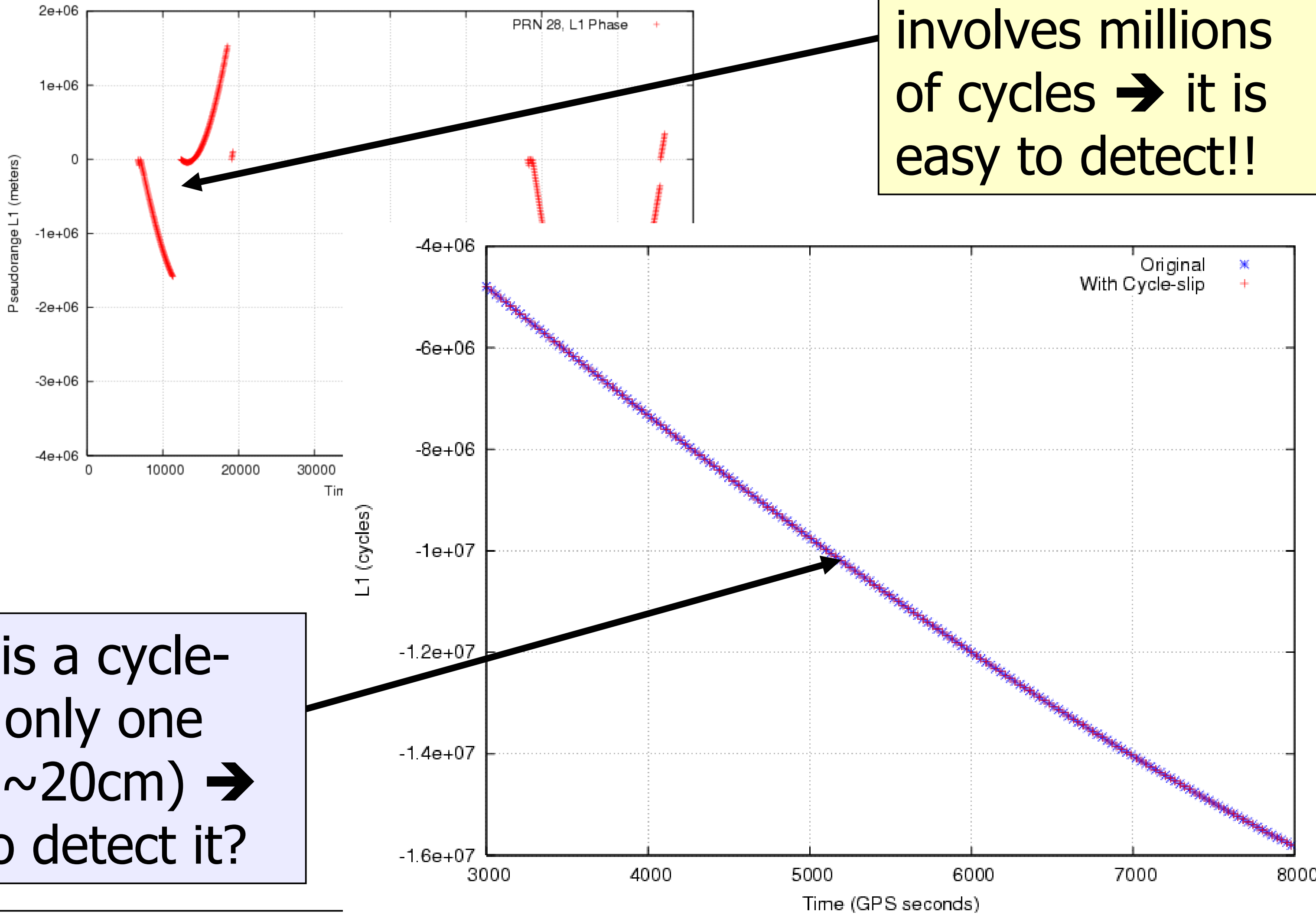
$$Lw_{sta}^{sat} = \rho_{sta}^{sat} + c \cdot (dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{w\,sta}^{\;sat} + k_{w\,sta} + k_w^{\;sat} + \lambda_w \boxed{N_w} + \varepsilon$$

The ambiguities Nw are INTEGERS!

No wind-up

Detecting cycle-slips

This cycle-slip involves millions of cycles ➔ it is easy to detect!!



There is a cycle-slip of only one cycle (~20cm) ➔ How to detect it?

Exercise 2:

a) Using the file 95oct18casa___r0.rnx, generate the "txt" file 95oct18casa.a (with data ordered in columns).

b) Insert a cycle-slip of "one wavelength" (19cm) in L1 measurement at t=5000 s (and no cycle-slip in L2).

c) Plot the measurements "L1, L1-P1, LC-PC, Lw-Pw and LI" and discuss which combination/s should be used to detect the cycle-slip.

Resolution:

a)

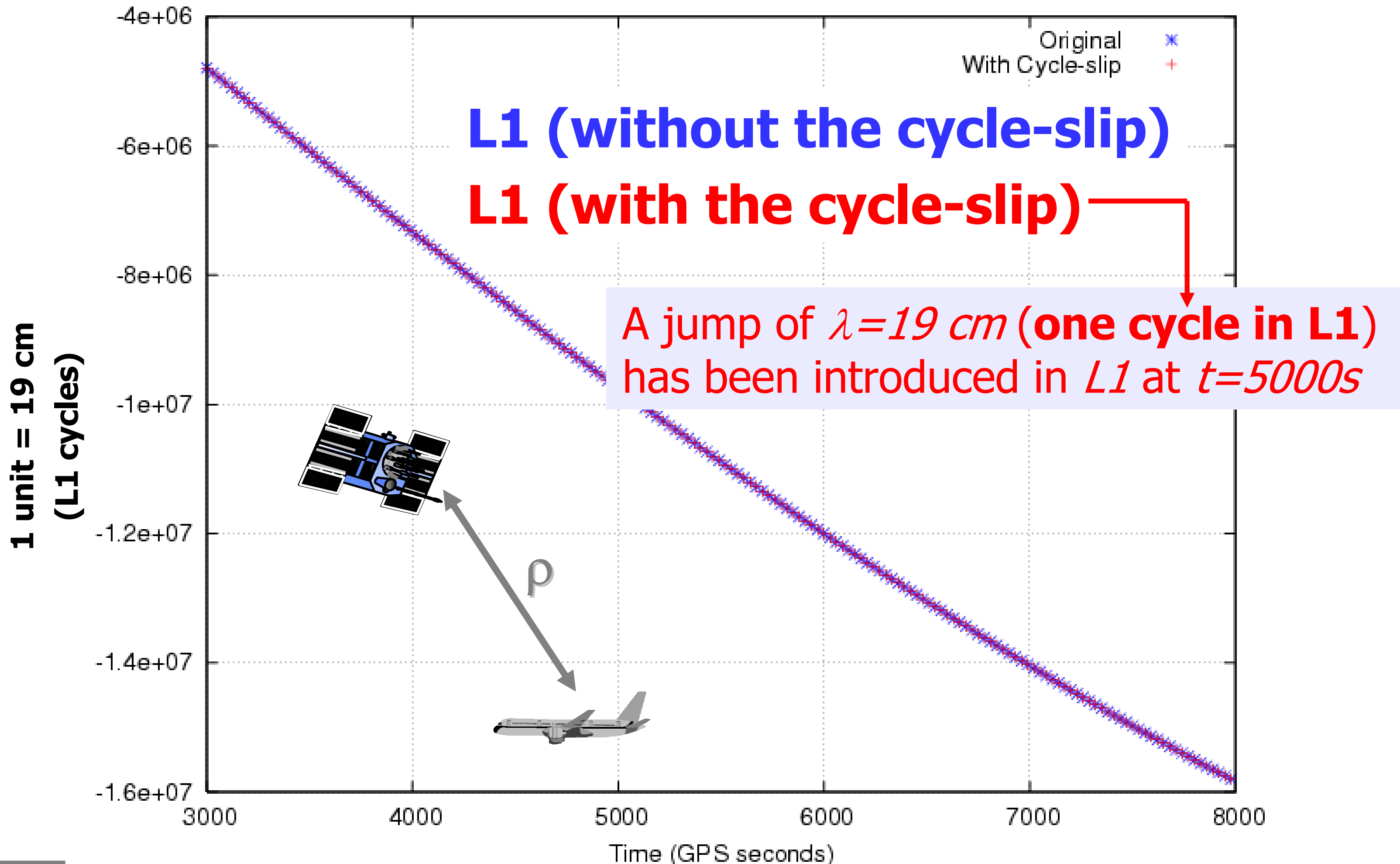
```
cat 95oct18casa___r0.rnx| rnx2txt > 95oct18casa.a
```

b)

```
cat 95oct18casa.a | gawk '{if ($4==18)
                              print $3,$5,$6,$7,$8}' > s18.org
cat s18.org | gawk '{if ($1>=5000) $2=$2+0.19;
  printf "%s %f %f %f %f \n", $1,$2,$3,$4,$5}' > s18.cl
```

c) See next plots:

The geometry "ρ" is the dominant term in the plot. The variation of "ρ" in 1 sec may be hundreds of meters, many times greater than the cycle-slip *(19 cm)* ➔ **the variation of ρ shadows the cycle-slip!**



$$L_{1sta}^{sat} = \boxed{\rho_{sta}^{sat}} + c \cdot (dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + k_{1sta} + k_1^{sat} + \lambda_1 N_1 + w_1 + \varepsilon$$

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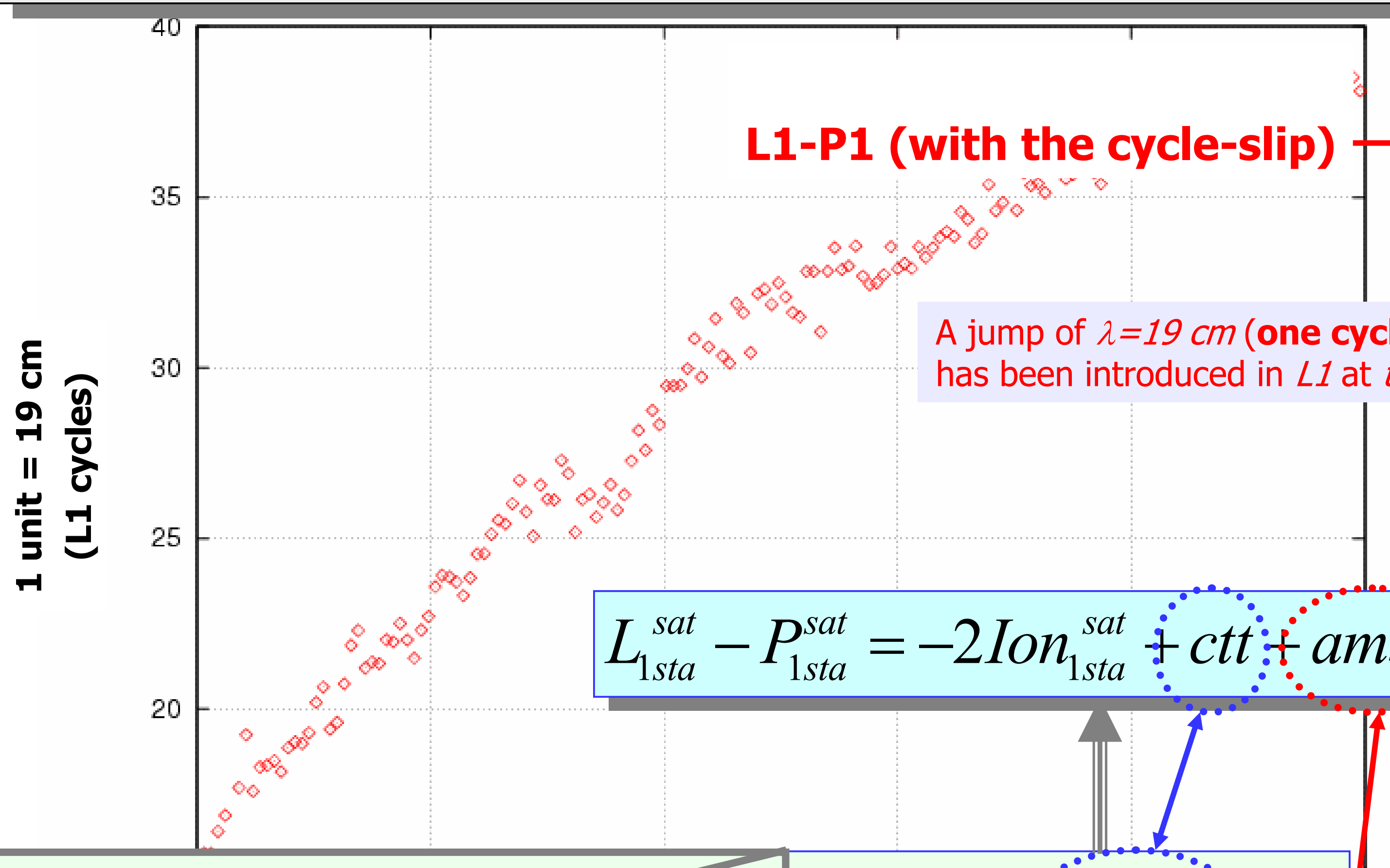
The geometry and clock offsets have been removed.

The trend is due to the Ionosphere. The *P1* code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



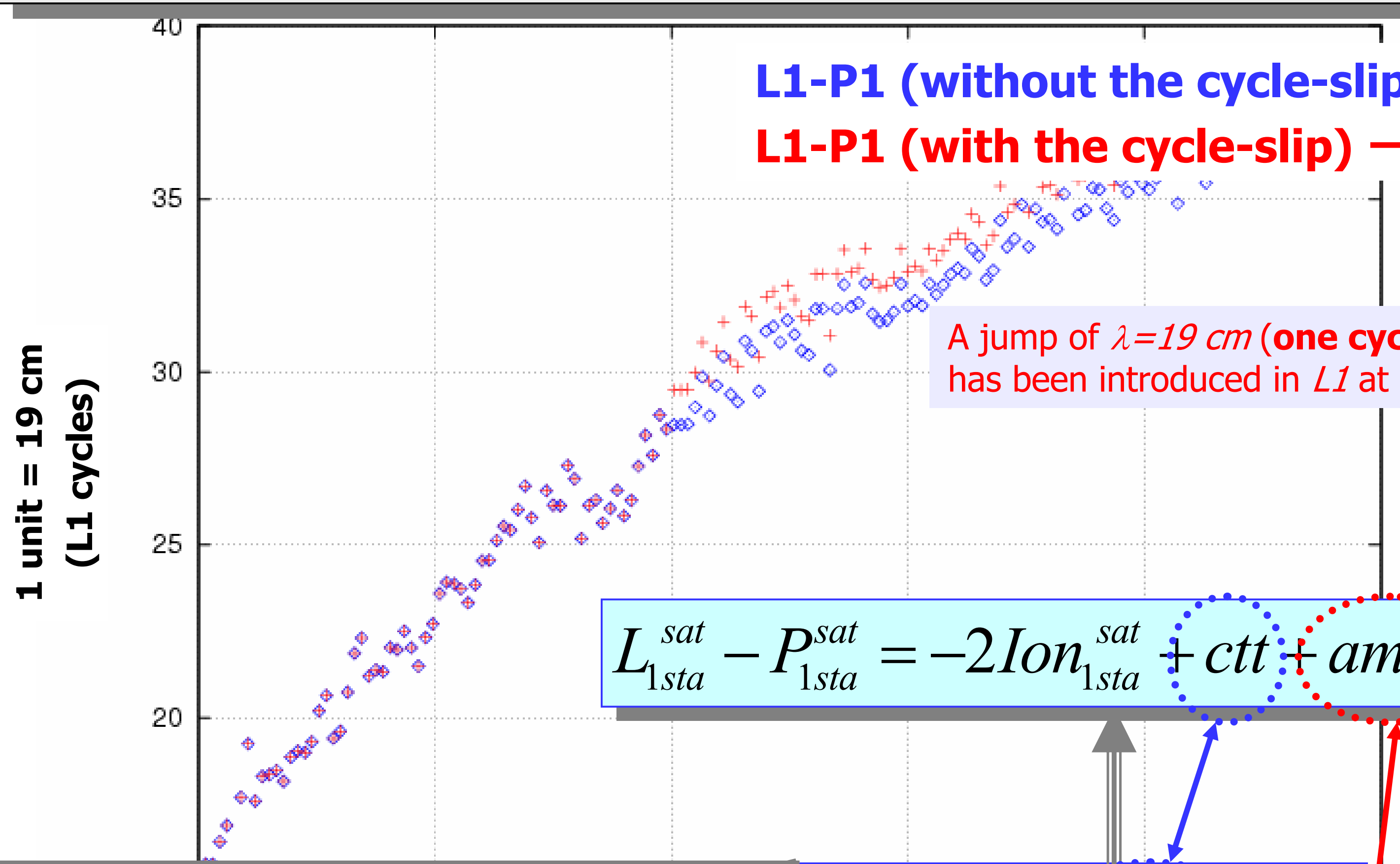
$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + \varepsilon$$

$$P_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\,(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon$$

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\,(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} - Ion_{1sta}^{sat} + k_{1sta} + k_1^{sat} + \lambda_1 N_1 + w_1 + \varepsilon$$

The geometry and clock offsets have been removed.

The trend is due to the Ionosphere. The *P1* code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + \varepsilon$$

$$P_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\,(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon$$

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\,(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} - Ion_{1sta}^{sat} + k_{1sta} + k_1^{sat} + \lambda_1 N_1 + w_1 + \varepsilon$$

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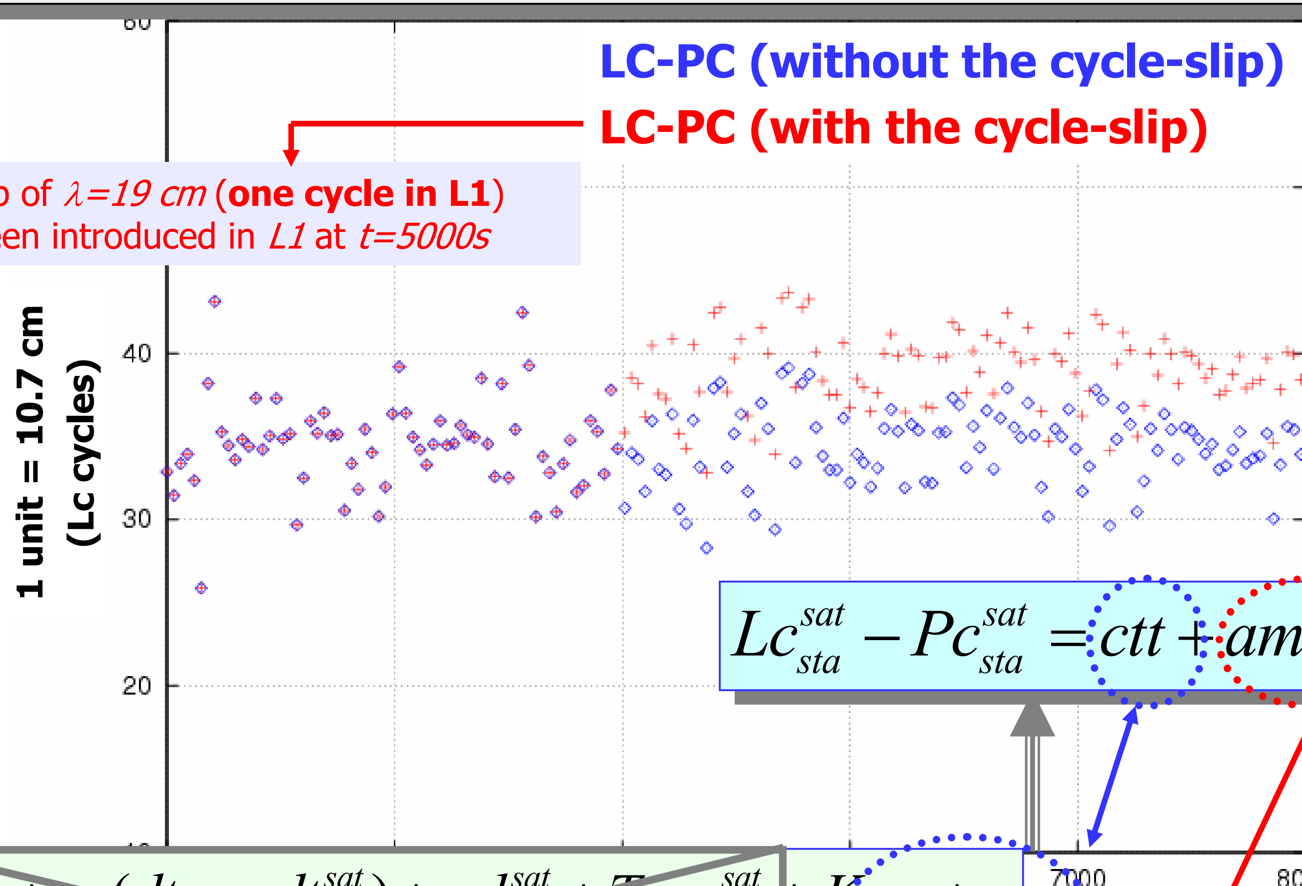
The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. The *P1* code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



A jump of $\lambda=19$ *cm* (**one cycle in L1**) has been introduced in *L1* at *t=5000s*

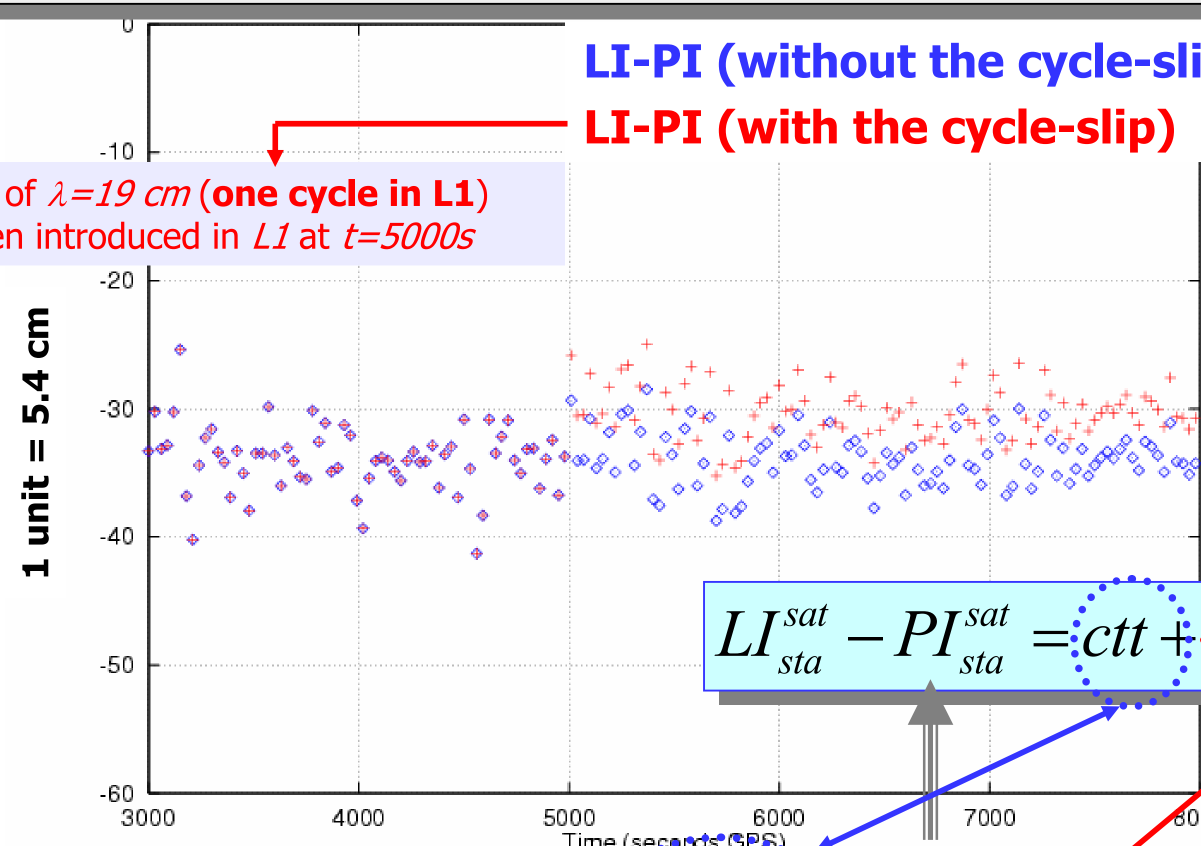
$$Lc_{sta}^{sat}-Pc_{sta}^{sat}=ctt+ambig+\varepsilon$$

$$Pc_{sta}^{sat}=\cancel{\rho_{sta}^{sat}+c\cdot(dt_{sta}-dt^{sat})+rel_{sta}^{sat}+Trop_{sta}^{sat}}+Kc_{sta}+\varepsilon$$

$$Lc_{sta}^{sat}=\cancel{\rho_{sta}^{sat}+c\cdot(dt_{sta}-dt^{sat})+rel_{sta}^{sat}+Trop_{sta}^{sat}}+kc_{sta}+kc^{sat}+\lambda_c N_c+w_c+\varepsilon$$

The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. *The P1 code noise also shadows the cycle-slip*, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



$$LI_{sta}^{sat} - PI_{sta}^{sat} = ctt + ambig + \varepsilon$$

$$PI_{sta}^{sat} = \cancel{Ion_I} + K_{I\,sta} + K_I^{sat} + \varepsilon$$

$$LI_{sta}^{sat} = \cancel{Ion_I} + k_{I_{sta}} + k_I^{sat} + \lambda_1 N_1 - \lambda_2 N_2 + w_I + \varepsilon$$

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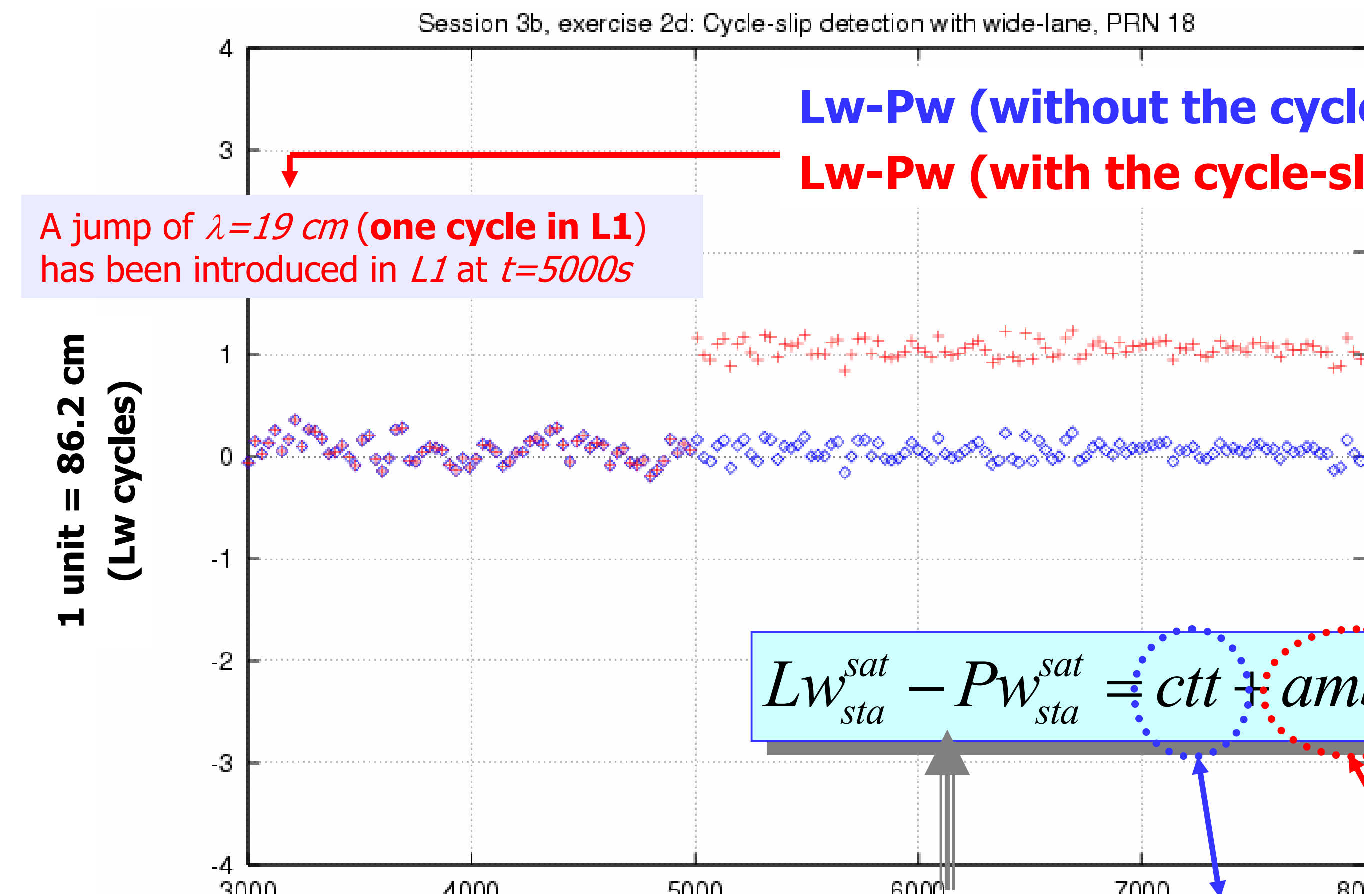
The geometry , clock offsets and iono have been removed.

There is a constant pattern plus noise. The *Pw* code noise is under one cycle of *Lw*. Thence, the cycle-slip is clearly detected



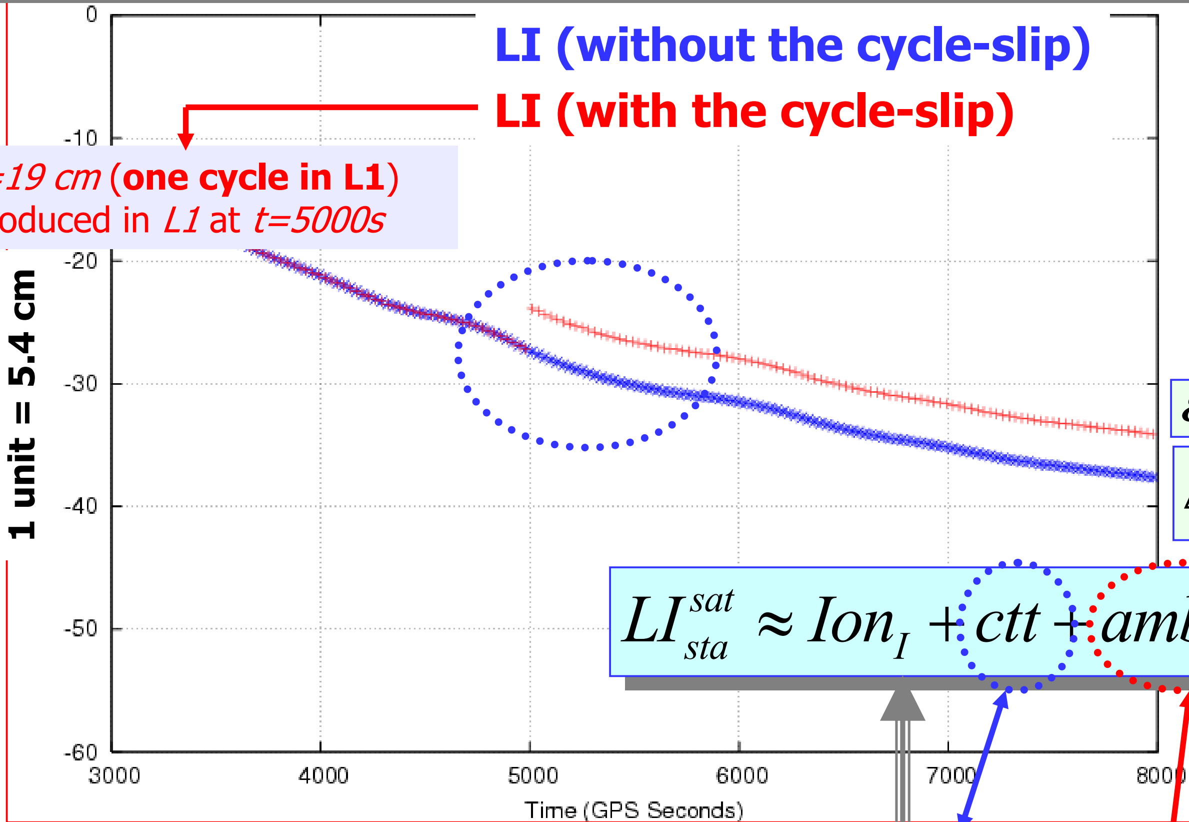
$$Lw_{sta}^{sat} - Pw_{sta}^{sat} = ctt + ambig + \varepsilon$$

$$Pw_{sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{w\,sta}^{\;sat}} + K_{w\,sta} + K_{w}^{\;sat} + \varepsilon$$

$$Lw_{sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat} + Ion_{w\,sta}^{\;sat}} + k_{w\,sta} + k_{w}^{\;sat} + \lambda_{w} N_{w} + \varepsilon$$

The geometry and clock offsets have been removed.

The trend is due to the Iono. The *L1* code noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19\ cm$ Thence, the cycle-slip is detected.



A jump of $\lambda = 19\ cm$ (**one cycle in L1**) has been introduced in *L1* at *t=5000s*

$\varepsilon \square\ mm$

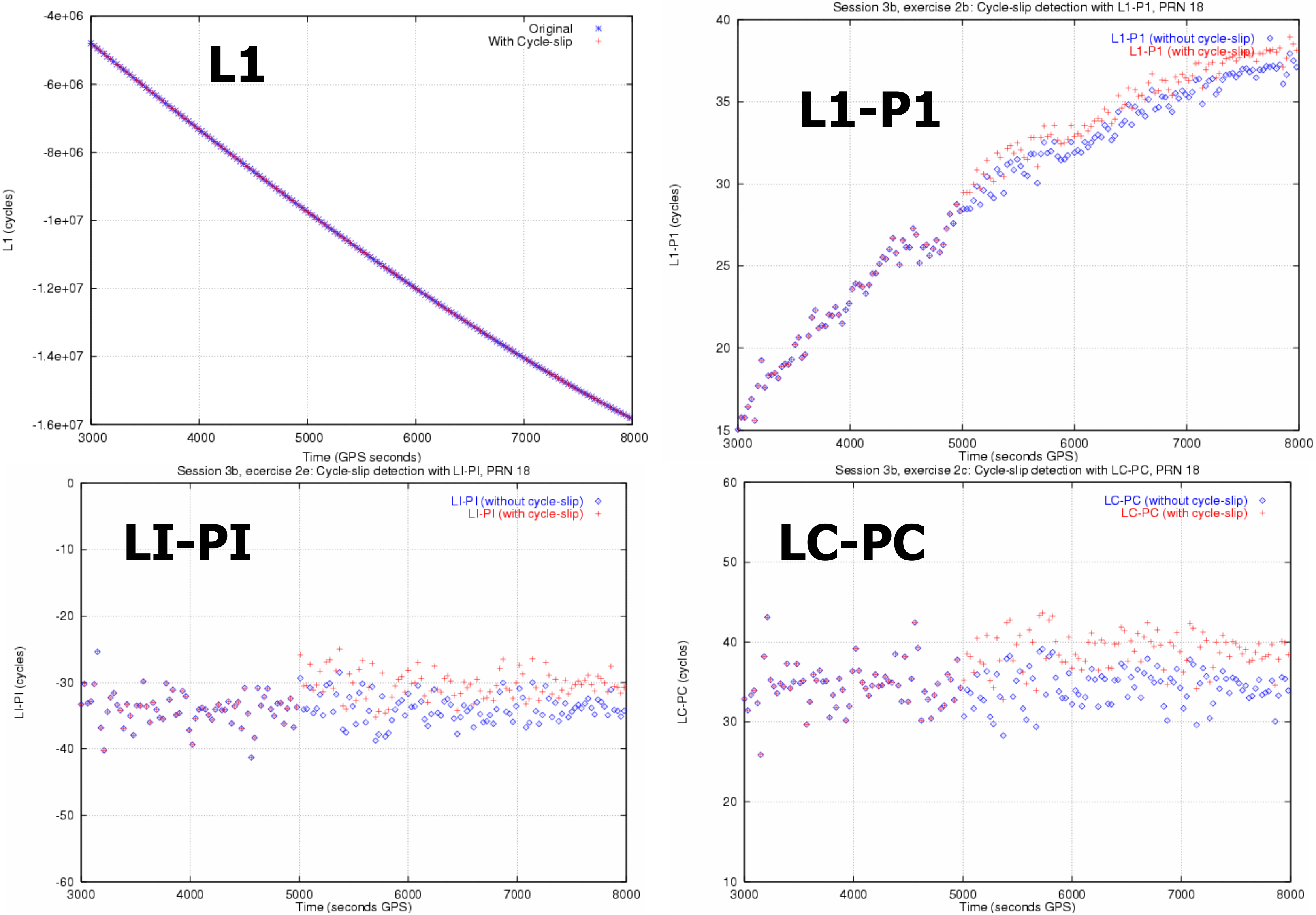
$\Delta w_I <<$

$$LI_{sta}^{sat} \approx Ion_I + ctt + ambig$$

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} - Ion_{1sta}^{sat} + k_{1sta} + k_1^{sat} + \lambda_1 N_1 + w_1 + \varepsilon$$

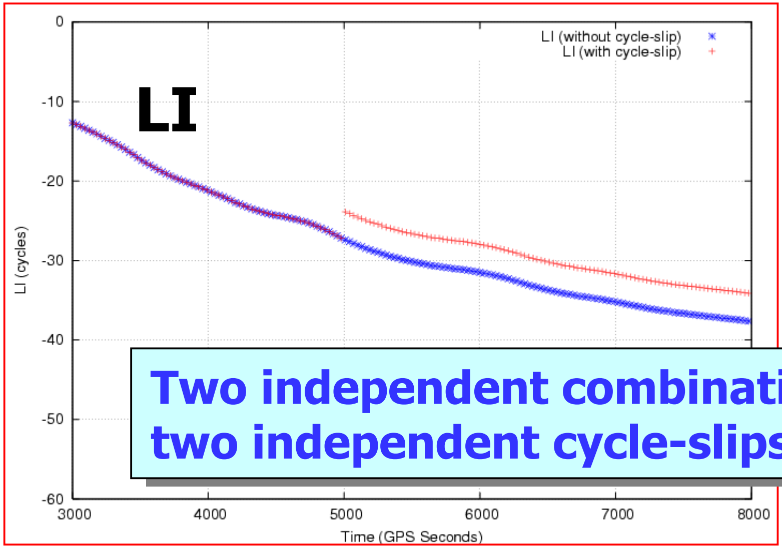
$$L_{2sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + rel_{sta}^{sat} + Trop_{sta}^{sat}} - Ion_{2sta}^{sat} + k_{2sta} + k_2^{sat} + \lambda_2 N_2 + w_2 + \varepsilon$$

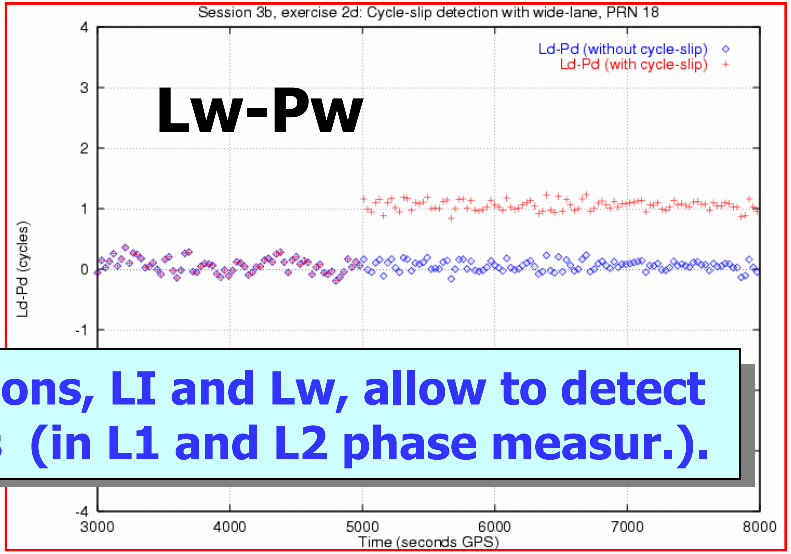
Summary



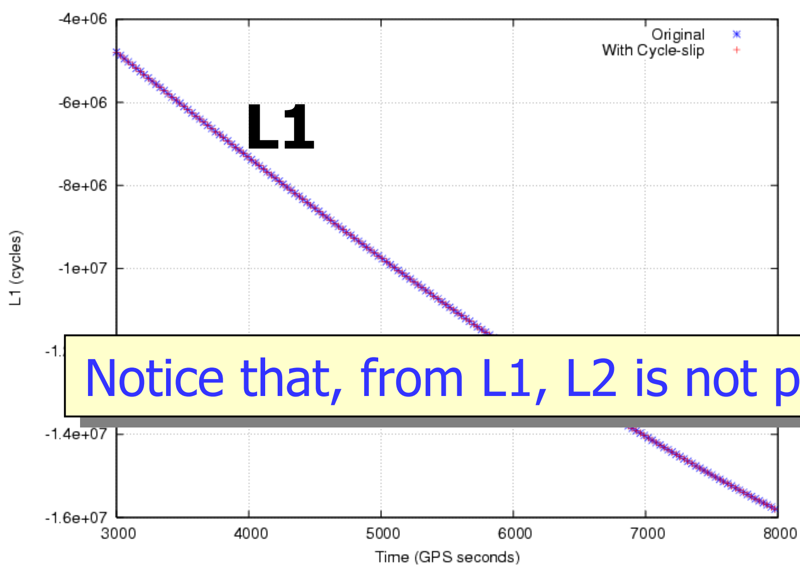
L1
Original
With Cycle-slip
L1 (cycles)
Time (GPS seconds)
Session 3b, exercise 2b: Cycle-slip detection with L1-P1, PRN 18
L1-P1
L1-P1 (without cycle-slip)
L1-P1 (with cycle-slip)
L1-P1 (cycles)
Time (seconds GPS)
Session 3b, ecercise 2e: Cycle-slip detection with LI-PI, PRN 18
LI-PI
LI-PI (without cycle-slip)
LI-PI (with cycle-slip)
LI-PI (cycles)
Time (seconds GPS)
Session 3b, exercise 2c: Cycle-slip detection with LC-PC, PRN 18
LC-PC
LC-PC (without cycle-slip)
LC-PC (with cycle-slip)
LC-PC (cyclos)
Time (seconds GPS)

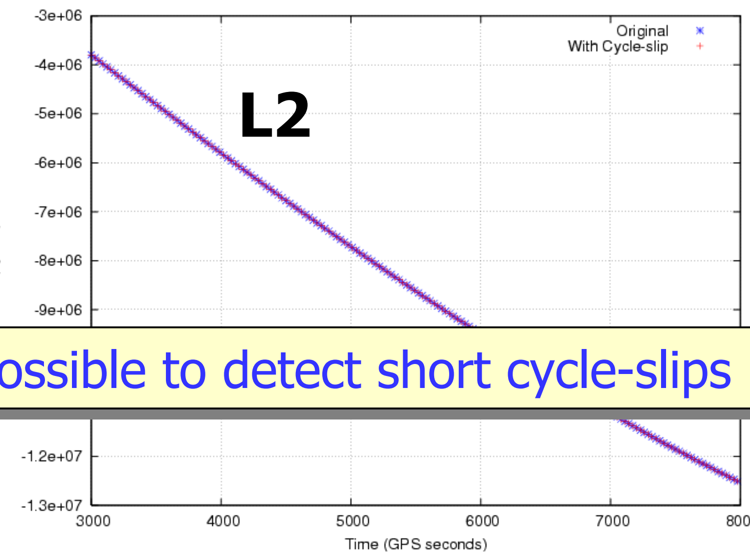
The cycle-slips are detected by the Ionospheric combination (LI=L1-L2) and the Melbourne Wübbena (W=Lw-Pw)





Two independent combinations, LI and Lw, allow to detect two independent cycle-slips (in L1 and L2 phase measur.).





Notice that, from L1, L2 is not possible to detect short cycle-slips

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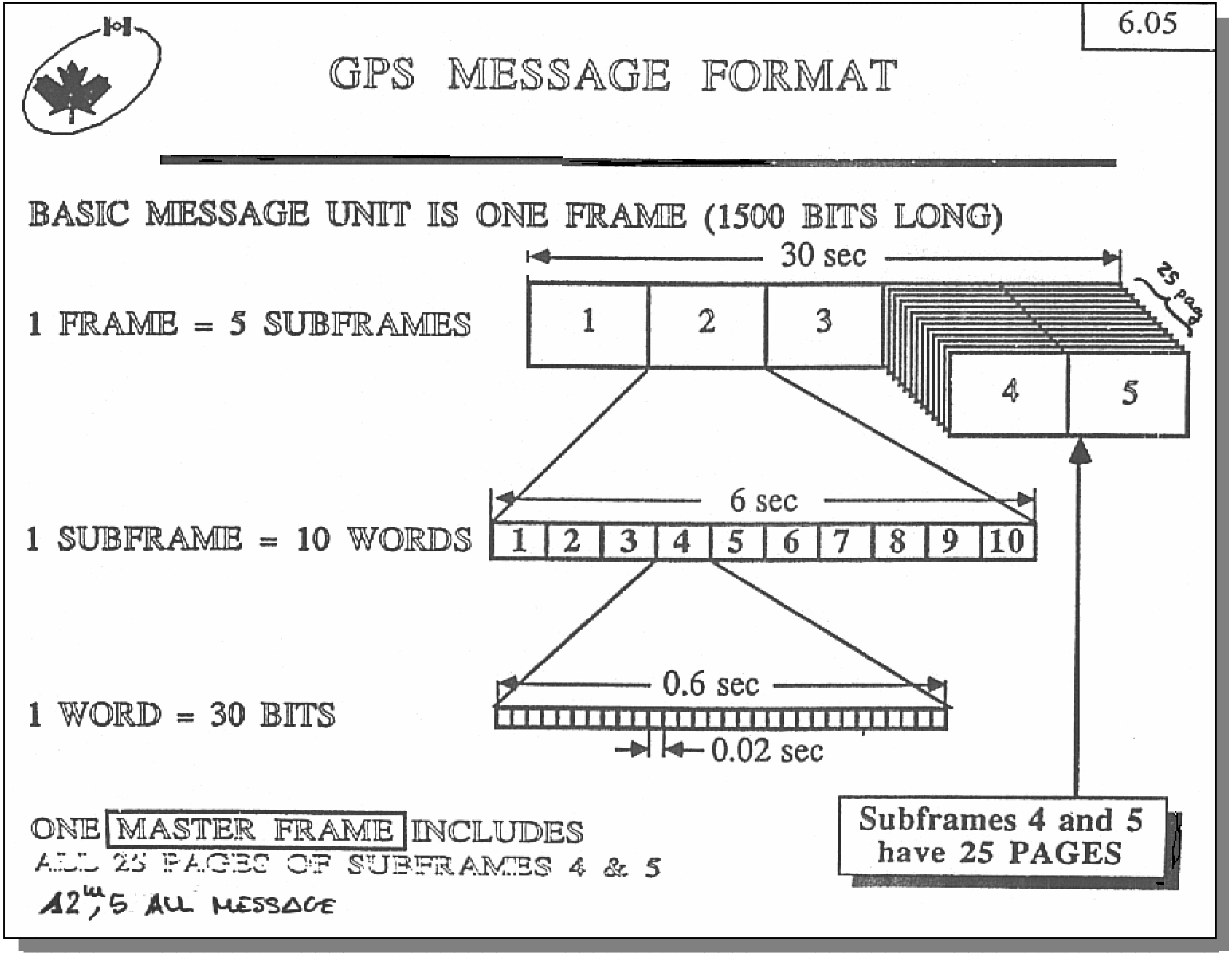
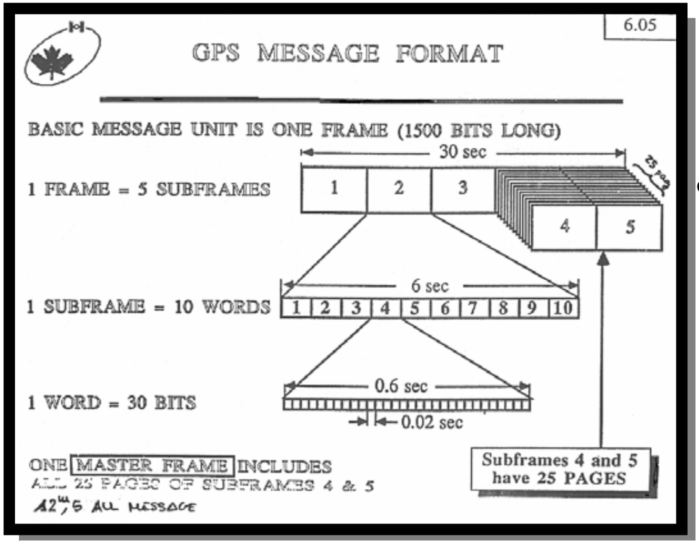
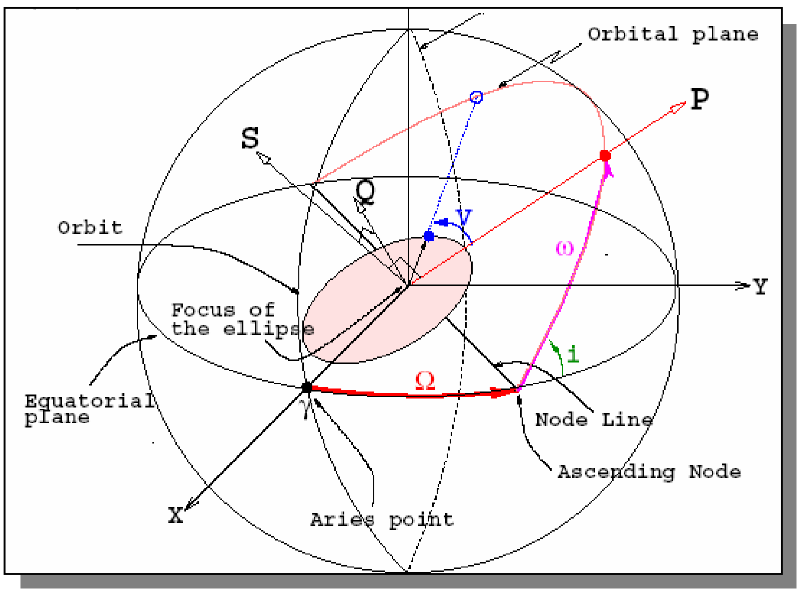
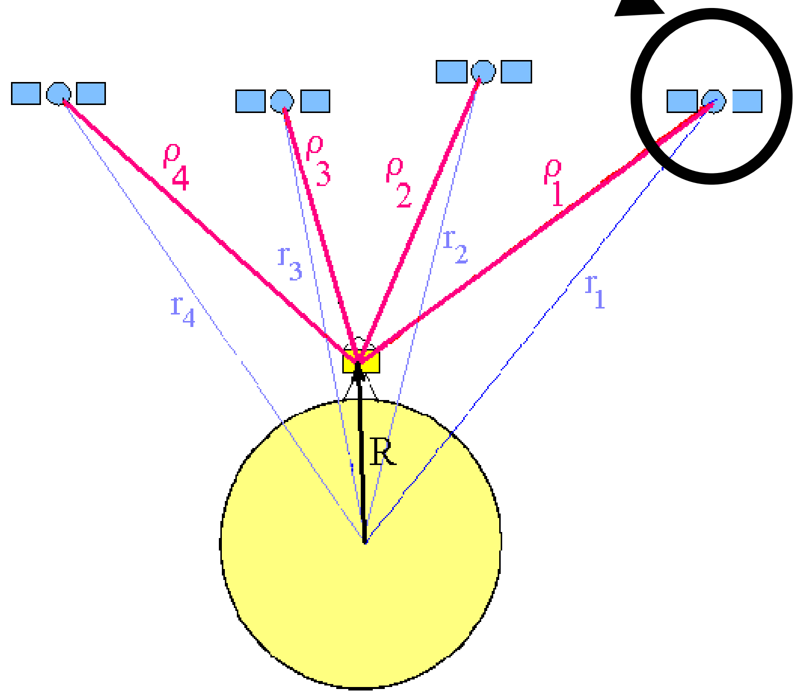
Lesson 4
Satellite coordinates


6.05
GPS MESSAGE FORMAT
BASIC MESSAGE UNIT IS ONE FRAME (1500 BITS LONG)
30 sec
1 FRAME = 5 SUBFRAMES
1
2
3
4
5
6 sec
1 SUBFRAME = 10 WORDS
1 2 3 4 5 6 7 8 9 10
0.6 sec
1 WORD = 30 BITS
0.02 sec
ONE MASTER FRAME INCLUDES
ALL 25 PAGES OF SUBFRAMES 4 & 5
Subframes 4 and 5
have 25 PAGES



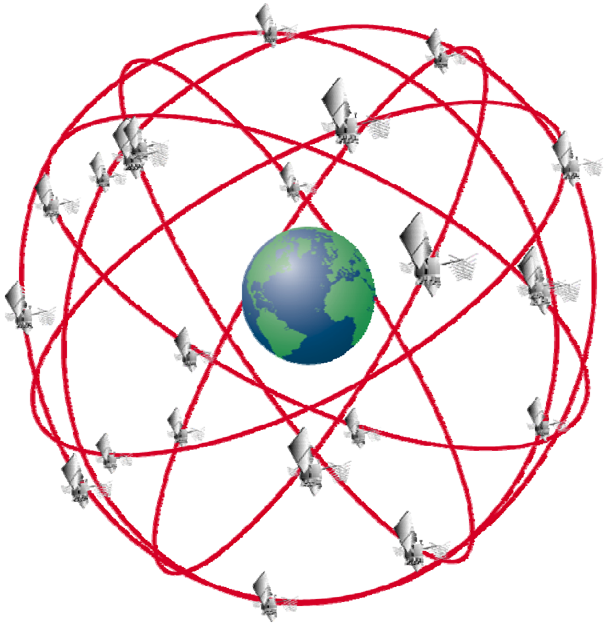
The GPS navigation message provides pseudo-Keplerian elements to compute satellite coordinates

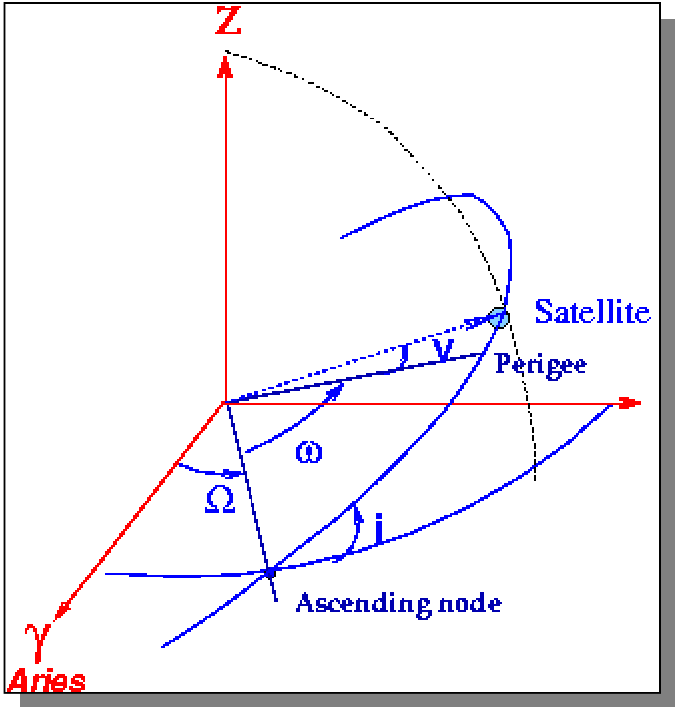




(X, Y, Z, Vx, Vy, Vz) → (a, e, i, Ω, ω, V)

6 values are needed (x,y,z,vx,vy,vz) to provide the position and velocity of a body. They can be map into the **six Keplerian elements** (a, e, i, Ω, ω, V), which provides the "natural" representation of the orbit!

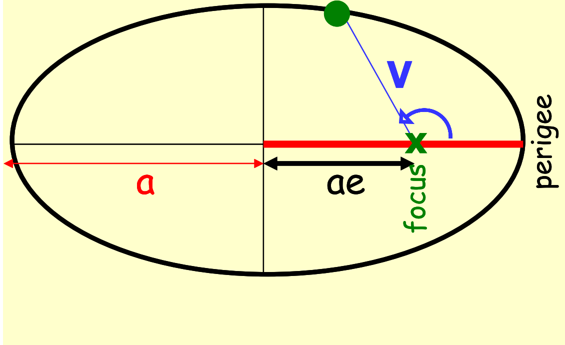


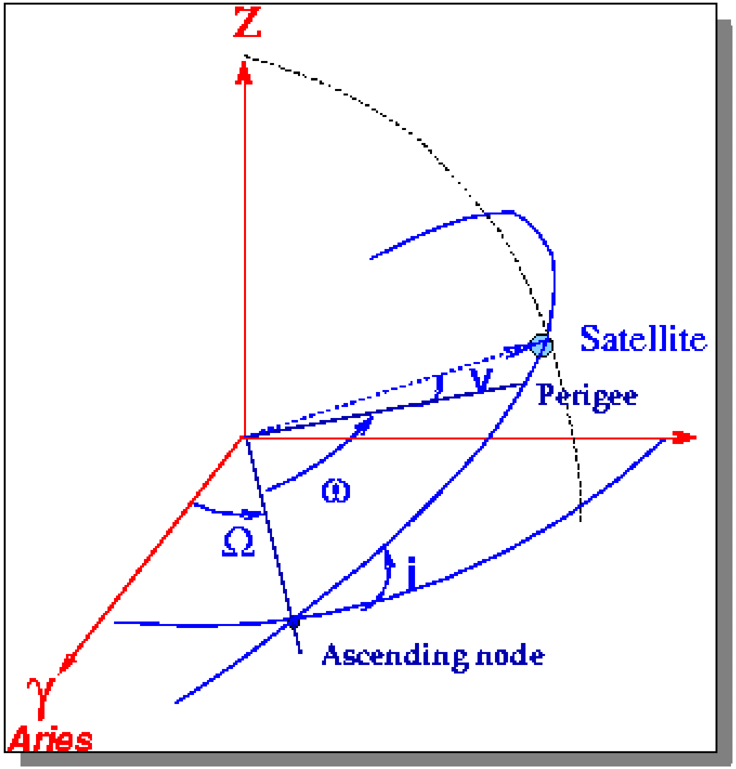


Hernández-Pajares M., Juan M., Sanz J, Salazar D., Ramos-Bosch P.

$(a, e, i, \Omega, \omega, V)$

- a, e: orbit shape
- i, Ω, ω: orbit orientation
- V: position in the orbit





a: semi major axis

e: eccentricity

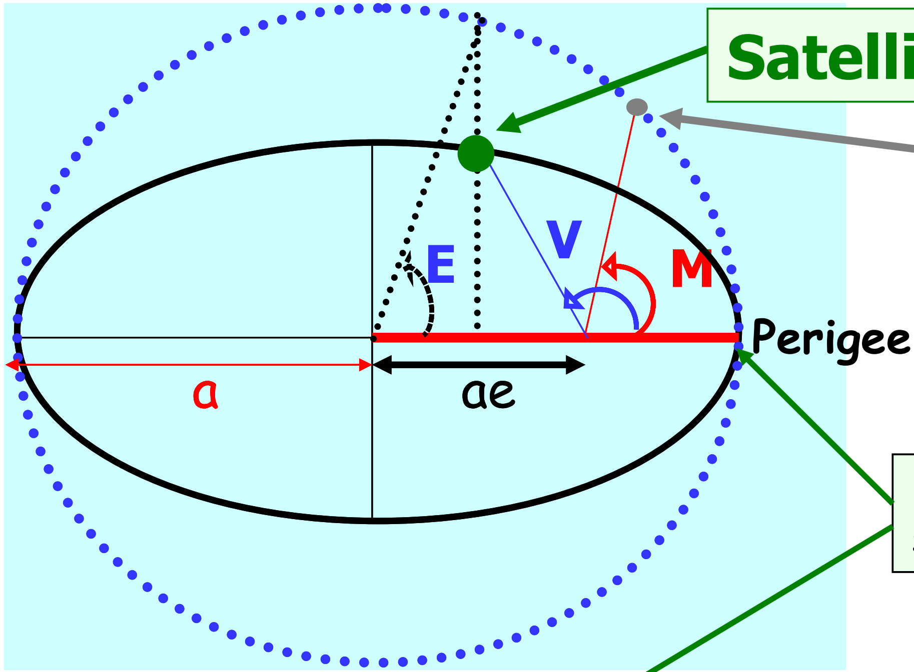
i: inclination

Ω: argument of ascending node

ω: argument of perigee

V: true anomaly

Satellite → True anomaly (*V*)



Fictitious body moving at velocity *n=2π/P=ctt.*
→ **Mean anomaly (*M*)**

T_0 : time of passage by satellite's perigee

$$t \rightarrow \boxed{\begin{array}{c} T_0 \\ n = \dfrac{2\pi}{P} \\ \mu, a, e \end{array}} \rightarrow V(t) \quad \Leftrightarrow$$

$$M(t) = n(t - T_0) \quad ; \quad n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

$$E(t) = M(t) + e \sin E(t)$$

$$V(t) = 2\arctan\left[\sqrt{\frac{1+e}{1-e}} \tan\frac{E(t)}{2}\right]$$

Calculation of osculatrix orbital elements from position and velocity (rv2ele_orb.f)

$$\boxed{(x, y, z, v_x, v_z, v_z) \Rightarrow (a, e, i, \Omega, \omega, M)}$$

$$\vec{c} = \vec{r} \times \vec{v} \Longrightarrow p = \frac{c^2}{\mu} \Longrightarrow p$$
$$\mathrm{v}^2 = \mu(2/r - 1/a) \Longrightarrow a$$
$$\mathrm{p} = \mathrm{a}\ (1 - \mathrm{e}^2) \Longrightarrow e$$

$$\vec{c} = c\vec{S} \Longrightarrow \Omega = \arctan(-c_x/c_y); \quad i = \arccos(c_z/c) \Longrightarrow \Omega, i$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cos(\omega+V) - \sin\Omega\sin(\omega+V)\cos i \\ \sin\Omega\cos(\omega+V) + \cos\Omega\sin(\omega+V)\cos i \\ \sin(\omega+V)\sin i \end{pmatrix} \Longrightarrow \omega + V$$

$$r = \frac{p}{1 + e\cos(V)} \Longrightarrow \omega, V$$

$$\tan(E/2) = \left(\frac{1-e}{1+e}\right)^{1/2} \tan(V/2) \quad ; \quad M = E - e\sin E \Longrightarrow M$$

Calculation of position and velocity from orbital elements (ele_orb2rv.f, orb2xyz.f)

$$(a, e, i, \Omega, \omega, \underbrace{T; t}_{V}) \Rightarrow (x, y, z, v_x, v_z, v_z)$$

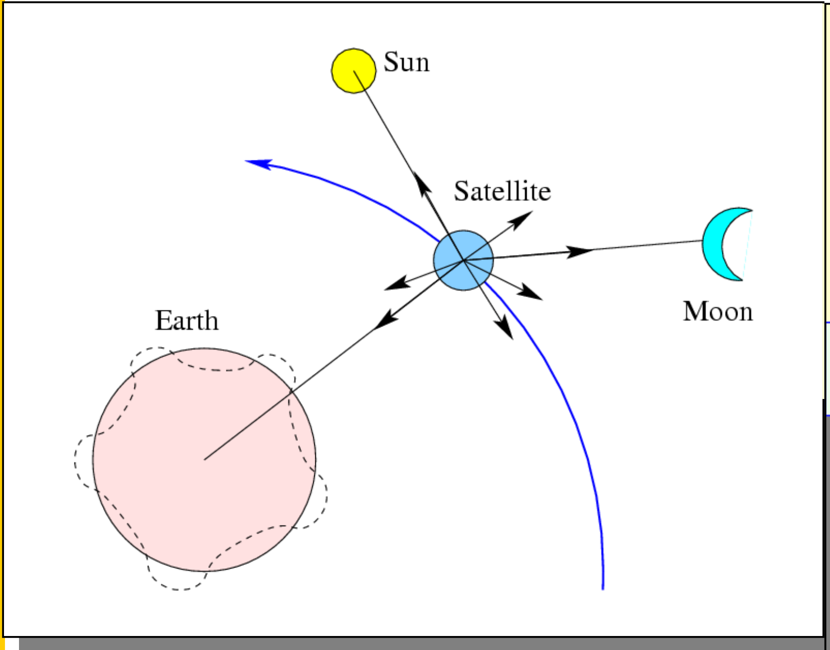
$$\begin{array}{ccccccc} t & \Longrightarrow & M & \Longrightarrow & E & \Longrightarrow & (r, V) \\ & M = n(t-T) & & M = E - e\sin E & & r = a(1 - e\cos E) & \\ & & & & & \tan(V/2) = (\frac{1+e}{1-e})^{1/2}\tan(E/2) & \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} \; ; \; \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{na^2}{r}\{\vec{Q}(1-e^2)^{1/2}\cos E - \vec{P}\sin E\}$$

Where:

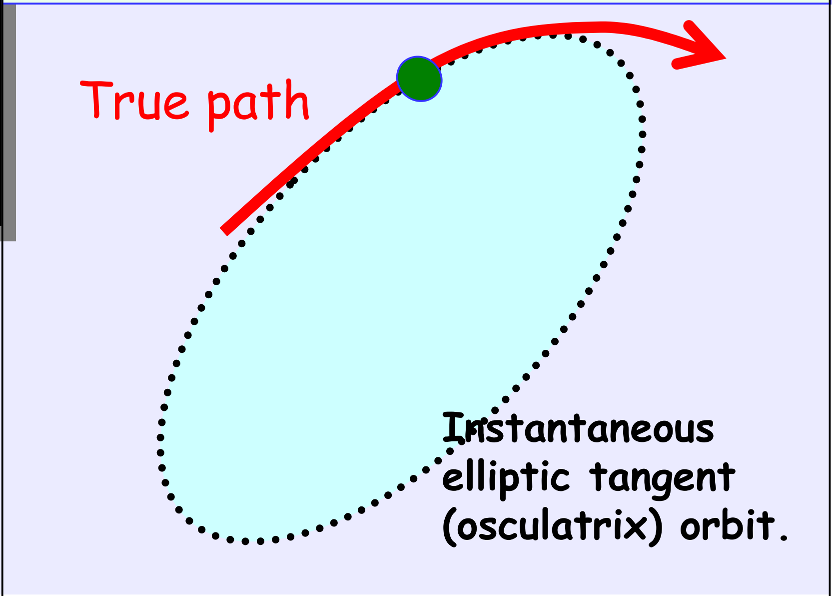
$$\begin{aligned} \mathrm{R} &= \mathrm{R}_3(-\Omega) R_1(-i) R_3(-\omega) = \\ &= \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \mathrm{P}_x & \mathrm{Q}_x & \mathrm{S}_x \\ \mathrm{P}_y & \mathrm{Q}_y & \mathrm{S}_y \\ \mathrm{P}_z & \mathrm{Q}_z & \mathrm{S}_z \end{pmatrix} = [\vec{P}\; \vec{Q}\; \vec{S}] \end{aligned}$$

Due to the non-spherical nature of gravitational potential, the attraction of the sun and moon, the solar radiation pressure, etc., **the true satellite path deviates from the elliptic orbit**.



At any time an elliptical orbit tangent to the true path can be defined. This is the "osculatrix orbit", whose Keplerian elements vary with time "t":

$$a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$$



Exercise 3: Orbital elements variation:

File 1995-10-18.eci contains the precise position and velocities of GPS satellites every 5 minutes for October 18th, 1995. **[from JPL/NASA server: ftp://sideshow.jpl.nasa.gov/pub/gipsy_products]**

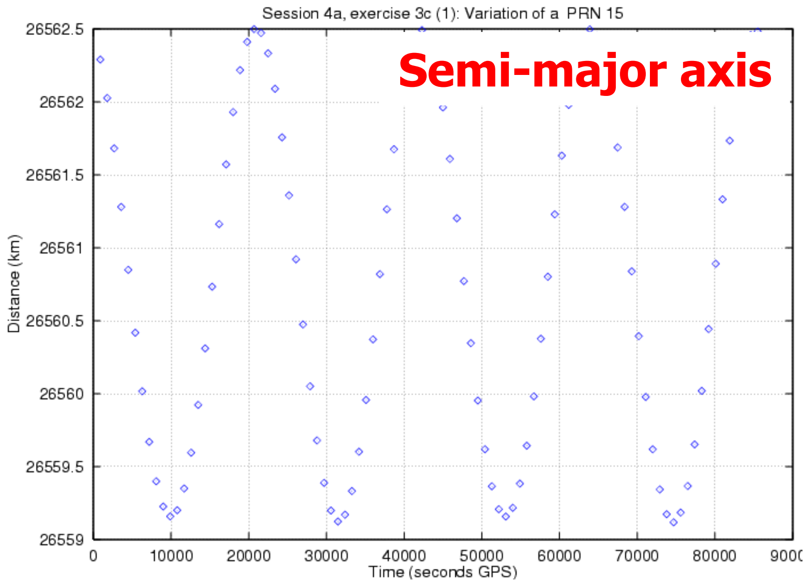
a) Use program "**rv2ele_orb**" to compute the instantaneous orbital elements for each epoch in the file. That is: **(X, Y, Z, Vx, Vy, Vz) → (*a, e, i, Ω, ω, V*)**

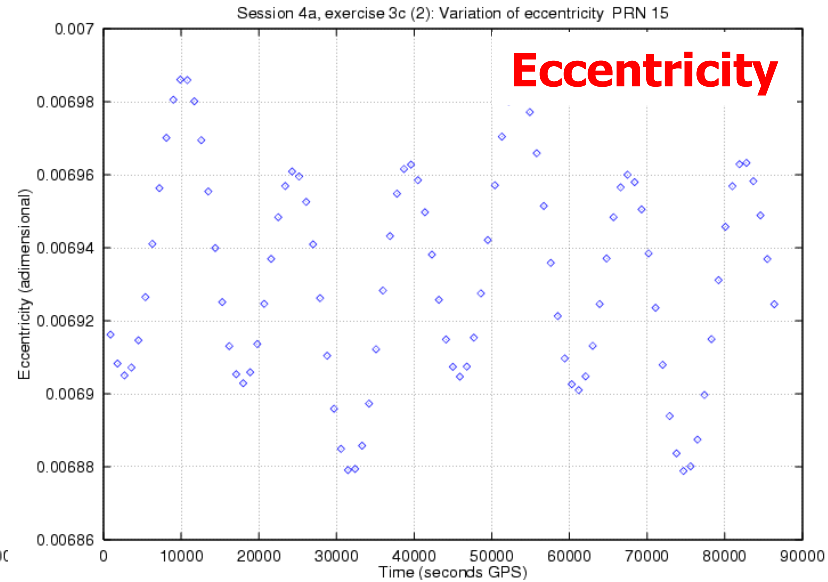
b) Plot the orbital elements in function of time to show their variation: ***a(t), e(t), i(t), Ω (t), ω(t), V(t)***

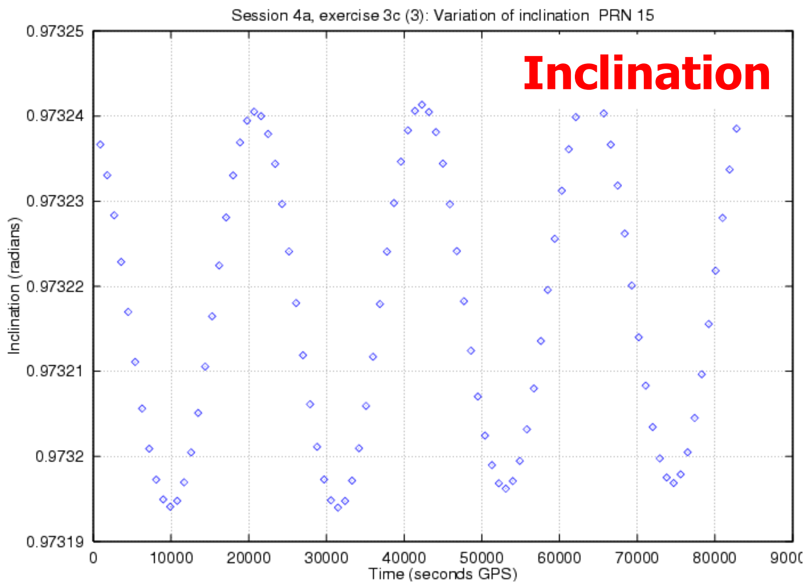
Solution:

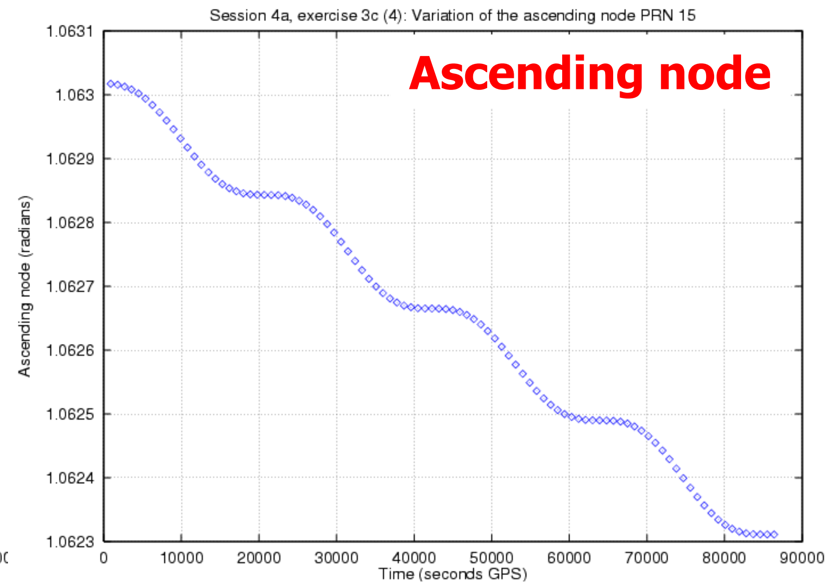
a) **cat 1995-10-18.eci|rv2ele_orb> orb.dat**

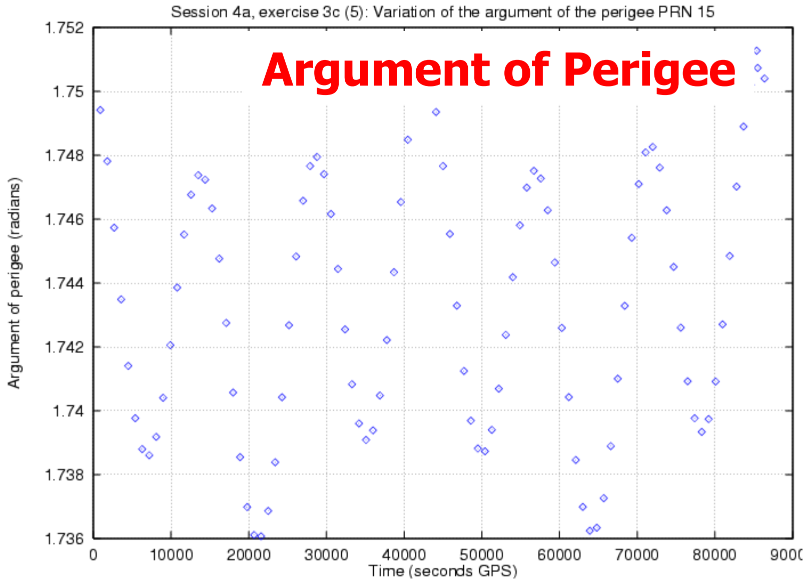
b) See the following plots

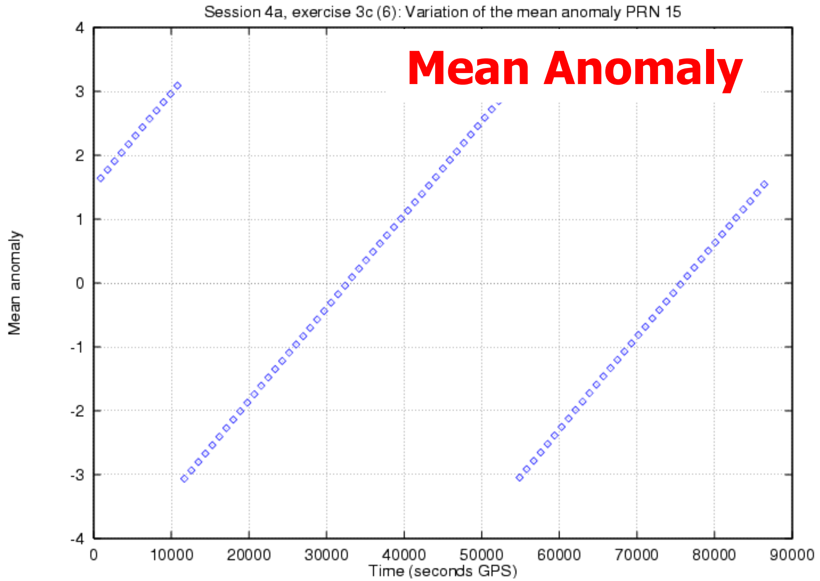

Session 4a, exercise 3c (1): Variation of a PRN 15
Semi-major axis
Distance (km)
Time (seconds GPS)


Session 4a, exercise 3c (2): Variation of eccentricity PRN 15
Eccentricity
Eccentricity (adimensional)
Time (seconds GPS)


Session 4a, exercise 3c (3): Variation of inclination PRN 15
Inclination
Inclination (radians)
Time (seconds GPS)


Session 4a, exercise 3c (4): Variation of the ascending node PRN 15
Ascending node
Ascending node (radians)
Time (seconds GPS)


Session 4a, exercise 3c (5): Variation of the argument of the perigee PRN 15
Argument of Perigee
Argument of perigee (radians)
Time (seconds GPS)
1.752
1.75
1.748
1.746
1.744
1.742
1.74
1.738
1.736
0
10000
20000
30000
40000
50000
60000
70000
80000
9000


Session 4a, exercise 3c (6): Variation of the mean anomaly PRN 15
Mean Anomaly
Mean anomaly
Time (seconds GPS)
4
3
2
1
0
-1
-2
-3
-4
0
10000
20000
30000
40000
50000
60000
70000
80000
90000

Ephemerids in navigation message:

Parameter	Explanation
$IODE$	Series number of ephemerides data
t_{oe}	Ephemerides reference epoch
$\sqrt{a}$	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω	Ascending node's right ascension
Δn	Mean motion difference
$\dot{i}$	rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
c_{uc}, c_{us}	Latitude argument correction
c_{rc}, c_{rs}	Orbital radius correction
c_{ic}, c_{is}	Inclination correction

In order to calculate WGS84 satellite coordinates, you should apply de following algorithm [GPS/SPS-SS, table 2-15] (see in the book FORTRAN subroutine orbit.f, annex IV)

RINEX ephemeris file

```
     2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
XPRINT v1.1         gAGE                00/08/17 09:31:37   PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE                               COMMENT
    +1.7695E-08 +2.2352E-08 -1.1921E-07 -1.1921E-07         ION ALPHA
    +1.1878E+05 +1.4746E+05 -1.3107E+05 -3.2768E+05         ION BETA
    +1.955777406693E-08+1.598721155460E-14   405504     1064 DELTA_UTC: A0,A1,T,W
     13                                                     LEAP SECONDS
                                                            END OF HEADER

03 00  5 30 10  0 40.0+7.855705916882E-06+3.524291969370E-12+0.000000000000E+00
    +1.010000000000E+02+6.500000000000E+01+5.456298524109E-09+5.530285585107E-01   Mo
    +3.475695848465E-06+1.308503560722E-03+2.641230821609E-06+5.153678266525E+03   e, √a
    +2.088000000000E+05+1.117587089539E-08+7.472176136643E-01-1.862645149231E-09   TOE, Ω
    +9.412719852649E-01+3.163750000000E+02+1.125448382894E+00-8.826796182859E-09   io, ω
    +1.239337382719E-10+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00
    +4.000000000000E+00+0.000000000000E+00-4.190951585770E-09+6.130000000000E+02   TGD
    +2.044980000000E+05+0.000000000000E+00+0.000000000000E+00+0.000000000000E+00

06 00  5 30 10  0  0.0+1.636799424887E-06+0.000000000000E+00+0.000000000000E+00
    +6.000000000000E+01+5.100000000000E+01+5.198073527168E-09-5.601816471398E-01
    +2.635642886162E-06+6.763593177311E-03+2.468004822731E-06+5.153726325989E+03
    +2.088000000000E+05+1.862645149231E-08+7.894129138508E-01+8.195638656616E-08
    +9.487675576456E-01+3.229687500000E+02-2.409256713064E+00-8.734292400447E-09
    +4.714481929846E-11+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00
```

Computation of satellite coordinates from navigation message (orbit.f)

- **Computation of t_k time since ephemerids reference epoch t_{oe} (t and t_{oe} are given in GPS seconds of week):**

$$t_k = t - t_{oe}$$

- **Computation of mean anomaly M_k for t_k,**

$$M_k = M_0 + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n \right) t_k$$

- **Iterative resolution of Kepler's equation in order to compute eccentric anomaly E_k:**

$$M_k = E_k - e \sin E_k$$

- **Calculation of true anomaly v_k:**

$$v_k = \arctan \left(\frac{\sqrt{1 - e^2} \sin E_k}{\cos E_k - e} \right)$$

- **Computation of latitude argument u_k from perigee argument W, true anomaly v_k and corrections c_{uc} and c_{us}:**

$$u_k = \omega + v_k + c_{uc} \cos 2\left(\omega + v_k\right) + c_{us} \sin 2\left(\omega + v_k\right)$$

- **Computation of radial distance r_k, taking into consideration corrections c_{rc} and c_{rs}:**

$$r_k = a\left(1-2\cos E_k\right)+c_{rc}\cos 2\left(\omega+v_k\right)+c_{rs}\sin 2\left(\omega+v_k\right)$$

- **Calculation of orbital plane inclination i_k from inclination i_o at reference epoch t_{oe} and corrections c_{ic} and c_{is} :**

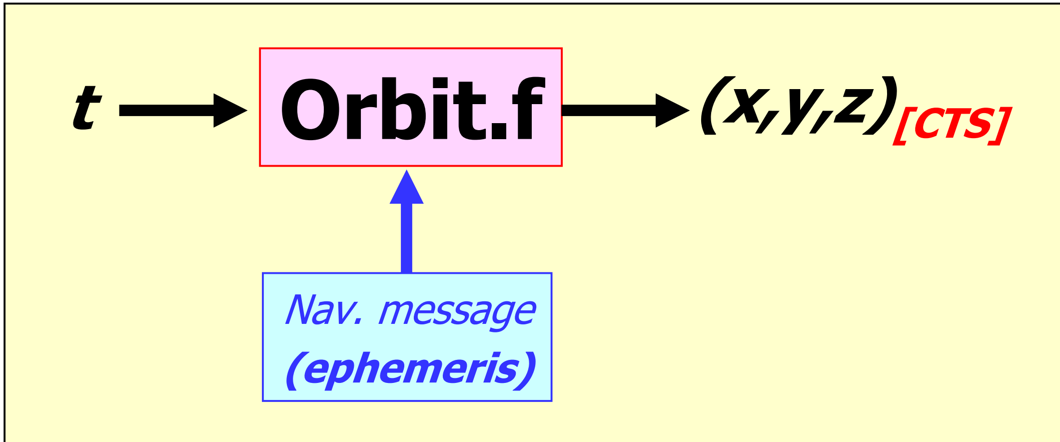
$$i_k = i_0 + it_k + c_{ic}\cos 2\left(\omega+v_k\right)+c_{is}\sin 2\left(\omega+v_k\right)$$

- **Computation of ascending node longitude Ω_k (Greenwich), from longitude Ω_0 at start of GPS week, corrected from apparent variation of sidereal time at Greenwich between start of week and and reference time $t_k=t-t_{oe}$, and also corrected from change of ascending node longitude since reference epoch t_{oe}.**

$$\Omega_k = \Omega_0 + \left(\Omega-\omega_E\right)t_k - \omega_E t_{oe}$$

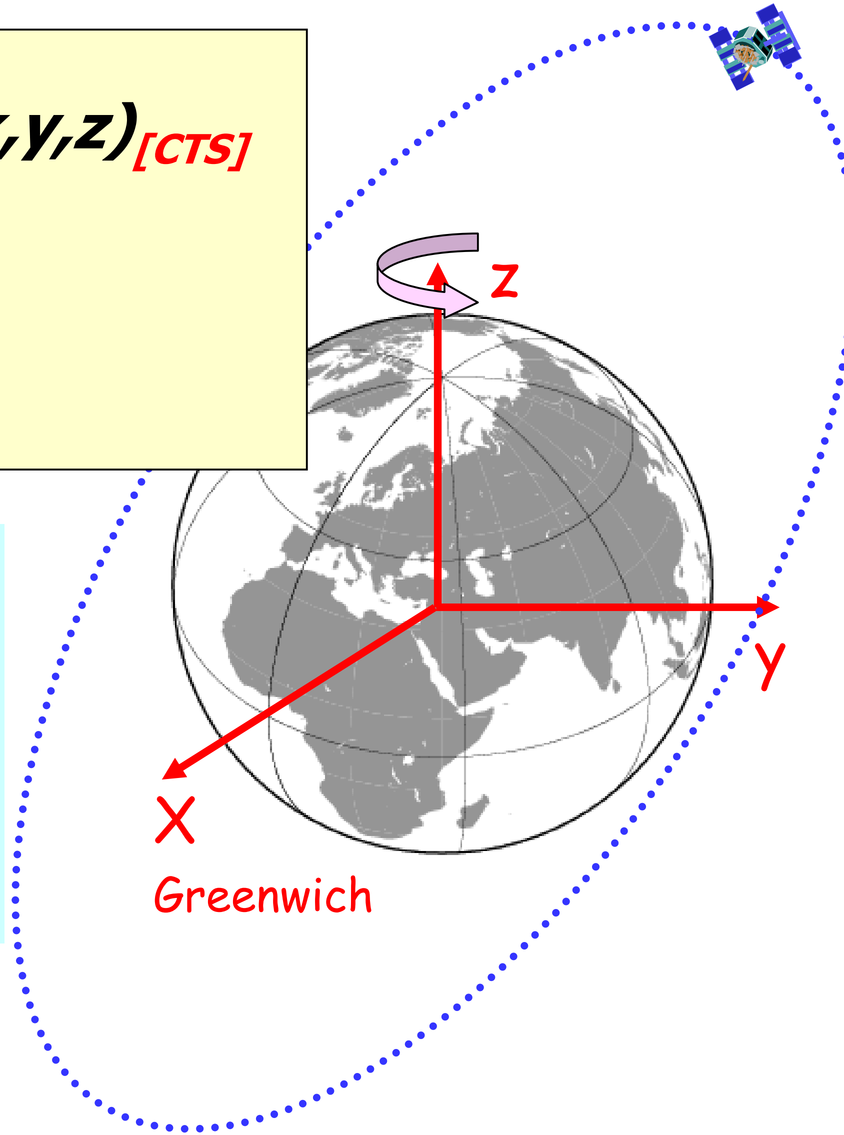
- **Calculation of coordinates in CTS system, applying three rotations (around u_k, i_k, Ω_k) :**

$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = \mathbf{R}_3(-\Omega_k)\mathbf{R}_1(-i_k)\mathbf{R}_3(-u_k)\begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$



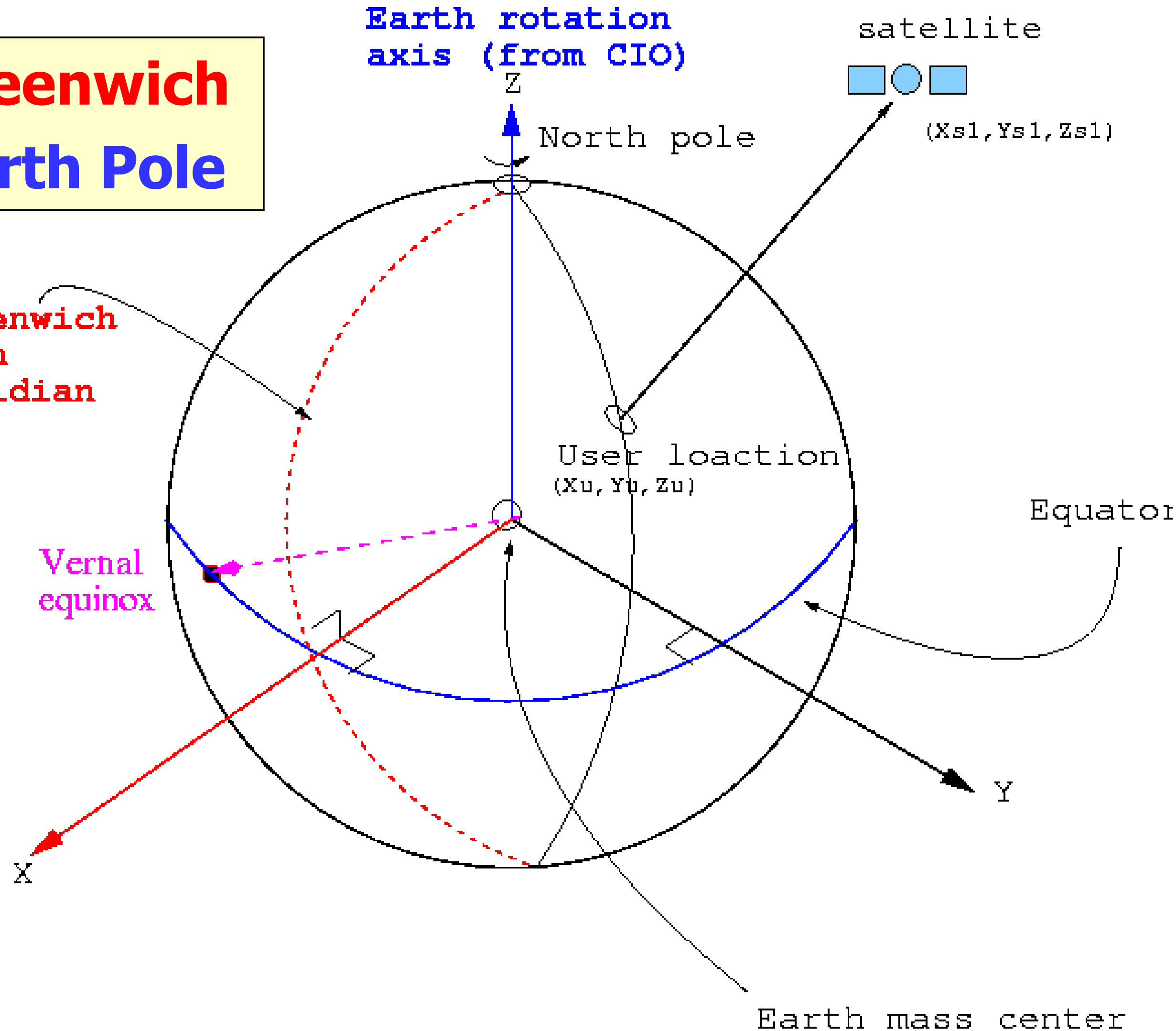
Conventional Terrestrial System **(CTS):**

Earth-Fixed System →
the reference system rotates with Earth.



The program orbit.f provides the satellite coordinates in a Earth fixed system **(CTS)**

X → Greenwich
Z → North Pole



Exercise 4: Orbits and clocks accuracy (S/A=on)

The file "eph.on" contains satellite coordinates (x,y,z) and clocks, computed from the navigation message of GPS satellites for March 23th, 1999. (with S/A=on)

[the coordinates has been computed using the program orbit.f]

The file "sp3.on" contains precise coordinates and clocks of GPS satellites for March 23th, 1999

[Provided by the IGS server ftp://igscb.jpl.nasa.gov/igscb/product]

Plot the error of broadcast orbits and clocks and discuss the results.

Solution:

See the following plots

ERROR in coordinates and clock S/A=on

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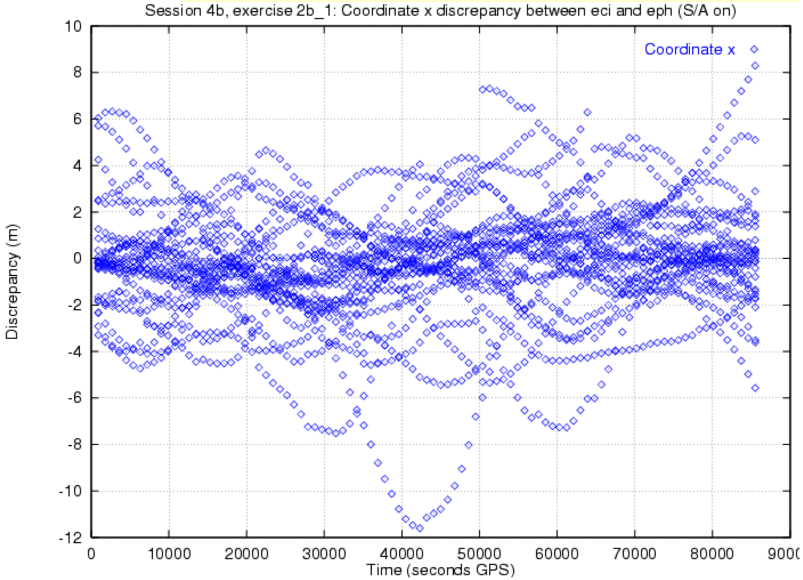
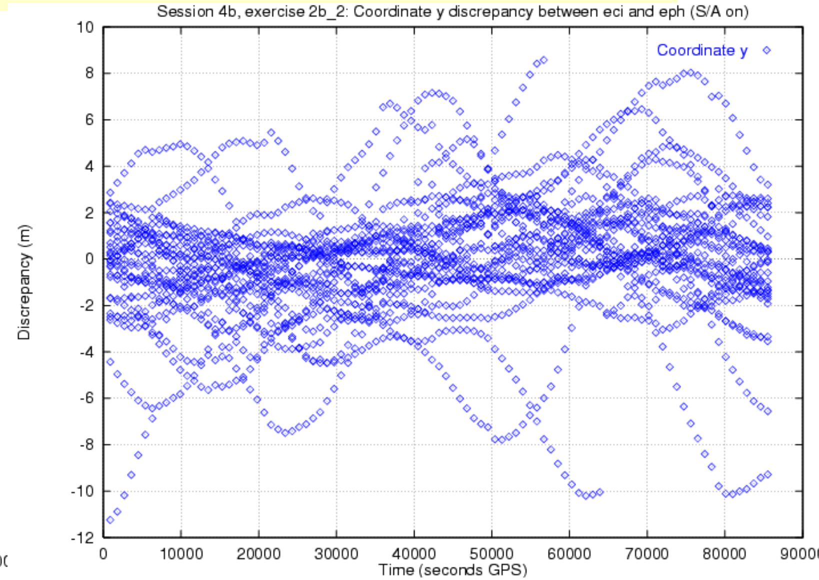
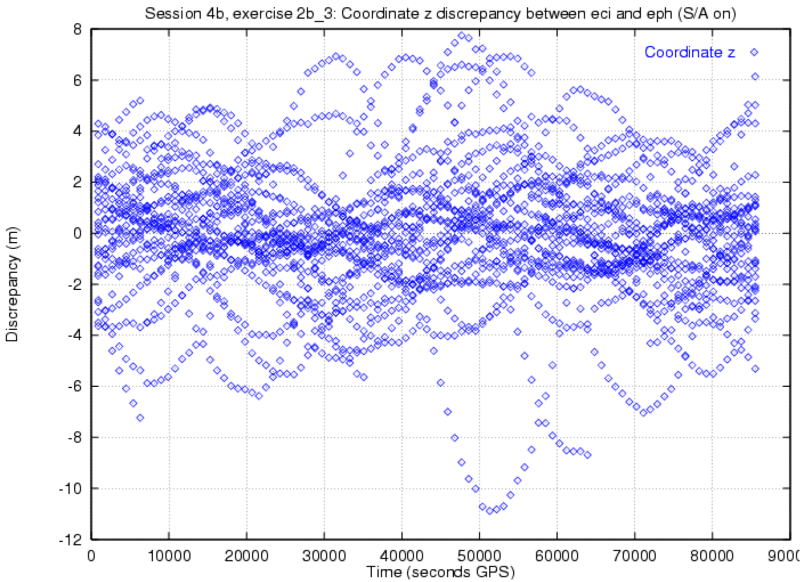
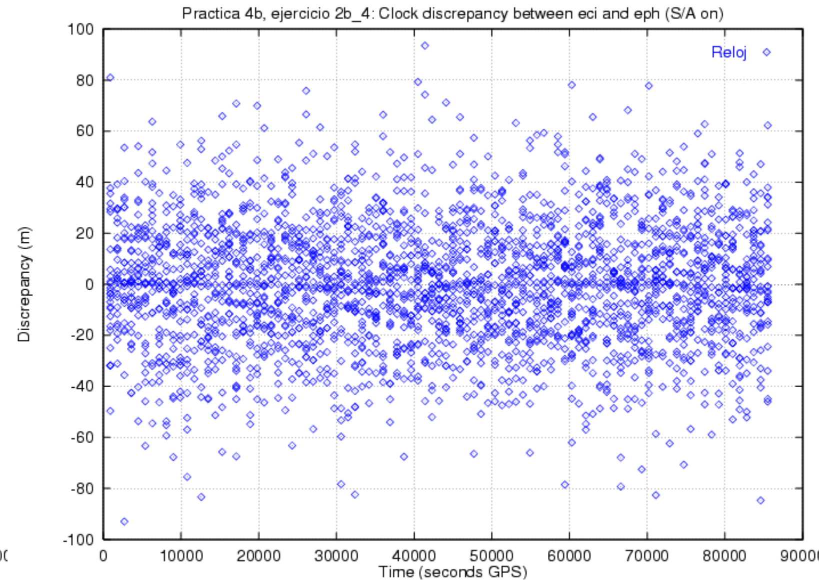








Exercise 5: Orbits and clocks accuracy (S/A=off)

The file “eph.off” contains satellite coordinates (x,y,z) and clocks, computed from the navigation message of GPS satellites for May 15th, 2000 (with S/A=off)

[the coordinates has been computed using the program orbit.f]

The file “sp3.off” contains precise coordinates and clocks of GPS satellites for May 15th, 2000

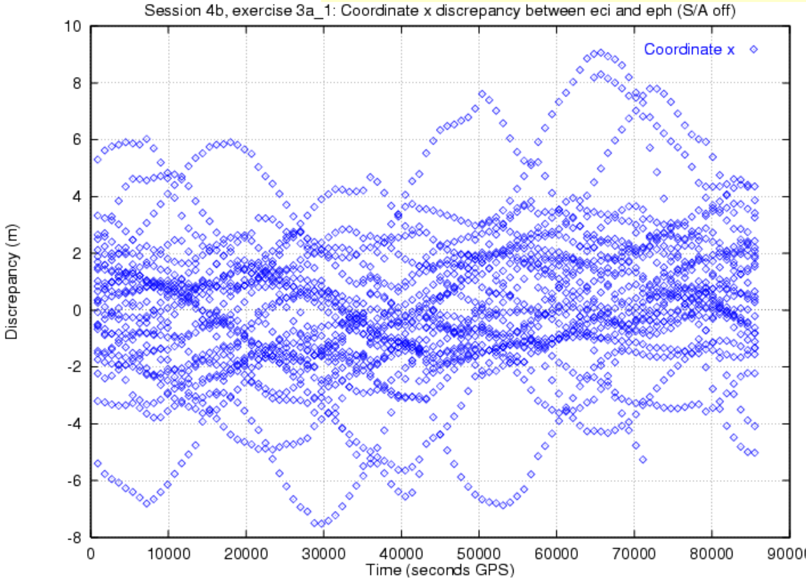
[Provided by the IGS server ftp://igscb.jpl.nasa.gov/igscb/product]

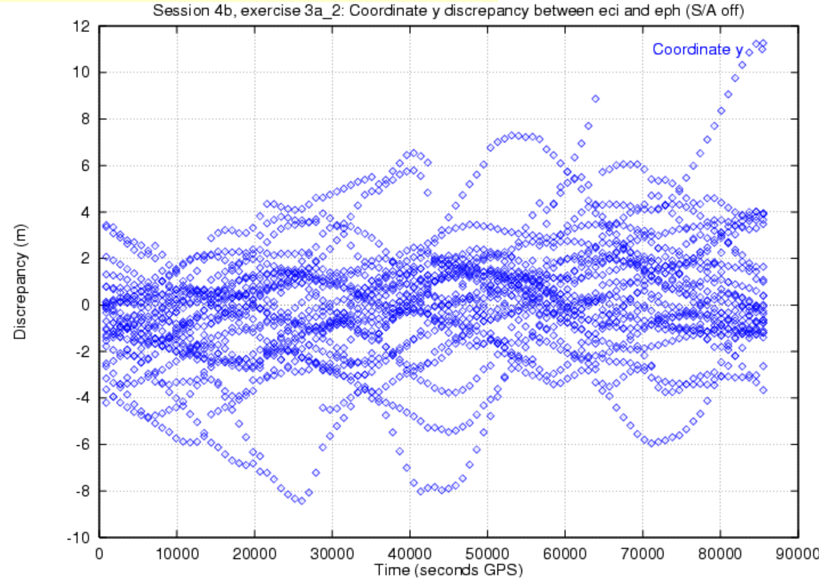
Plot the error of broadcast orbits and clocks and discuss the results.

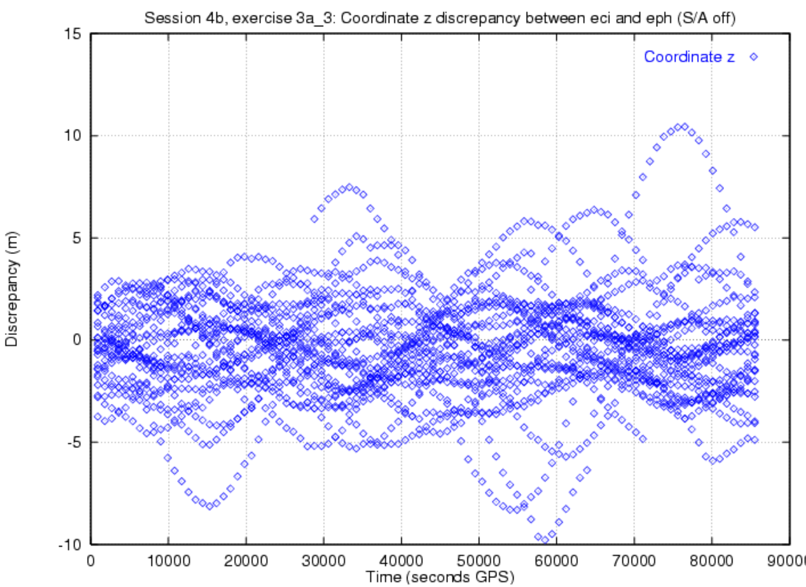
Solution:

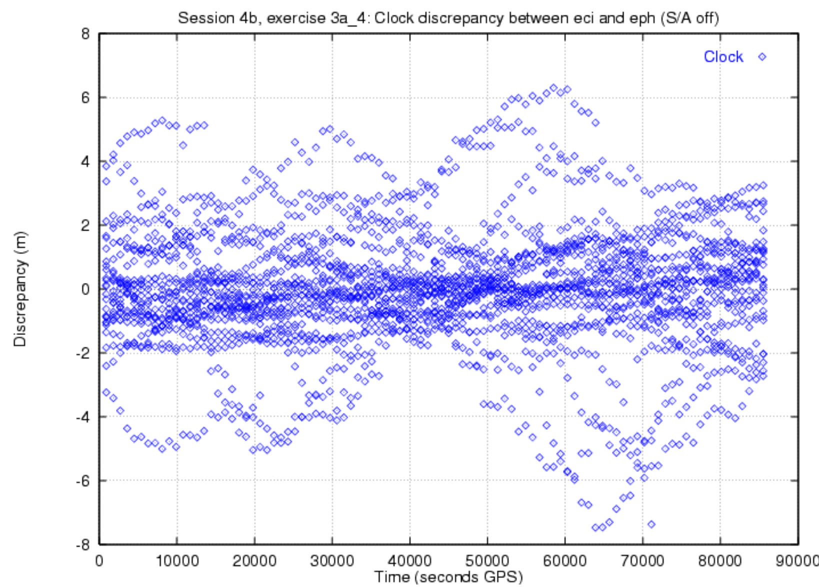
See the following plots.

ERROR in coordinates S/A=off









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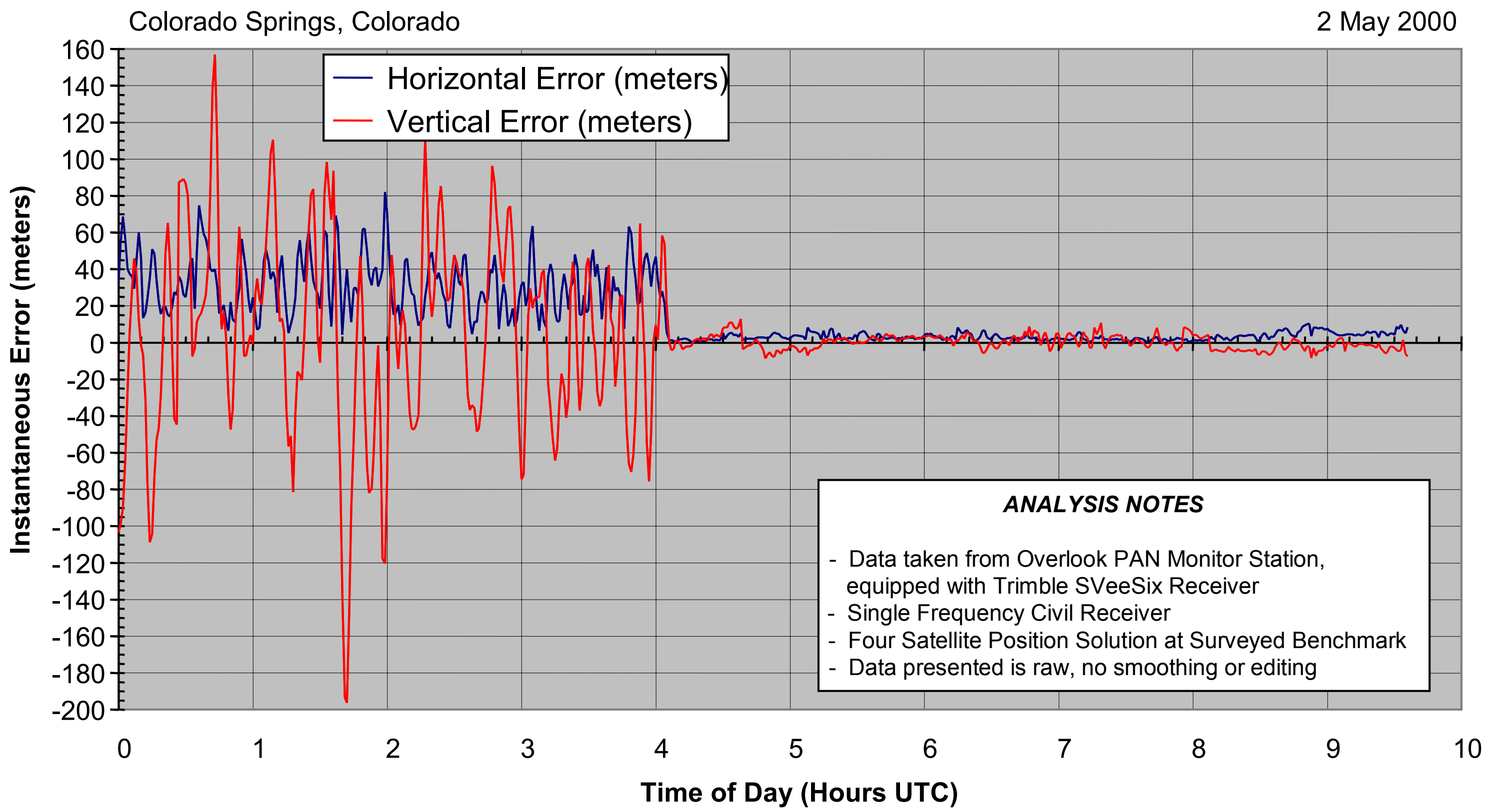
Selective Availability (S/A): Intentional degradation of satellite clocks and broadcast ephemeris. (from 25 March, 1990)

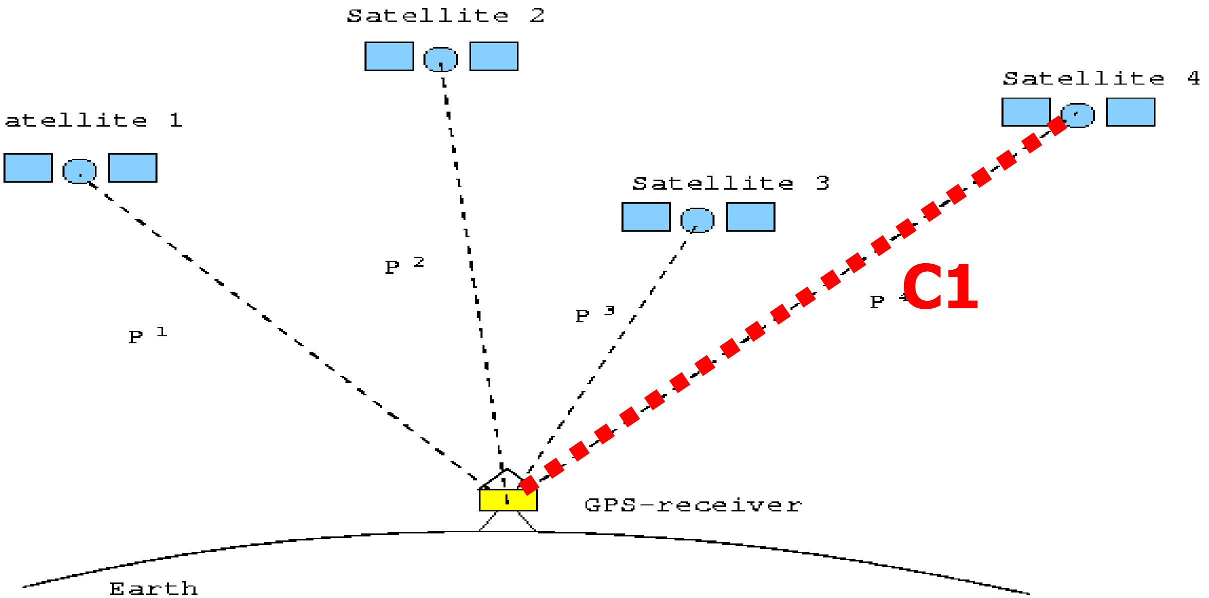
GPS Before and After S/A was switched off



Lesson 5

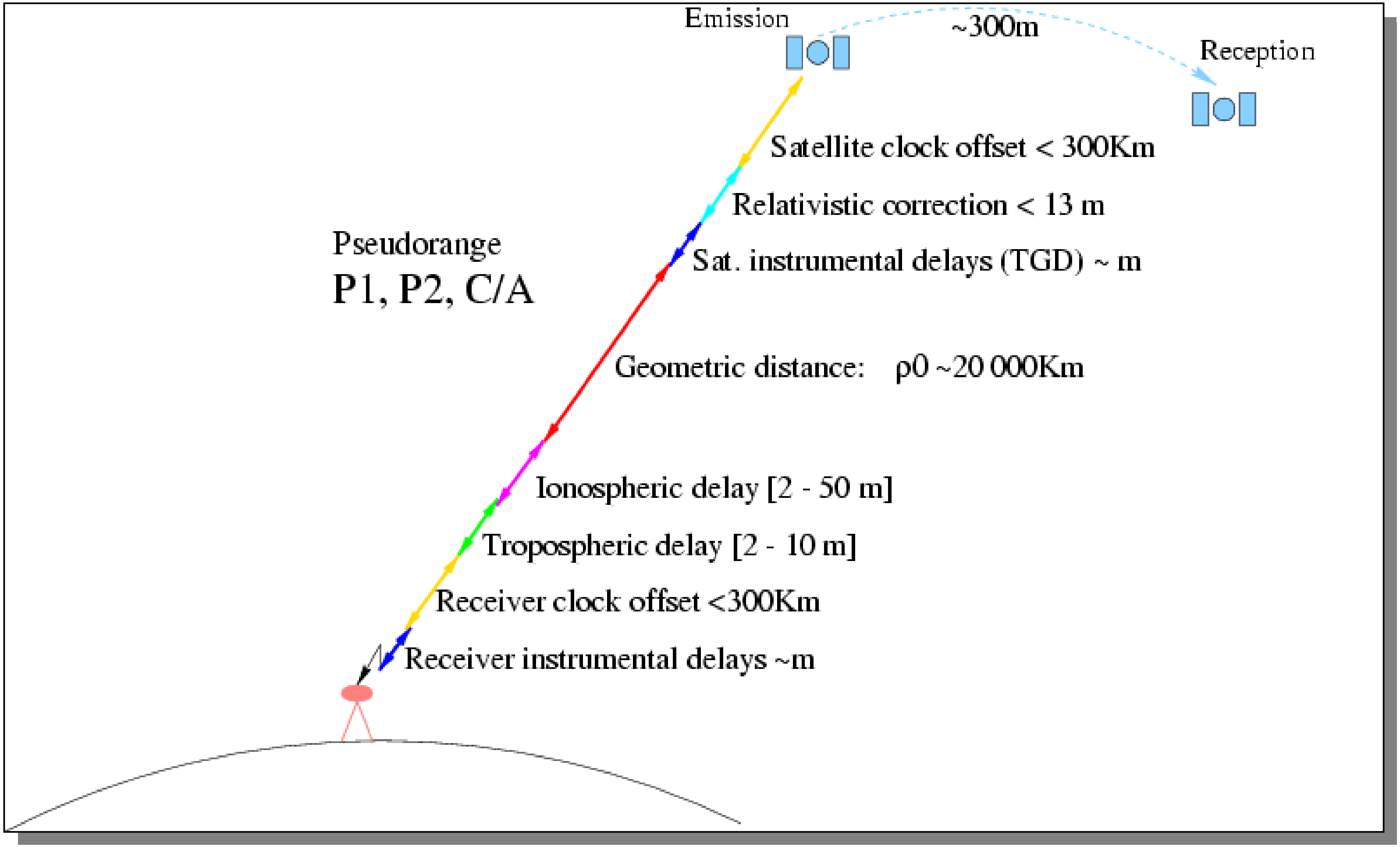
Model

Pseudorange modeling (code)



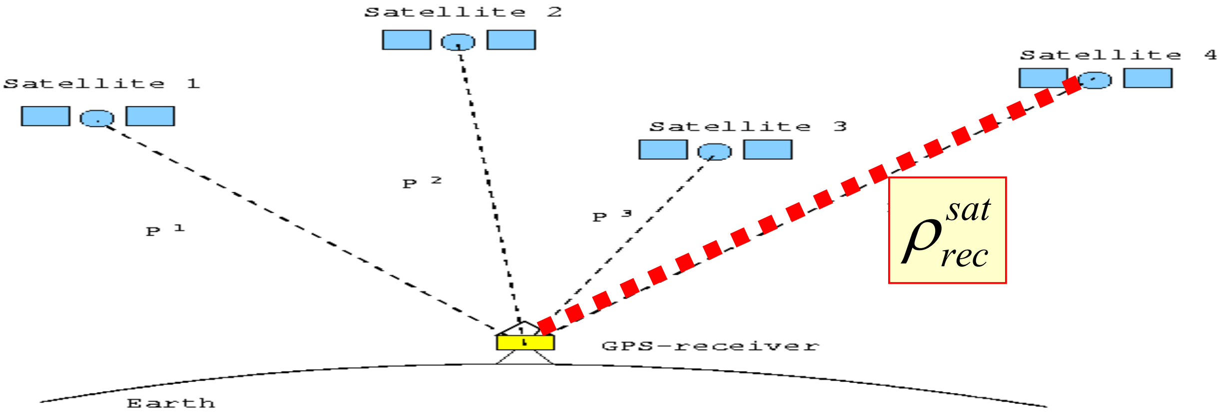
The pseudorange modeling is based in the GPS Standard Positioning Service Signal Specification (GPS/SPS-SS).

$$C1_{1rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + K_{1}^{sat} + \varepsilon$$



$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + K_{1}^{sat} + \varepsilon$$

Geometric range



Euclidean distance between satellite coordinates at emission time and receiver coordinates at reception time.

$$\rho_{rec}^{sat} = \sqrt{\left(x^{sat} - x_{rec}\right)^2 + \left(y^{sat} - y_{rec}\right)^2 + \left(z^{sat} - z_{rec}\right)^2} =$$

Of course, receiver coordinates are not known (is the target of this problem). But ....

$$C1_{1rec}^{sat} = \boxed{\rho_{rec}^{sat}} + c \cdot (dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + K_1^{sat} + \varepsilon$$

$$\rho_{rec}^{sat} = \sqrt{\left(x^{sat} - x_{rec}\right)^2 + \left(y^{sat} - y_{rec}\right)^2 + \left(z^{sat} - z_{rec}\right)^2}$$

Of course, receiver coordinates $(x_{rec}, y_{rec}, z_{rec})$ are not known (they are the target of this problem). But, we can always assume that an "approximate position $(x_{0_{rec}}, y_{0_{rec}}, z_{0_{rec}})$ is known" (it can be computed using the Bancroft's method –see next lesson--):

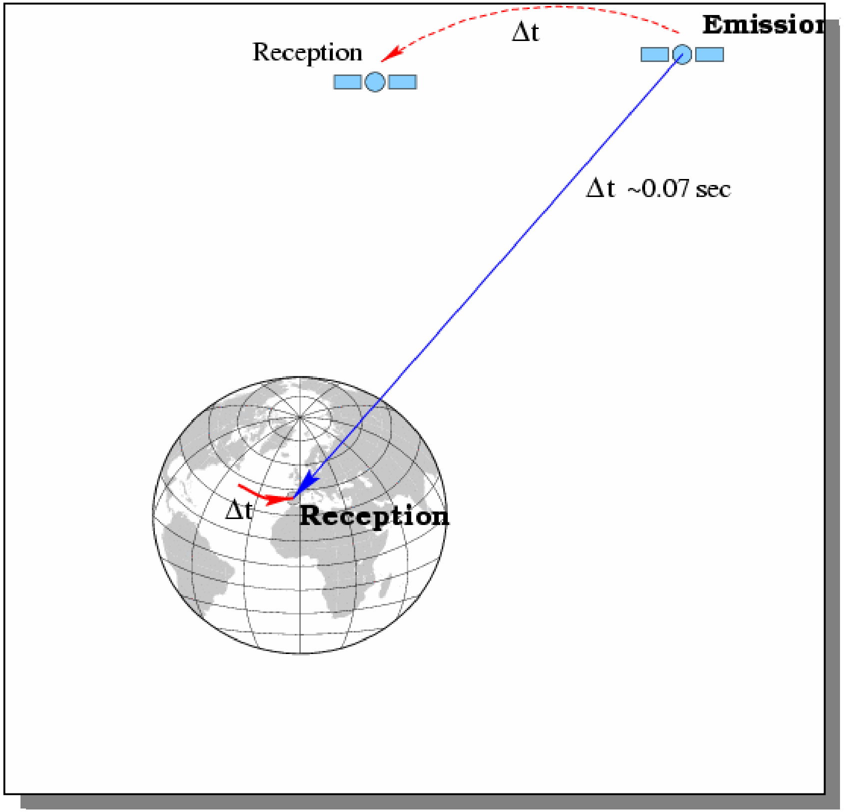
Thence, as it will be shown in next lesson, the navigation problem will consist on:

1.- To start from an approximate value for receiver position $(x_{0_{rec}}, y_{0_{rec}}, z_{0_{rec}})$ (it can be computed with Bancroft's method)

2.- With the pseudorange measurements and the navigation equations, compute the correction $(dx_{rec}, dy_{rec}, dz_{rec})$ to have a more precise position of the receiver.

$$(x_{rec}, y_{rec}, z_{rec}) = (x_{0_{rec}}, y_{0_{rec}}, z_{0_{rec}}) + (dx_{rec}, dy_{rec}, dz_{rec})$$

Satellite coordinates **at emission time** (rec2ems.f)



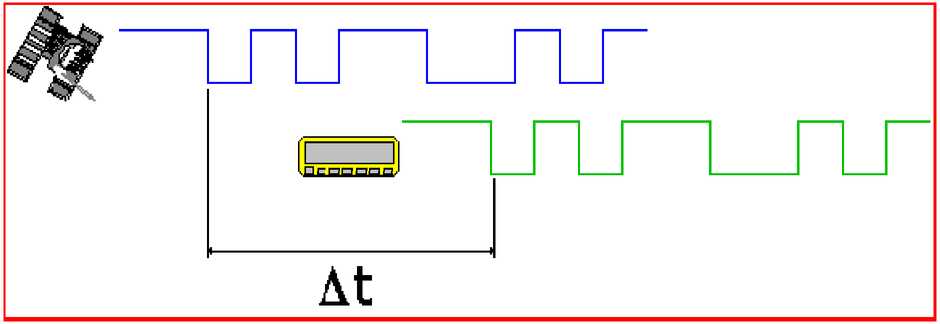
- The GPS signal travels from satellite coordinates at emission time (t^{ems}) to receiver coordinates at reception time (t_{rec}).

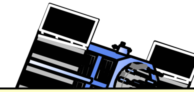
- The satellite can move several hundreds of meters from t^{ems} to t_{rec}.

- The receiver time-tags are given at reception time and in the receiver clock time.

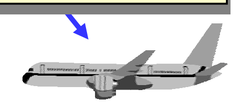
An algorithm is needed to compute the satellite coordinates at **emission time** "in the GPS system time" from reception time in the receiver time tags.

Emission time in the GPS system time *T[ems]*





The satellite offset clock dt^S can be computed from the navigation message



$$C1 = c\,\Delta t = c\,[t_{rec}(T_R) - t^{ems}(T^S)]$$

As it is known, the pseudorange measurements link the "emission time (t^{ems})" in satellite clock (T^S) with reception time (t_{rec}) in receiver clock (T_R) (receiver time tags).

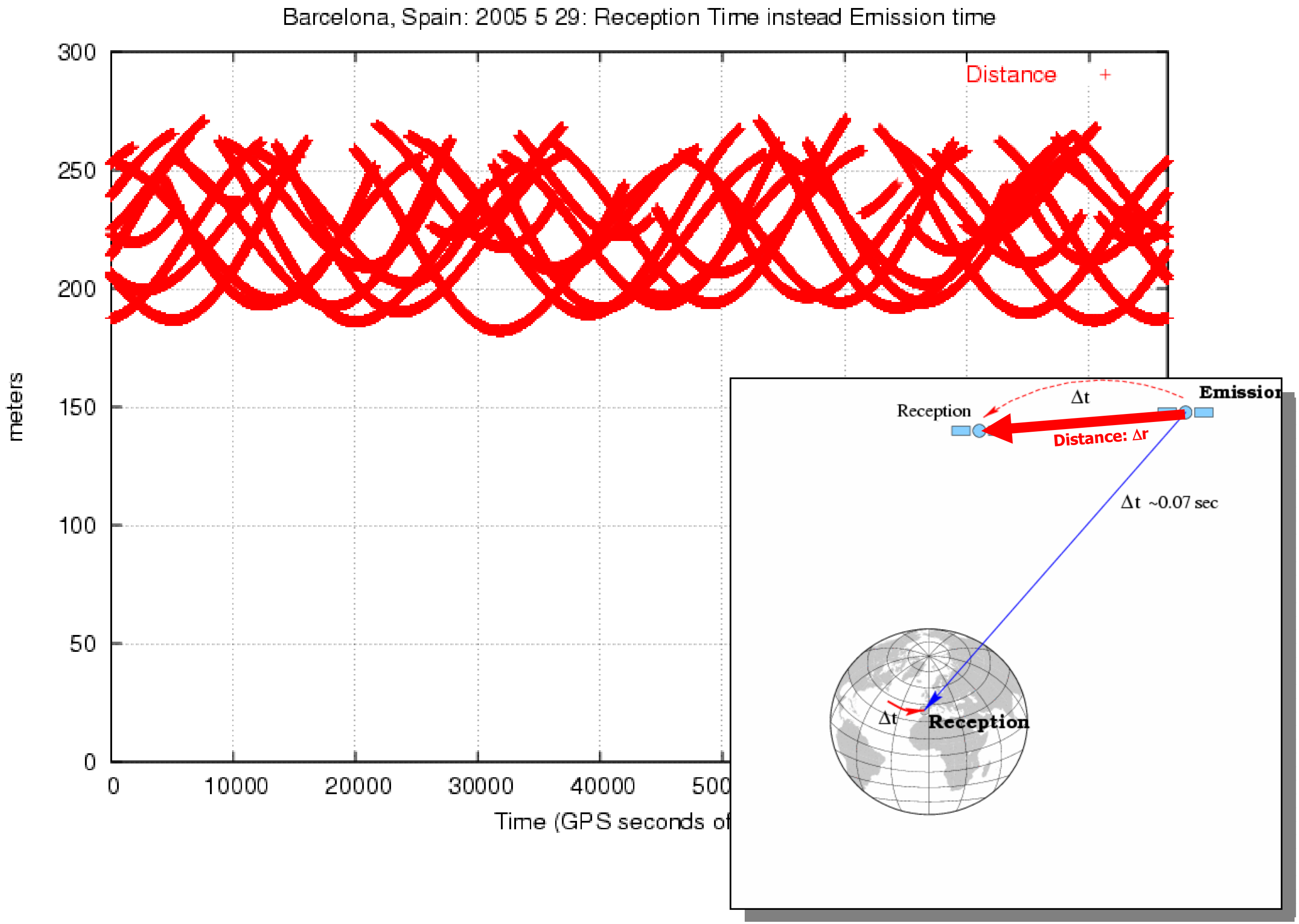
Thence, the emission time in the satellite clock is:

$$t^{ems}(T^S) = t_{rec}(T_R) - C1/c$$

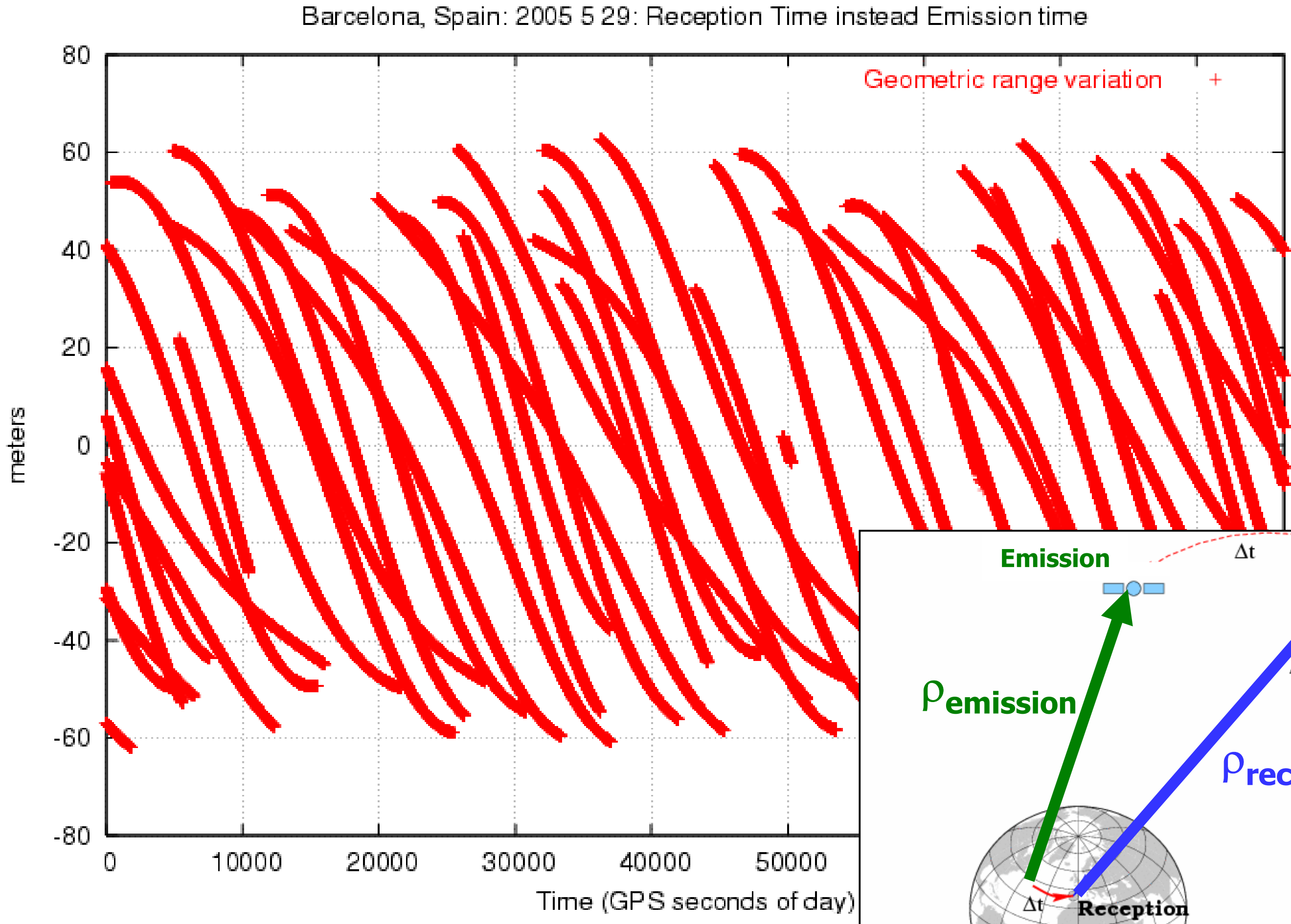
Finally, since $dt^S = t^S - T$ is the time offset between satellite clock (t^S) and **GPS system time *(T)*,** thence:

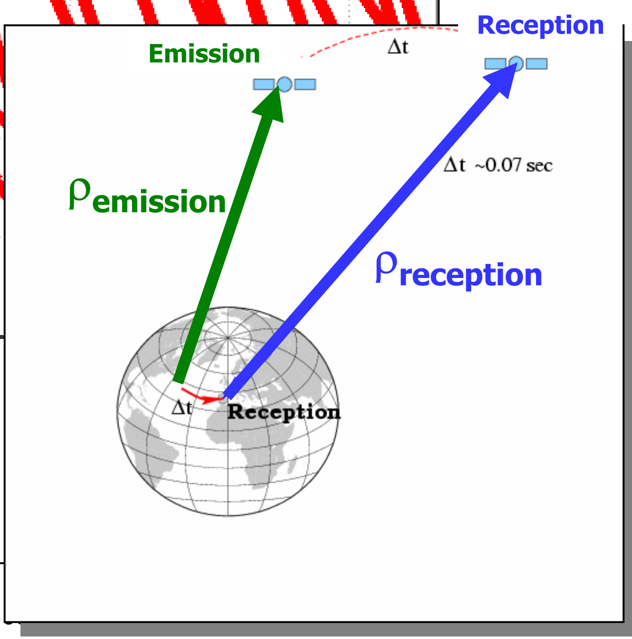
$$T[ems] = t^{ems}(T^S) - dt^S = t_{rec}(T_R) - (C1/c + dt^S)$$

Distance: Δr

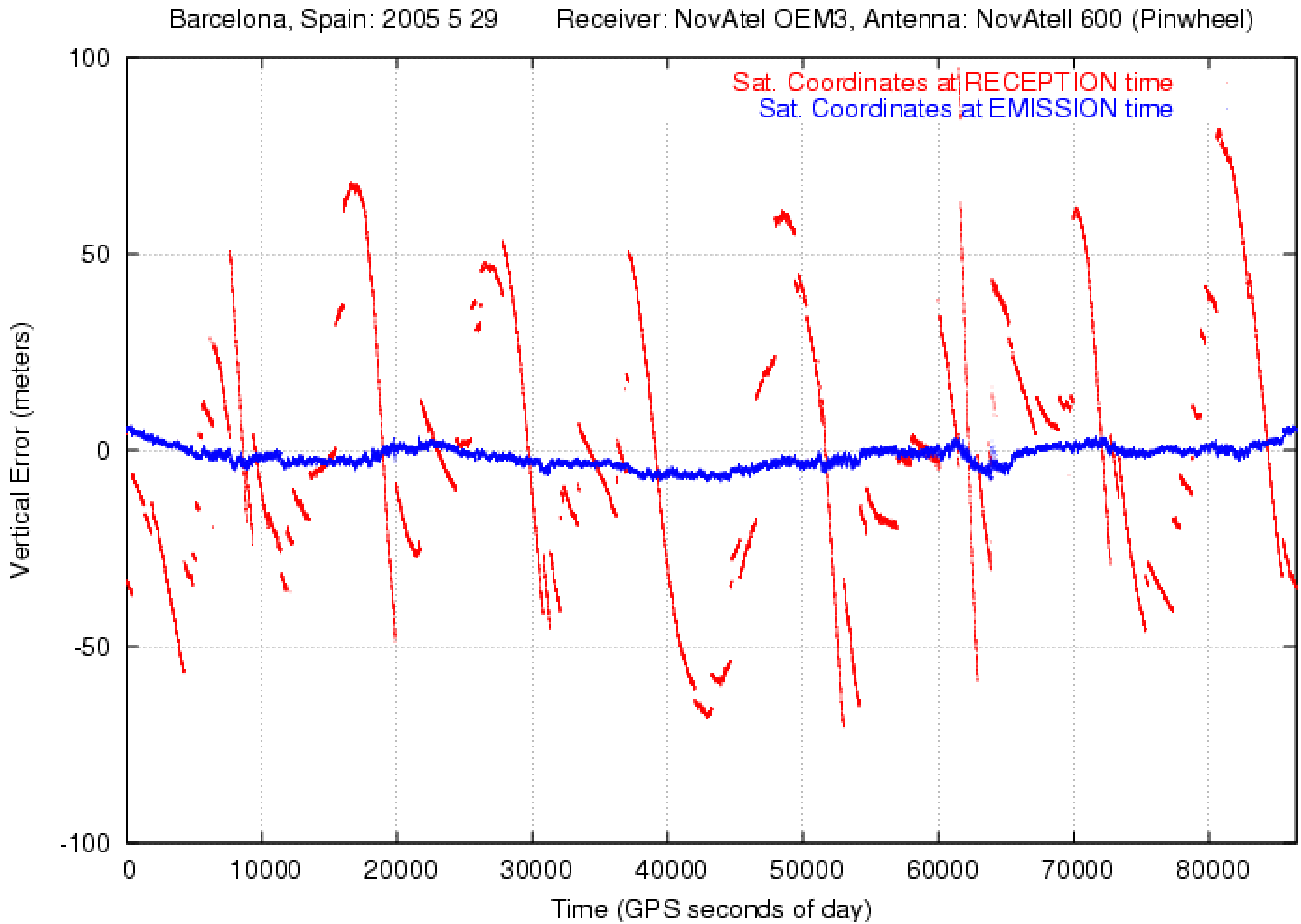


Variation in range: $\Delta\rho = \rho_{emission} - \rho_{reception}$

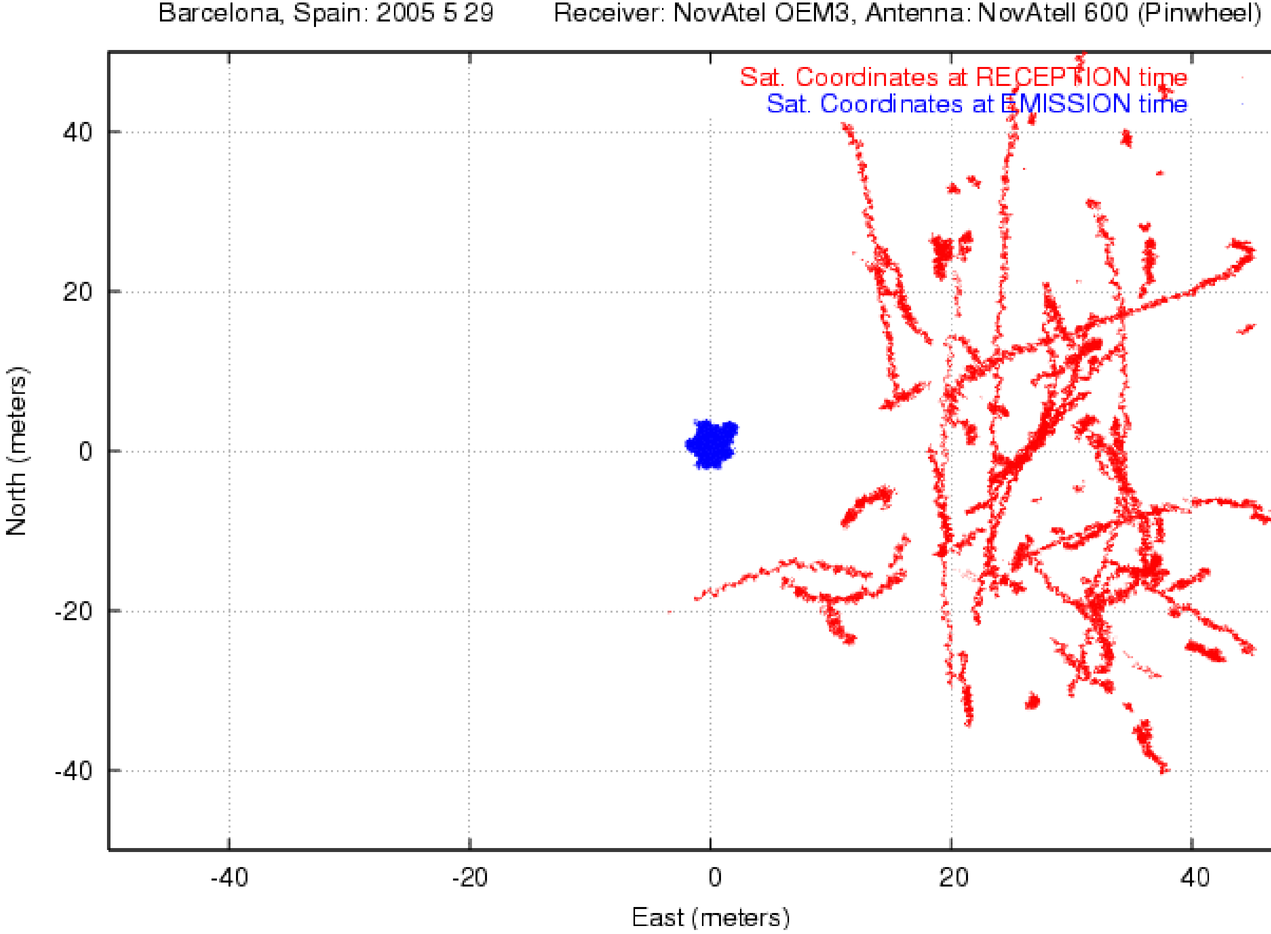




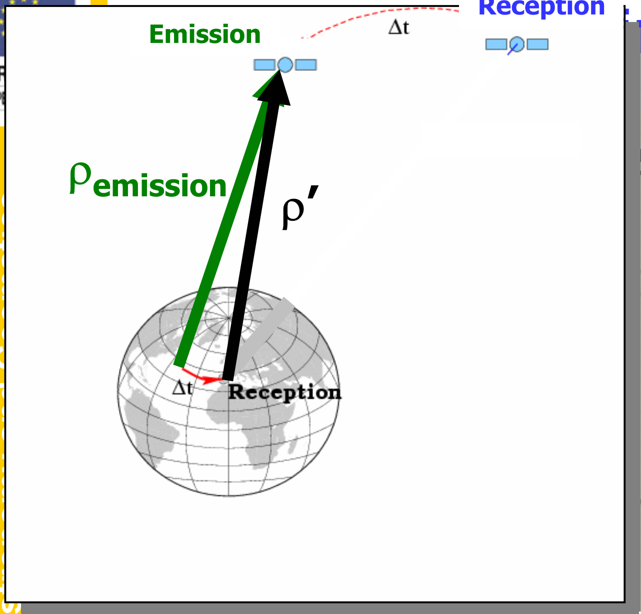
Vertical error comparison



Horizontal error comparison



...nates computation **at emission time**



...ed by the GPS/SPS-SS (**orbit.f**) supplies satellite ... Earth-Fixed reference frame. To compute the ...es

See **rec2ems.f**

... the following algorithm can be applied:

...e-tags, compute emission time in GPS system

$$T[ems] = t_{rec}(T_R) - (C1/c + \boldsymbol{dt^S})$$

2. Compute satellite coordinates at emission time *T[ems]*

$$T[ems] \rightarrow \textbf{[orbit]} \rightarrow (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$$

3. Account for Earth rotation during traveling time from emission to reception "$\boldsymbol{\Delta t}$" *(CTS reference system at reception time is used to build the navigation equations).*

$$(X^{sat}, Y^{sat}, Z^{sat})_{CTS[reception]} = R_3(\omega_E\, \boldsymbol{\Delta t}) . (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$$

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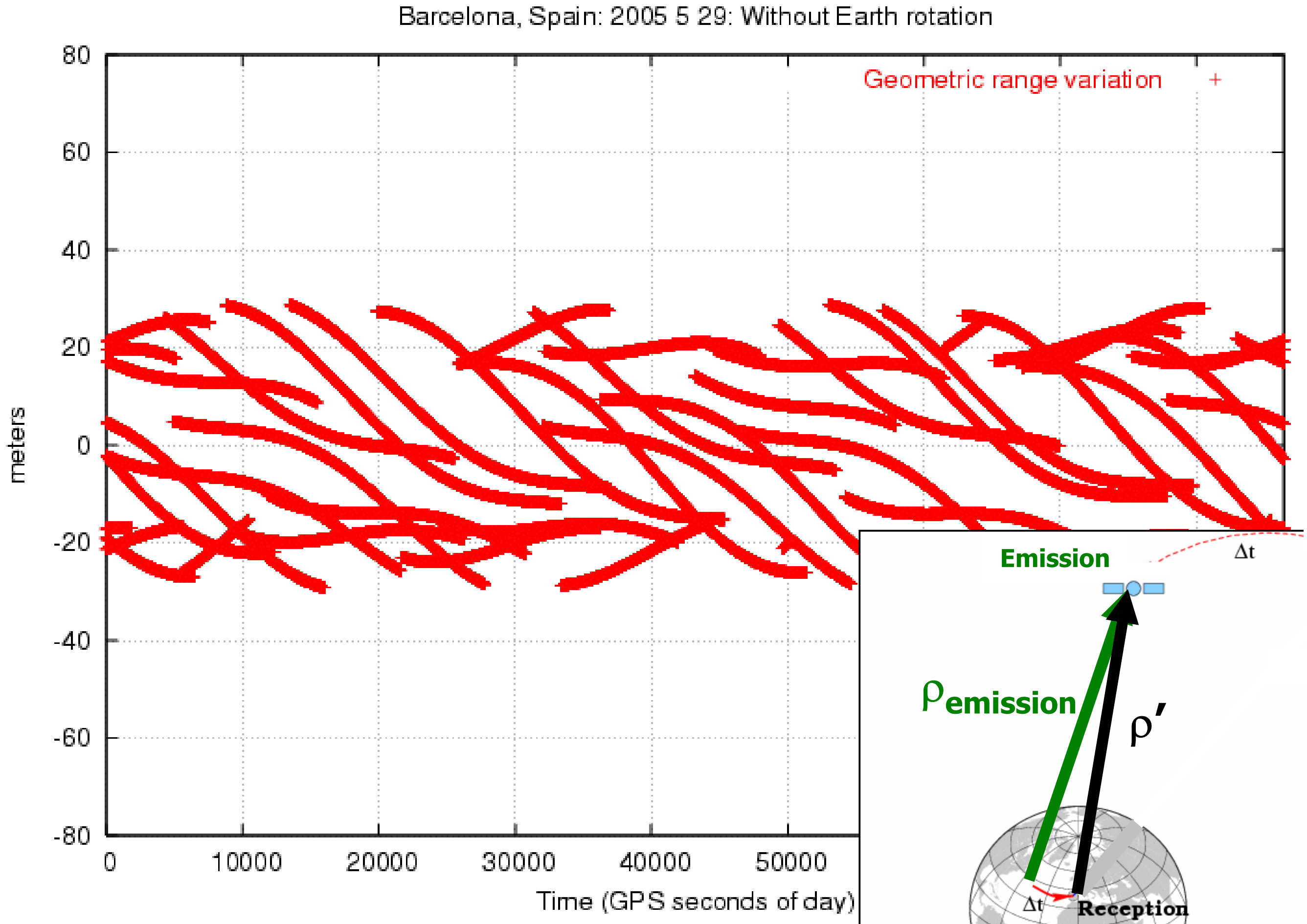
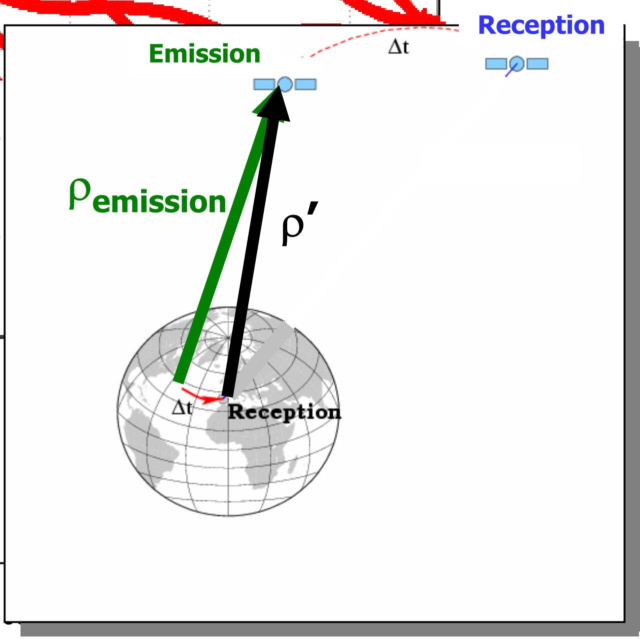
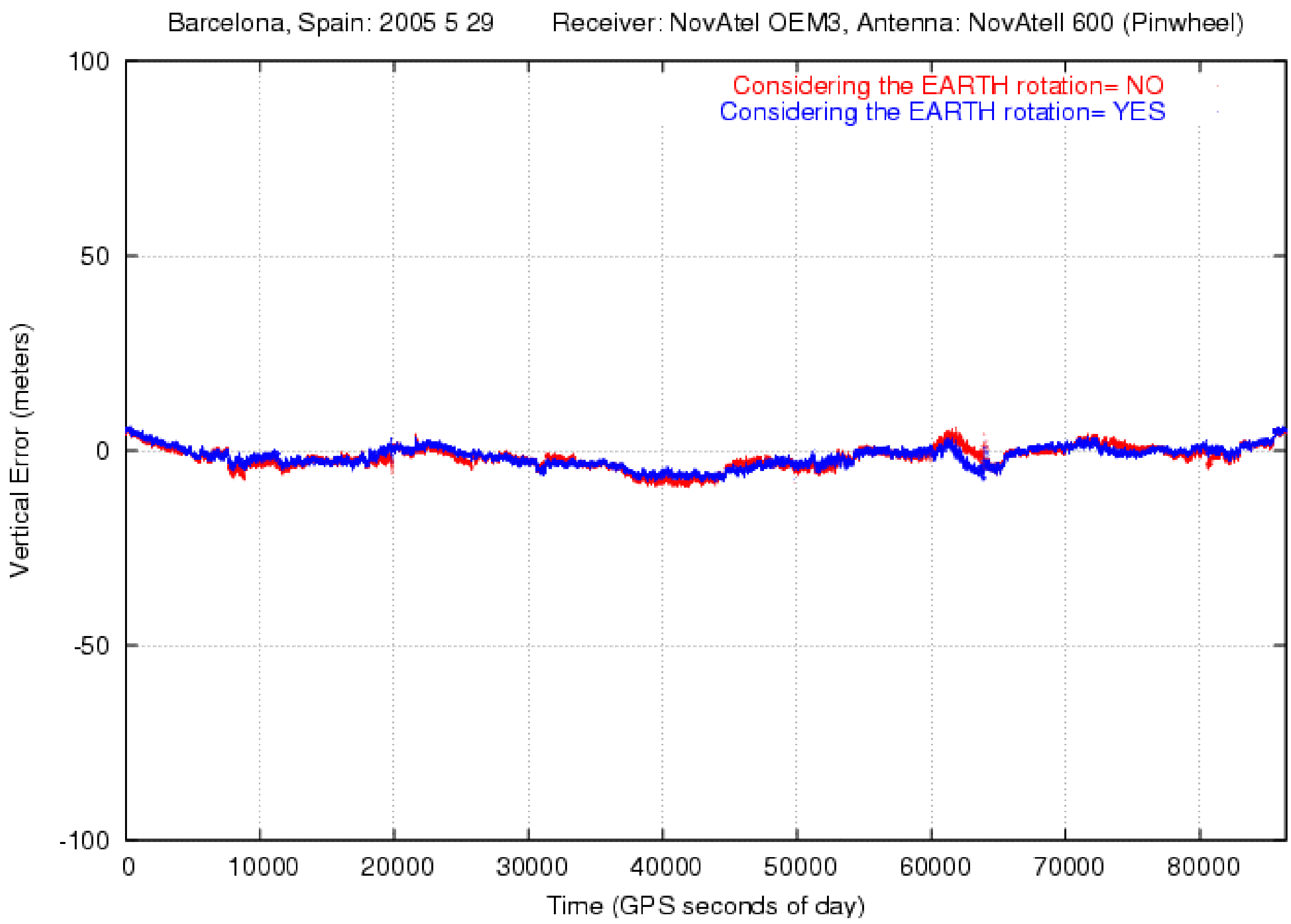
Variation in range: $\Delta\rho = \rho' - \rho_{emission}$

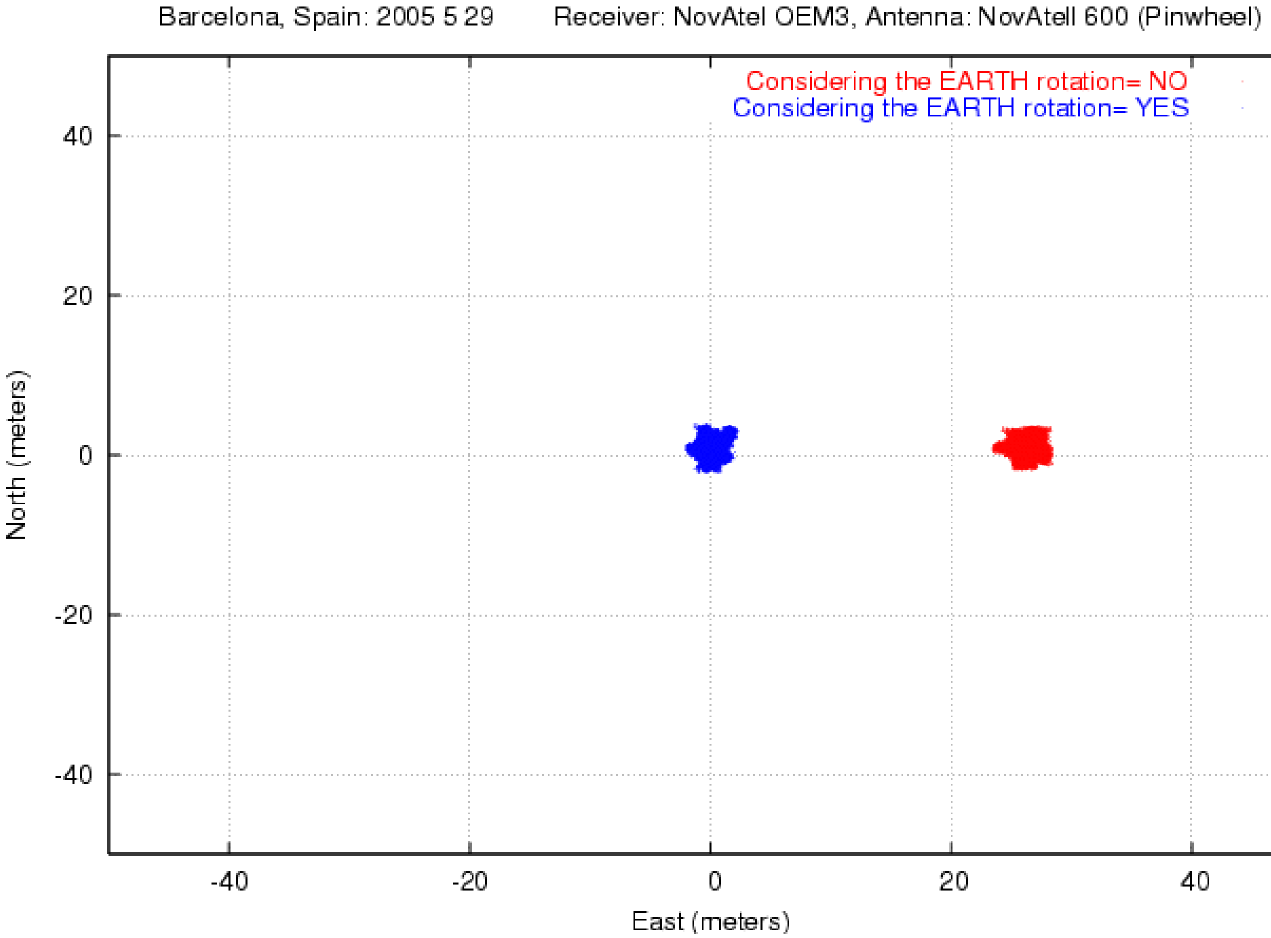




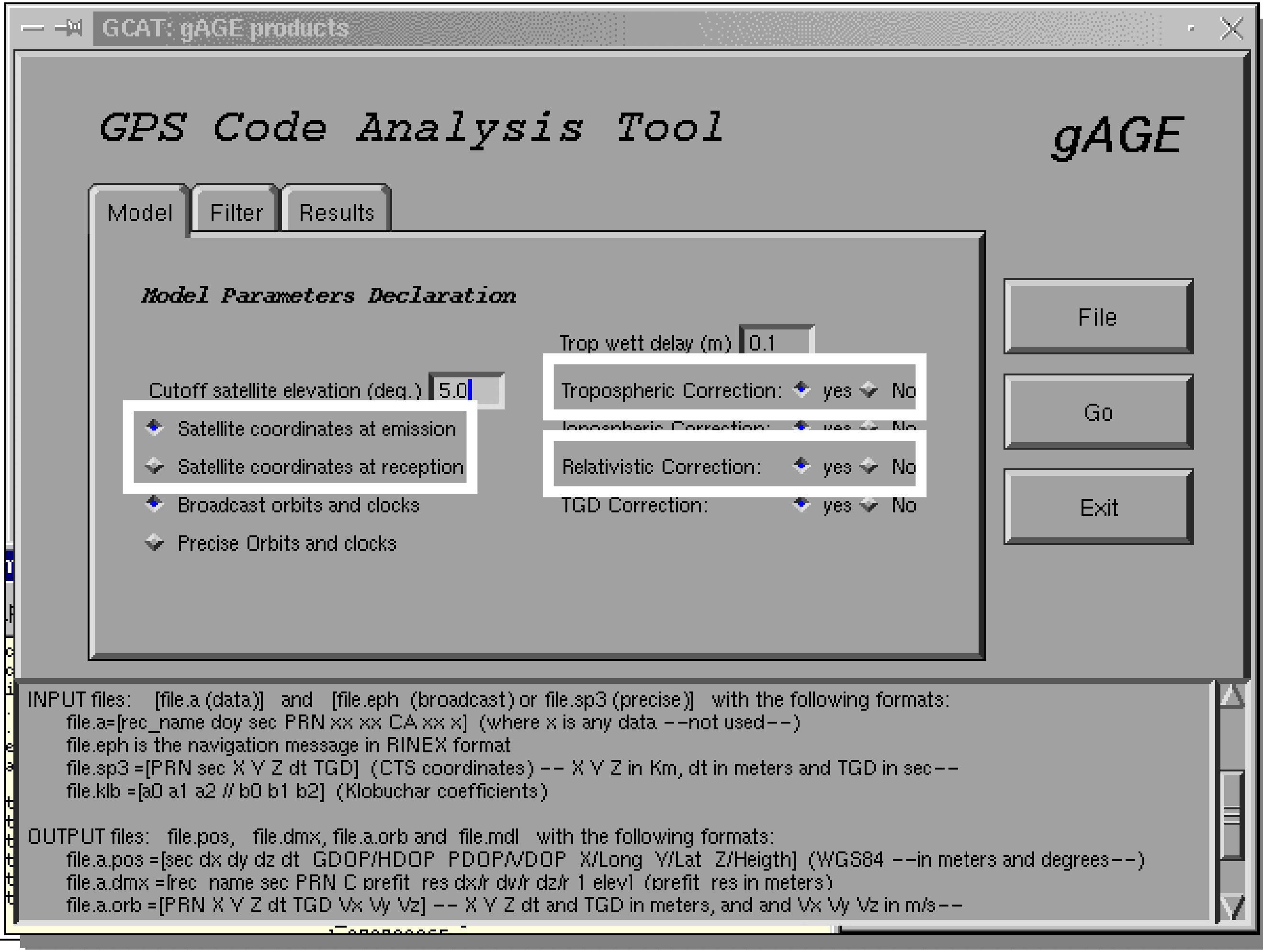
Vertical error comparison



Horizontal error comparison



Exercise 6: Using the GCAT program, compute satellite coordinates at emission time and at reception time. Plot the module of the vector difference between both positions (use October 13th, 1998 data files).



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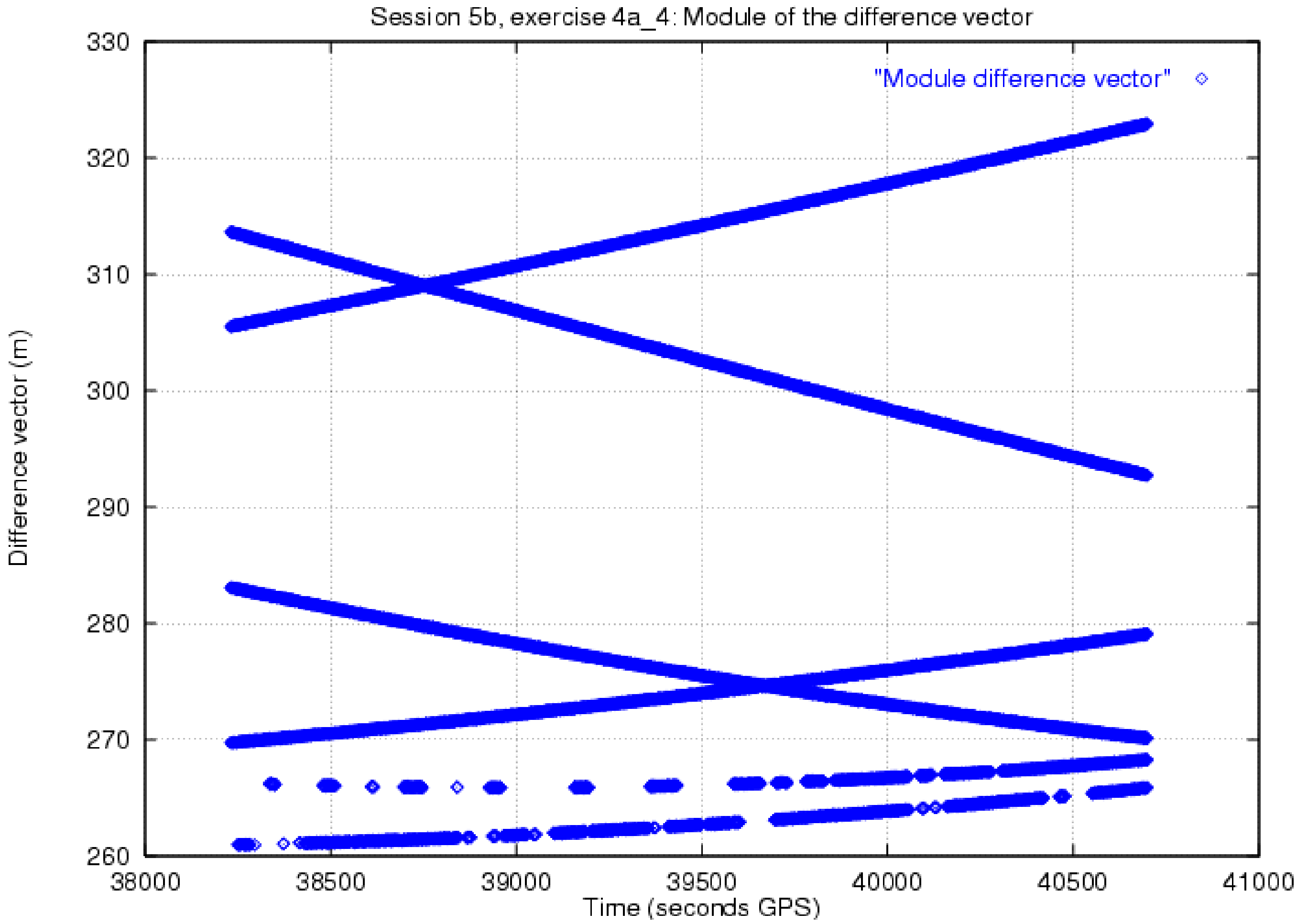

Satellite and receiver clock offsets

- They are time-offsets between satellite/receiver time and GPS system time (provided by the ground control segment):

- The receiver clock offset (dt_{rec}) is estimated together with receiver coordinates.

- Satellite clock offset (dt^{sat}) may be computed from navigation message:

$$dt^{sat} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2$$

$$C1_{1rec}^{sat} = \rho_{rec}^{sat} + c \cdot \boxed{(dt_{rec} - dt^{sat})} + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + K_1^{sat} + \varepsilon$$

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$$dt^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2$$

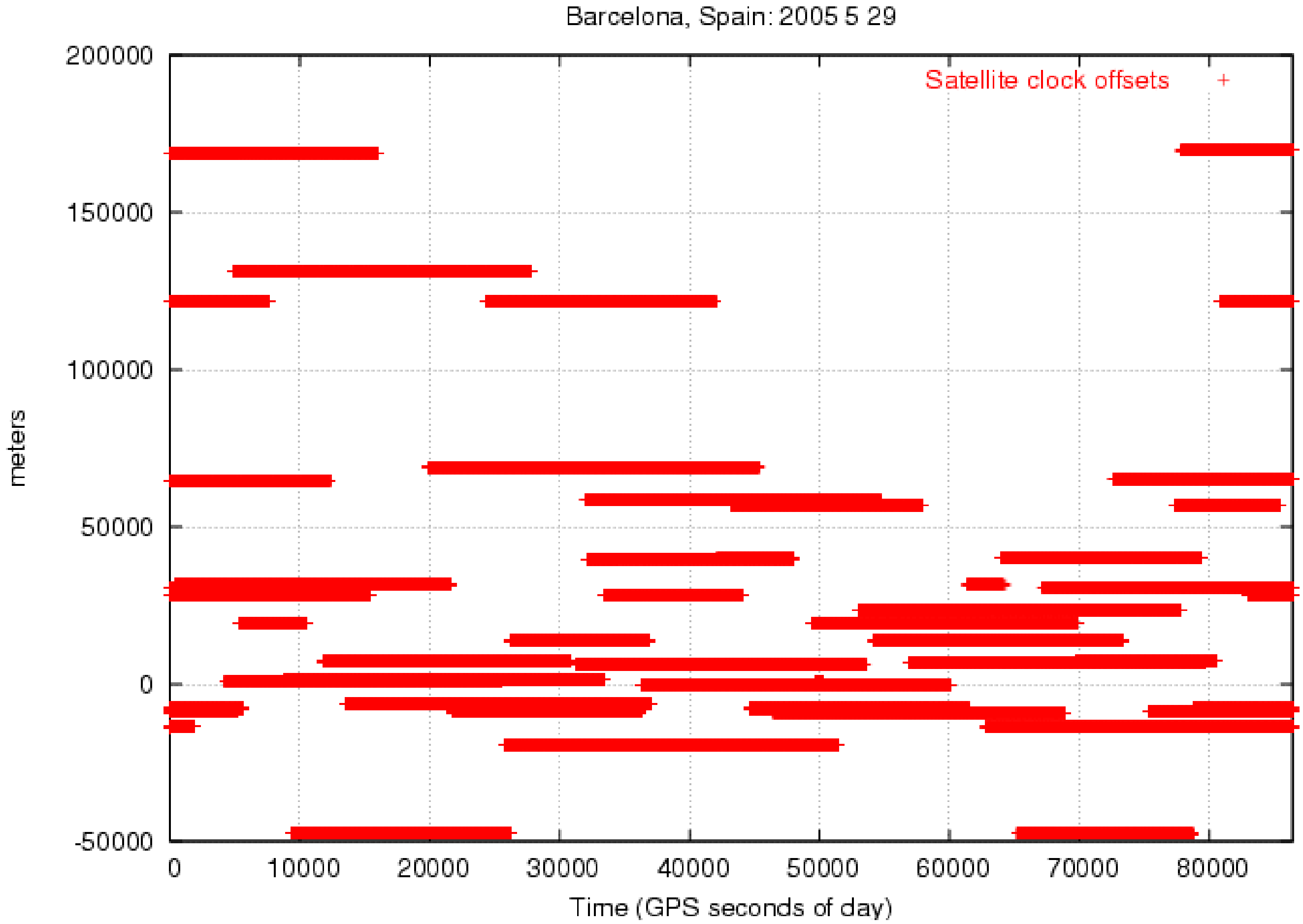
t0 a0 a1 a2

YY MM DD H M S

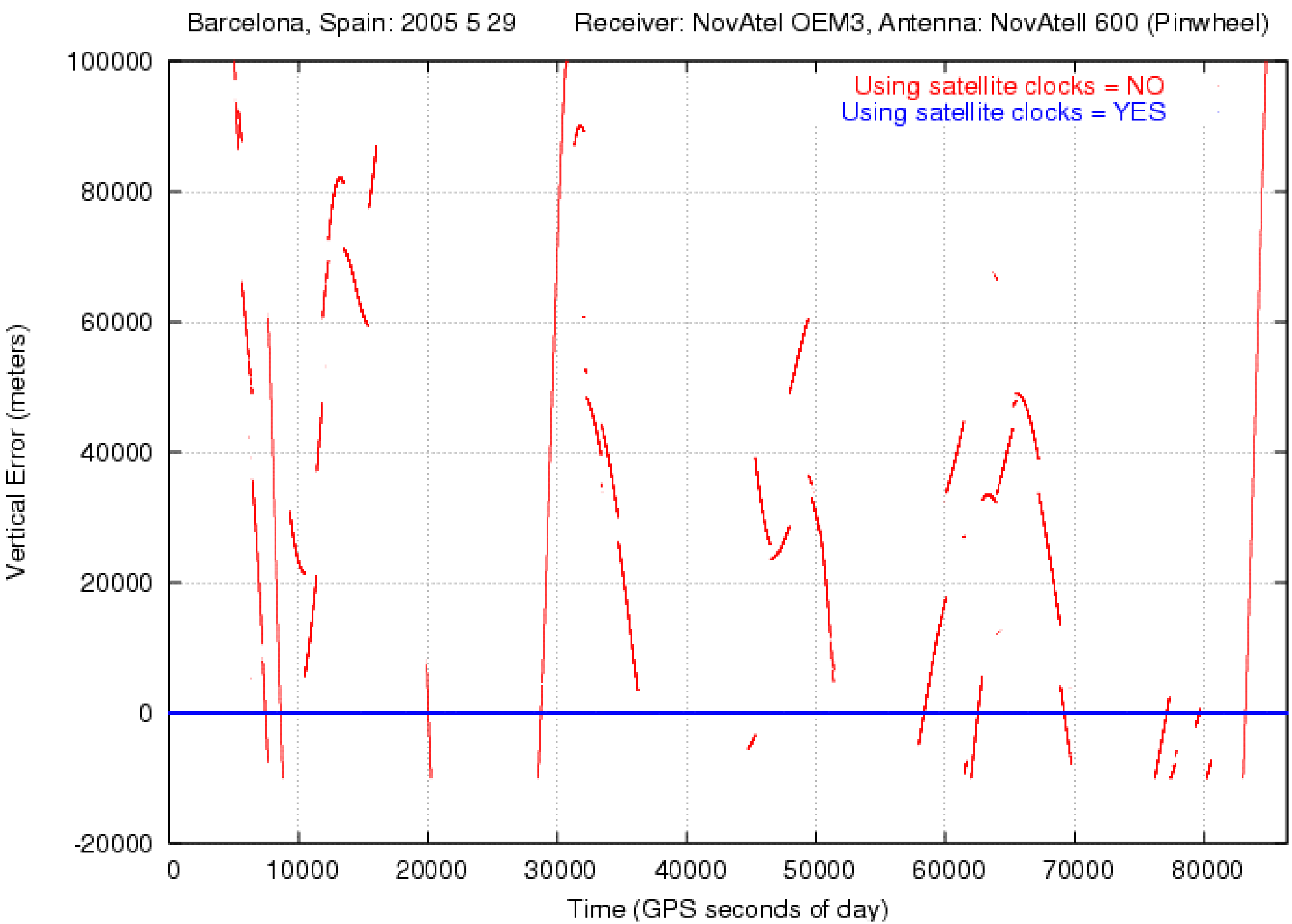
PRN

```
 2              NAVIGATION DATA     GPS                 RINEX VERSION / TYPE
srx/v1.8.1.4        BAI             95/10/19 03:18:35   PGM / RUN BY / DATE
CASA                                                    COMMENT
 -2444431.2031 -4428688.6270  3875750.1442              COMMENT
                                                        END OF HEADER
14 95 10 18 00 51 44.0 1.129414886236D-05 1.136868377216D-13 0.000000000000D+00
    1.730000000000D+02-5.175000000000D+01 4.375182243902D-09-5.836427291652D-01
   -2.712011337280D-06 2.427505562082D-03 8.568167686462D-06 5.153718931198D+03
    2.623040000000D+05 4.470348358154D-08 1.698435481558D+00 1.676380634308D-08
    9.636381916043D-01 2.153437500000D+02 3.056960010495D+00-8.030691653399D-09
   -5.178787145843D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.730000000000D+02
    2.592180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

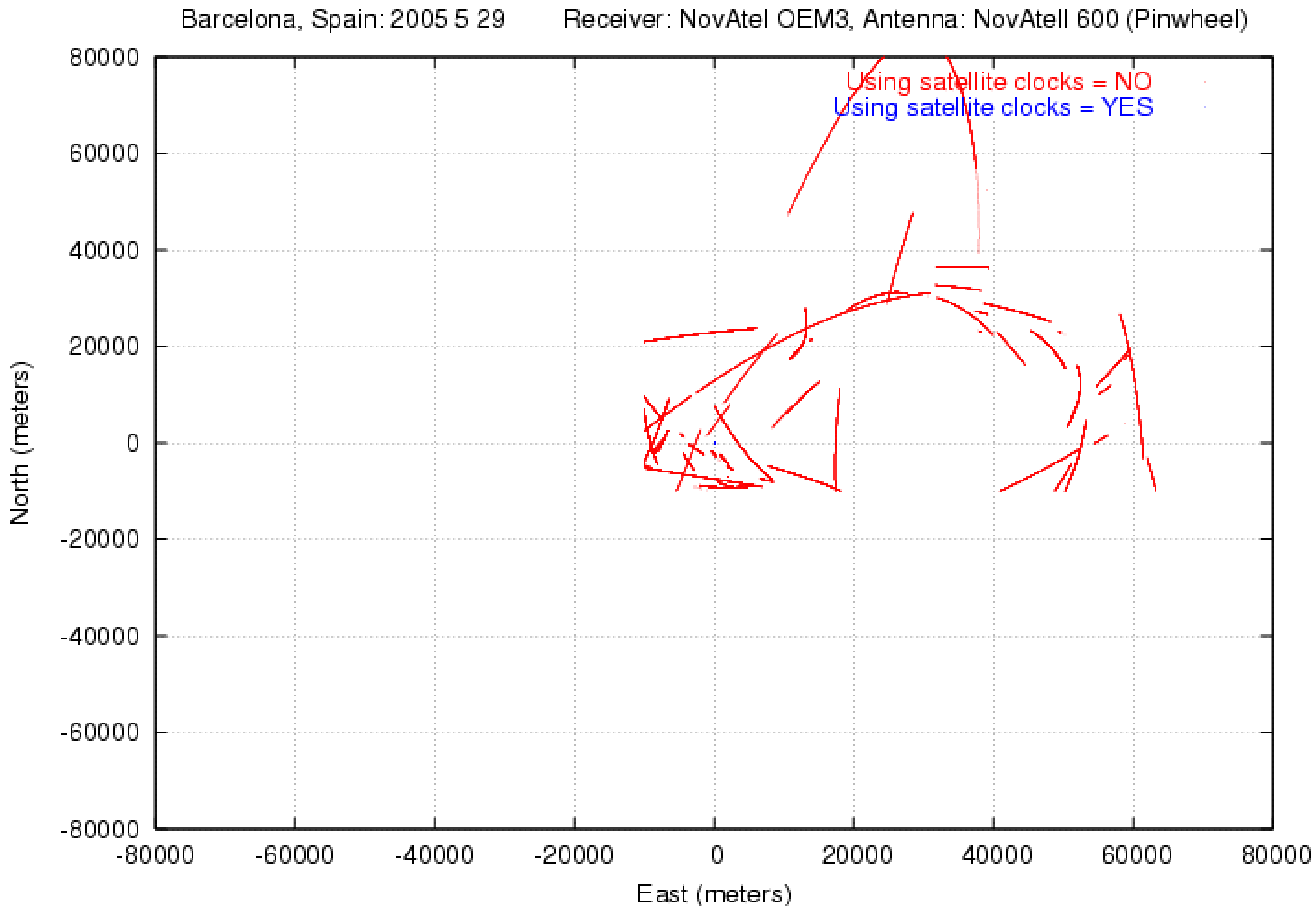
Range variation: satellite clocks



Vertical error comparison



Horizontal error comparison



Relativistic correction (rel_i^j)

- A constant component depending only on nominal value of satellite's orbit major semi-axis, being corrected modifying satellite's clock oscillator frequency*:

$$\frac{f_0' - f_0}{f_0} = \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{\Delta U}{c^2} \simeq -4.464 \cdot 10^{-10}$$

- A periodic component due to orbit eccentricity (to be corrected by user receiver):

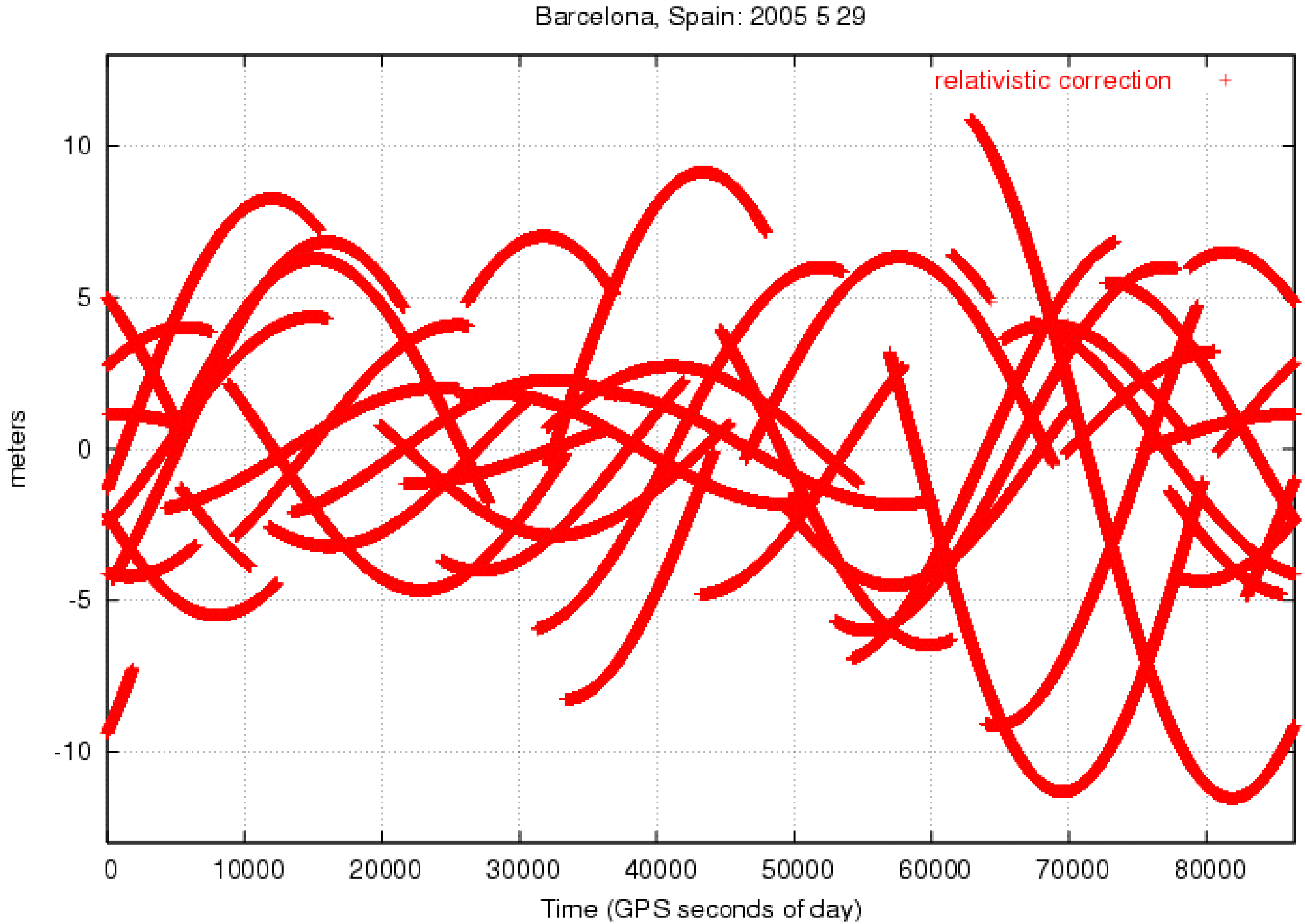
$$rel = 2\frac{\sqrt{\mu a}}{c} e \sin(E) = 2\frac{\mathbf{r}\cdot\mathbf{v}}{c} \ (meters)$$

Being μ=*3.986005 10^{14} (m^3/s^2)* universal gravity constant, *c =299792458 (m/s)* light speed in vacuum, *a* is orbit's major semi-axis, *e* is its eccentricity, *E* is satellite's eccentric anomaly, and *r* and *v* are satellite's geocentric position and speed in an inertial system.

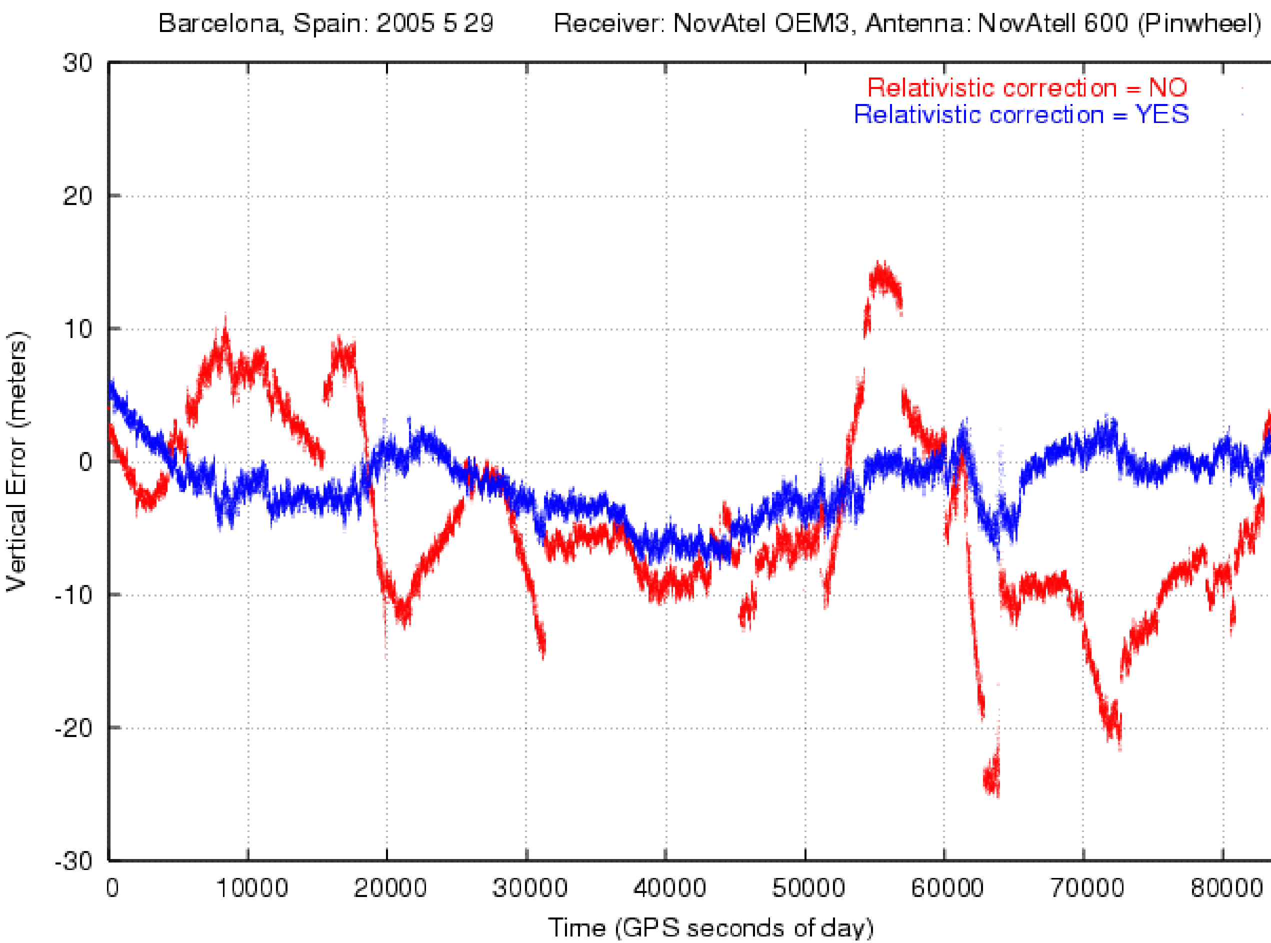
being f_0 = 10.23 MHz, we have *Δf=4.464 10^{-10} f_0= 4.57 10^{-3} Hz

so satellite should use *f'o=10.22999999543 MHz*.

Range variation: relativistic correction



Vertical error comparison



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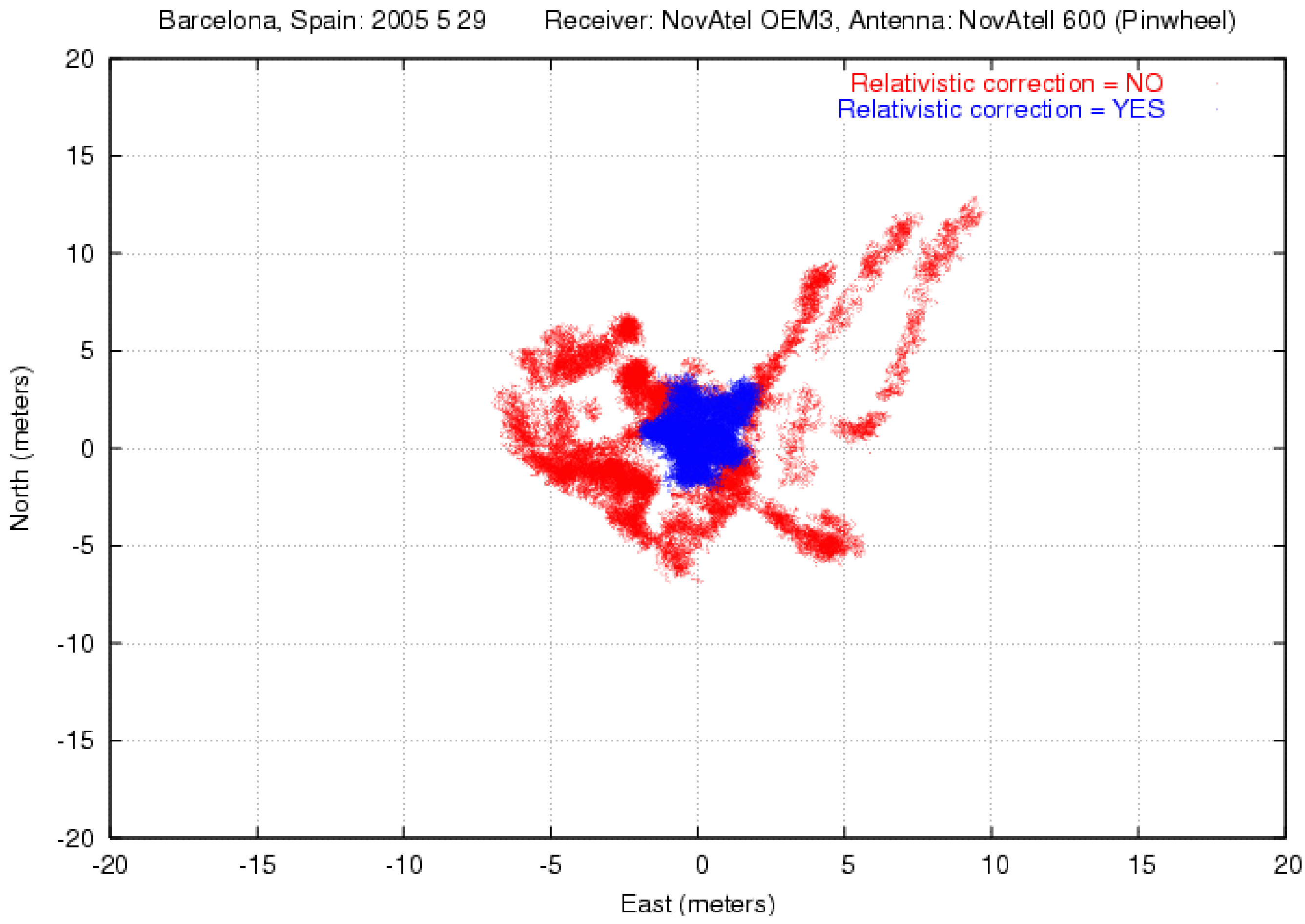
EUROPEAID CO-OPERATION OFFICE

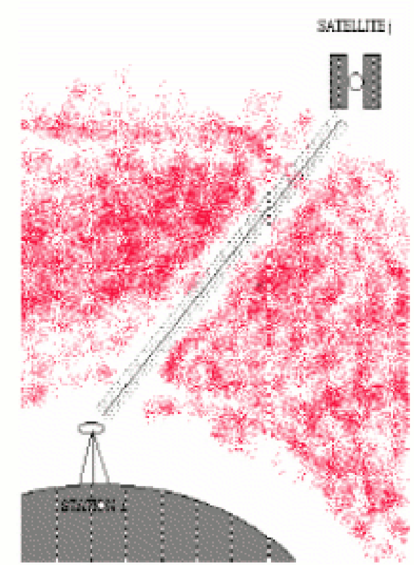
Horizontal error comparison



Ionospheric Delay αI_i^j

As a first approach, ionospheric delay depends on frequency as given by:

$$\delta_{ion} = \frac{40.3}{f^2} I$$



Where I is number of electrons per area unit in the direction of observation, or STEC (*Slant Total Electron Content*)

$$I = \int N_e ds$$

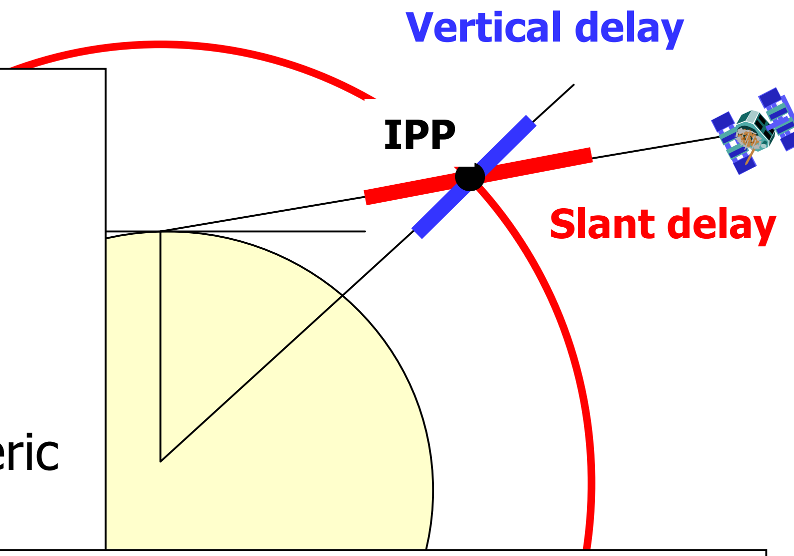
- For two-frequency receivers, it may be cancelled (99.9%) using ionosphere-free combination

$$LC = \frac{f_1^2 L1 - f_2^2 L2}{f_1^2 - f_2^2}$$

- For one-frequency receivers, it may be corrected (about 60%) using Klobuchar model (defined in GPS/SPS-SS), whose parameters are sent in navigation message. (See program klob.f)

$$C1_{1rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + Trop_{rec}^{sat} + \boxed{Ion_{1rec}^{sat}} + K_{1rec} + K_1^{sat} + \varepsilon$$

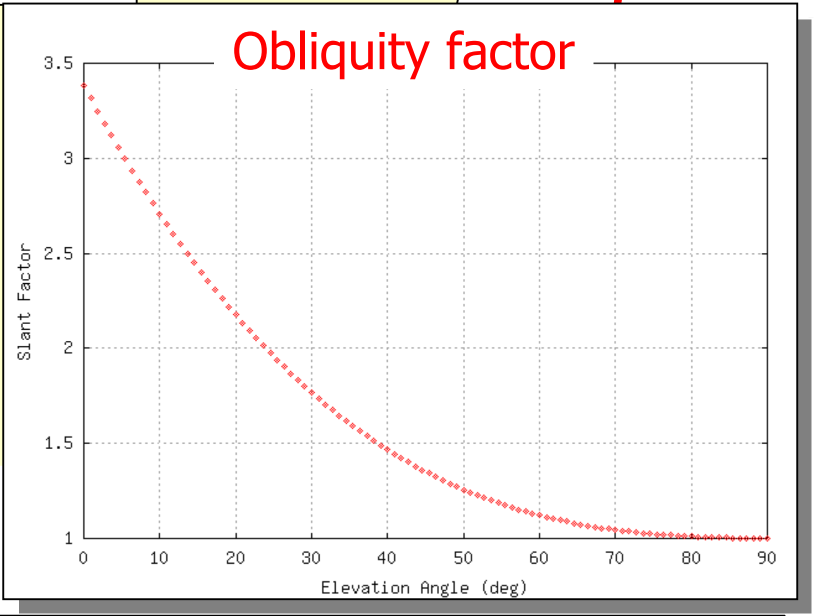
Klobuchar model (klob.f)



It was designed to minimize user computational complexity.

- Minimum user computer storage
- Minimum number of coefficients transmitted on satellite-user link
- At least 50% overall RMS ionospheric error reduction worldwide.

- It is assumed that the electron content is concentrated in a thin layer at 350 Km in height.
- The slant delay is computed from the vertical delay at the ionospheric Pierce Point (IPP), multiplying by the obliquity factor.



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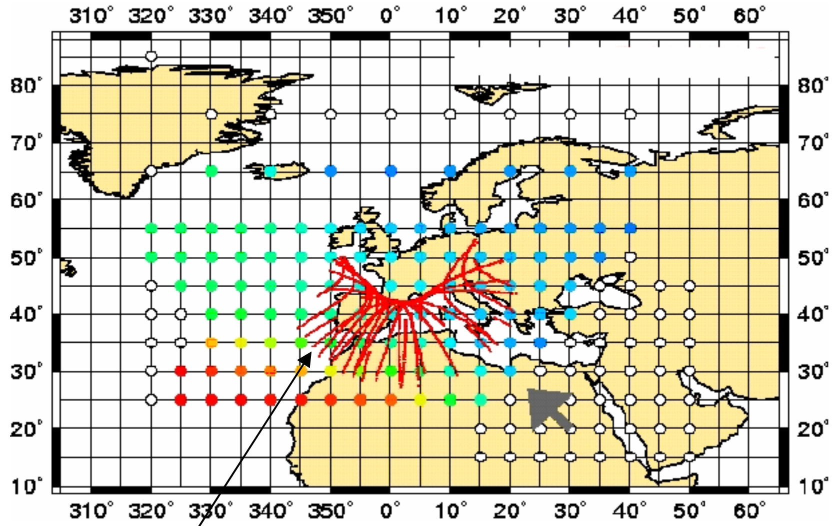
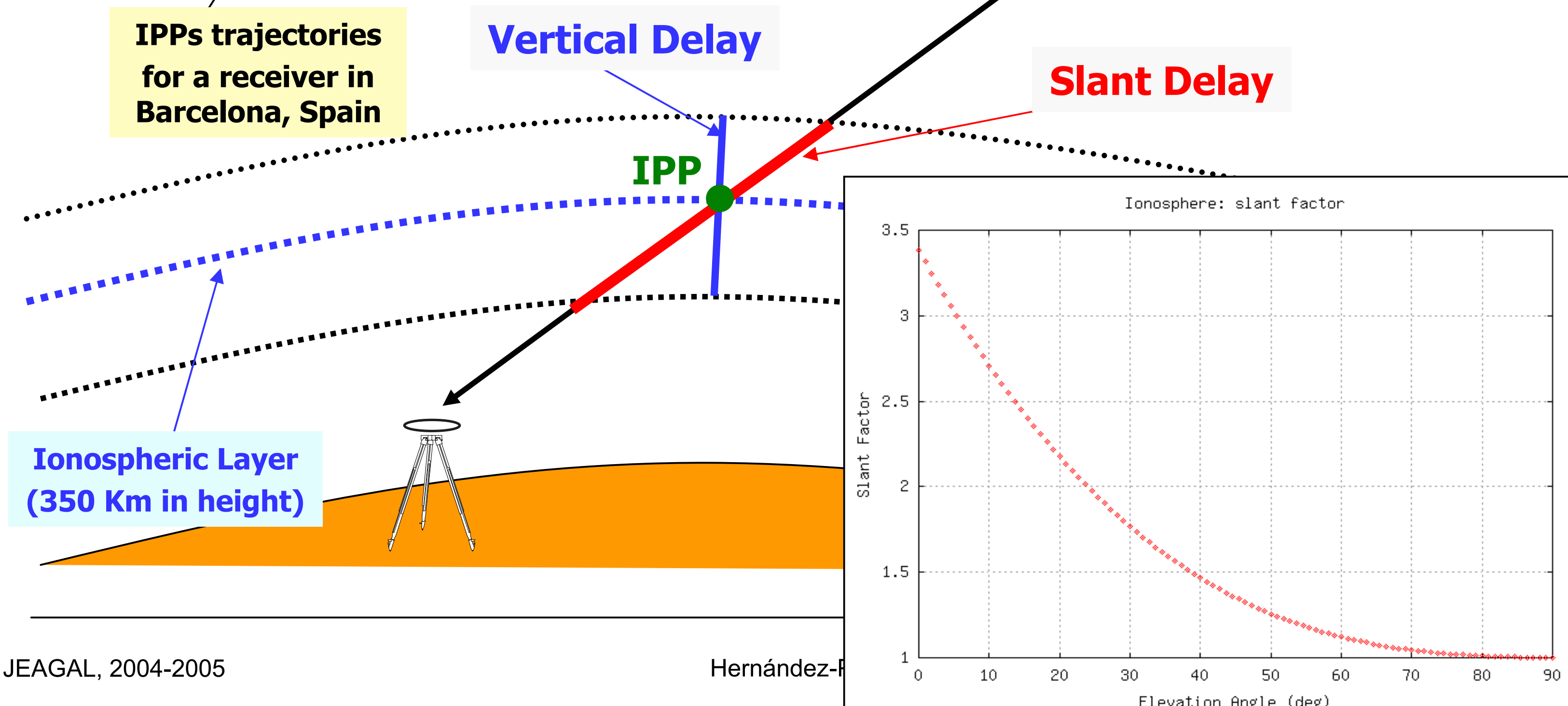
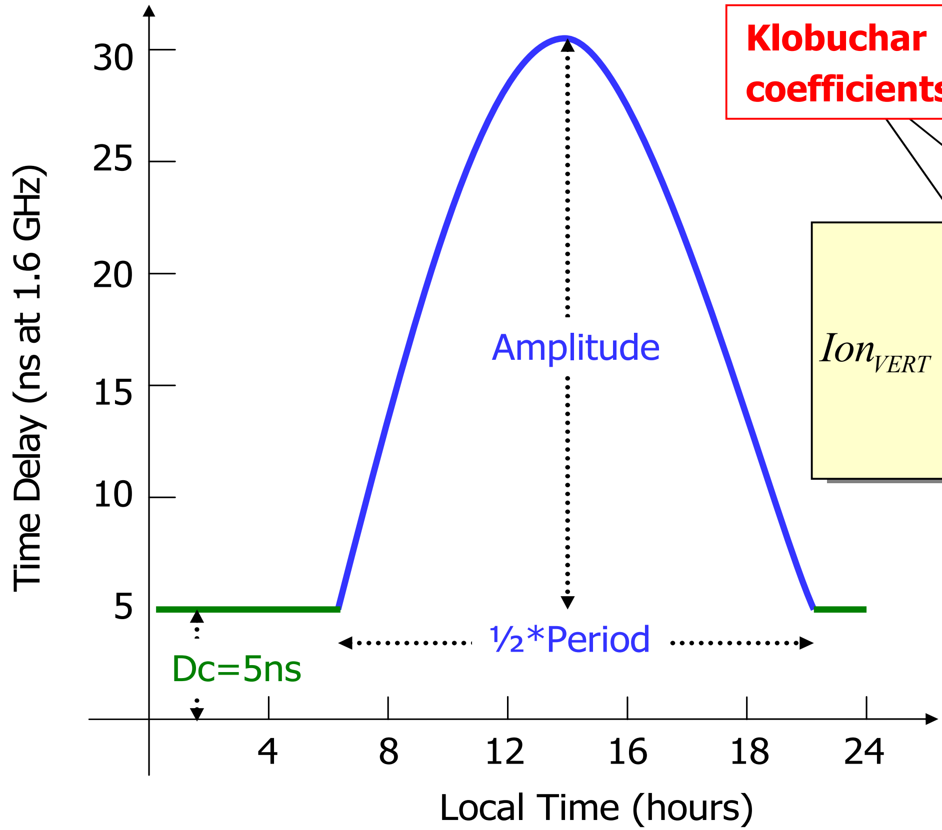
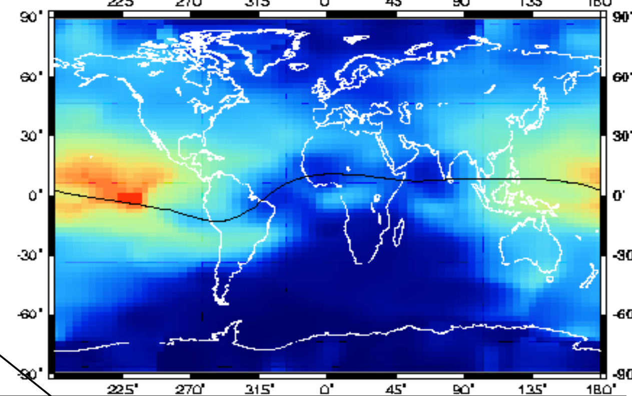
IONOSPHERIC PIERCE POINTS (IPP)





Klobuchar model





Klobuchar coefficients

$$Ion_{VERT} = \begin{cases} DC + A\cos\left[\dfrac{2\pi(t-\Phi)}{P}\right] & (day) \\ DC \quad ; \; if \left[\dfrac{2\pi(t-\Phi)}{P}\right] > \dfrac{\pi}{2} & (night) \end{cases}$$

Being :

$$A = \sum_{n=0}^{3} \alpha_n \varphi^n \quad ; \quad P = \sum_{n=0}^{3} \beta_n \varphi^n$$

φ = Geomagnetic Latitude

$$Ion_{SLANT} = Ion_{VERT} \; m(elev)$$

$$m(elev) = 1 + 16\left(0.53 - elev/\pi\right)^3$$

Where:

DC= 5ns

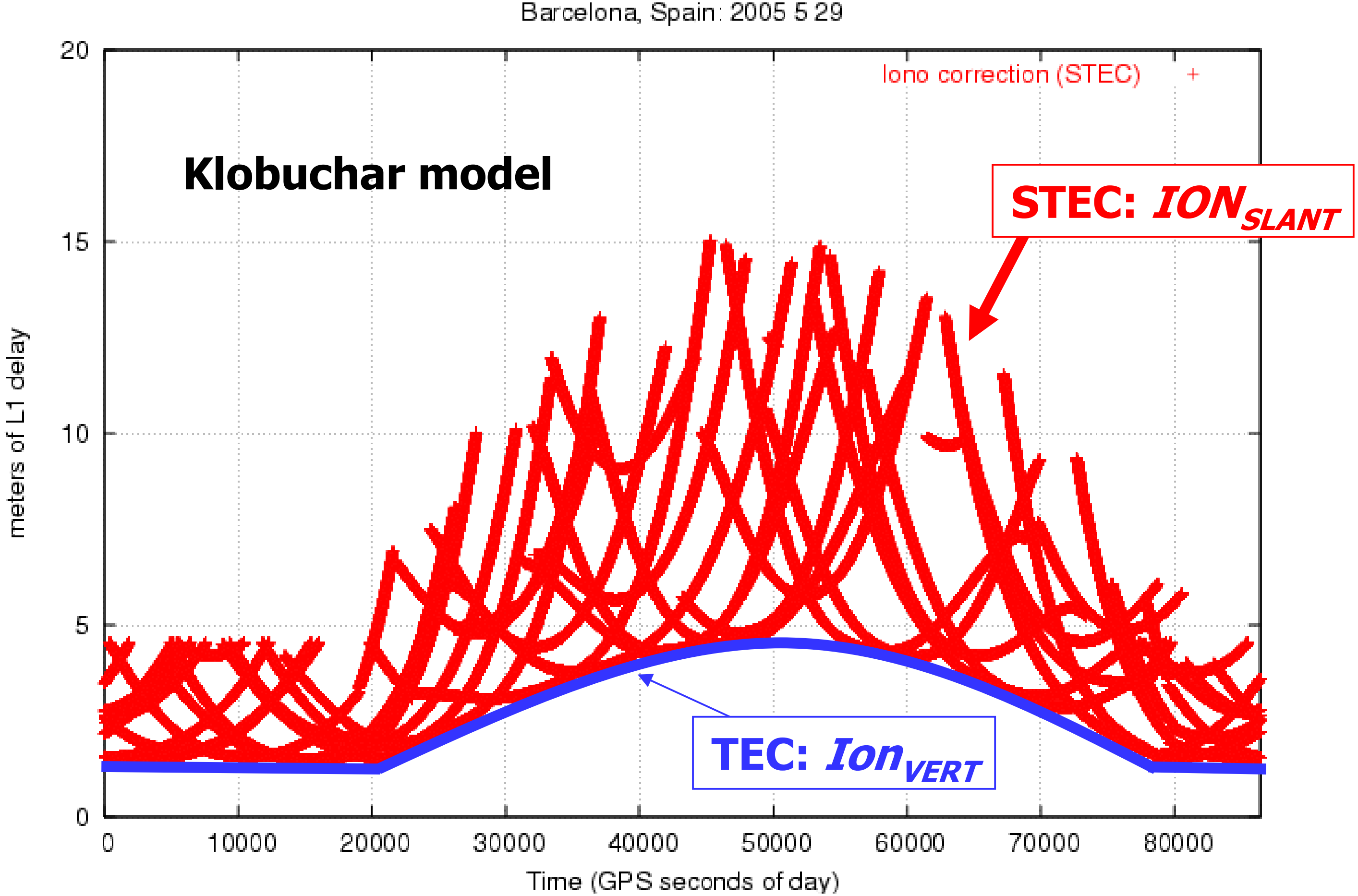
Φ= 14 **(ctt. phase offset)**

t = Local Time

$$(\text{time}, \underbrace{r_{sta}, r^{sat}}_{elev,\ \phi}, \alpha 0, \alpha 1, \alpha 2, \alpha 3, \beta 0, \beta 1, \beta 2, \beta 3) \rightarrow [\text{Klob}] \rightarrow \text{Iono}$$

```
     2              NAVIGATION DATA                         RINEX VERSION / TYPE
CCRINEXN V1.5.2 UX  CDDIS               24-MAR- 0 00:23     PGM / RUN BY / DATE
IGS BROADCAST EPHEMERIS FILE                                COMMENT
    0.3167D-07  0.4051D-07 -0.2347D-06  0.1732D-06          ION ALPHA
   -0.2842D+05 -0.2150D+05 -0.1096D+06  0.4301D+06          ION BETA
   -0.121071934700D-07-0.488498130835D-13   319488     1002 DELTA-UTC: A0,A1,T,W
    13                                                      LEAP SECONDS
                                                            END OF HEADER
 1 99  3 23  0  0  0.0 0.783577561379D-04 0.113686837722D-11 0.000000000000D+00
    0.191000000000D+03-0.106250000000D+01 0.487163149444D-08-0.123716752769D+01
   -0.540167093277D-07 0.476544268895D-02 0.713579356670D-05 0.515433833885D+04
    0.172800000000D+06-0.260770320892D-07-0.850753478531D+00 0.763684511185D-07
    0.957259887797D+00 0.241437500000D+03-0.167990552187D+01-0.823998608564D-08
    0.174650132022D-09 0.100000000000D+01 0.100200000000D+04 0.000000000000D+00
    0.320000000000D+02 0.000000000000D+00 0.465661287308D-09 0.191000000000D+03
    0.172800000000D+06 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

Range variation: Ionospheric correction



$$Ion_{SLANT} = Ion_{VERT}\ m(elev)$$

$$m(elev) = 1 + 16\left(0.53 - elev/\pi\right)^3$$

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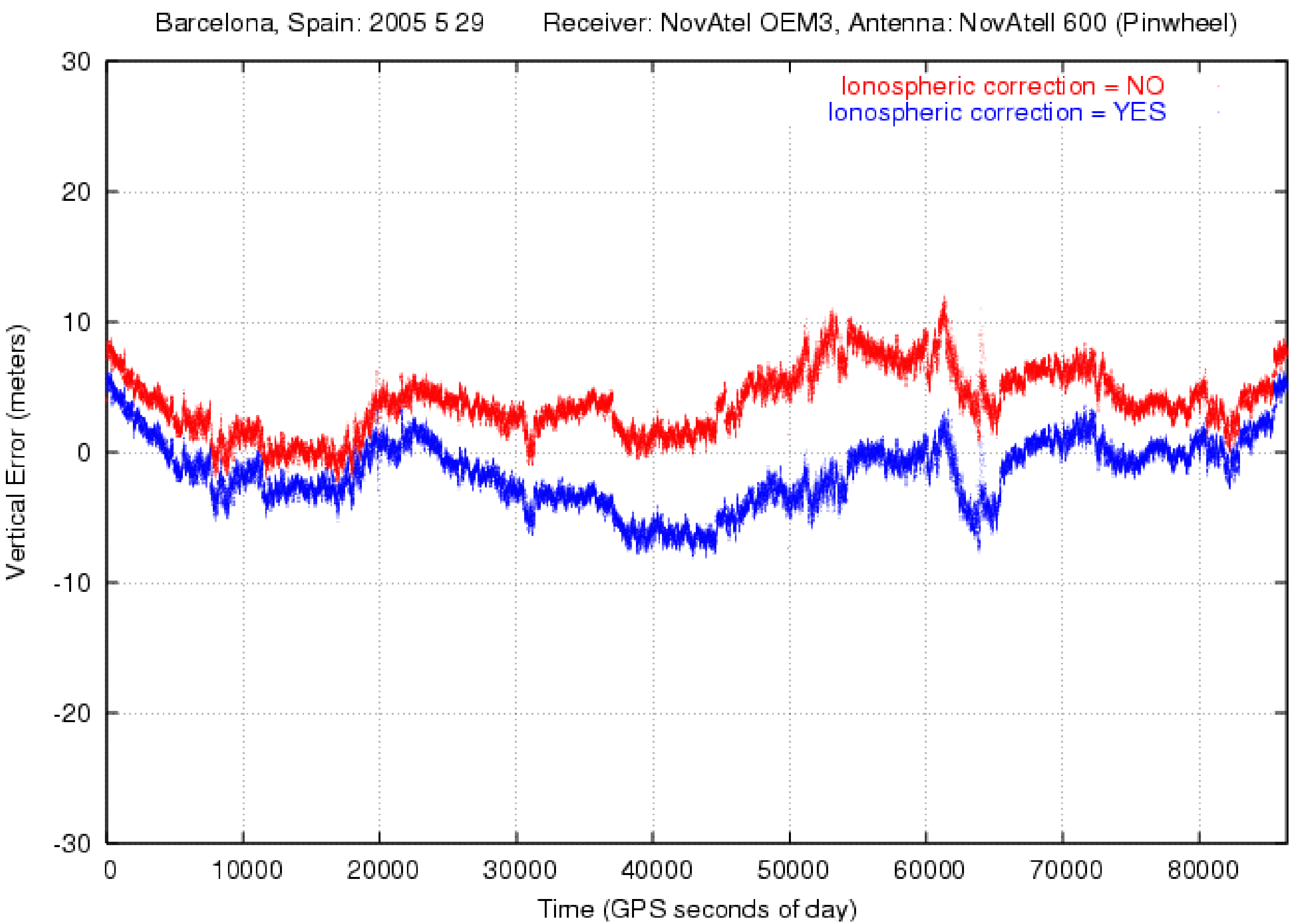
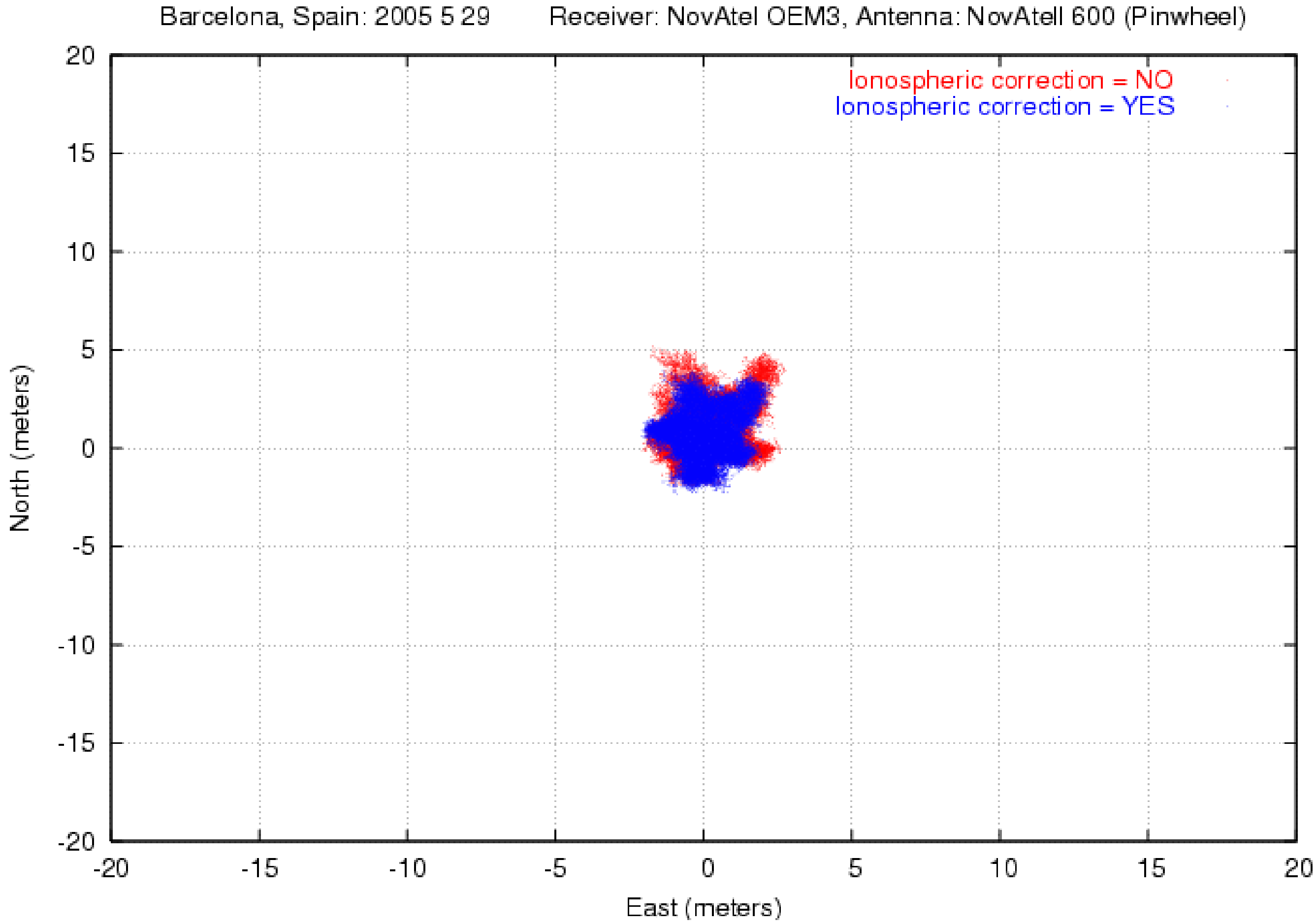
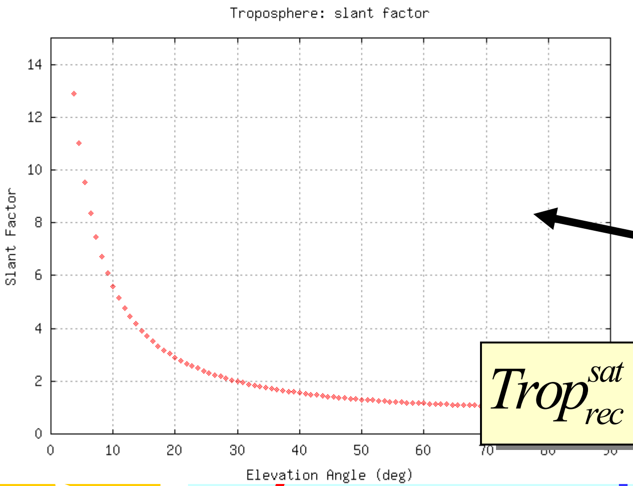
Vertical error comparison



Horizontal error comparison



Tropospheric Delay



end on frequency and affects both the code
es in the same way

led **(about 90%)** by:

$$Trop_{rec}^{sat} = (d_{dry} + d_{wet}) \cdot m(elev)$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

- ***d_{dry}*** corresponds to the vertical delay of the dry atmosphere (basically oxygen and nitrogen in hydrostatical equilibrium) ➔ It can be modeled as an **ideal gas**.
- ***d_{wet}*** corresponds to the vertical delay of the wet component (water vapor) ➔ **difficult to model.**

A simple model is:

$$d_{dry} = 2.3\exp(-0.116 \cdot 10^{-3} H) \quad meters$$
$$d_{wet} = 0.1m \quad ;[H : heigh]\, over\ the\ sea\ level$$

A more accurate model for *d_{dry} and d_{wet}* is provided for SBAS receivers in RTCA-Do229C. This model depends on the latitude and the day-of-year, being interpolated over a table of several meteorological parameters.

More sophisticated models uses two different mappings (for wet and dry)

$$C1_{1rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + \boxed{Trop_{rec}^{sat}} + Ion_{1rec}^{sat} + K_{1rec} + K_1^{sat} + \varepsilon$$

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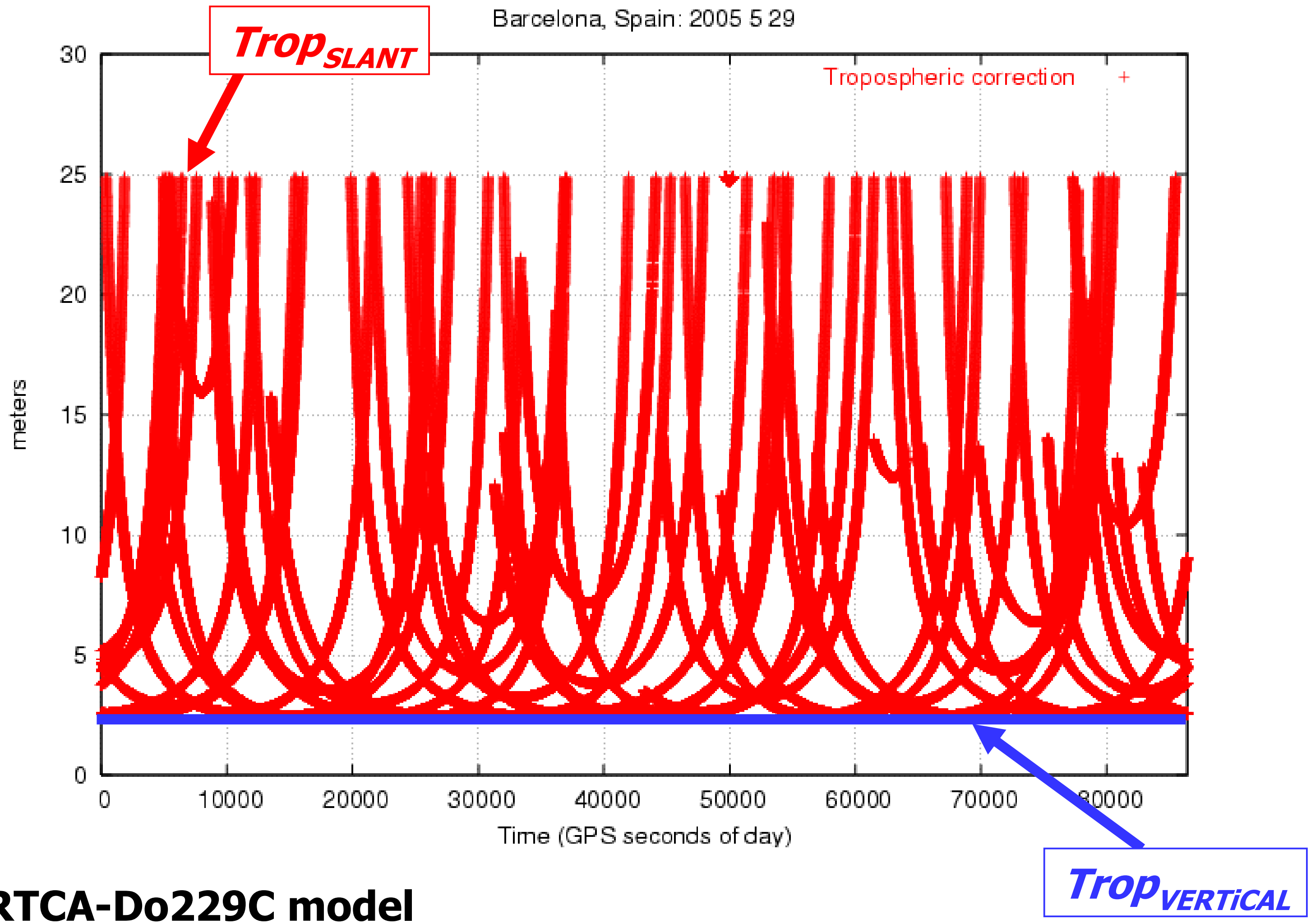
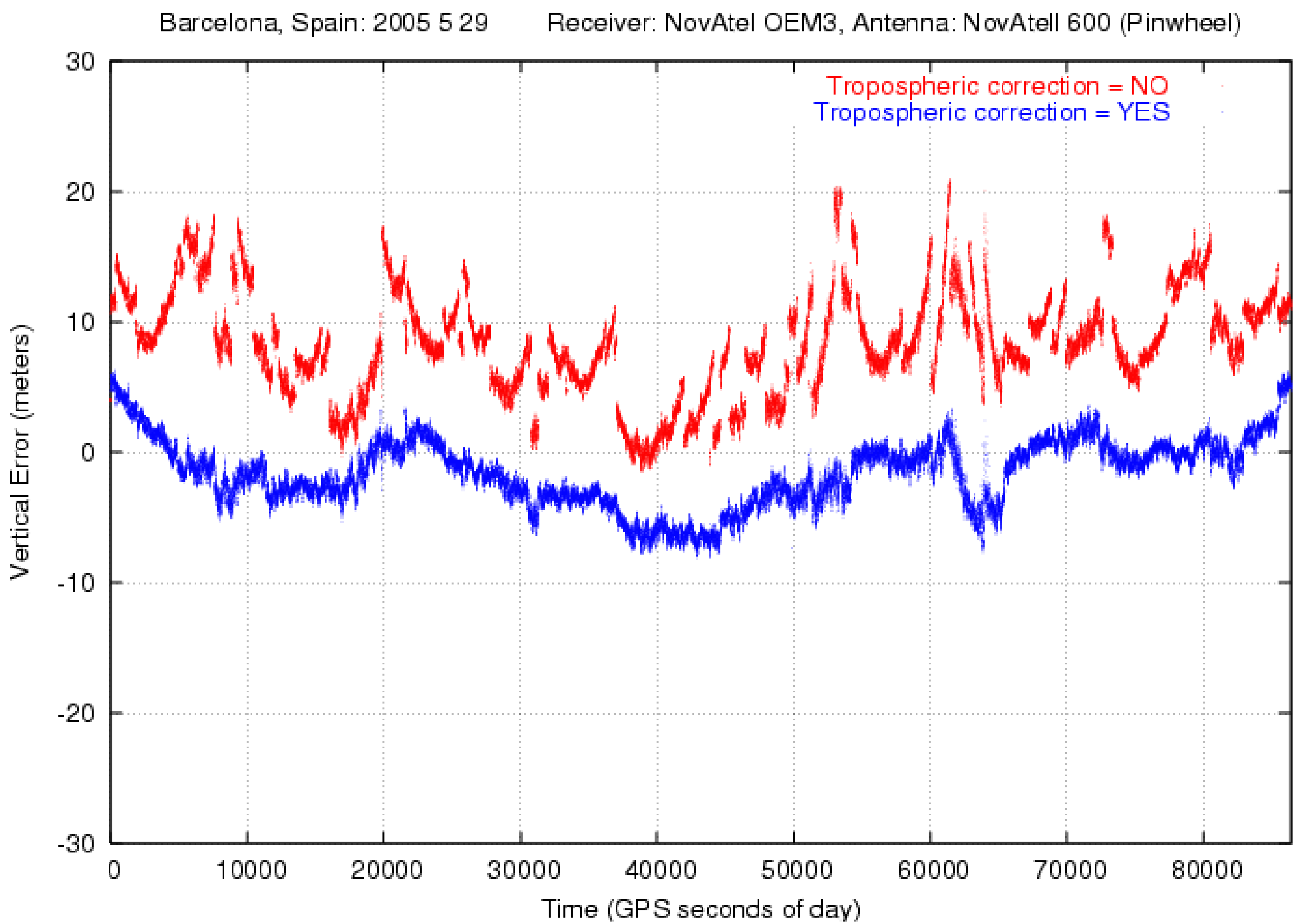
Range variation: Tropospheric correction

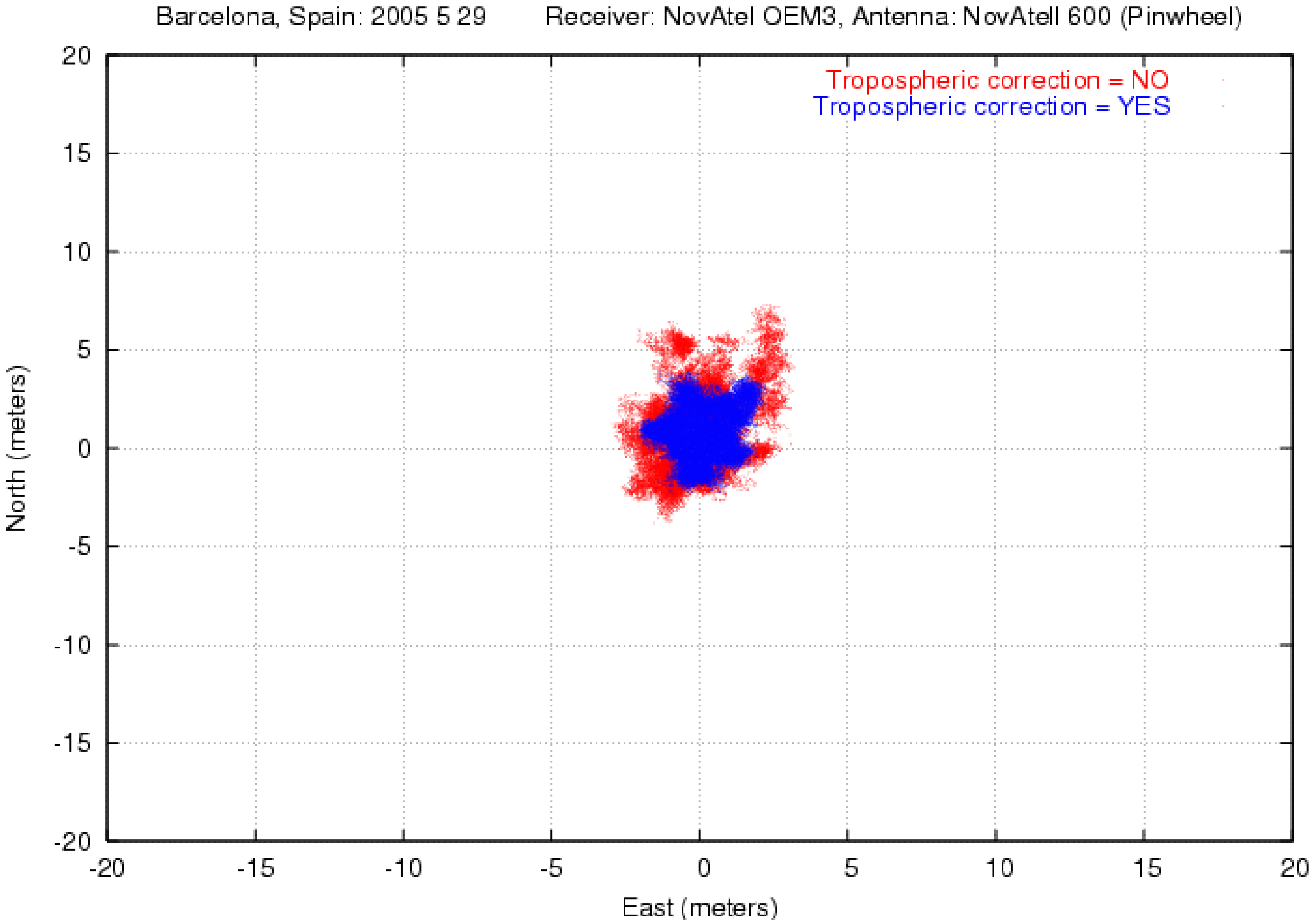


RTCA-Do229C model

Vertical error comparison



Horizontal error comparison



Instrumental Delays

Some sources for these delays are antennas, cables, as well as several filters used in both satellites and receivers.

They are composed by a delay corresponding to satellite and other to receiver, depending on frequency:

$$
\begin{aligned}
K1_{rec}^{sat} &= K1_{rec} - TGD^{sat} \\
K2_{rec}^{sat} &= K2_{rec} - \frac{f_1^2}{f_2^2} TGD^{sat}
\end{aligned}
$$

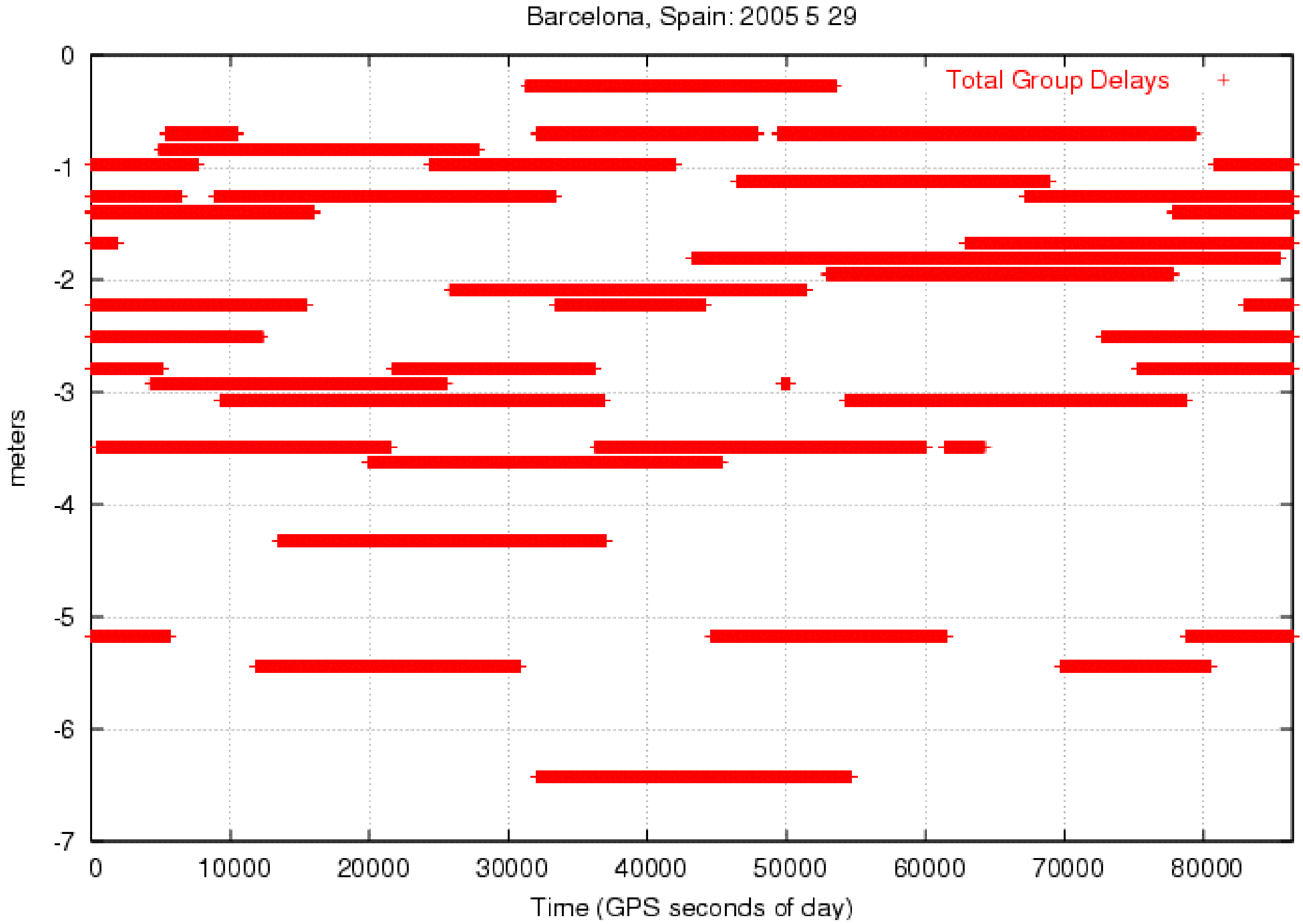
- $R1_{rec}$ may be assumed as zero (including it in receiver clock offset).
- TGD^{sat} is transmitted in satellite's navigation message (*Total Group Delay*)

According to ICD GPS-2000, control segment monitors satellite timing, so TGD cancels out when using free-ionosphere combination. That is why we have that particular equation for K2

TGD

$$
C1_{1rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + \boxed{K_{1rec} + K_1^{sat}} + \varepsilon
$$

Range variation: Instrumental delays (TGD)



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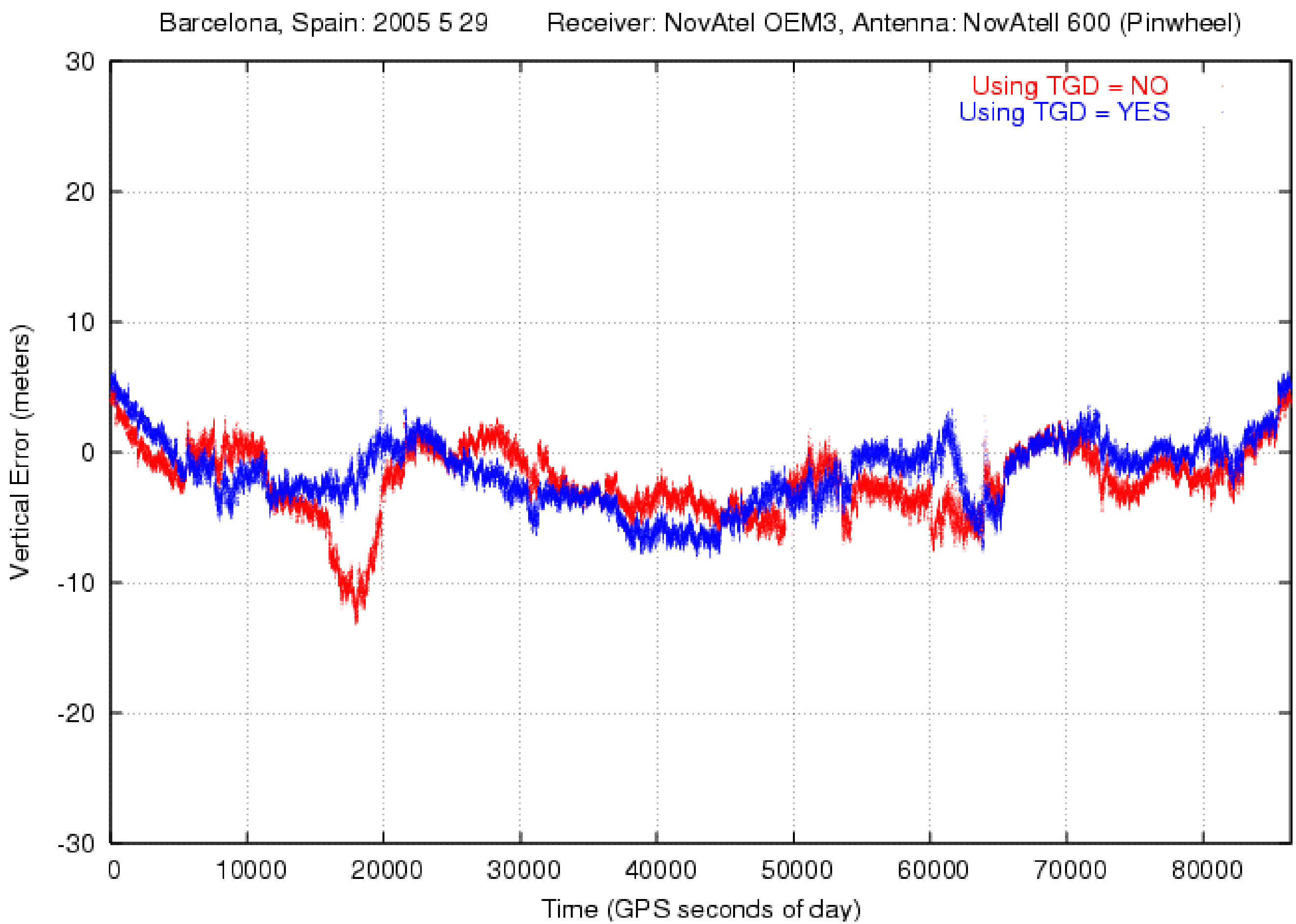
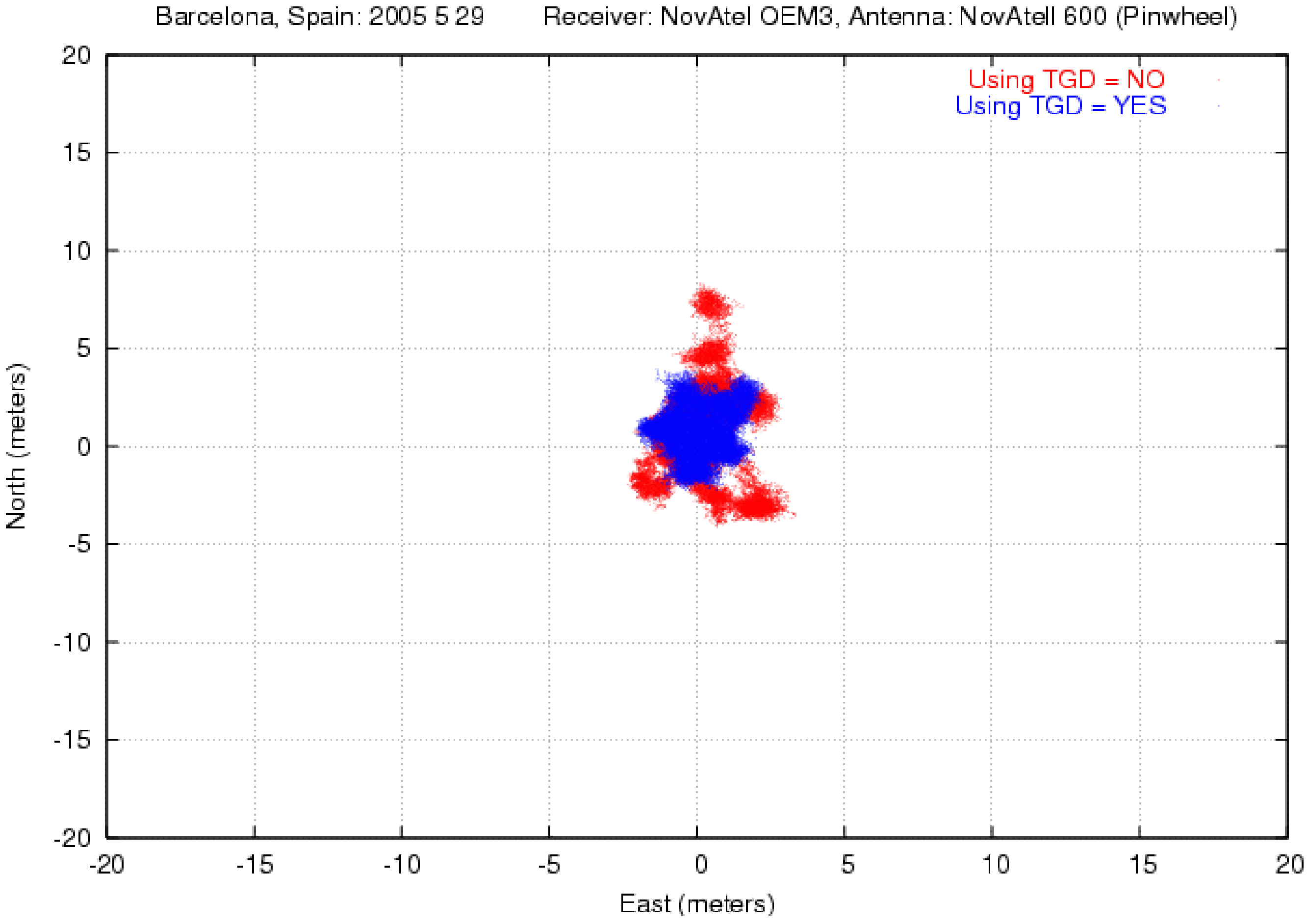
Vertical error comparison



Horizontal error comparison



Measurement noise (thermal noise)

Antispoofing (A/S):
The code P is encrypted to Y.
→ Only the code C at frequency L1 is available.

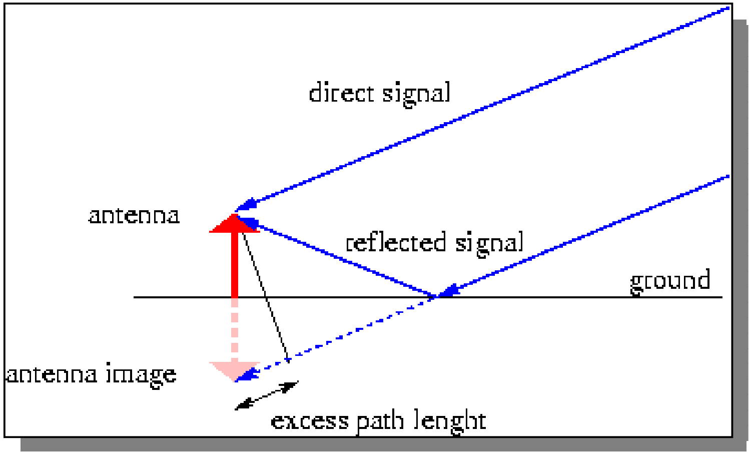
	Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
Code measurements			
C1	300 m	3 m	**Unambiguous** but noisier
P1 (Y1): encrypted	30 m	30 cm	
P2 (Y2): encrypted	30 m	30 cm	
Phase measurements			
L1	19.05 cm	2 mm	**Precise** but ambiguous
L2	24.45 cm	2 mm	

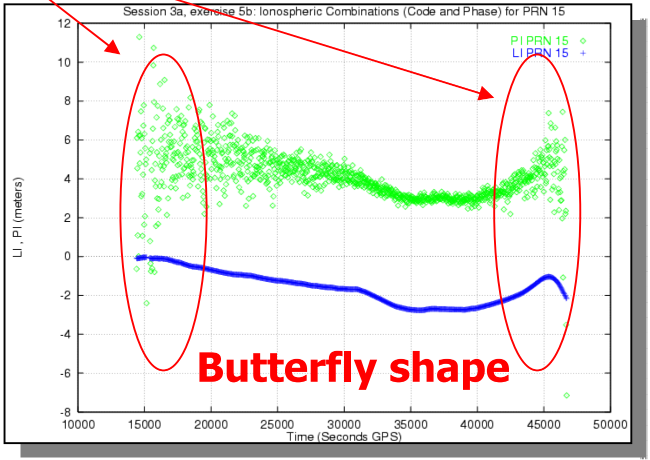
[*] codes may be smoothed with the phases in order to reduce noise (i.e., C1 smoothed with L1 → 50 cm noise)

$$C1_{1rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_{1rec} + K_1^{sat} + \boxed{\varepsilon}$$

Multipath

- **One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.**
- **It affects both code and carrier phase measurements, and it is more important at low elevation angles.**

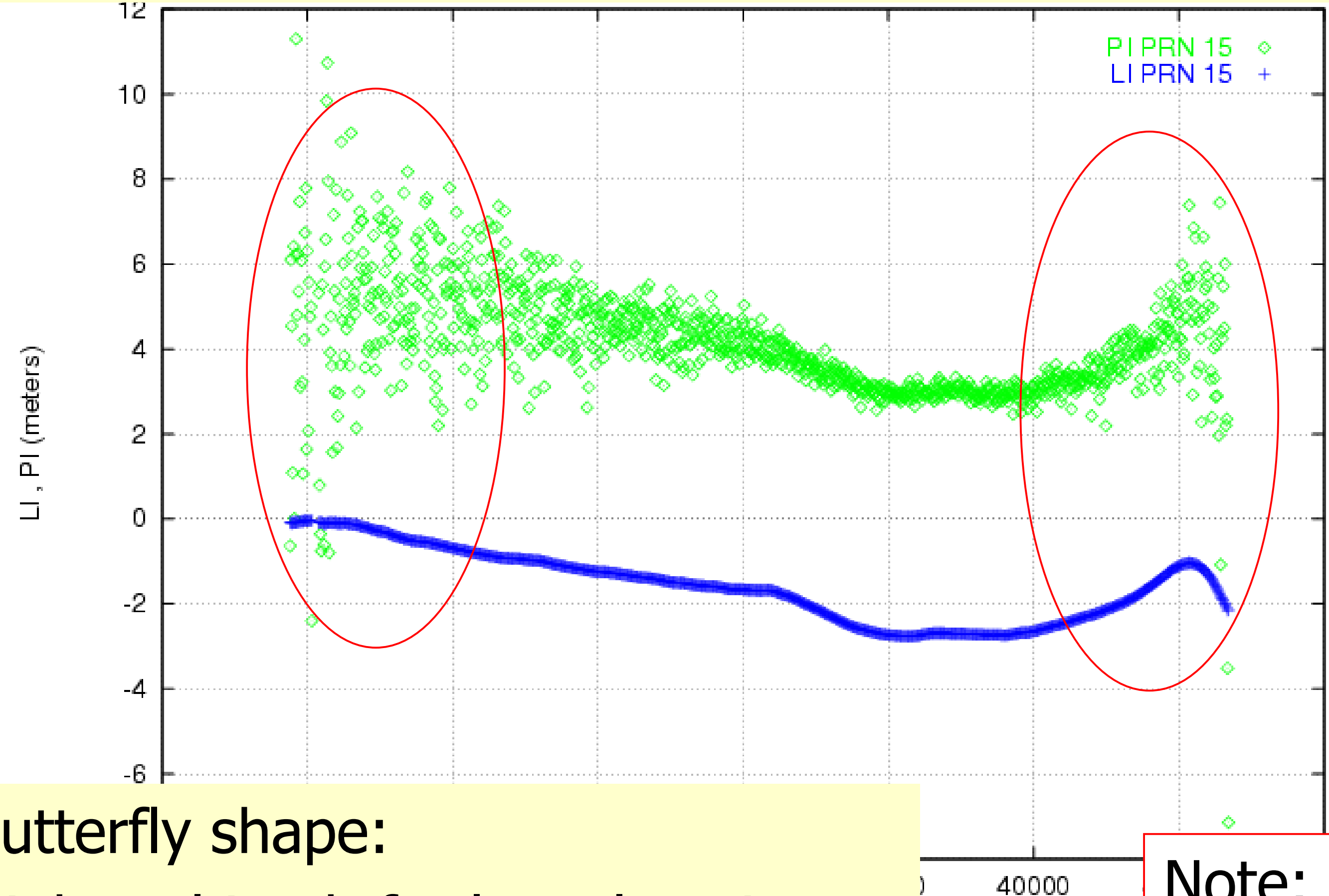




- **Code: up to 1.5 chip-length ➔ up to 450m for C1** [theoretically]
  Typically: less than 2-3 m.
- **Phase: up to $\lambda/4$ ➔ up to 5 cm for L1 and L2** [theoretically]
  Typically: less than 1 cm

Exercise 7:

Plot code and phase ionospheric combination for satellite PRN 15 of file 97jan09coco___r0.rnx and discuss the results.



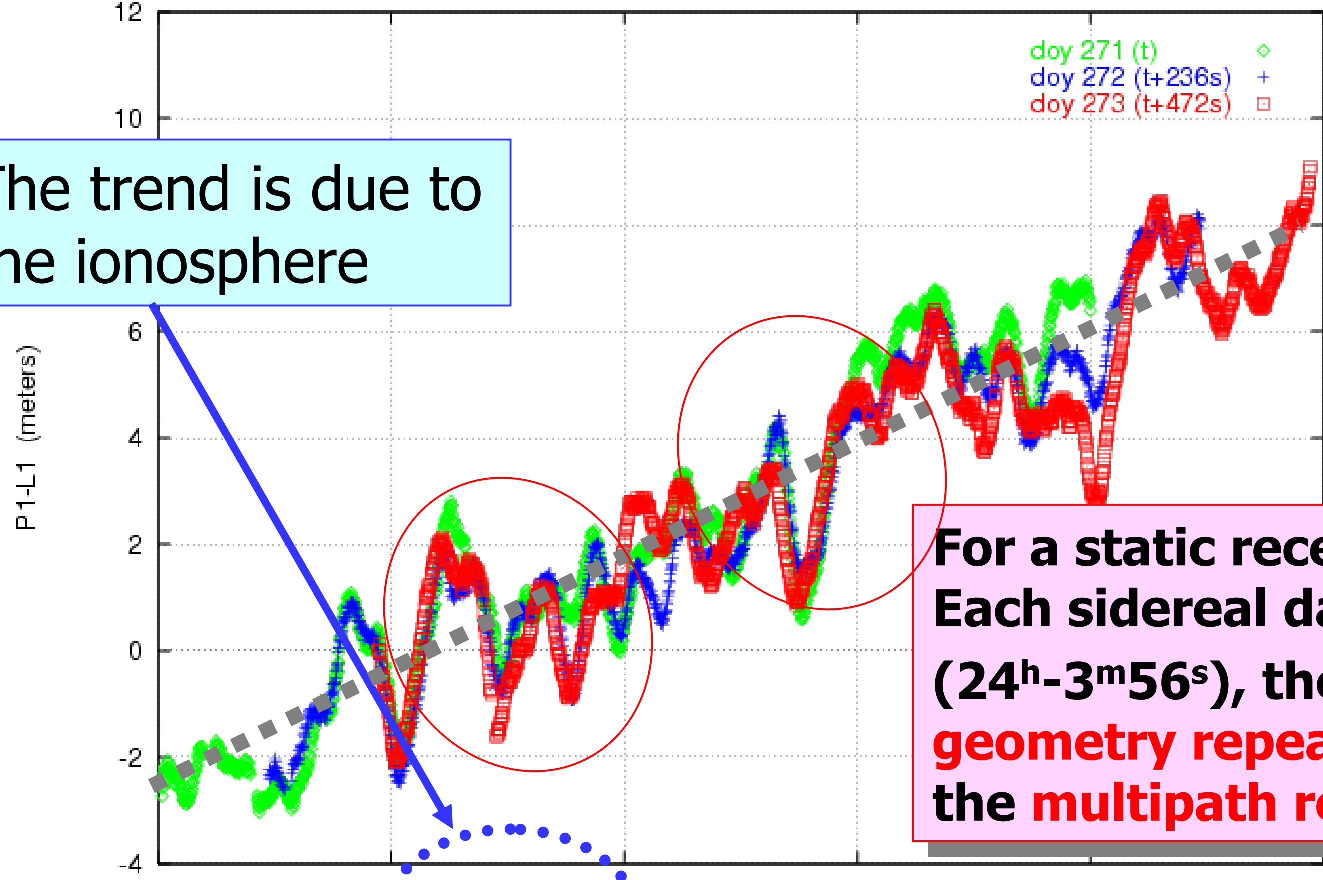
Butterfly shape:

High multipath for low elevation rays (when satellite rises and sets)

Note: A/S=on

Exercise 8:

Files gage2710.98.a, 2720.98.b and gage2730.98.a contain 1-second measurements collected by a static receiver in three consecutive days. Plot the combination P1-L1 and identify the multipath (note: shift the plots 3^m56^s= 236 sec each day)

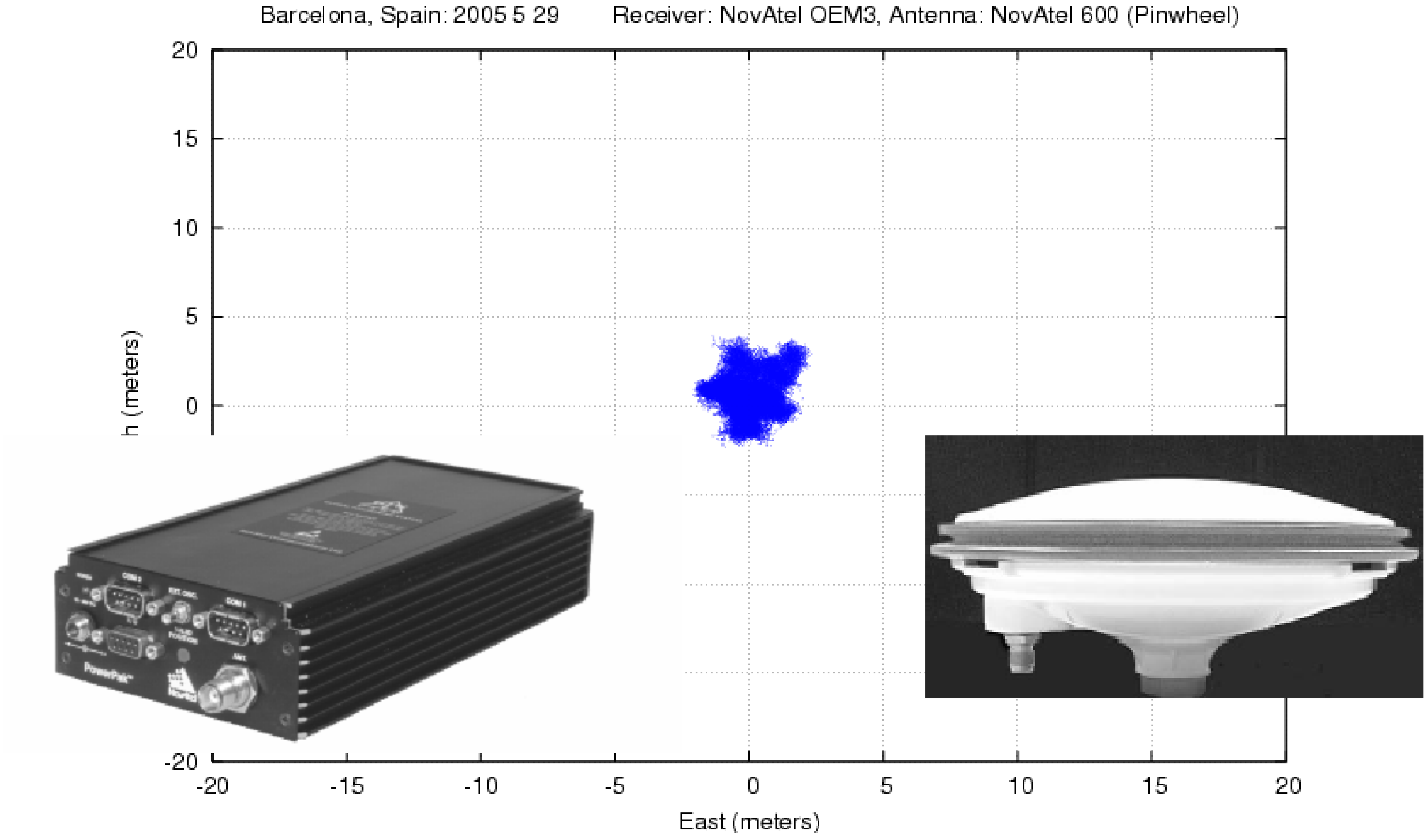


The trend is due to the ionosphere

For a static receiver: Each sidereal day is (24^h-3^m56^s), the geometry repeats → the multipath repeats

$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + [\boxed{Multipath} + noise]$$

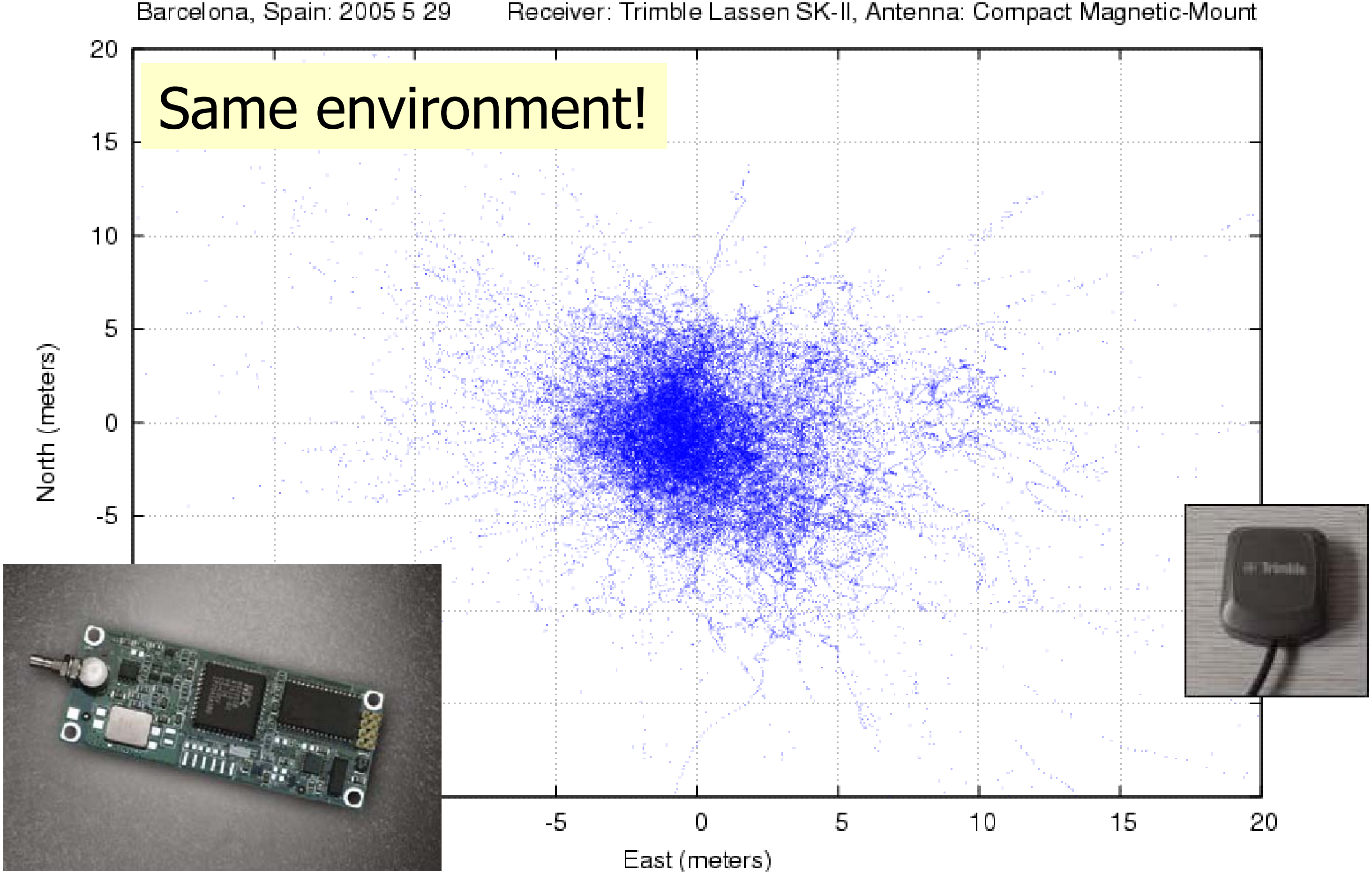
Receiver and multipath noise



GPS standalone (C1 code)

12,000 $

Receiver and multipath noise



GPS standalone (C1 code)

300 $

Exercise 9: Computation of modeled pseudorange

Using data of files 13oct98.rnx and 13oct98.eph, compute "by hand" the modeled pseudorange for satellite PRN 14 at t=38230 sec (10h37m10s).

$$PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cdt^{sat} + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_1^{sat}$$

Follow these steps:

1. Select orbital elements closer to 38230
2. Compute satellite clock offset
3. Compute satellite-receiver aprox. geometric range

   3.1 Compute emission time from receiver (reception) time-tags and code pseudorange.

   3.2 Compute satellite coordinates at emission time

   3.3 Compute approximate geometric range.
4. Compute satellite Instrumental delay (TGD):
5. Compute relativistic correction
6. Compute tropospheric delay
7. Compute ionospheric delay
8. Compute modeled pseudorange from previous values:

$$\boxed{PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cdt^{sat} + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_1^{sat}}$$

1. Selection of orbital elements: From file 13oct98.eph, select the last transmitted navigation message block before instant t=38230 s (10h37m10s).

Transmission time:
979 208818 → 10h 0m 18s

PRN

14 98 10 13 12 0 0	+5.65452501178E-06	+9.09494701773E-13	+0.000000000000E+00
+1.28000000000E+02	-6.10000000000E+01	+4.38125402624E-09	+8.19804251360E-01
-3.31364572048E-06	+1.09227513894E-03	+5.67547976971E-06	+5.15379510116E+03
+2.16000000000E+05	-6.33299350738E-08	[illegible]	-3.725290298462E-09
+9.73658001335E-01	+2.74031250000E+02	[illegible]	-8.081050495434E-09
[illegible]	+1.00000000000E+00	+9.79000000000E+02	+0.000000000000E+00
[illegible]	+0.00000000000E+00	-2.32830643654E-09	+1.280000000000E+02
+2.08818000000E+05	+0.00000000000E+00	+0.00000000000E+00	+0.000000000000E+00

GPS week

GPS sec of week

2. Satellite clock offset computation: From file 13oct98.eph, compute satellite clock offset at time t=3830 s for PRN14:

PRN $\quad t_0 \quad a_0 \quad a_1 \quad a_2$

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

t = 38230 sec

t_0= 12h 0m 0s= 43200 s

$$dt^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 = 5.65\ 10^{-6}\ s$$

$$PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - \boxed{cdt^{sat}} + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_1^{sat}$$

3. Satellite-receiver geometric range computation:

Use the following values (4789031, 176612, 4195008) as approximate coordinates.

3.1: Emission time computation from receiver time-tag and code pseudorange:

$$T[ems] = t_{rec}(T_R) - (C1/c + dt^{sat})$$

Measurement file 13oct98.a	→	Pseudorange *C1* at receiver time-tag t=38230: *C1*= 23585247.703 m
Ephemeris file 13oct98.eph	→	Satellite clock offset at t=38230 sec dt^{sat}= 5.65 10^{-6} sec (see previous results)

Thence, the emission time in GPS satellite clock is:

$$T[ems] = 38230 - (23585247.703/c + 5.65\ 10^{-6}) =$$

$$= 38229.9213224$$ **(where $c=299792458$)**

3.2: Satellite coordinates at emission time pseudorange:

T[emission]= 38229.921 sec → **Orbit.f** → X= 11453479.346, Y= 122468524.004, Z= 8245076.145 *CTS [emission]*

↑ Use the selected ephemeris for PRN14 (from file 13oct98.eph)

The previous coordinates are given in an Earth-fixed reference frame (CTS) at *t=T[emission]=* 38229.921 s.

This reference frame rotates by un amount "$\omega_E \Delta t$" during traveling time Δt*=T[reception]-T[emission].*

$$(X^{sat},Y^{sat},Z^{sat})_{CTS[reception]} = R_3(\omega_E\,\Delta \boldsymbol{t}).(X^{sat},Y^{sat},Z^{sat})_{CTS[emission]}$$

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$$(X^{sat},Y^{sat},Z^{sat})_{CTS[reception]} = R_3(\omega_E\,\Delta t).(X^{sat},Y^{sat},Z^{sat})_{CTS[emission]}$$

$$\boxed{\begin{pmatrix} 11453350.377 \\ 122468589.797 \\ 8245076.145 \end{pmatrix}_{CTS[reception]}} = \begin{pmatrix} \cos(\omega_E\Delta t) & \sin(\omega_E\Delta t) & 0 \\ -\sin(\omega_E\Delta t) & \cos(\omega_E\Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 11453479.346 \\ 122468524.004 \\ 8245076.145 \end{pmatrix}_{CTS[emission]}$$

$$\omega_E\Delta t = -5.74\cdot 10^{-6}\,rad. \quad (where \quad \omega_E = 7.2921151467\cdot 10^{-5}\,rad/\sec)$$

$$\Delta t = -\frac{\rho_{rec}^{sat}}{c} = -0.079\sec$$

$$\rho_{rec}^{sat} = \sqrt{\left(x^{sat}-x_{rec}\right)^2+\left(y^{sat}-y_{rec}\right)^2+\left(z^{sat}-z_{rec}\right)^2} \approx 23616673.3m$$

$$(x,y,z)^{satellite} \approx (11453479,\ 22468524,\ 8245076)$$

$$(x,y,z)_{receiver} \approx (4789031,\ \ 176612,\ 4195008)$$

An approximate value is enough to compute Δt.

Note: Both satellite and receiver coordinates must be given in the same reference system!

➔the CTS[reception] will be used to build navigation equations.

3.2: Geometric range computation

The geometric range between satellite coordinates at emission time and the "approximate position of the receiver" at reception time *(both coordinates given in the same reference system [for instance the CTS system at reception time])* is computed by:

$$\rho_{0,receiver}^{satellite}=\sqrt{\left(x^{sat}-x_{0,rec}\right)^2+\left(y^{sat}-y_{0,rec}\right)^2+\left(z^{sat}-z_{0,rec}\right)^2}=23616699.124m$$

$$(x,y,z)^{satellite}=(11453350.2771,\,22468589.7975,\,8245076.1448)_{CTS[reception]}$$

$$\boxed{(x_0,y_0,z_0)_{receiver}}=(4789031,\;176612,\,4195008)_{CTS[reception]}$$

"Approximate" receiver coordinates at reception time.

$$PR[\mathrm{mod}]_{1rec}^{sat}=\boxed{\rho_{0,rec}^{sat}}-cdt^{sat}+rel_{rec}^{sat}+Trop_{rec}^{sat}+Ion_{1rec}^{sat}+K_1^{sat}$$

4. Satellite Instrumental delay (TGD): From file 13oct98.eph, compute the Total Group Delay for PRN14:

TGD (in sec)

PRN

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

TGD= -2.32830643654E-09 * c= -0.69801 m

$$PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cdt^{sat} + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + \boxed{K_1^{sat}}$$

5. Relativistic correction:

PRN

e

sqrt(a)

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

T[emission] = 38229.921 s → **Orbit.f** → ***E* =0.095 rad. (eccentric anomaly)**

$$rel_{receiver}^{satellite} = 2\frac{\sqrt{\mu a}}{c} e \sin(E) = 0.07m$$

$$\mu = 3.986005 \cdot 10^{14} \quad m^3 s^{-2}$$

$$c = 299792458 \quad m\, s^{-1}$$

$$PR[\text{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cdt^{sat} + \boxed{rel_{rec}^{sat}} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_1^{sat}$$

6. Tropospheric correction

$$Trop_{rec}^{sat} = \left(d_{dry} + d_{wet}\right) m(elev) = 6.76m$$

$$d_{dry} = 2.3e^{-0.116 \cdot 10^{-3} H} = 2.3m$$

$$d_{wet} = 0.1m$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

See klob.f

$$\boxed{elev} = 20.57\frac{\pi}{180} = 0.359 rad$$

$$\boxed{H = 160m} \quad (heigh\ \ over\ the\ ellipsoid)$$

$(x,y,z)_{sta}$ ➔[car2geo] ➔ $(Lon, Lat, \boxed{H})_{sta}$

$$PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cdt^{sat} + rel_{rec}^{sat} + \boxed{Trop_{rec}^{sat}} + Ion_{1rec}^{sat} + K_1^{sat}$$

7. Ionospheric correction

(time, r_{sta}, r^{sat}, $\alpha0,\alpha1,\alpha2,\alpha3,\beta0,\beta1,\beta2,\beta3$) → [Klob] → Iono=10.26m

```
 2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
XPRINT v1.1          gAGE               00/06/04 17:36:23   PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE                               COMMENT
   +1.9558E-08 +0.0000E+00 -1.1921E-07 +0.0000E+00          ION ALPHA
   +1.2288E+05 -1.6384E+04 -2.6214E+05 +1.9661E+05          ION BETA
   -8.381903171539E-09-1.421085471520E-14    405504     979 DELTA_UTC: A0,A1,T,W
    12                                                      LEAP SECONDS
                                                            END OF HEADER
```

$$t = 38230\,\text{sec}$$

$$(x,y,z)^{satellite} = (11453350.2771,\ 22468589.7975,\ 8245076.1448)_{CTS[reception]}$$

$$(x_0,y_0,z_0)_{receiver} = (4789031,\ 176612,\ 4195008)_{CTS[reception]}$$

Approximate values for receiver or satellite coordinates are enough

$$PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cdt^{sat} + rel_{rec}^{sat} + Trop_{rec}^{sat} + \boxed{Ion_{1rec}^{sat}} - K_1^{sat}$$

7. Compute the modeled pseudorange.

$$PR[\mathrm{mod}]_{1rec}^{sat} = \rho_{0,rec}^{sat} - cDt^{sat} + rel_{rec}^{sat} + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + K_1^{sat}$$

$$\rho_{0,rec}^{sat} = 23616699.124m$$

$$cdt^{sat} = 5.65\,10^{-6}\,c = 1693.828m$$

$$rel_{rec}^{sat} = 0.071m$$

$$Trop_{rec}^{sat} = 6.760m$$

$$Ion_{1rec}^{sat} = 10.260m$$

$$K_1^{sat} = -0.698m$$

$$\rightarrow \quad PR[\mathrm{mod}]_{1rec}^{sat} = 23615021.689\mathrm{m}$$

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Prefit residual:

Is the difference between measured and modeled pseudorange

$$\boxed{Pref_{rec}^{sat} = C1_{rec}^{sat} - PR_1[mod]_{rec}^{sat}} = \boxed{\rho_{rec}^{sat} - \rho_{0,rec}^{sat} + c \cdot Dt_{rec} + K_{1rec} + \varepsilon}$$

In the previous example (PRN14 at t = 38230 s):

Pref= 23585247.703 - 23615021.689 = -29773.986 m

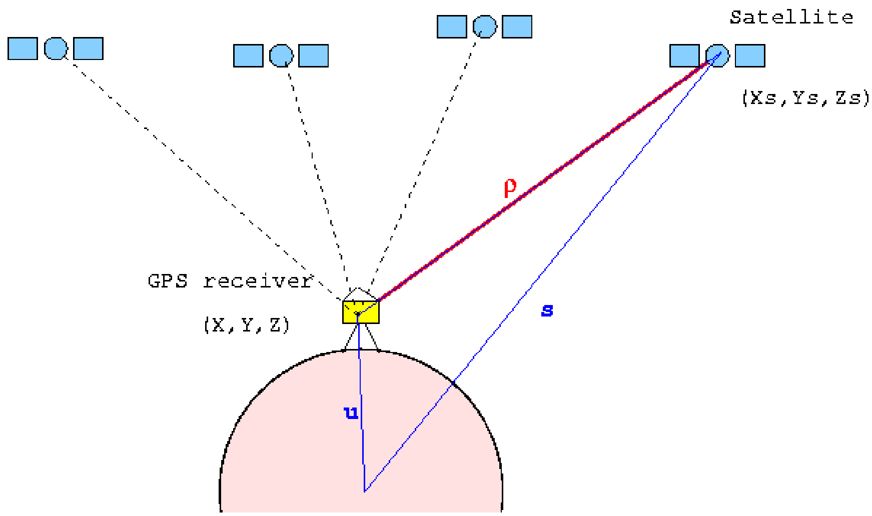
Previously calculated

From measurement file

Lesson 6

Navigation Equations

Solving navigation equations



Input:

- *Pseudoranges* (receiver-satellite j): P^j
- Navigation message. In particular:
  - *satellite position* when transmitting signal: $r^j = (x^j, y^j, z^j)$
  - *offsets* of satellite clocks: dt^j

  (j = 1,2,...n)   (n>=4)

Unknowns:

- receiver *position*: $r = (x, y, z)$
- receiver *clock* offset: DT

For each satellite in view

rel.+ Iono+Tropo+TGD

$$C1_i^j = \rho_i^j + c\cdot(Dt_i - Dt^j) + \sum \delta_k + \varepsilon$$

$$= \rho_{io}^j + \frac{x_{io}-x^j}{\rho_{io}^j}dx_i + \frac{y_{io}-y^j}{\rho_{io}^j}dy_i + \frac{z_{io}-z^j}{\rho_{io}^j}dz_i + c\left(Dt_i - Dt^j\right) + \sum\delta_k$$

where:

$$dx_i = x_i - x_{io} \quad ; \quad dy_i = y_i - y_{io} \quad ; \quad dz_i = z_i - z_{io}$$

Prefit-residuals (Prefit)

$$C1_i^j - \left(\rho_{io}^j + cDt^j - \sum\delta_k\right) = \frac{x_{io}-x^j}{\rho_{io}^j}dx_i + \frac{y_{io}-y^j}{\rho_{io}^j}dy_i + \frac{z_{io}-z^j}{\rho_{io}^j}dz_i + cDt_i$$

measurement

computed

unknown

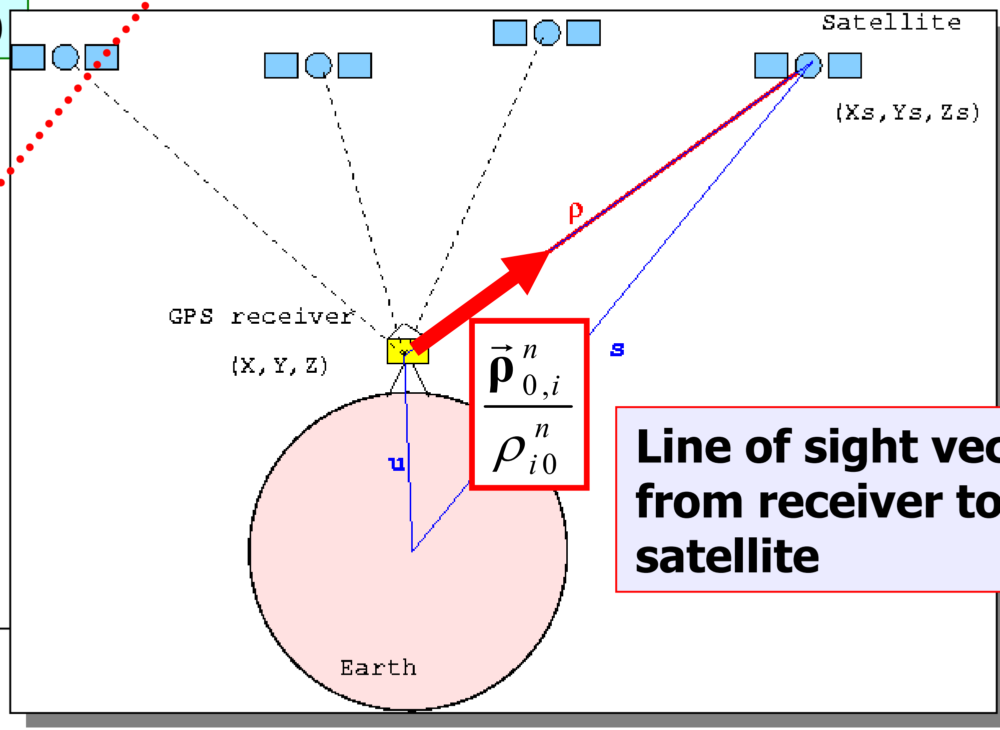
For all sat. in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \cdots\cdots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \dfrac{x_{io}-x^1}{\rho_{io}^1} & \dfrac{y_{io}-y^1}{\rho_{io}^1} & \dfrac{z_{io}-z^1}{\rho_{io}^1} & 1 \\ \dfrac{x_{io}-x^2}{\rho_{io}^2} & \dfrac{y_{io}-y^2}{\rho_{io}^2} & \dfrac{z_{io}-z^2}{\rho_{io}^2} & 1 \\ & \cdots\cdots\cdots & & \\ \dfrac{x_{io}-x^n}{\rho_{io}^n} & \dfrac{y_{io}-y^n}{\rho_{io}^n} & \dfrac{z_{io}-z^n}{\rho_{io}^n} & 1 \end{bmatrix} \begin{bmatrix} dx_i \\ dy_i \\ dz_i \\ cDT_i \end{bmatrix}$$

Observations
(measured/computed)

Geometry of rays

$$-\frac{\vec{\boldsymbol{\rho}}_{0,i}^{\,n}}{\rho_{i0}^{n}}$$



$$\frac{\vec{\boldsymbol{\rho}}_{0,i}^{\,n}}{\rho_{i0}^{n}}$$

Line of sight vector from receiver to satellite

COMMENTS:

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \cdots\cdots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{io}-x^1}{\rho_{io}^1} & \frac{y_{io}-y^1}{\rho_{io}^1} & \frac{z_{io}-z^1}{\rho_{io}^1} & 1 \\ \frac{x_{io}-x^2}{\rho_{io}^2} & \frac{y_{io}-y^2}{\rho_{io}^2} & \frac{z_{io}-z^2}{\rho_{io}^2} & 1 \\ & \cdots\cdots\cdots & & \\ \frac{x_{io}-x^n}{\rho_{io}^n} & \frac{y_{io}-y^n}{\rho_{io}^n} & \frac{z_{io}-z^n}{\rho_{io}^n} & 1 \end{bmatrix} \begin{bmatrix} dx_i \\ dy_i \\ dz_i \\ cDT_i \end{bmatrix}$$

Of course, receiver coordinates $(x_{rec}, y_{rec}, z_{rec})$ are not known (they are the target of this problem). But we can always assume that an "approximate position $(x_{0_{rec}}, y_{0_{rec}}, z_{0_{rec}})$ is known":

That is:

The navigation problem will consist on:

1.- To start from an approximate value for receiver position $(x_{0_{rec}}, y_{0_{rec}}, z_{0_{rec}})$ (it can be computed with Bancroft's method)

2.- With the pseudorange measurements and the navigation equations, to compute the correction $(dx_{rec}, dy_{rec}, dz_{rec})$ to have a more precise position of receiver.

$$(x_{rec}, y_{rec}, z_{rec}) = (x_{0_{rec}}, y_{0_{rec}}, z_{0_{rec}}) + (dx_{rec}, dy_{rec}, dz_{rec})$$

Note:

An (approximate) initial receiver position *(x_0, y_0, z_0)* is assumed to be known in former equations!!

The method of Bancroft (see next lesson) allows to compute approximate receiver coordinates from pseudorange measurements and satellite coordinates.

$T = t_0$: Compute an approximate initial position using "Bancroft" algorithm ➔ ***$r_0(t_o) = (x_0, y_0, z_0)$***

$T = t_n$: take as an approximate position ***$r_0(t_n) = r(t_{n-1})$***, the coordinates computed in the previous epoch.

Bancroft to initialize $r_0(t_o)$

Linearized model: from t_{n-1} to t_n

$r_0(t_n) = r(t_{n-1}) \rightarrow r(t_n)$

Solving the equations

For all sat. in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{io}-x^1}{\rho_{io}^1} & \frac{y_{io}-y^1}{\rho_{io}^1} & \frac{z_{io}-z^1}{\rho_{io}^1} & 1 \\ \frac{x_{io}-x^2}{\rho_{io}^2} & \frac{y_{io}-y^2}{\rho_{io}^2} & \frac{z_{io}-z^2}{\rho_{io}^2} & 1 \\ & \dots\dots\dots & & \\ \frac{x_{io}-x^n}{\rho_{io}^n} & \frac{y_{io}-y^n}{\rho_{io}^n} & \frac{z_{io}-z^n}{\rho_{io}^n} & 1 \end{bmatrix} \begin{bmatrix} dx_i \\ dy_i \\ dz_i \\ cDT_i \end{bmatrix}$$

Observations
(measured/computed)

Geometry of rays

Unknowns

Thence, the basic linearized GPS measurement equation can be written as:

$$Y = \mathbf{A} X$$

This is a linear system with, in general, $\boxed{n \geq 4}$ equations which we can solve using LMS, WMS, Kalman filter,…

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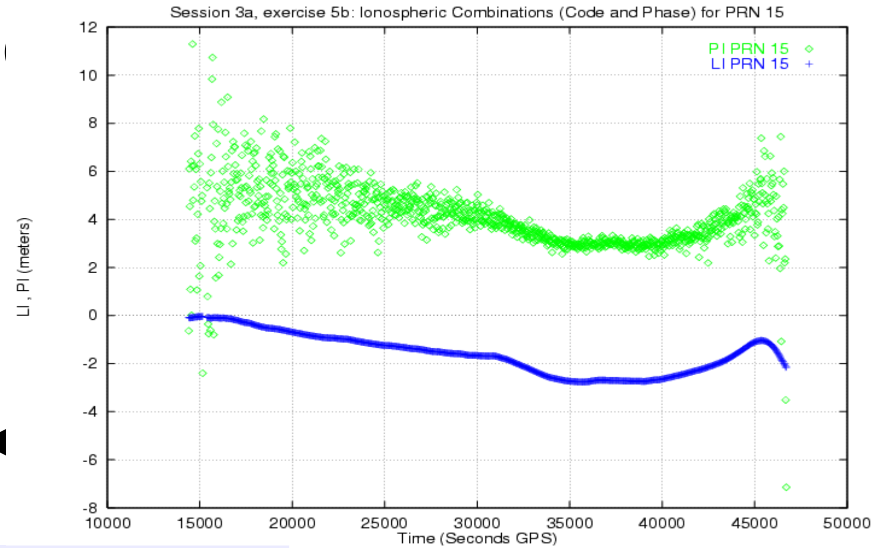
Let be the basic linearized

$$\boxed{Y = \mathbf{A}X}$$

- Least Squares solution:

$$\boxed{\hat{X} = \left(\mathbf{A}^t\mathbf{A}\right)^{-1}\mathbf{A}^t Y}$$



The **same error** is assumed in all measurements

- Weighted Least Squares solution

If the measurements have **different errors**, the equations can be weighted by matrix ***W***:

$$W = \begin{bmatrix} w_{y_1} & & 0 \\ & \ddots & \\ 0 & & w_{y_n} \end{bmatrix}$$

Uncorrelated errors are assumed

And the weighted least squares solution is:

$$\boxed{\hat{X} = \left(\mathbf{A}^t W \mathbf{A}\right)^{-1}\mathbf{A}^t W Y}$$

$$\min \left\| Y - \hat{Y} \right\|_W^2 = \min \left[\sum_i w_i \left(y_i - \hat{y}_i\right)^2 \right]$$

$$\hat{Y} = A\hat{X}$$

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Assuming that measurements Y have **random errors with zero mean and variance σ^2**, and assuming that error sources for each satellite are **uncorrelated** with error sources for any other satellite, the following weighted matrix may be used:

$$W = \begin{bmatrix} 1/\sigma_{y_1}^2 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_{y_n}^2 \end{bmatrix}$$

$$w_i = \frac{1}{\sigma_{y_i}^2} \rightarrow \sigma_{y_i}^2 \uparrow \quad \Rightarrow \quad w_i \downarrow$$

greater error → less weight

- Minimum Variance Solution:

Let be "P_Y" the error covariance matrix for measurements Y.

If the weighting matrix is taken as $W = P_Y^{-1}$, thence the Minimum Variance Solution is found:

$$\hat{X} = \left(\mathbf{A}^t P_Y^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^t P_Y^{-1} Y$$

And the error covariance matrix for the estimation X is:

$$P_{\hat{X}} = \left(\mathbf{A}^t P_Y^{-1} \mathbf{A}\right)^{-1}$$

Exercise 10:

With program GCAT and data files 13oct98.a, 13oct98.eph, and taking $(x_0,y_0,z_0)=[4789031 \;\; 176612 \; 4195008 \,]$:

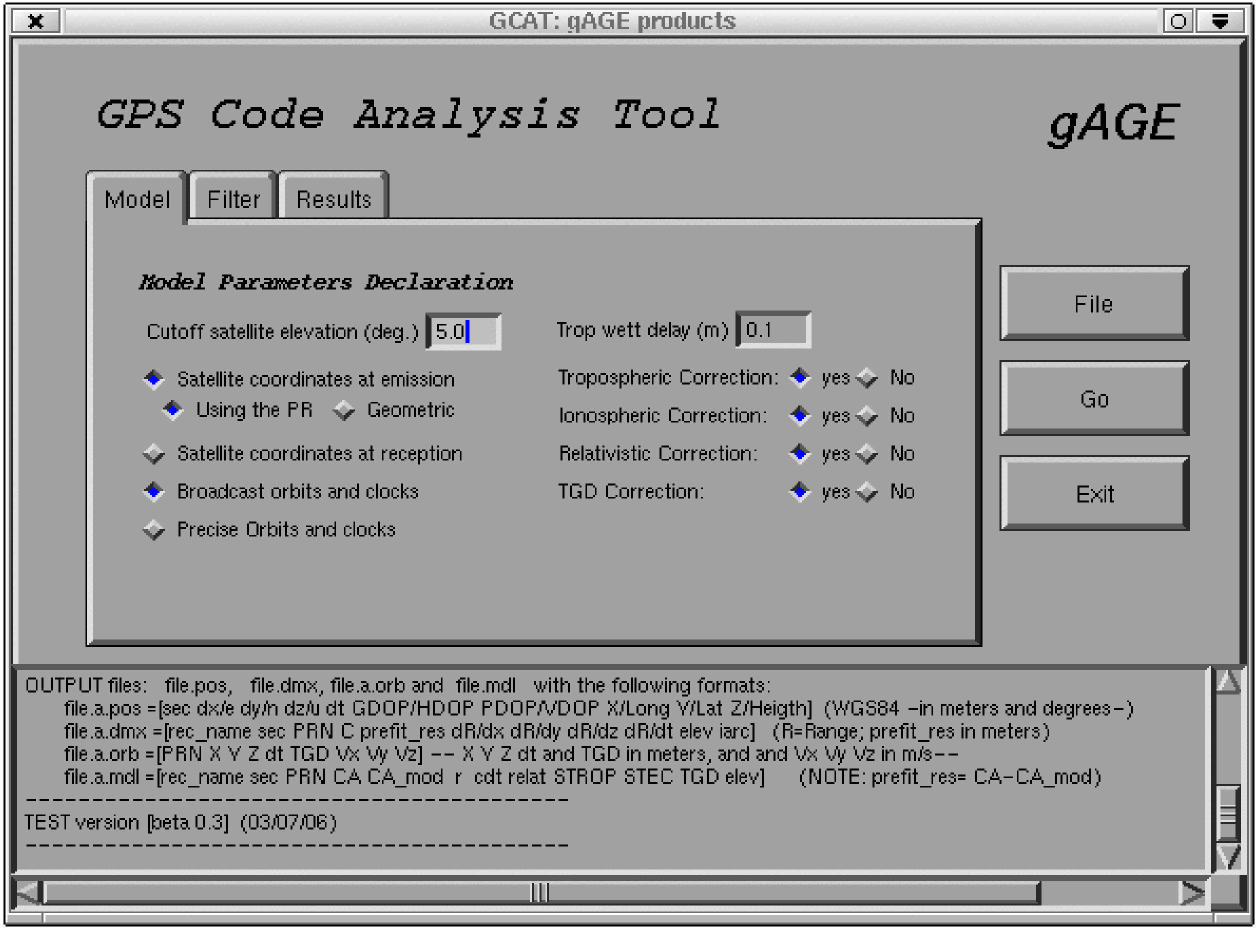
a) Generate data file "13oct98.a.dmx" with the prefit-residuals (Y) and the design matrix ($\boldsymbol{A}$) for $t = 38230\ s.$

b) Use these values to compute the navigation solution at that instant

$$X = inv(\boldsymbol{A}'*W*\boldsymbol{A})*\boldsymbol{A}'*Y \qquad \text{[note } X=(dx,dy,dz)\text{]}$$

c) Compute receiver position:

$$(x,y,z)= (x_0,y_0,z_0)+ (dx,dy,dz)$$


GCAT: gAGE products
GPS Code Analysis Tool
gAGE
Model
Filter
Results
Model Parameters Declaration
Cutoff satellite elevation (deg.) 5.0
Trop wett delay (m) 0.1
Satellite coordinates at emission
Using the PR
Geometric
Satellite coordinates at reception
Broadcast orbits and clocks
Precise Orbits and clocks
Tropospheric Correction: yes No
Ionospheric Correction: yes No
Relativistic Correction: yes No
TGD Correction: yes No
File
Go
Exit
OUTPUT files: file.pos, file.dmx, file.a.orb and file.mdl with the following formats:
file.a.pos =[sec dx/e dy/n dz/u dt GDOP/HDOP PDOP/VDOP X/Long Y/Lat Z/Heigth] (WGS84 -in meters and degrees-)
file.a.dmx =[rec_name sec PRN C prefit_res dR/dx dR/dy dR/dz dR/dt elev iarc] (R=Range; prefit_res in meters)
file.a.orb =[PRN X Y Z dt TGD Vx Vy Vz] -- X Y Z dt and TGD in meters, and and Vx Vy Vz in m/s--
file.a.mdl =[rec_name sec PRN CA CA_mod r cdt relat STROP STEC TGD elev] (NOTE: prefit_res= CA-CA_mod)
TEST version [beta 0.3] (03/07/06)

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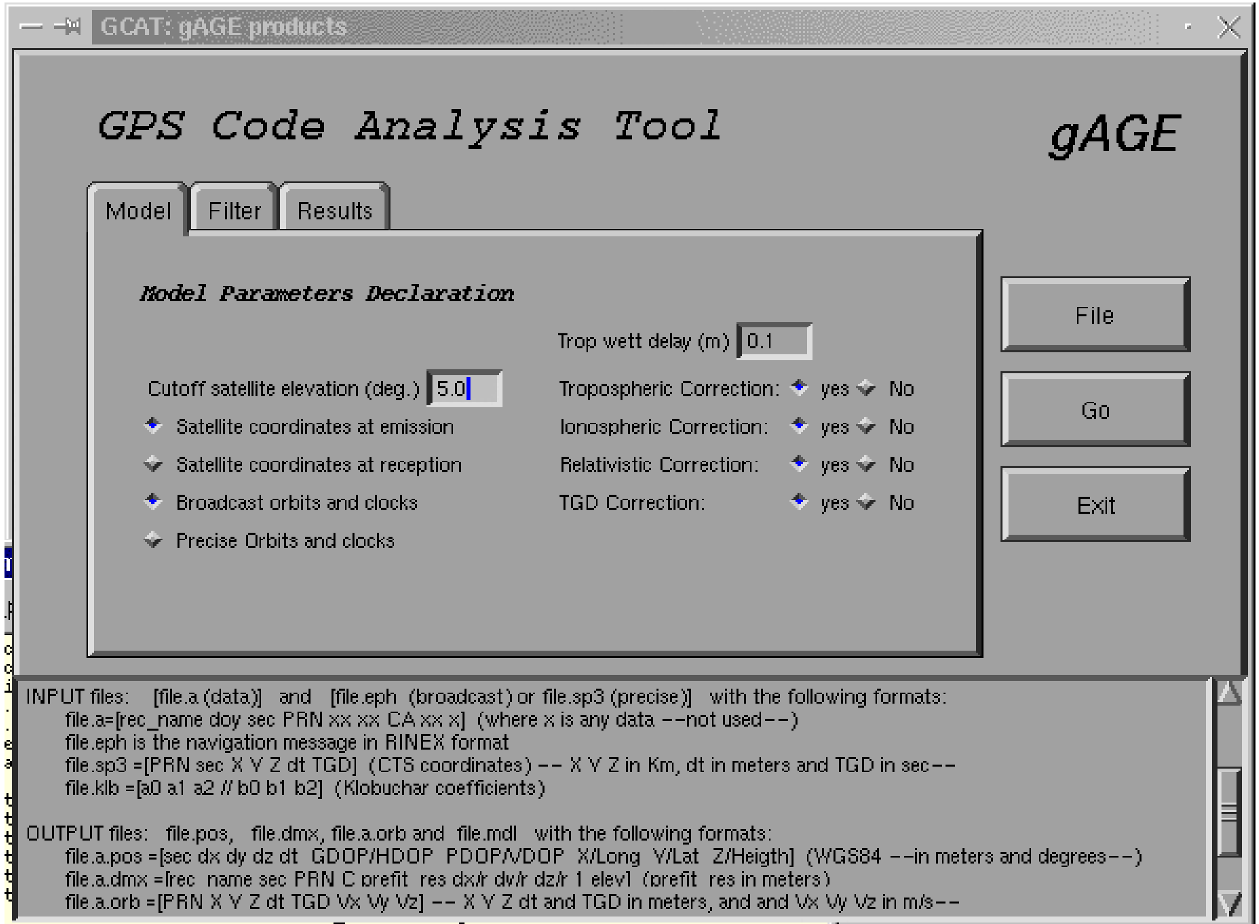

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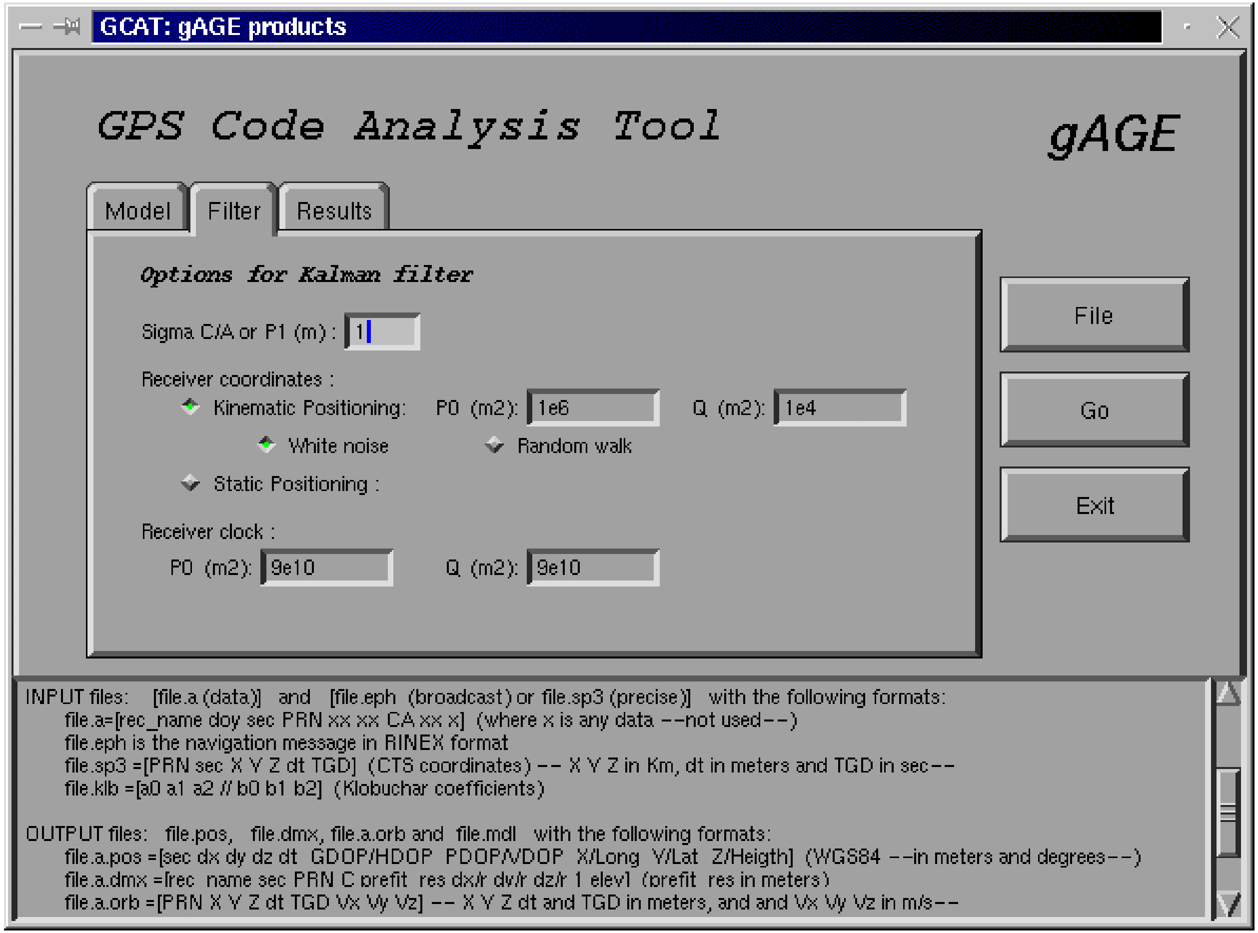
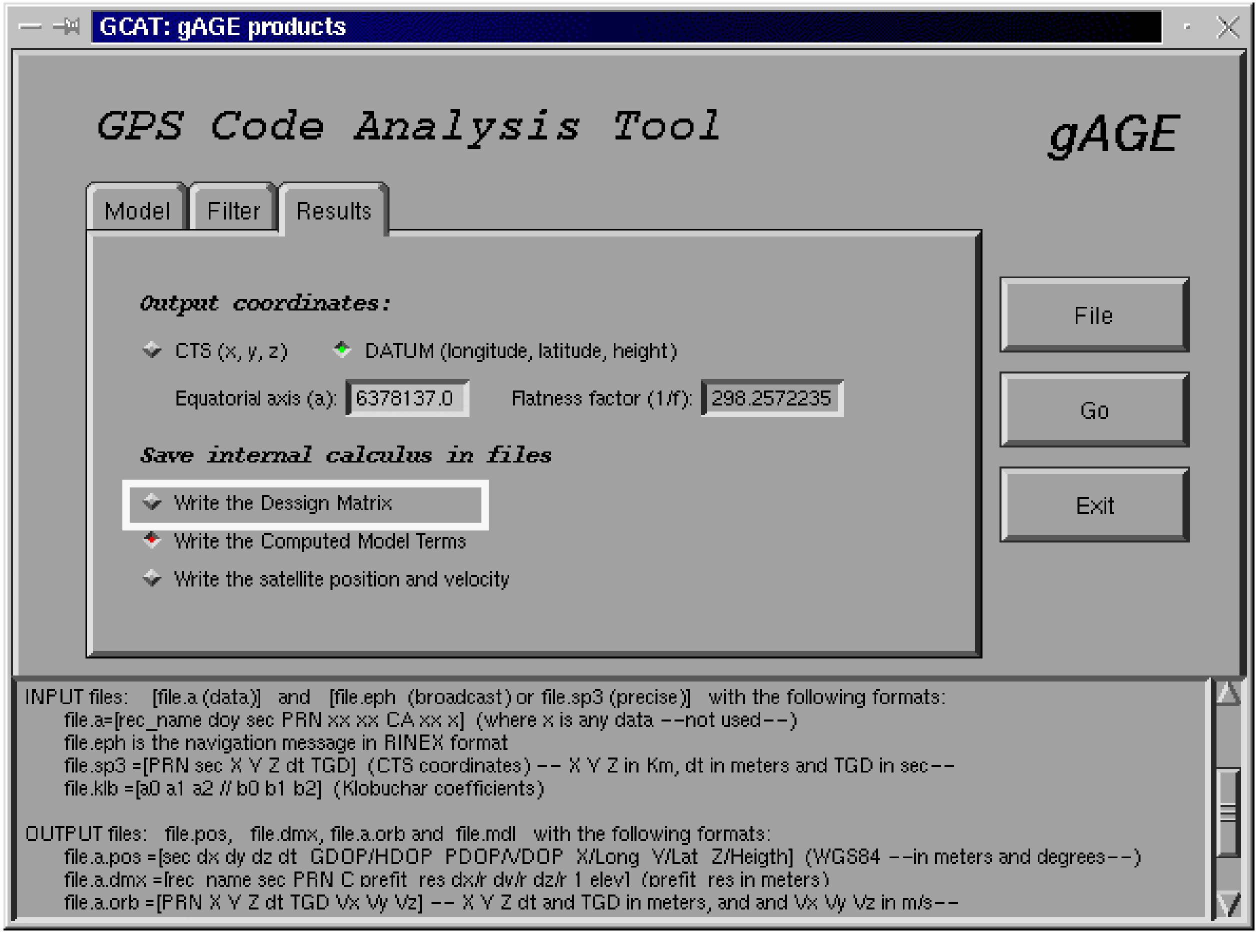



GCAT: gAGE products
GPS Code Analysis Tool
gAGE
Model
Filter
Results
Output coordinates:
CTS (x, y, z)
DATUM (longitude, latitude, height)
Equatorial axis (a): 6378137.0
Flatness factor (1/f): 298.2572235
Save internal calculus in files
Write the Design Matrix
Write the Computed Model Terms
Write the satellite position and velocity
File
Go
Exit
INPUT files: [file.a (data)] and [file.eph (broadcast) or file.sp3 (precise)] with the following formats:
file.a=[rec_name doy sec PRN xx xx CA xx x] (where x is any data --not used--)
file.eph is the navigation message in RINEX format
file.sp3 =[PRN sec X Y Z dt TGD] (CTS coordinates) -- X Y Z in Km, dt in meters and TGD in sec--
file.klb =[a0 a1 a2 // b0 b1 b2] (Klobuchar coefficients)
OUTPUT files: file.pos, file.dmx, file.a.orb and file.mdl with the following formats:
file.a.pos =[sec dx dy dz dt GDOP/HDOP PDOP/VDOP X/Long Y/Lat Z/Heigth] (WGS84 --in meters and degrees--)
file.a.dmx =[rec name sec PRN C prefit res dx/r dy/r dz/r 1 elev] (prefit res in meters)
file.a.orb =[PRN X Y Z dt TGD Vx Vy Vz] -- X Y Z dt and TGD in meters, and and Vx Vy Vz in m/s--

Resolution: a)

```
------------------------------ 13oct98.a.dmx ------------------------------
                                          x_sat-x0    y_sat-y0     z_sat-z0
rec     sec      PRN    Prefit-res        --------    --------     --------   sig2_y
                                            rho0        rho0         rho0
--------------------------------------------------------------------------------
gAGE 38230.000 27 C -29790.69346 -0.825386165  0.083009922  0.558432656  1
gAGE 38230.000 14 C -29773.88675 -0.282186738 -0.943907431 -0.171491711  1
gAGE 38230.000 16 C -29692.55363 -0.524933459 -0.498000630 -0.690246504  1
gAGE 38230.000 19 C -29730.62312 -0.958194420 -0.270768810  0.092453794  1
```

t=38230

```
Y=[-29790.69346]    A=[-0.825386165  0.083009922  0.558432656 1]
  [-29773.88675]      [-0.282186738 -0.943907431 -0.171491711 1]
  [-29692.55363]      [-0.524933459 -0.498000630 -0.690246504 1]
  [-29730.62312]      [-0.958194420 -0.270768810  0.092453794 1]

Py=[1 0 0 0]
   [0 1 0 0]
   [0 0 1 0]
   [0 0 0 1]
```

Resolution: b)

```
Y=[-29790.69346]   A=[-0.825386165  0.083009922  0.558432656 1]
  [-29773.88675]     [-0.282186738 -0.943907431 -0.171491711 1]
  [-29692.55363]     [-0.524933459 -0.498000630 -0.690246504 1]
  [-29730.62312]     [-0.958194420 -0.270768810  0.092453794 1]

                  Py=[1 0 0 0]
                     [0 1 0 0]
                     [0 0 1 0]
                     [0 0 0 1]
```

Note data set collected under S/A=on

```
                               [   -99.09]  ← dx
x=inv(A'*W*A)*A'*W*Y=          [     6.02]  ← dy
       where W=inv(Py)         [  -105.24]  ← dz
                               [-29814.21]  ← clock
```

Resolution: c)

$x = x_0 + dx$ = [4789031] + [-99.09] = [4788931.91]

$y = y_0 + dy$ = [176612] + [6.02] = [176618.02]

$z = z_0 + dz$ = [4195008] + [-105.24] = [4194902.76]

The program Gnav.f

(source code provided with the book)

The software code of **`Gnav.f`** emulates a GPS receiver.

It computes the navigation solution from RINEX measurement and ephemeris files.

It applies the Bancroft method "for cold start" and the linearized equations for navigating.

Kalman filtering:

It is based on computing the **weighted average between**:

- the measurement $\boxed{Y(n)}$ (i.e., at $t = t_n$)
- the prediction of the state $\boxed{\hat{X}^-(n)}$ from previous estimation $\boxed{\hat{X}(n-1)}$

1. Weighted average:

$$\begin{cases} Y(n) = A(n)\,X(n) \\ \boxed{\hat{X}^-(n) = X(n)} \end{cases}$$

Let's assume, that we have the prediction $\boxed{\hat{X}^-(n)}$, with $\boxed{P_{\hat{X}^-(n)}}$ thence, it can be used to **add an additional set of equations** to the measurement equation ***Y= A X***

$$\begin{bmatrix} Y(n) \\ \boxed{\hat{X}^-(n)} \end{bmatrix} = \begin{pmatrix} A(n) \\ I \end{pmatrix} X(n)$$

$$W = \begin{pmatrix} P_{Y(n)} & 0 \\ 0 & \boxed{P_{\hat{X}^-(n)}} \end{pmatrix}^{-1}$$

$$\begin{bmatrix} Y(n) \\ \hat{X}^-(n) \end{bmatrix} = \left(\boxed{\begin{matrix} A(n) \\ I \end{matrix}} \right) X(n) \qquad \boxed{W} = \begin{pmatrix} P_{Y(n)} & 0 \\ 0 & P_{\hat{X}^-(n)} \end{pmatrix}^{-1}$$

And the following solution of the previous equation system can be found with some elemental algebraic manipulations:

$$\hat{X} = \left(\mathbf{A}^t P_Y^{-1} \boxed{\mathbf{A}}\right)^{-1} \mathbf{A}^t \boxed{P_Y^{-1}} Y$$

$$P_{\hat{X}} = \left(\mathbf{A}^t P_Y^{-1} \mathbf{A}\right)^{-1}$$

$$\hat{X}(n) = P_{\hat{X}(n)} \left[A^t(n) P_{Y(n)}^{-1} Y(n) + P_{\hat{X}^-(n)}^{-1} \hat{X}^-(n) \right]$$

$$P_{\hat{X}(n)} = \left[A^t(n) P_{Y(n)}^{-1} A(n) + P_{\hat{X}^-(n)}^{-1} \right]^{-1}$$

2.- Prediction

Scalar case:

Let's $\hat{x}_{n-1}$ be the state at $t = n$ with variance $\sigma^2_{\hat{x}_{n-1}}$

The *simplest prediction model* is to assume that the prediction at $t = n$ is proportional to the state at $t = n\text{-}1$. That is:

$$\boxed{\hat{x}_n^- = \phi \, \hat{x}_{n-1}}$$

Thence, existing a linear relation between $\hat{x}_{n-1}$ and $\hat{x}_n^-$, the variance of the prediction will be:

$$\boxed{\sigma^2_{\hat{x}_n^-} = \phi^2 \, \sigma^2_{\hat{x}_{n-1}} + q^2}$$

An additional term is added to account for modeling error!

Generalization to the vector case:

$$\hat{x}_n^- = \phi \, \hat{x}_{n-1}$$

$$\sigma^2_{\hat{x}_n^-} = \phi^2 \, \sigma^2_{\hat{x}_{n-1}} + q^2$$

$$\begin{array}{ccc} x_n & \rightarrow & X(n) \\ \phi & \rightarrow & \Phi(n) \\ \sigma^2_{x_n} & \rightarrow & P_{X(n)} \\ q^2 & \rightarrow & Q(n) \end{array}$$

$\Phi(n)$: *transition matrix*
$Q(n)$: *process noise matrix*

$$\hat{X}^-(n) = \Phi(n-1) \cdot \hat{X}(n-1)$$

$$P_{\hat{X}^-(n)} = \Phi(n-1) \cdot P_{\hat{X}(n-1)} \cdot \Phi^t(n-1) + Q(n-1)$$

Kalman filter (see kalman.f)

Measurements

$$Y(n) \qquad P_{Y(n)}$$

$$\hat{X}^-(n) \qquad P_{\hat{X}^-(n)}$$

Prediction

$$\hat{X}^-(n) = \Phi \hat{X}(n-1)$$

$$P_{\hat{X}^-(n)} = \Phi P_{\hat{X}(n-1)} \Phi^t + Q$$

Estimation

$$\hat{X}(n) = P_{\hat{X}(n)} \cdot \left[A^t P^{-1}_{Y(n)} Y(n) + P^{-1}_{\hat{X}^-(n)} \hat{X}^-(n) \right]$$

$$P_{\hat{X}(n)} = \left[A^t P^{-1}_{Y(n)} A + P^{-1}_{\hat{X}^-(n)} \right]^{-1}$$

$$\hat{X}(n) \qquad P_{\hat{X}(n)}$$

$$\hat{X}(0) \qquad P_{\hat{X}(0)}$$

Initialization

Kalman filter (classical version)

Measurements

$$Y(n) \qquad P_{Y(n)}$$

$$\hat{X}^-(n) \qquad P_{\hat{X}^-(n)}$$

Estimation

$$\hat{X}(n) = \hat{X}^-(n) + K(n)\cdot\left[Y(n) - A(n)\cdot\hat{X}(n)\right]$$

$$K(n) = P_{\hat{X}^-(n)}\cdot A^t(n)\cdot\left[A(n)\cdot P_{\hat{X}^-(n)}\cdot A^t(n) - P_{Y(n)}\right]^{-1}$$

$$P_{\hat{X}(n)} = \left[I - K(n)\cdot A(n)\right]\cdot P_{\hat{X}^-(n)}$$

$$\hat{X}(n) \qquad P_{\hat{X}(n)}$$

Prediction

$$\hat{X}^-(n) = \Phi\cdot\hat{X}(n-1)$$

$$P_{\hat{X}^-(n)} = \Phi\cdot P_{\hat{X}(n-1)}\cdot\Phi^t + Q$$

$$\hat{X}(0) \qquad P_{\hat{X}(0)}$$

Initialization

Some simple examples to define matrices Φ and Q

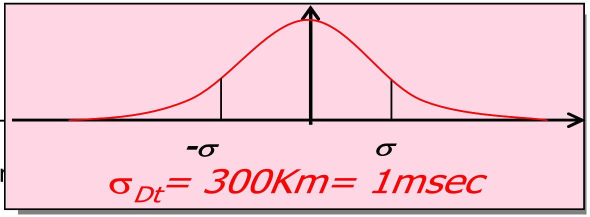
$$\hat{X}^-(n) = \Phi(n-1)\cdot\hat{X}(n-1)$$
$$P_{\hat{X}^-(n)} = \Phi(n-1)\cdot P_{\hat{X}(n-1)}\cdot\Phi^t(n-1) + Q(n-1)$$

a) Static positioning:

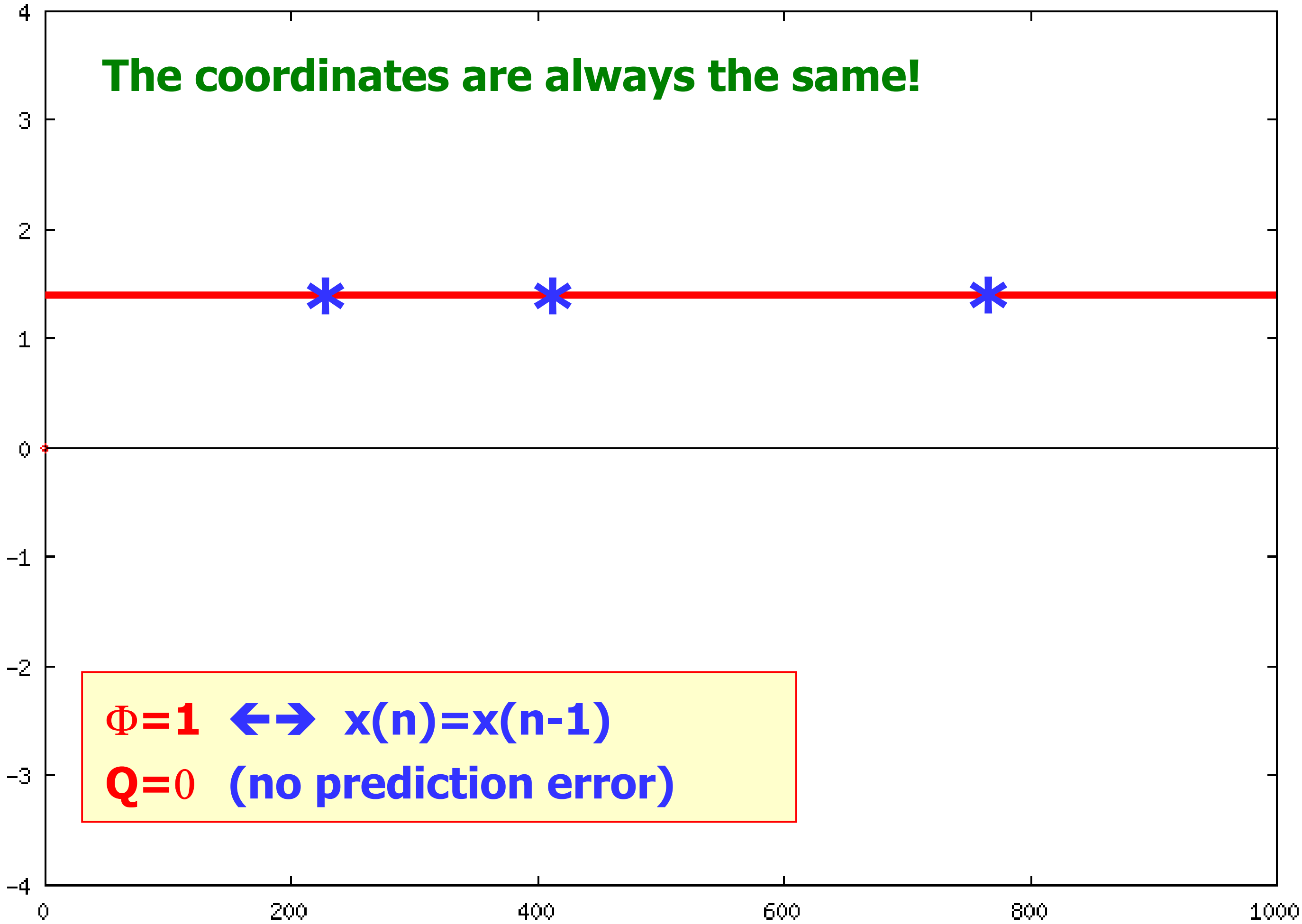
State vector to be determined is $X = (x_{rec}, y_{rec}, z_{rec}, dt_{rec})$ where coordinates $(x_{rec}, y_{rec}, z_{rec})$ are considered **constant** (because receiver is fixed) and clock offset dT_{rec} is treated as **white noise** with zero mean and variance σ^2_{Dt}. In these conditions, matrices have the form:

$$\mathbf{\Phi}(n) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \qquad \mathbf{Q}(n) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & \sigma^2_{DT} \end{pmatrix}$$

Being σ^2_{DT} process noise associated to clock offset (in some way, the uncertainty in clock value).



Constant



We can assure that, the next **x(n)** will be **the same as x(n-1).**

$$\hat{X}^-(n) = \Phi(n-1)\cdot\hat{X}(n-1)$$

$$P_{\hat{X}^-(n)} = \Phi(n-1)\cdot P_{\hat{X}(n-1)}\cdot\Phi^t(n-1) + Q(n-1)$$

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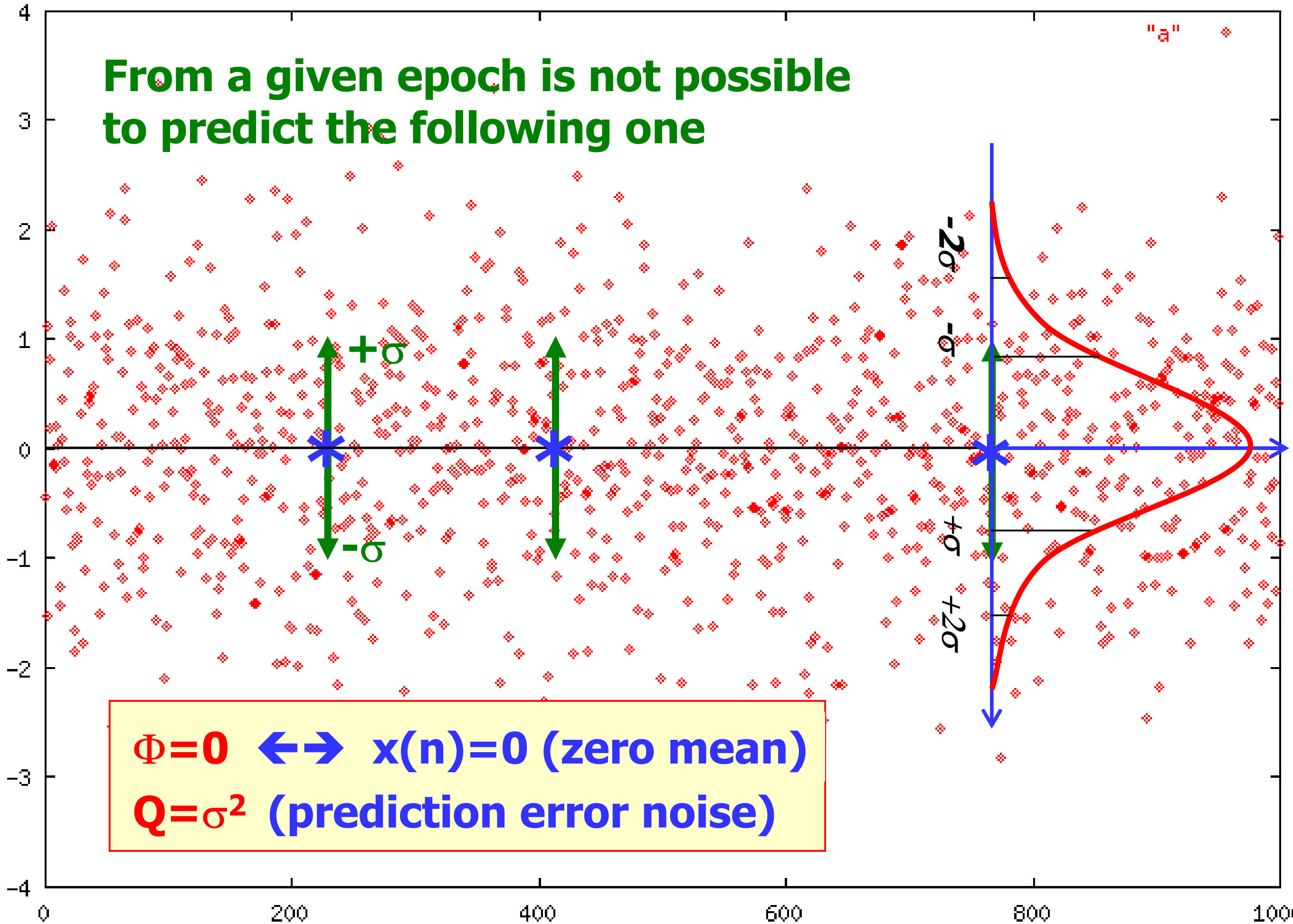
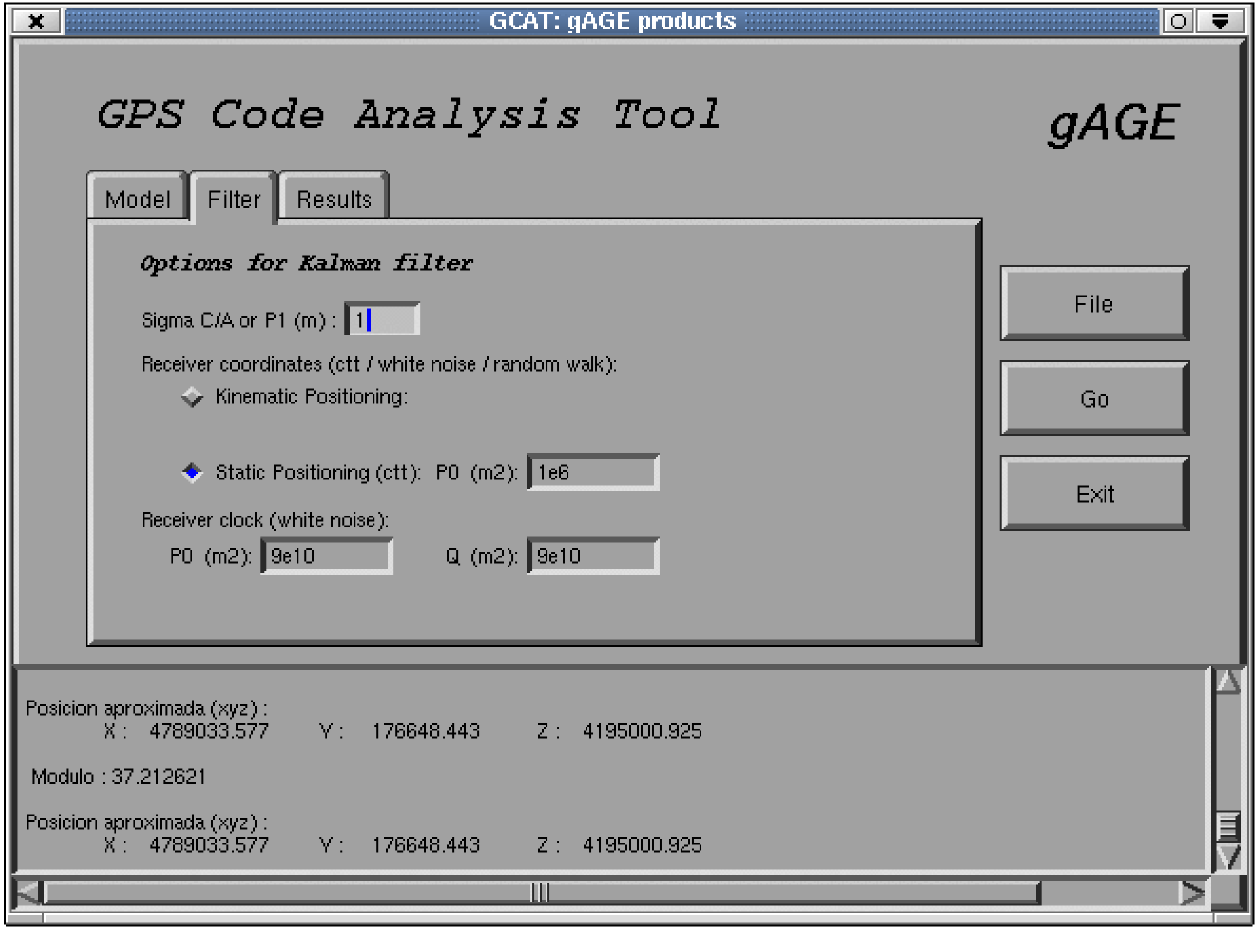
White Noise process N(0,1)



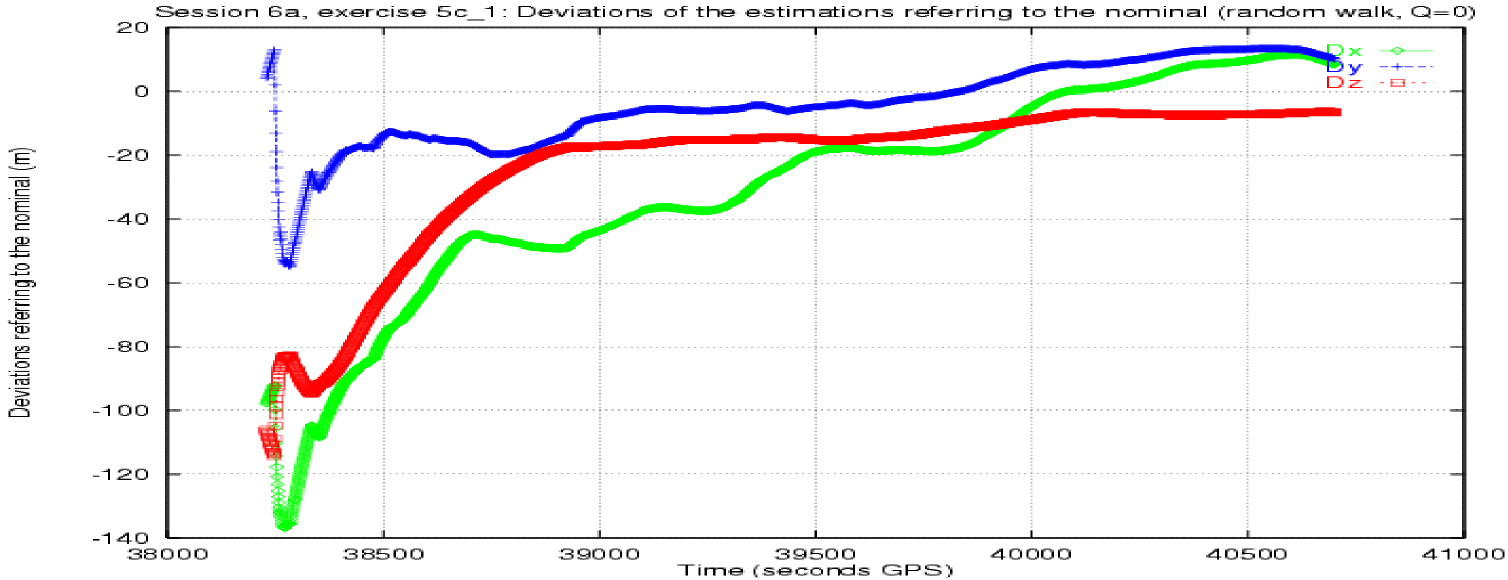
We only can assume that, the next **x(n)** can be **x(n)=0 with a confidence** σ.

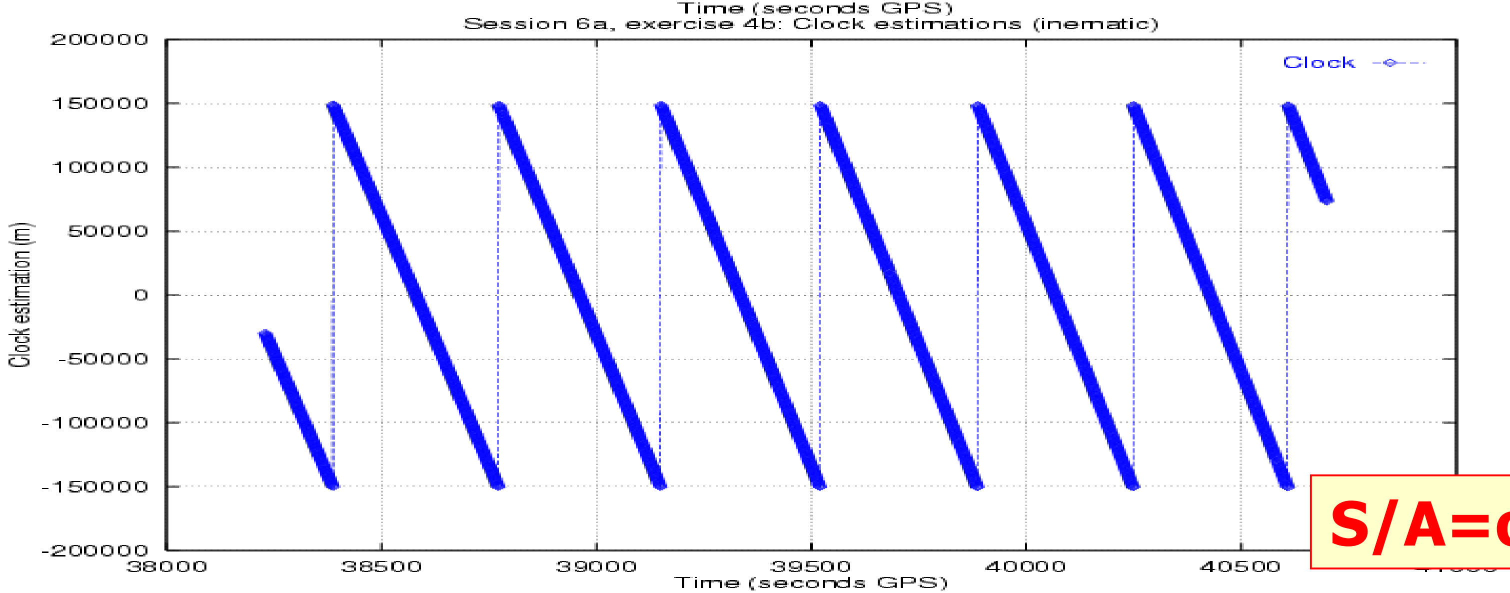
$$\hat{X}^-(n) = \Phi(n-1) \cdot \hat{X}(n-1)$$

$$P_{\hat{X}^-(n)} = \Phi(n-1) \cdot P_{\hat{X}(n-1)} \cdot \Phi^t(n-1) + Q(n-1)$$


GCAT: gAGE products
GPS Code Analysis Tool
gAGE
Model
Filter
Results
Options for Kalman filter
Sigma C/A or P1 (m) : 1
Receiver coordinates (ctt / white noise / random walk):
Kinematic Positioning:
Static Positioning (ctt): P0 (m2): 1e6
Receiver clock (white noise):
P0 (m2): 9e10
Q (m2): 9e10
File
Go
Exit
Posicion aproximada (xyz) :
X : 4789033.577 Y : 176648.443 Z : 4195000.925
Modulo : 37.212621
Posicion aproximada (xyz) :
X : 4789033.577 Y : 176648.443 Z : 4195000.925

Static positioning: constant coordinates and white noise clock





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b) Kinematic positioning

1) In case of a fast moving vehicle, coordinates will be modeled as **white noise** with zero mean, and the same rationale applies for clock offset:

$$\mathbf{\Phi}(n) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \qquad \mathbf{Q}(n) = \begin{pmatrix} \sigma^2_{dx} & & & \\ & \sigma^2_{dy} & & \\ & & \sigma^2_{dz} & \\ & & & \sigma^2_{DT} \end{pmatrix}$$

2) In case of a slow moving vehicle, coordinates may be modeled as **random walk**, process' spectral density $\dot{q} = \frac{d\sigma^2}{dt}$, and the clock as a **white noise**:

$$\mathbf{\Phi}(n) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \qquad \mathbf{Q}(n) = \begin{pmatrix} \dot{q}_{dx}\Delta t & & & \\ & \dot{q}_{dy}\Delta t & & \\ & & \dot{q}_{dz}\Delta t & \\ & & & \sigma^2_{DT} \end{pmatrix}$$

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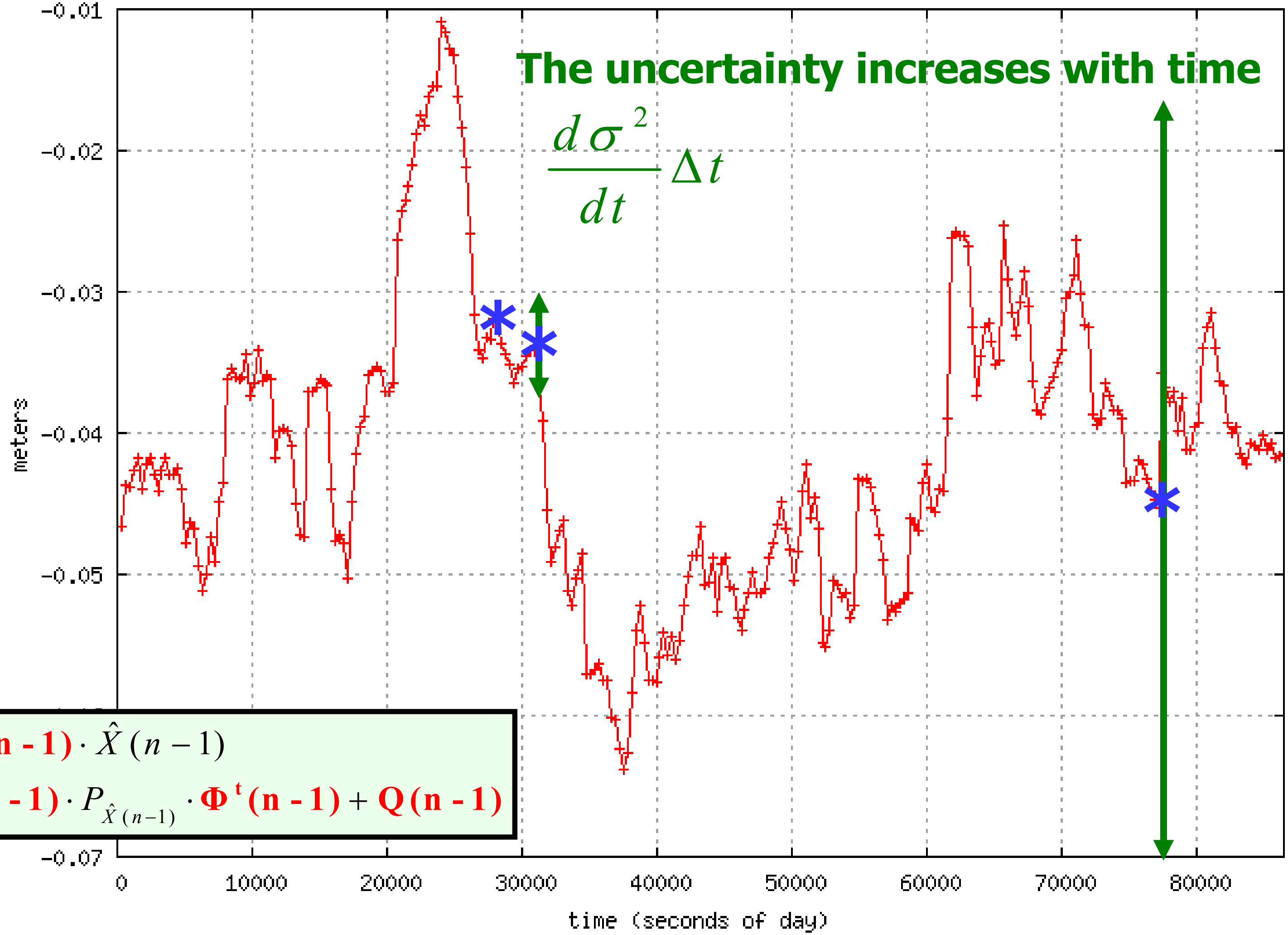
Random Walk process: it varies slowly



$$\hat{X}^-(n) = \Phi(n-1)\cdot\hat{X}(n-1)$$

$$P_{\hat{X}^-(n)} = \Phi(n-1)\cdot P_{\hat{X}(n-1)}\cdot\Phi^t(n-1) + Q(n-1)$$

$\Phi=1$ ←→ x(n)=x(n-1) (the same value is assumed)

$Q=(d\sigma^2/dt)*\Delta t$ (but, with prediction error noise increasing with time)

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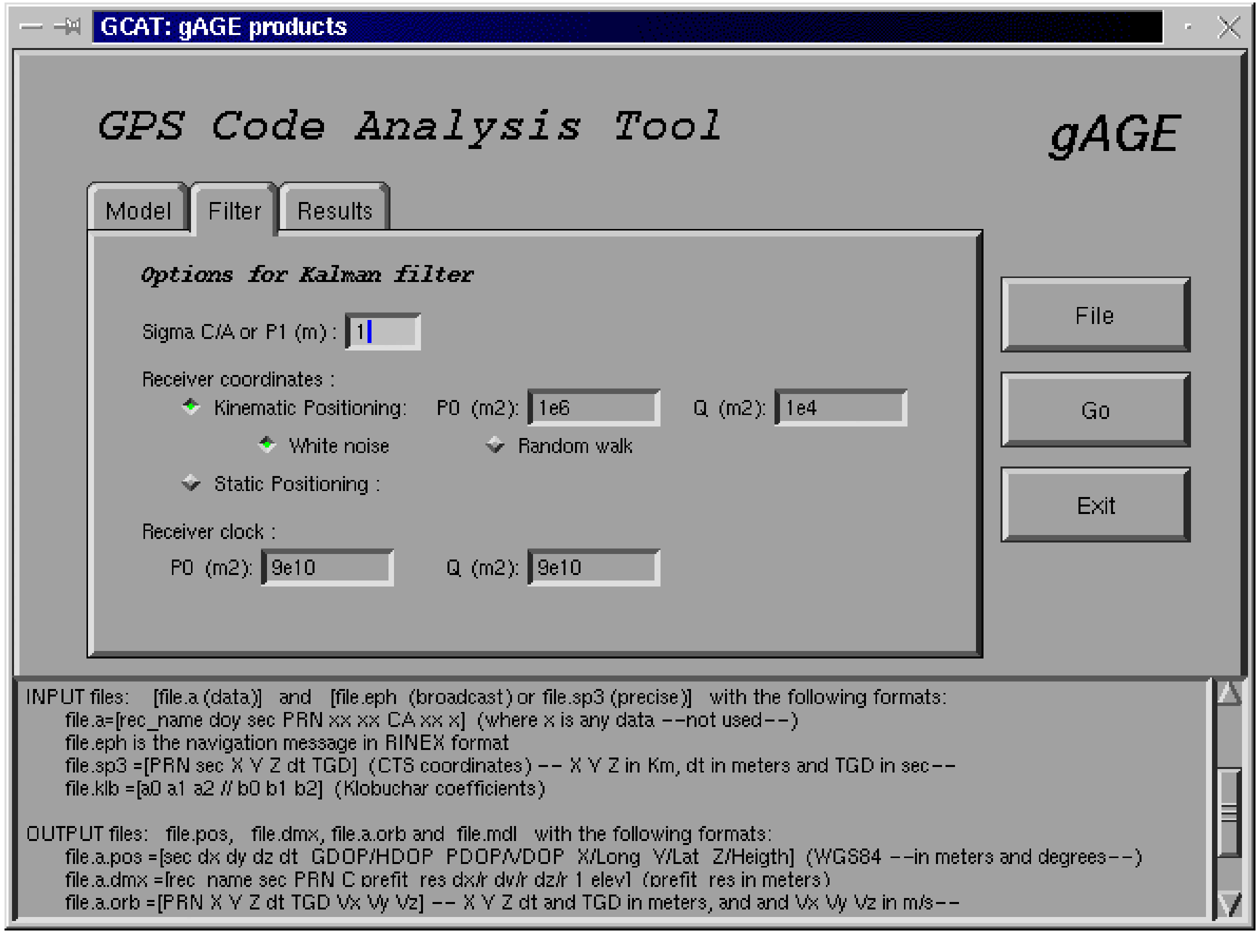
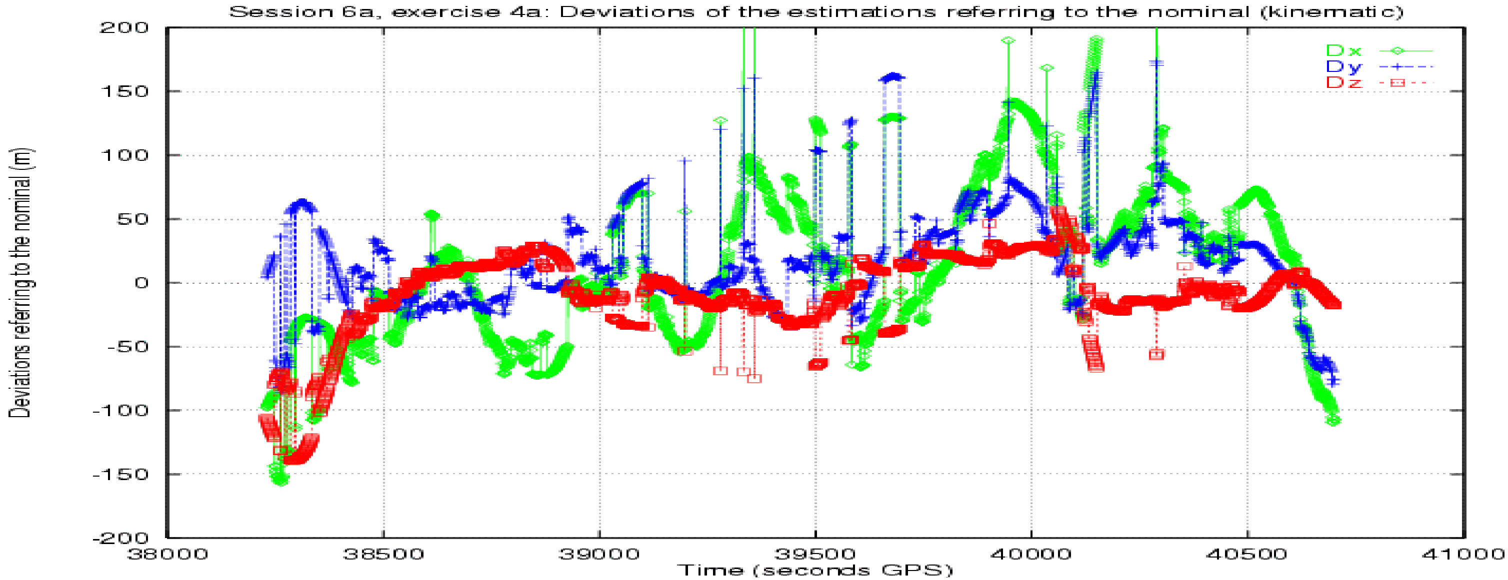
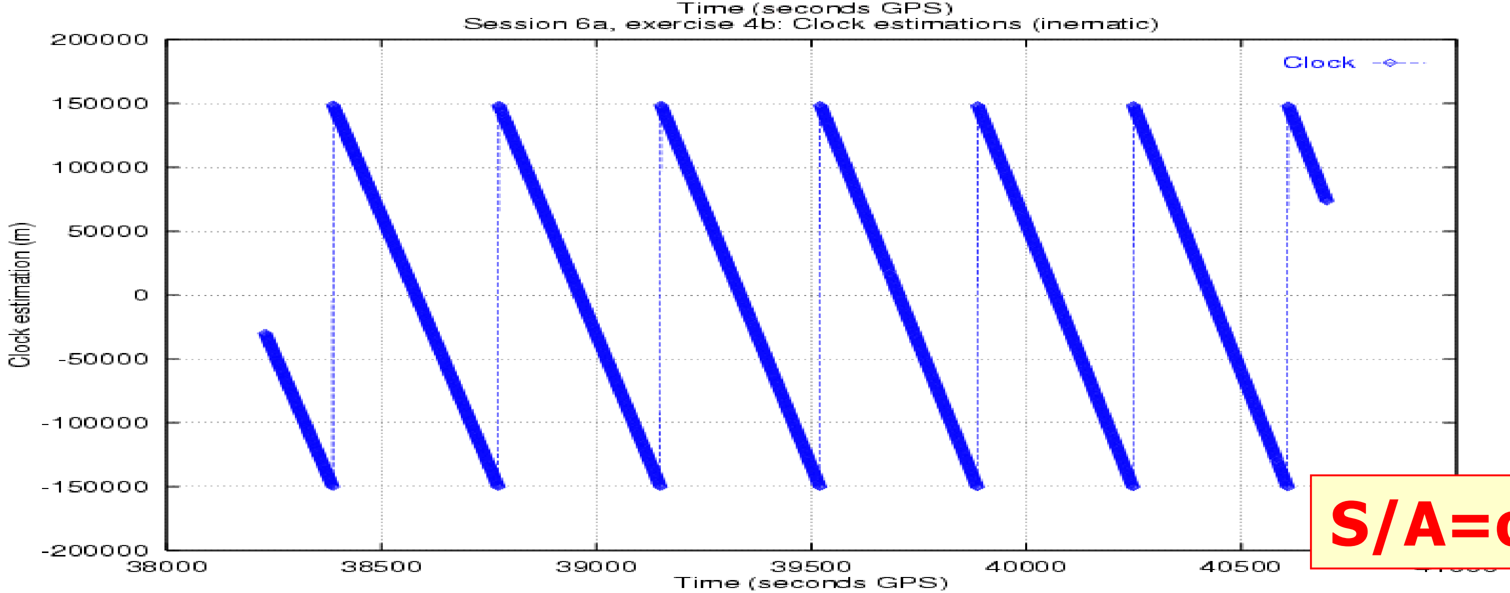


Pure Kinematic positioning: white noise coordinates and clock





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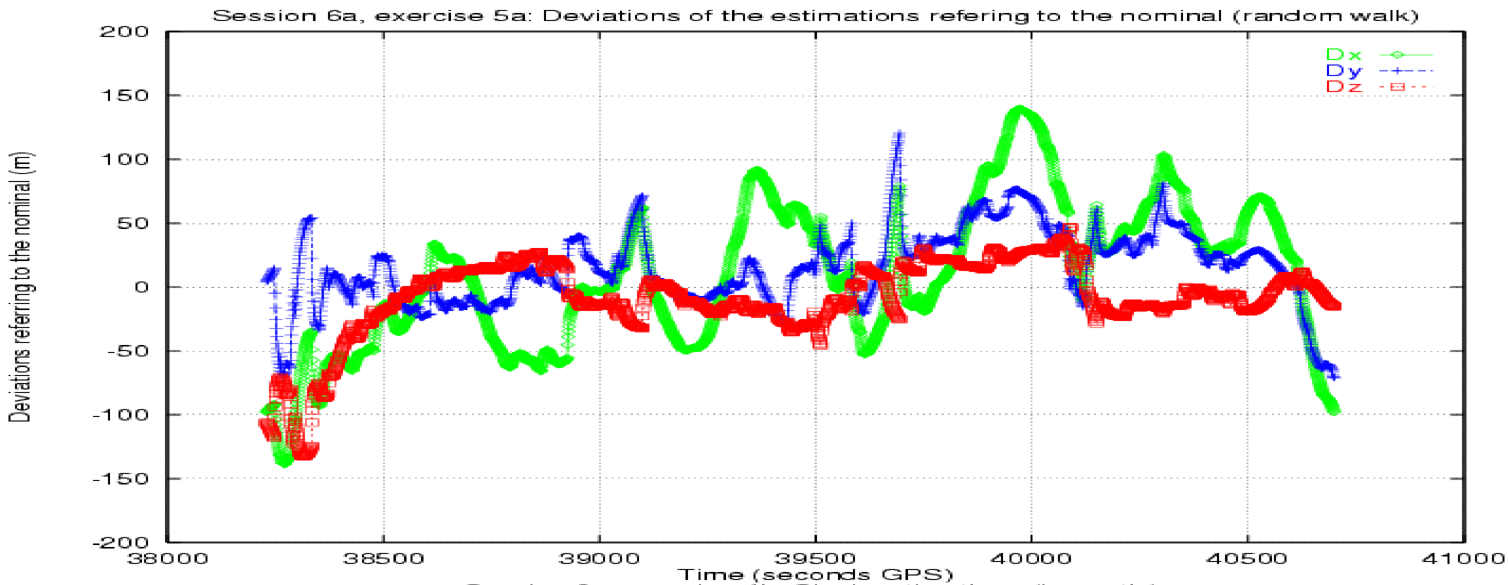
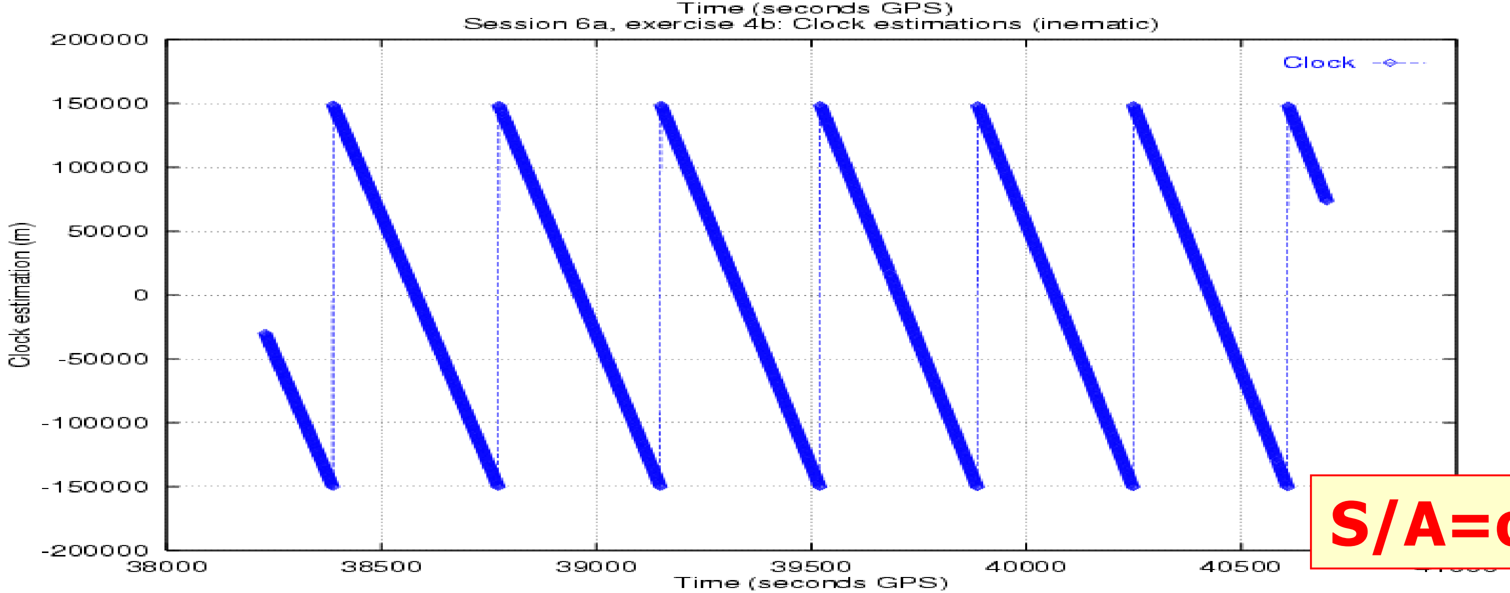
Kinematic positioning: random walk noise coordinates and white noise clock





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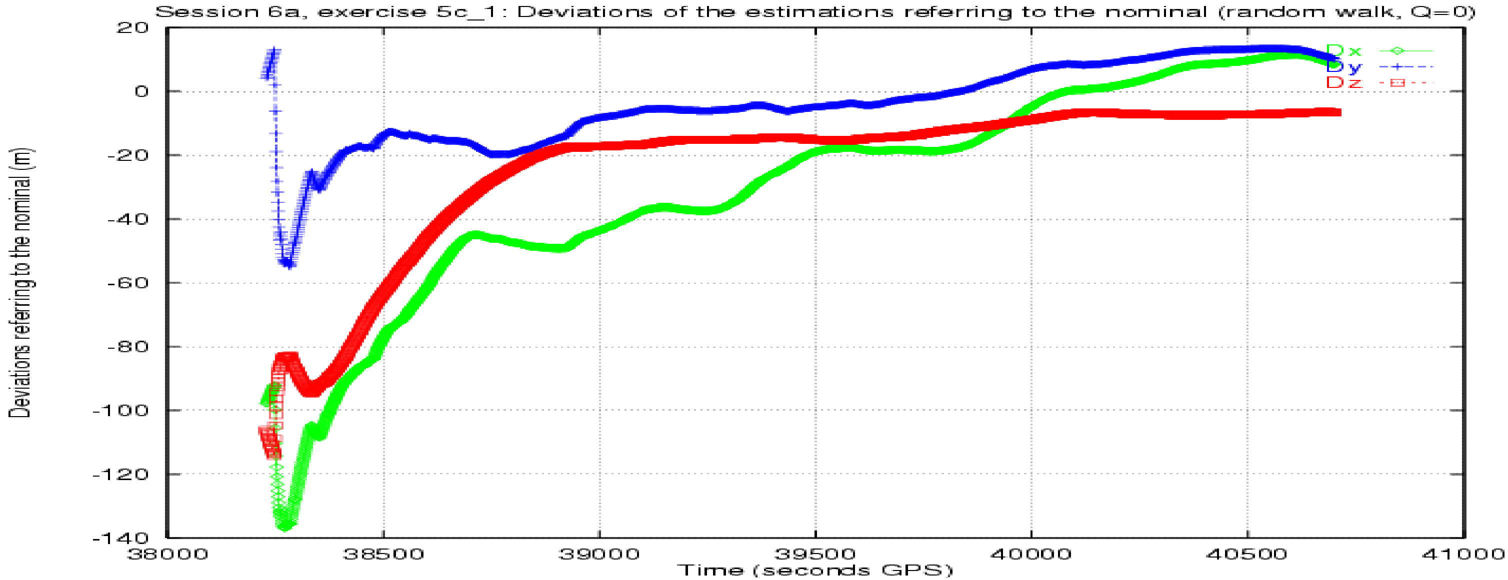
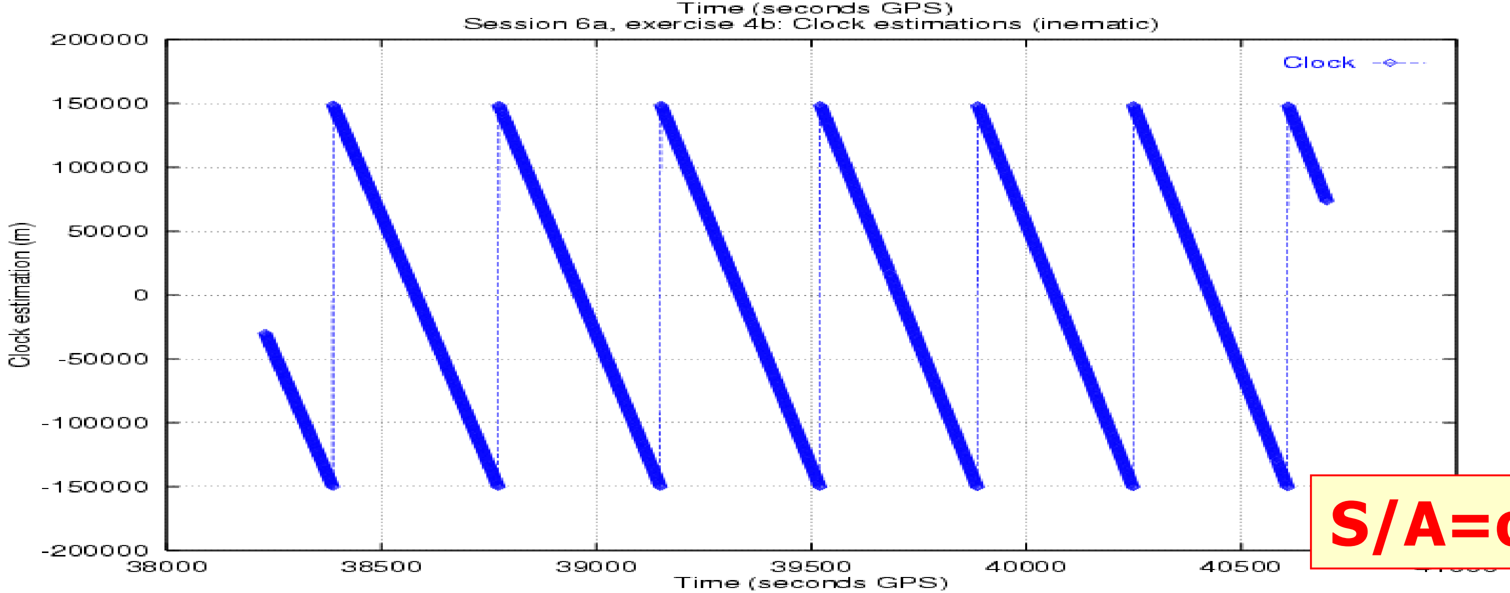
Positioning: Random walk coordinates with Q=0 and white noise clock





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Annex

Bancroft method for direct estimation of receiver position and clock offset

Bancroft method allows to get a direct solution of receiver coordinates and clock offset *(x,y,z,DT)*, without any "a priori" knowledge. So, this method may be used to obtain the initial value (x_0, y_0, z_0) for navigation equations (cold start)

$$\left\{\begin{array}{l} P^1 = \sqrt{(x-x^1)^2 + (y-y^1)^2 + (z-z^1)^2} + cDT \\ \vdots \\ P^n = \sqrt{(x-x^n)^2 + (y-y^n)^2 + (z-z^n)^2} + cDT \end{array}\right. \Rightarrow \begin{bmatrix} x \\ y \\ z \\ cDT \end{bmatrix} = \mathbf{M}\mathbf{B}^{-1}(\lambda\mathbf{1} + \mathbf{a})$$

Where:

$$\langle \mathbf{B}^{-1}\mathbf{1}, \mathbf{B}^{-1}\mathbf{1} \rangle \lambda^2 + 2\left[\langle \mathbf{B}^{-1}\mathbf{1}, \mathbf{B}^{-1}\mathbf{a} \rangle - 1\right]\lambda + \langle \mathbf{B}^{-1}\mathbf{a}, \mathbf{B}^{-1}\mathbf{a} \rangle = 0$$

$$\mathbf{B} = \begin{pmatrix} x^1 & y^1 & z^1 & P^1 \\ \vdots & \vdots & \vdots & \vdots \\ x^n & y^n & z^n & P^n \end{pmatrix} \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad a_j = \frac{1}{2}\left\langle \begin{bmatrix} x^j \\ y^j \\ z^j \\ P^j \end{bmatrix}, \begin{bmatrix} x^j \\ y^j \\ z^j \\ P^j \end{bmatrix} \right\rangle$$

Satellite coordinates

Measured pseudorange

$$\langle \mathbf{a}, \mathbf{b} \rangle = [a_1,, a_2, a_3, a_4] \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Comments:

- To improve the estimation, the measured pseudorange can be substituted by:

$$\boxed{P^j} \rightarrow \boxed{P^j + dt^j - rel^j - TGD^j}$$

Nevertheless, to compute the ionospheric and tropospheric delay, the receiver coordinates are needed.

- Thence, after computing the approximate receiver coordinates, a second iteration is needed to compute the solution using all the corrections (which is done using the linearized equations).

Lesson 7

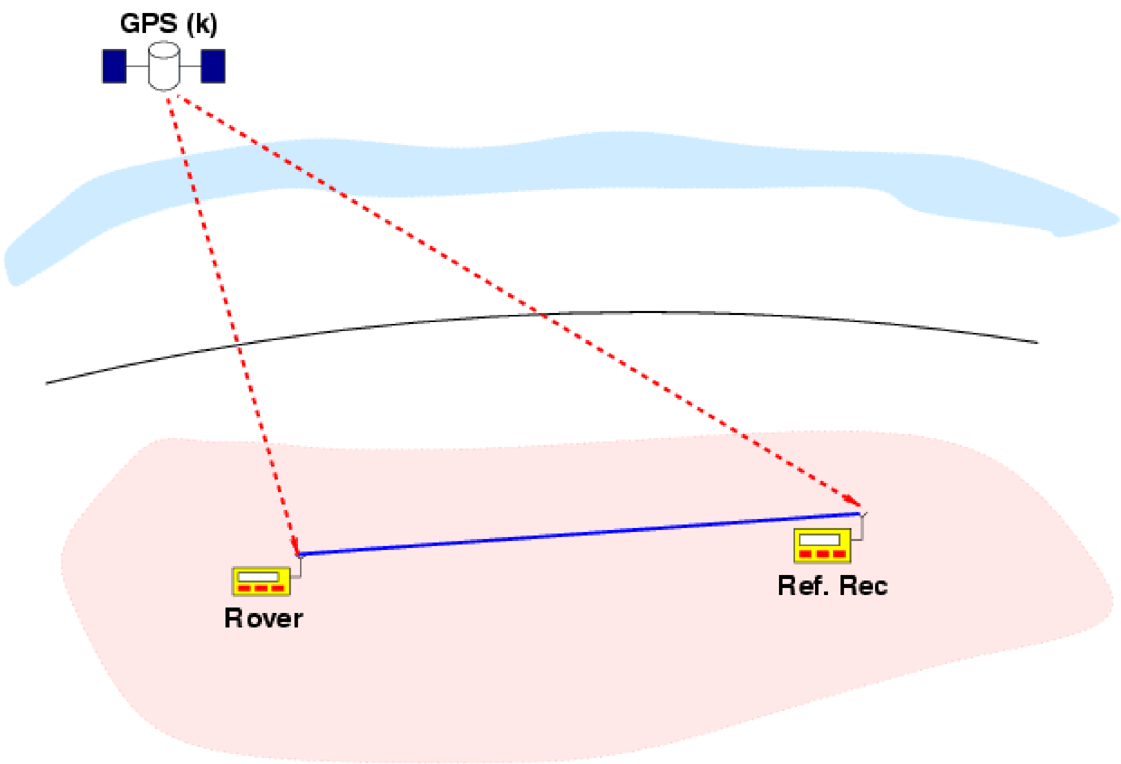
Code and phase differential positioning.

Floating versus fixing ambiguities

Differential Positioning

A receiver (rover) is positioned regarding to another receiver (reference) whose coordinates are known.

It allows to cancel out most of the common errors between both receivers, improving dramatically the positioning accuracy.



ERORS on the Signal

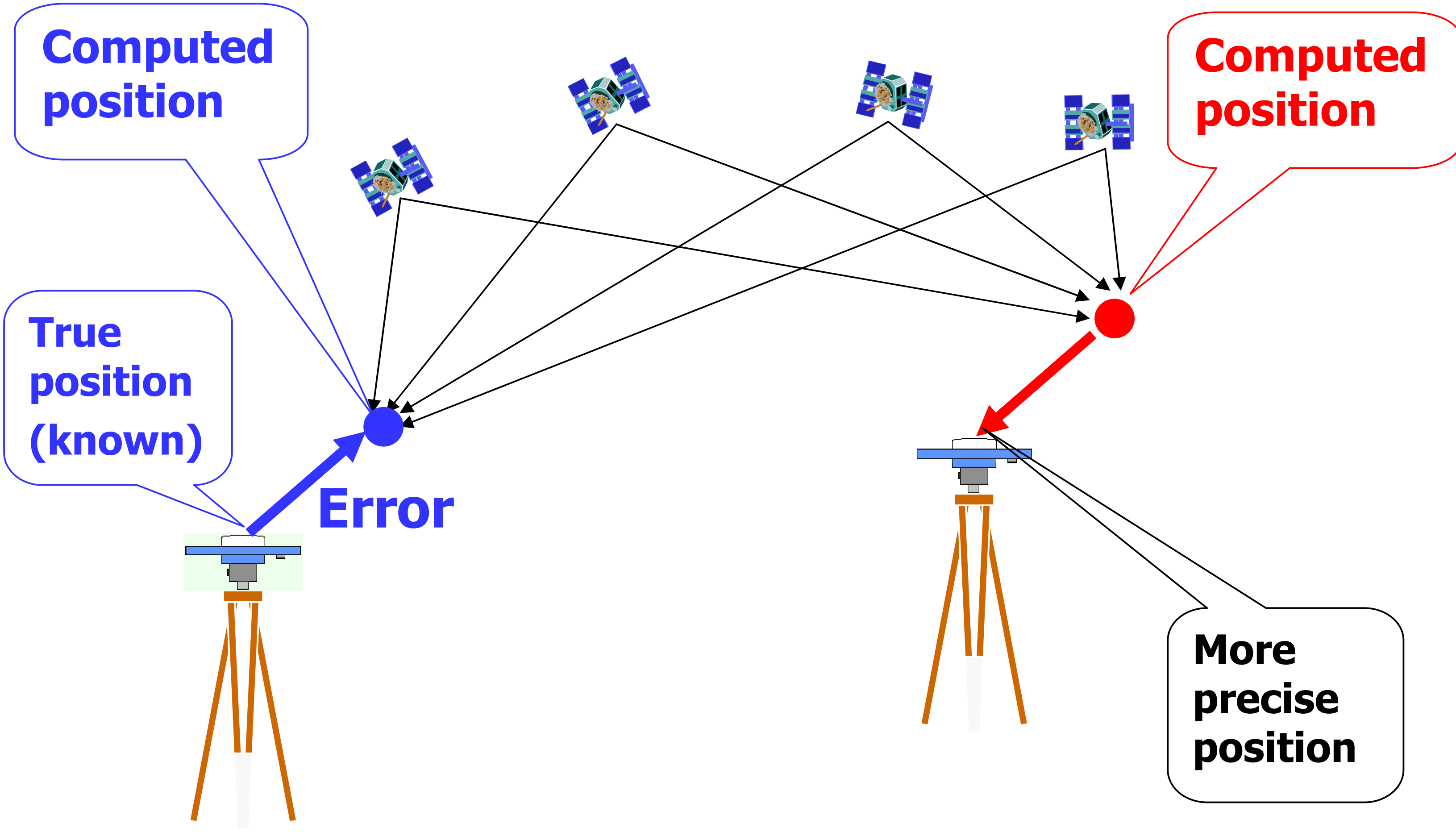
- Space Segment Errors:
  - Clock errors — Common
  - Ephemeris errors
- Propagation Errors
  - Ionospheric delay
  - Tropospheric delay — Weak spatial correlation

Ephemeris errors and Ionospheric delay — Strong spatial correlation

- Local Errors
  - Multipath
  - Receiver noise

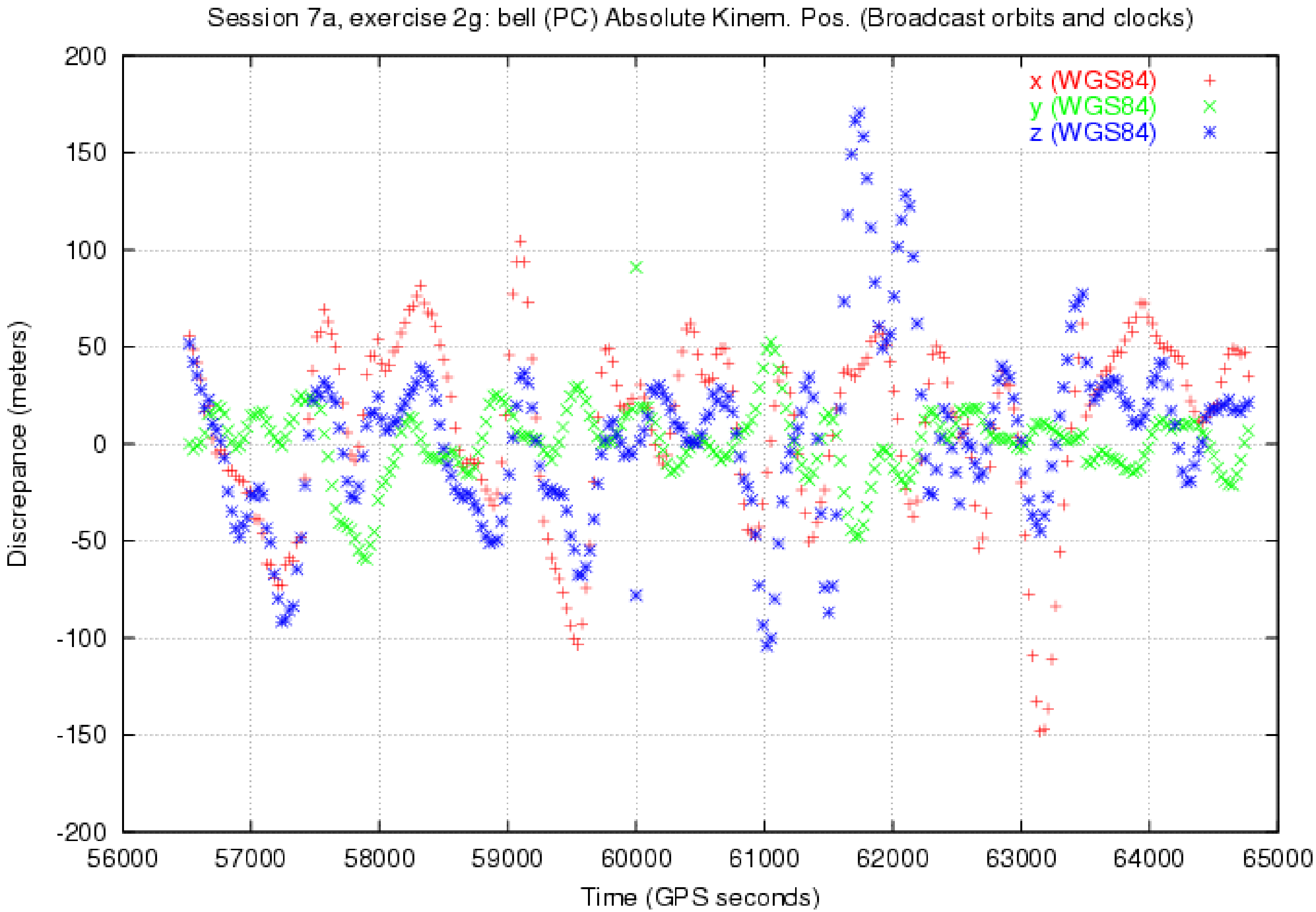
Multipath and Receiver noise — No spatial correlation

DIFFERENTIAL GPS (DGPS) with code



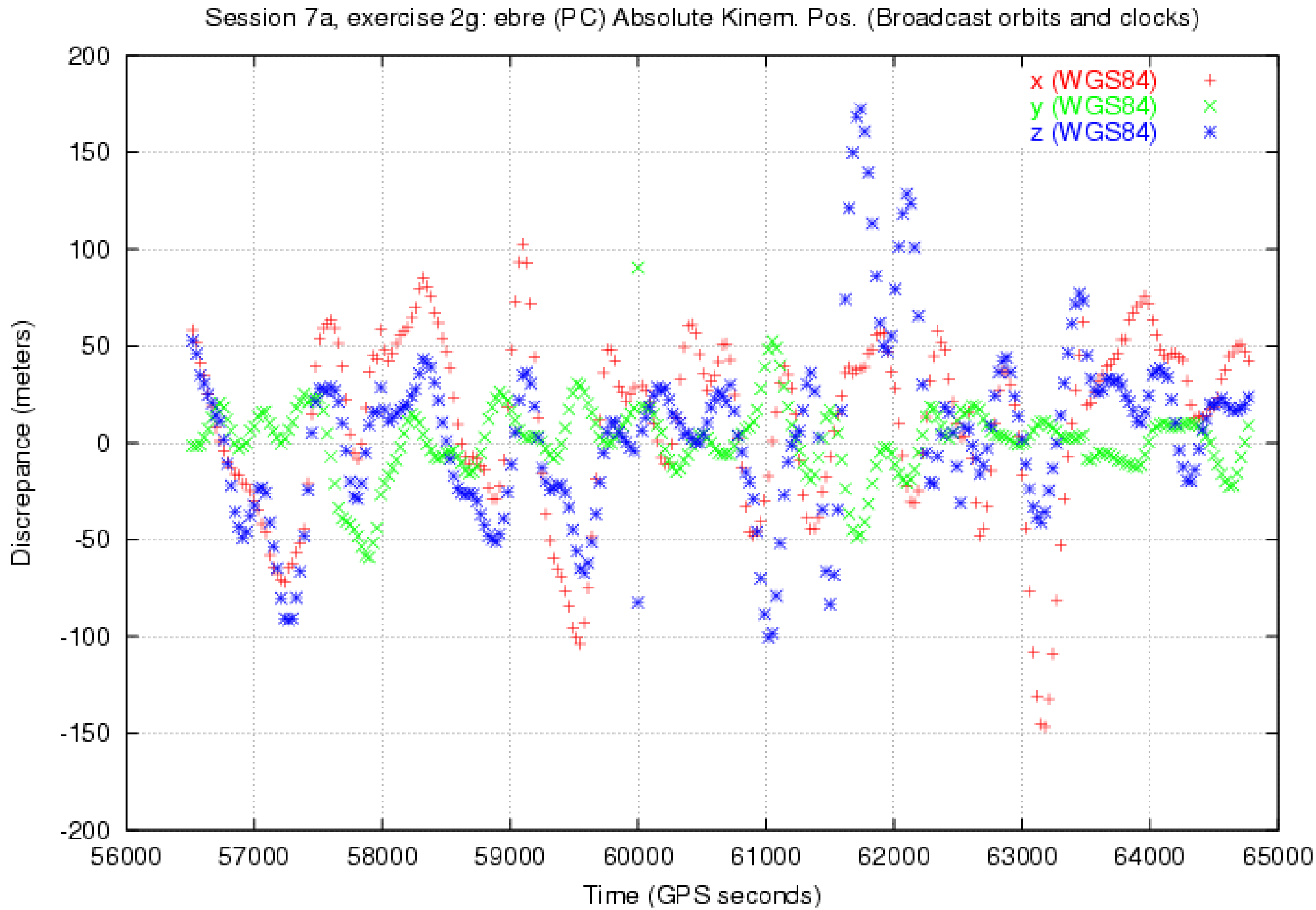
In short baselines, similar errors are expected

bell: Absolute positioning (S/A=on)



Positioning error in station "bell" (S/A=on)

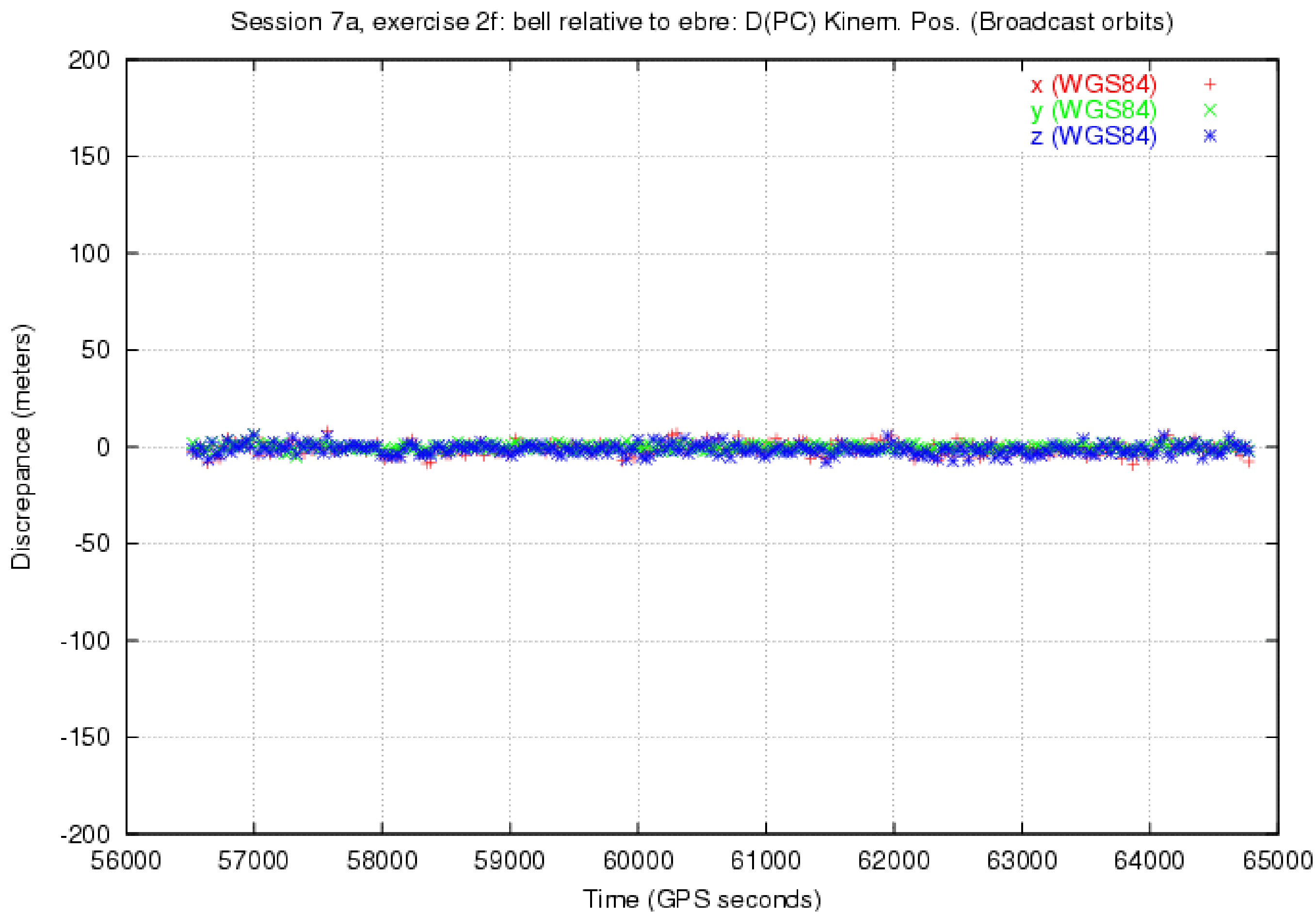
ebre: Absolute positioning (S/A=on)



Basically, the same errors are found in station "ebre", 100Km far from "bell"

(the same satellites are used in the navigation solution)

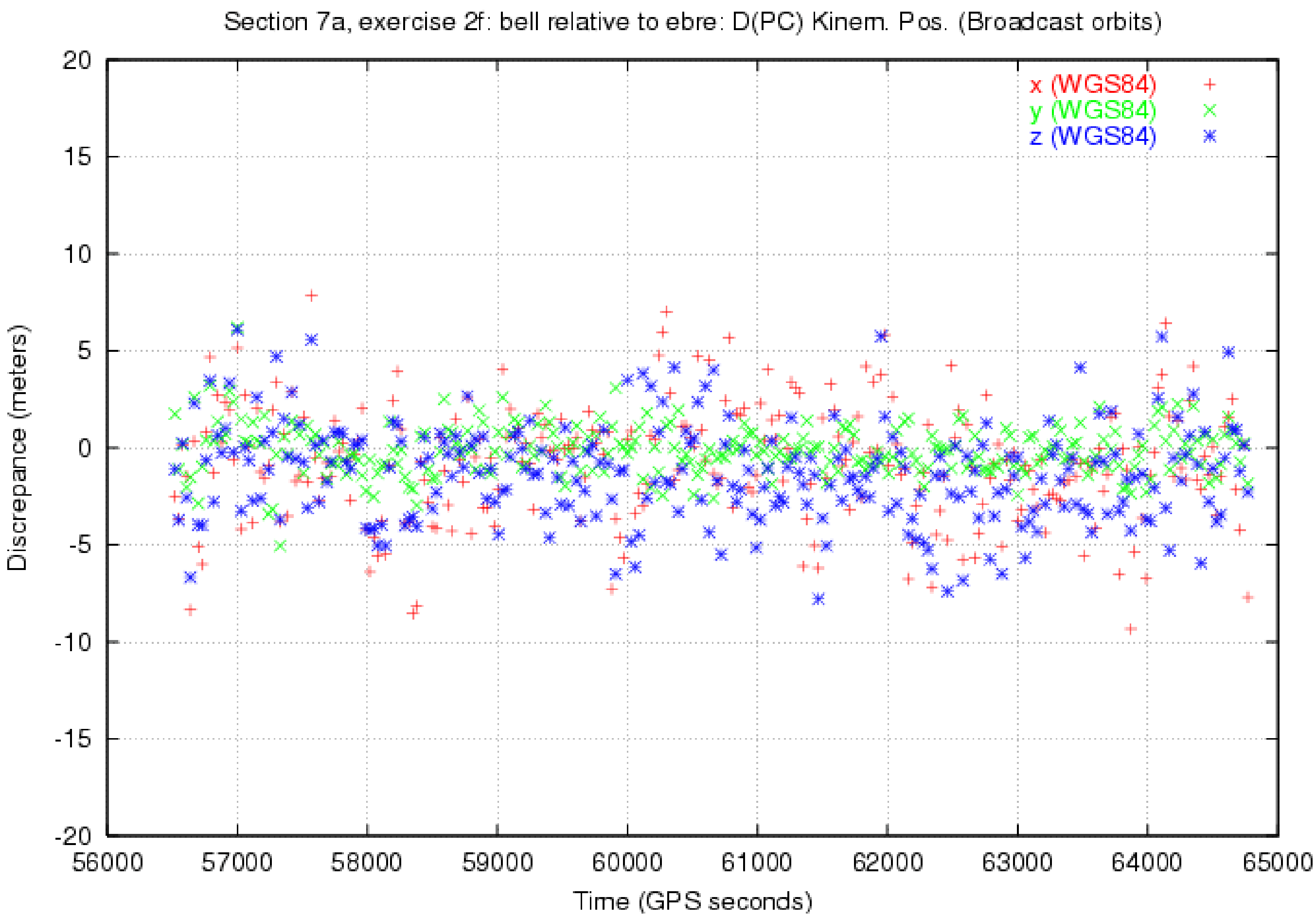
bell-ebre (S/A=on)



Most of the errors cancels out when computing the difference between errors in “ebre” and “bell”.

(the same satellites are used in the navigation solution)

bell-ebre (S/A=on)



Zoom of previous plot

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In the previous example, the differential error has been cancelled in the "position" domain.

But:

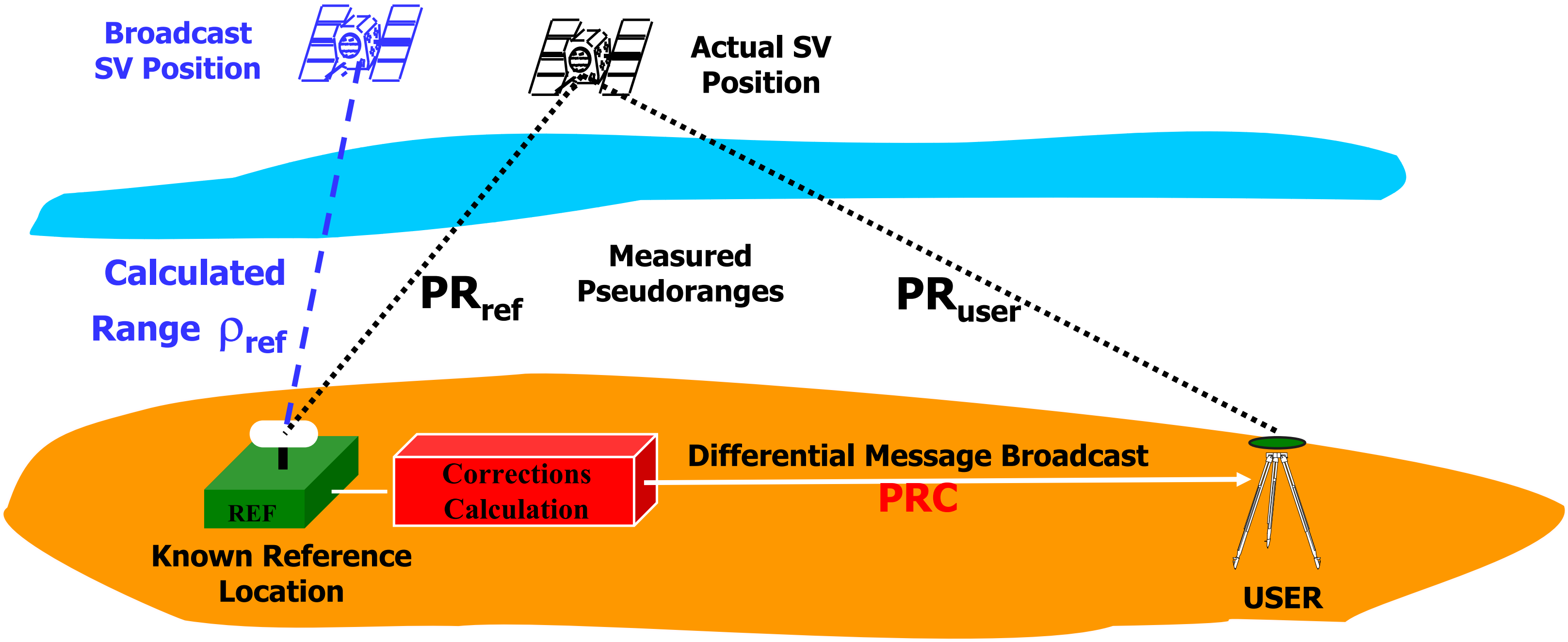
- It requires to use the same satellites in both stations.

Thence, is much better to solve the problem in the "pseudorange" domain than in the "position" domain.

Concept:

- A reference station (its exact position is known) computes a differential correction for each satellite in view.
- The user applies this correction to remove most part of the common errors in pseudorange.

Range Differential Correction Calculation



– The first receiver, in a reference station, can calculate these errors knowing its exact location (corrections "PRC" calculated by the ground station): $PRC = PR_{ref} - \rho_{ref}$

– The second receiver (the user) will use these corrections to adjust its own measurements and increase the accuracy of these measurements: $PR_{user} - PRC$

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1. Differential positioning (with simple differences)

Let's P^j_{rov} and P^j_{ref} be code measurements of rover (rov) and reference station (ref), respectively, for satellite "*j*". Thence:

$$Prefit^j_{rov} = \frac{x_{0,rov} - x^j}{\rho^j_{0,rov}} dx_{rov} + \frac{y_{0,rov} - y^j}{\rho^j_{0,rov}} dy_{rov} + \frac{z_{0,rov} - z^j}{\rho^j_{0,rov}} dz_{rov} + cDT_{rov} + \varepsilon^j_{rov}$$

$$Prefit^j_{ref} = 0 + 0 + 0 + cDT_{ref} + \varepsilon^j_{ref}$$

Introducing the following notation to symbolize the simple difference of measurements $\Delta \square^j \equiv \square^j_{rov} - \square^j_{ref}$, the difference of previous equation can be written as:

$$\Delta Prefit^j = \frac{x_{0,rov} - x^j}{\rho^j_{0,rov}} dx_{rov} + \frac{y_{0,rov} - y^j}{\rho^j_{0,rov}} dy_{rov} + \frac{z_{0,rov} - z^j}{\rho^j_{0,rov}} dz_{rov} + \Delta(cDT) + \Delta\varepsilon^j$$

And, considering all satellites in view from both receivers, the following equation system can be written. This system is similar to that of absolute positioning, but the relative clock between both receivers is instead estimated: $\Delta(cDT)=cDT_{rov}-cDT_{ref}$

$$\begin{bmatrix} \Delta Prefit^1 \\ \Delta Prefit^2 \\ \cdots\cdots \\ \Delta Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{o,rov}-x^1}{\rho^1_{0,rov}} & \frac{y_{o,rov}-y^1}{\rho^1_{0,rov}} & \frac{z_{o,rov}-z^1}{\rho^1_{0,rov}} & 1 \\ \frac{x_{o,rov}-x^2}{\rho^2_{0,rov}} & \frac{y_{o,rov}-y^2}{\rho^2_{0,rov}} & \frac{z_{o,rov}-z^2}{\rho^2_{0,rov}} & 1 \\ & \cdots\cdots\cdots & & \\ \frac{x_{o,rov}-x^n}{\rho^n_{0,rov}} & \frac{y_{o,rov}-y^n}{\rho^n_{0,rov}} & \frac{z_{o,rov}-z^n}{\rho^n_{0,rov}} & 1 \end{bmatrix} \begin{bmatrix} dx_{rov} \\ dy_{rov} \\ dz_{rov} \\ \boxed{\Delta(cDT)} \end{bmatrix}$$

This system can be solved applying the same mathematic tools than in absolute positioning (LMS, WMS, Kalman filtering,...)

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Residual GPS Pseudorange Errors

	Without DGPS Correction		Zero Basline Zero Latency DGPS		Decorrelation with Latency		Geographic Decorrelation (m/100 Km)
ERRORS	**Bias (meters)**	**Random (meters)**	**Bias (meters)**	**Random (meters)**	**Velocity (m/s)**	**Acceleration (m/s2)**	
Receiver Noise	0.5	0.2	0.5	0.3	0.0	0.0	0.0
Multipath	0.3 to 3.0	0.2 to 1.0	0.4 to 3.0	0.2 to 1.0	0.0	0.0	0.0
Satellite Clock (S/A=off)	2.0	0.0	0.0	0.10	0.02	0.004	0.0
Satellite Clock (S/A=on)	21.0	0.1	0.0	0.14	0.21	0.004	0.0
Ephemeris (S/A=off)	10.0 (extreme)	0.0	0.0	0.0	negl.	negl.	<0.05
Ephemeris (S/A=on)	100.0 (extreme)	0.0	0.0	0.0	<0.01	<0.001	<0.5
Ionosphere (raw iono)	2 to 10 (times obliquity)	<0.1 (times obliquity)	0.0	<0.14	0.092	negli.	<0.2
Troposphere	2 (times obliquity)	<0.1 (times obliquity)	0.0	<0.14	negl.	negli.	<0.05

2. Differential positioning with double differences

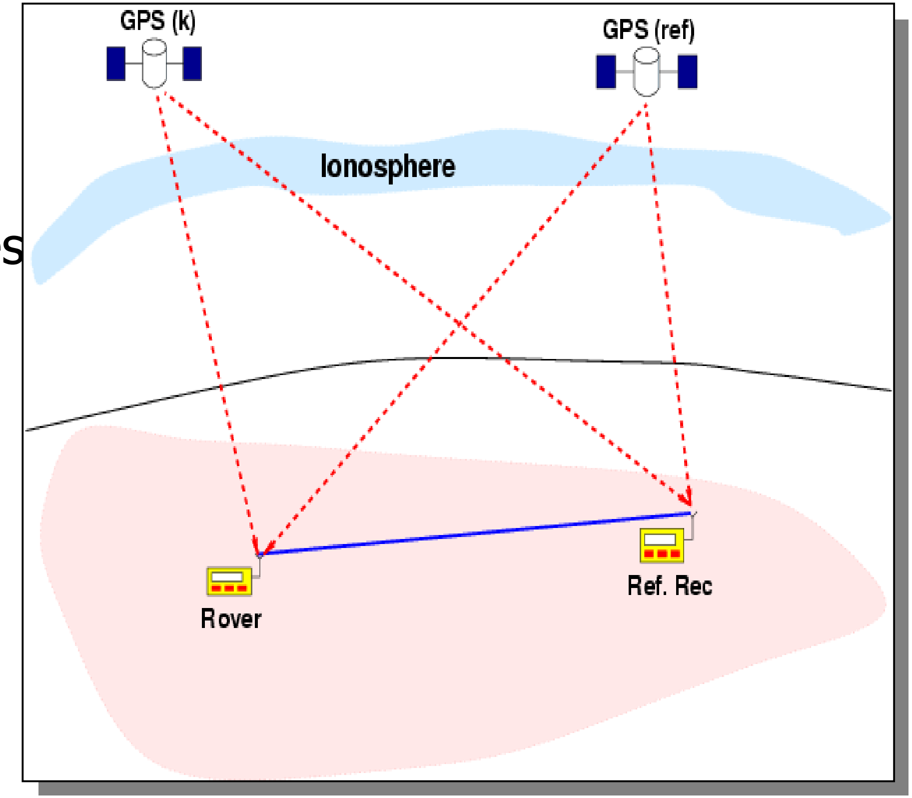
Taking one station and one satellite as a reference, the double differences between satellites and stations can be computed from the single ones

$$\Delta \square^{\bullet} \equiv \square^{\bullet}_{rov} - \square^{\bullet}_{ref}$$

$$\nabla \square_{\bullet} \equiv \square^{j}_{\bullet} - \square^{R}_{\bullet}$$

$$\Delta\nabla \square \equiv \Delta \square^{j} - \Delta \square^{R} =$$

$$= \nabla \square_{rov} - \nabla \square_{ref}$$



Thence:

$$\Delta\nabla \square^{j} = \left[\square^{j}_{rov} - \square^{j}_{ref}\right] - \left[\square^{R}_{rov} - \square^{R}_{ref}\right] = \left[\square^{j}_{rov} - \square^{R}_{rov}\right] - \left[\square^{j}_{ref} - \square^{R}_{ref}\right]$$

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Double differences of measurement equations:

$$Prefit_{rov}^{j} = \frac{x_{0,rov} - x^{j}}{\rho_{0,rov}^{j}} dx_{rov} + \frac{y_{0,rov} - y^{j}}{\rho_{0,rov}^{j}} dy_{rov} + \frac{z_{0,rov} - z^{j}}{\rho_{0,rov}^{j}} dz_{rov} + \cancel{cDT_{rov}} + \varepsilon_{rov}^{j}$$

$$Prefit_{rov}^{R} = \frac{x_{0,rov} - x^{R}}{\rho_{0,rov}^{R}} dx_{rov} + \frac{y_{0,rov} - y^{R}}{\rho_{0,rov}^{R}} dy_{rov} + \frac{z_{0,rov} - z^{R}}{\rho_{0,rov}^{R}} dz_{rov} + \cancel{cDT_{rov}} + \varepsilon_{rov}^{R}$$

$$Prefit_{ref}^{j} = 0 + 0 + 0 + \cancel{cDT_{ref}} + \varepsilon_{ref}^{j}$$

$$Prefit_{ref}^{R} = 0 + 0 + 0 + \cancel{cDT_{ref}} + \varepsilon_{ref}^{R}$$

$$\Delta\nabla Prefit^{j} = \nabla\left[\frac{x_{0,rov} - x^{j}}{\rho_{0,rov}^{j}}\right] dx_{rov} + \nabla\left[\frac{y_{0,rov} - y^{j}}{\rho_{0,rov}^{j}}\right] dy_{rov} + \nabla\left[\frac{z_{0,rov} - z^{j}}{\rho_{0,rov}^{j}}\right] dz_{rov} + \Delta\nabla\varepsilon^{j}$$

Where:

$$\Delta\nabla Prefit^{j} \equiv \left[Prefit_{rov}^{j} - Prefit_{rov}^{R}\right] - \left[Prefit_{ref}^{j} - Prefit_{ref}^{R}\right]$$

$$\nabla\left[\frac{x_{0,rov} - x^{j}}{\rho_{0,rov}^{j}}\right] \equiv \frac{x_{0,rov} - x^{j}}{\rho_{0,rov}^{j}} - \frac{x_{0,rov} - x^{R}}{\rho_{0,rov}^{R}} \quad ; \quad \cdots$$

And applying the double differences to the measurement equations, it follows that:

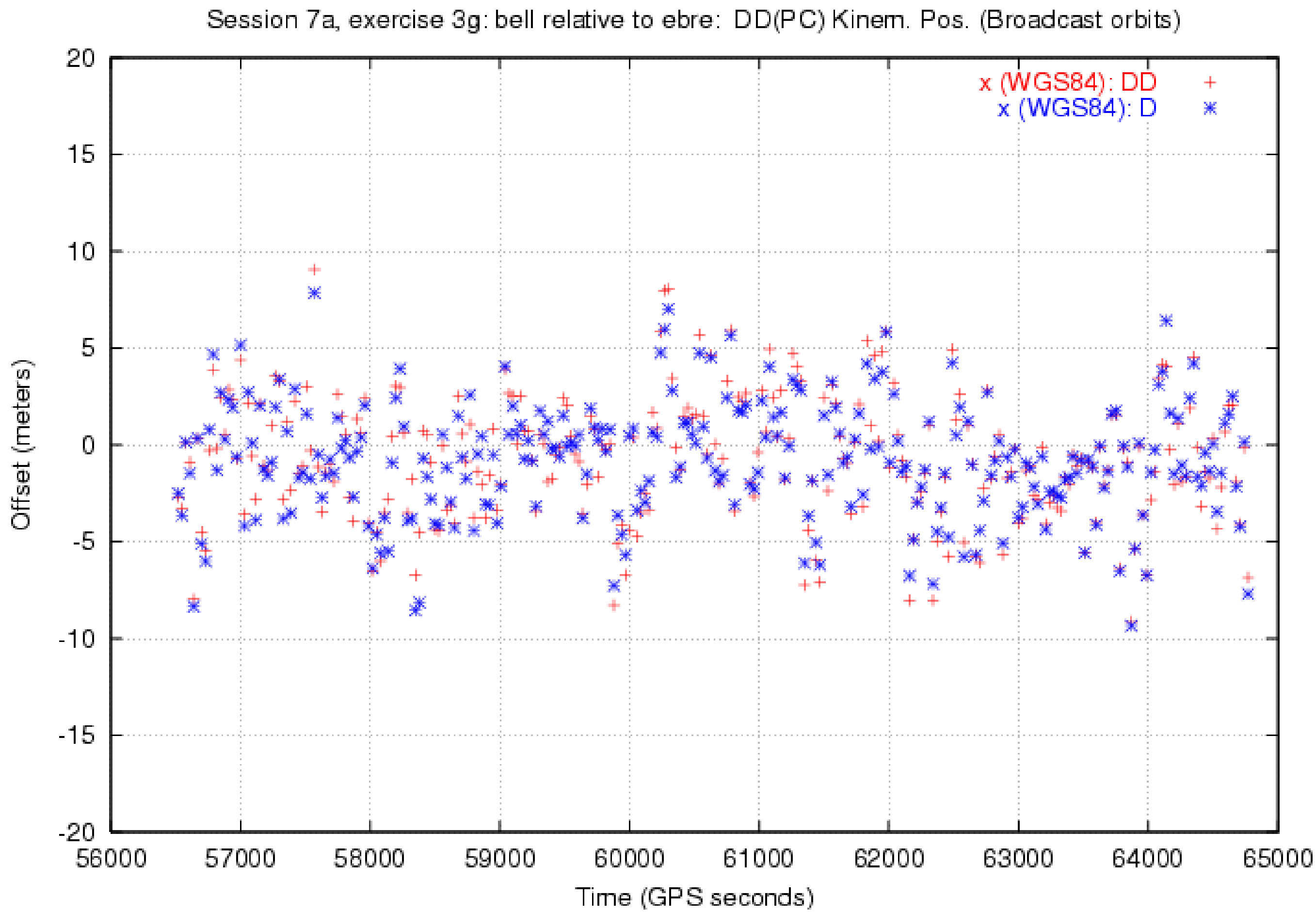
$$\Delta\nabla Prefit^j = \nabla\left[\frac{x_{0,rov}-x^j}{\rho^j_{0,rov}}\right]dx_{rov} + \nabla\left[\frac{y_{0,rov}-y^j}{\rho^j_{0,rov}}\right]dy_{rov} + \nabla\left[\frac{z_{0,rov}-z^j}{\rho^j_{0,rov}}\right]dz_{rov} + \Delta\nabla\varepsilon^j$$

where: $\Delta\nabla(cDT)=\nabla\left[\Delta\left(cDT\right)\right]=0$

Considering all satellites in view from both receivers, the following equation system may be written, where the clock term is cancelled.

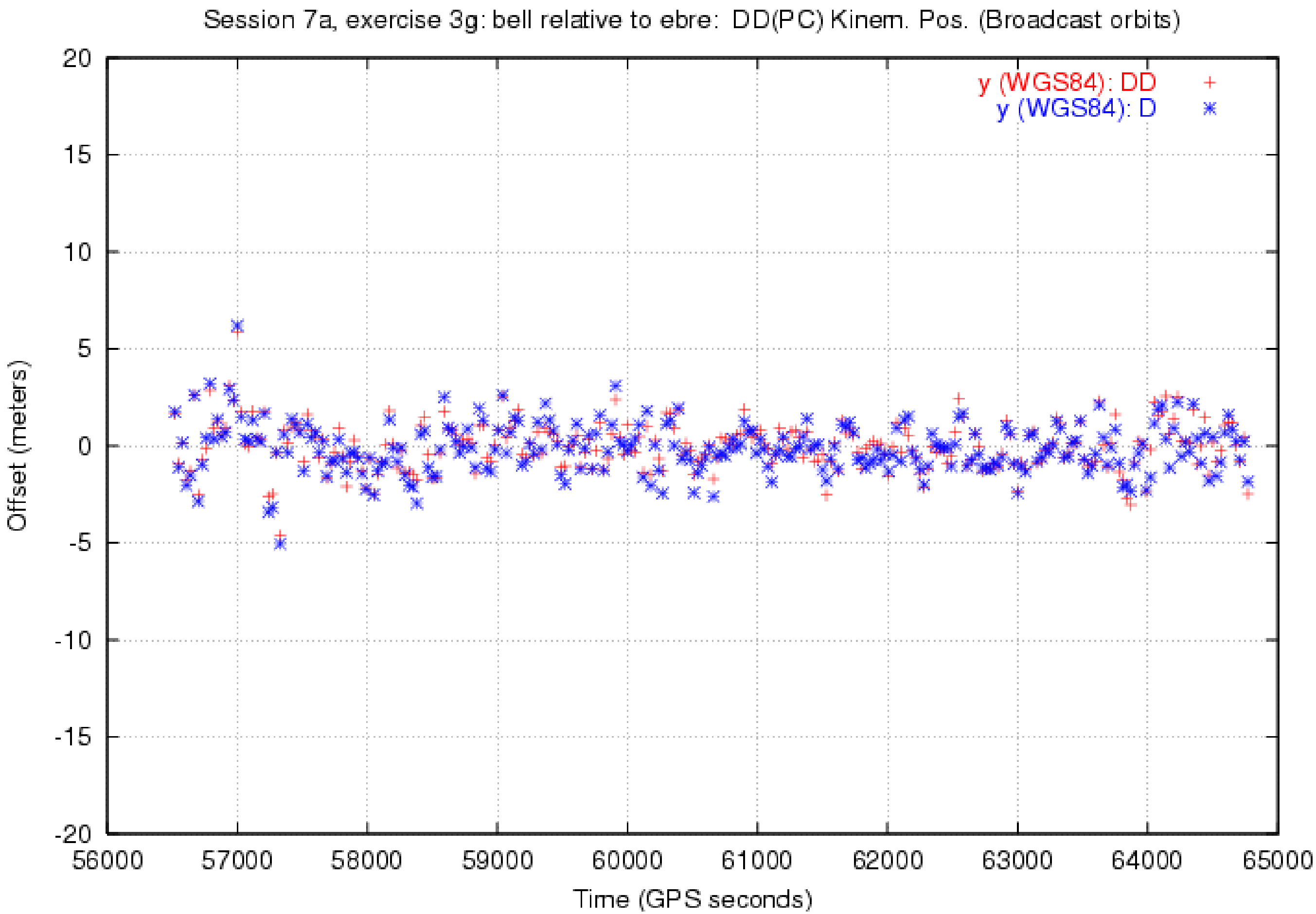
$$\begin{bmatrix} \Delta\nabla Prefit^1 \\ \Delta\nabla Prefit^2 \\ \cdots\cdots \\ \Delta\nabla Prefit^n \end{bmatrix} = \begin{bmatrix} \nabla\left[\frac{x_{o,rov}-x^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^1_{0,rov}}\right] \\ \nabla\left[\frac{x_{o,rov}-x^2}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^2}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^2}{\rho^2_{0,rov}}\right] \\ & \cdots\cdots\cdots & \\ \nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] \end{bmatrix} \begin{bmatrix} dx_{rov} \\ dy_{rov} \\ dz_{rov} \end{bmatrix}$$

Double versus Single difference: X (WGS-84)



Positioning error in station "bell" (S/A=on)

Double versus Single difference: Y (WGS-84)



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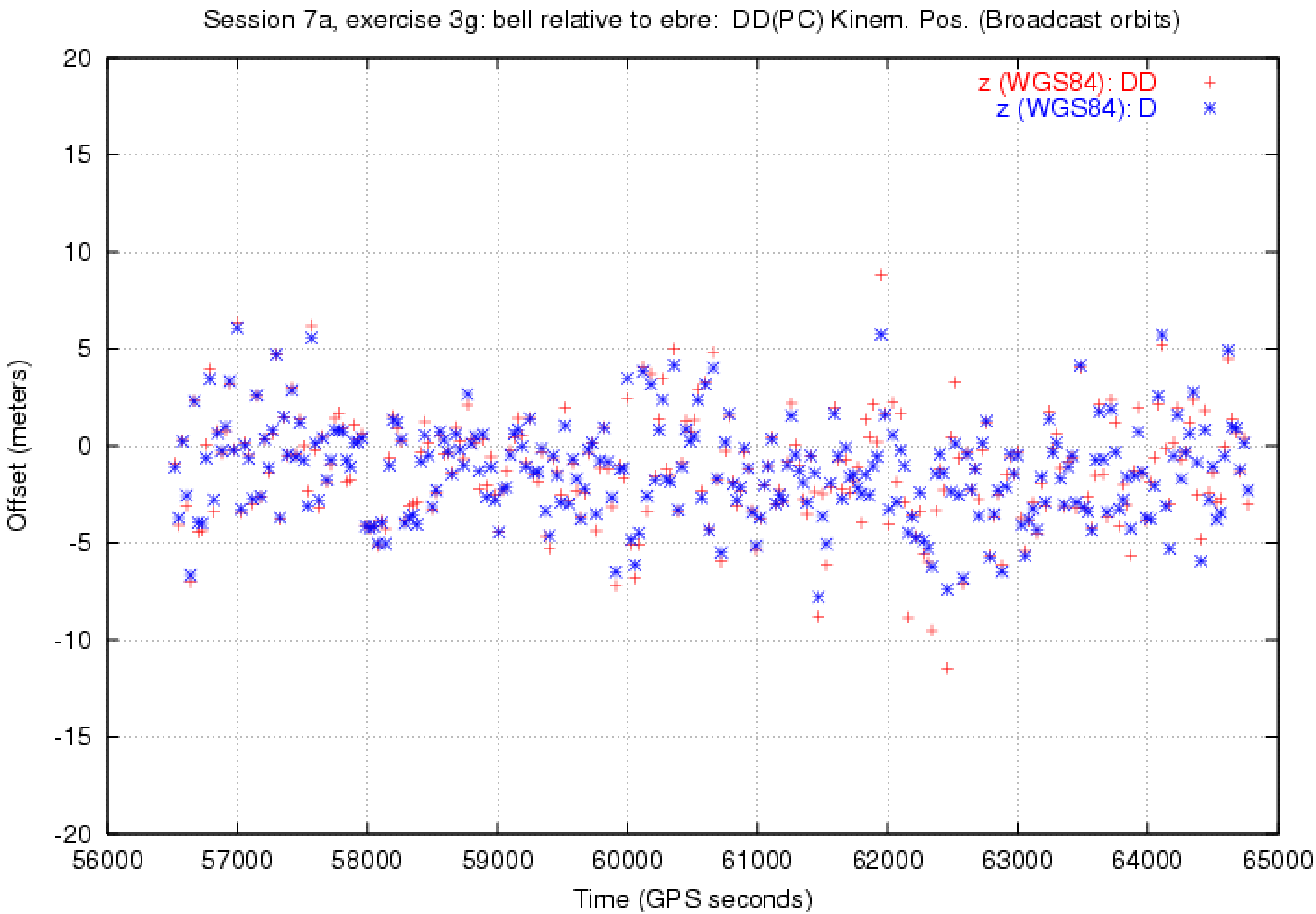
Double versus Single difference: Z (WGS-84)



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As it has been seen in the previous plots, double differences and single differences performs similar when positioning with code pseudoranges.

Nevertheless, doubles differences have an important application when positioning with carrier phases, in particular for the fixing ambiguities techniques.

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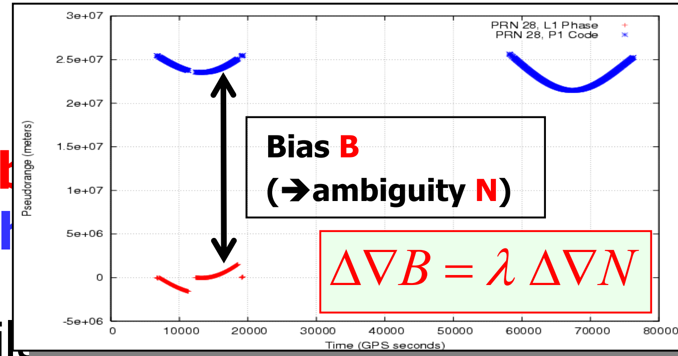
3. Carrier phase positioning

3.1 Differential positioning with doub… and phase measurements (floating th…



Phase measurements are modeled in simil… taking into account the phase **ambiguities** ***$\Delta\nabla B$***, which must be estimated together with rover coordinates *(dx,dy,dz)*.

$$
\begin{bmatrix}
\Delta\nabla Prefit(P)^1 \\
\Delta\nabla Prefit(L)^1 \\
\cdots\cdots \\
\Delta\nabla Prefit(P)^n \\
\Delta\nabla Prefit(L)^n
\end{bmatrix}
=
\begin{pmatrix}
\nabla\left[\frac{x_{o,rov}-x^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^1_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\nabla\left[\frac{x_{o,rov}-x^1}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^2_{0,rov}}\right] & 0 & \cdots & \underset{(k)}{1} & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
\nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & \underset{(l)}{1} & \cdots & 0
\end{pmatrix}
\begin{bmatrix}
dx_{rov} \\ dy_{rov} \\ dz_{rov} \\ \Delta\nabla B_1 \\ \vdots \\ \Delta\nabla B_k \\ \vdots \\ \Delta\nabla B_l \\ \vdots \\ \Delta\nabla B_s
\end{bmatrix}
$$

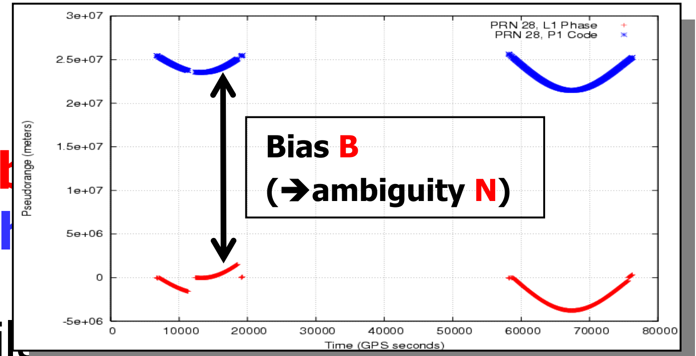
B

$$P^{sat}_{1sta} = \rho^{sat}_{sta} + c\cdot(dt_{sta} - dt^{sat}) + rel^{sat}_{sta} + Trop^{sat}_{sta} + Ion^{sat}_{1sta} + K_{1sta} + K_1^{sat} + \varepsilon$$

$$L^{sat}_{1sta} = \rho^{sat}_{sta} + c\cdot(dt_{sta} - dt^{sat}) + rel^{sat}_{sta} + Trop^{sat}_{sta} - Ion^{sat}_{1sta} + \boxed{k_{1sta} + k_1^{sat} + \lambda_1 N_1} + w_1 + \varepsilon$$

3. Carrier phase positioning

3.1 Differential positioning with doub… and phase measurements (floating th…



Phase measurements are modeled in simil… taking into account the phase **ambiguities** $\Delta\nabla B$, which must be estimated together with rover coordinates *(dx,dy,dz)*.

$$\begin{bmatrix} \Delta\nabla Prefit(P)^1 \\ \Delta\nabla Prefit(L)^1 \\ \cdots\cdots \\ \Delta\nabla Prefit(P)^n \\ \Delta\nabla Prefit(L)^n \end{bmatrix} = \begin{pmatrix} \nabla\left[\frac{x_{o,rov}-x^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^1_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \nabla\left[\frac{x_{o,rov}-x^1}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^2_{0,rov}}\right] & 0 & \cdots & \underset{(k)}{1} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & \underset{(l)}{1} & \cdots & 0 \end{pmatrix} \begin{bmatrix} dx_{rov} \\ dy_{rov} \\ dz_{rov} \\ \Delta\nabla B_1 \\ \vdots \\ \Delta\nabla B_k \\ \vdots \\ \Delta\nabla B_l \\ \vdots \\ \Delta\nabla B_s \end{bmatrix}$$

This system can be solved with a **Kalman filter**, assuming **ambiguities** $\Delta\nabla B$ constant along continuous carrier phase arcs, and white noise at cycle-slips.

The tropospheric delay can be also estimated as a random walk

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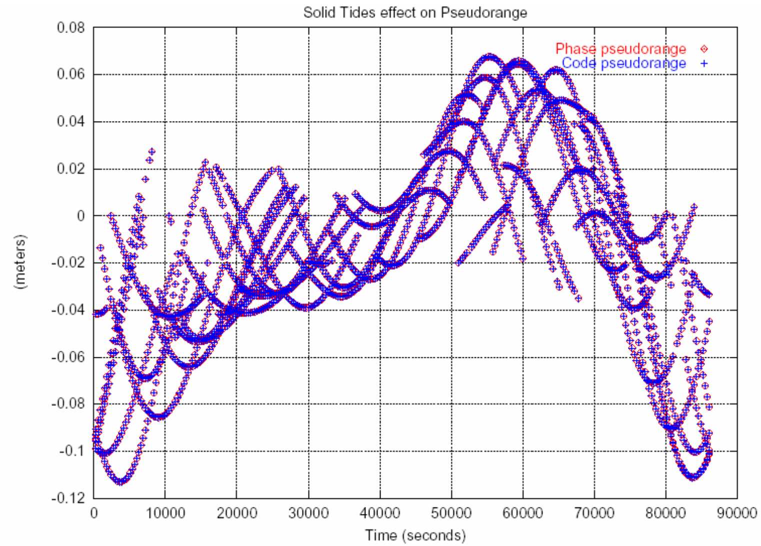
Comments:

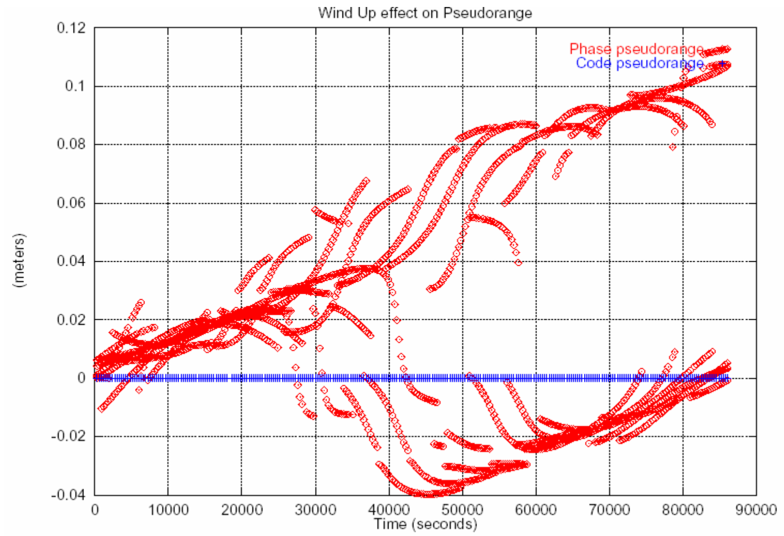
More accurate modeling for phase positioning is needed. It should involve:

- **Phase Wind-up** (the GPS signal is polarized wave)
- **Antenna Phase center** (satellite, receiver)
- **To estimate tropospheric delay** (wet: random walk)
- **To adjust broadcast orbits** (long baselines)
- **Tidal Effects** (in particular solid Earth Tides)

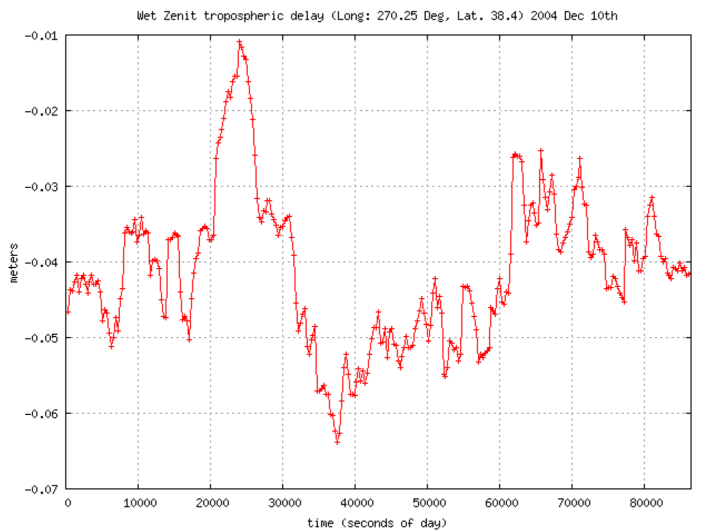
Solid Earth tides



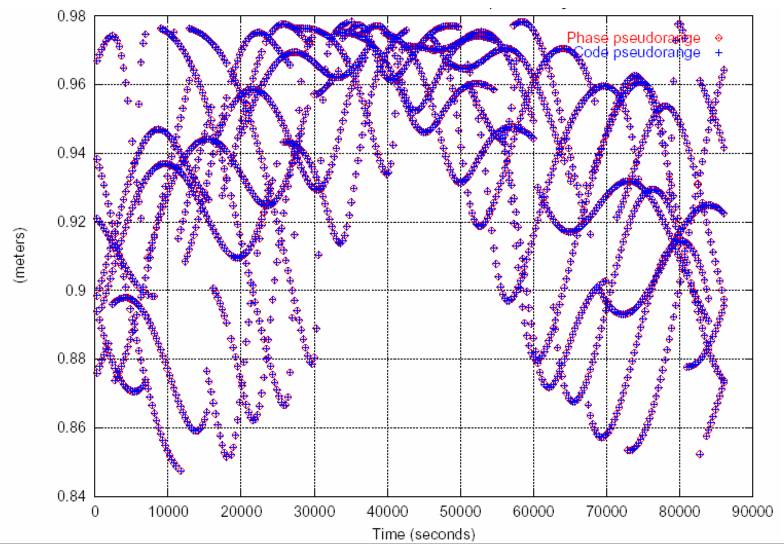
Wind-Up (Lc)



Wet Zenith Tropospheric delay

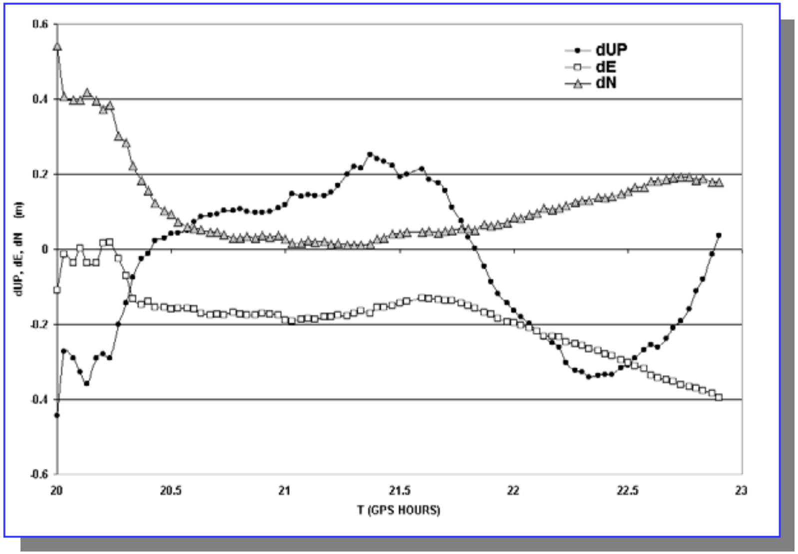


Antenna Phase center (satellite)

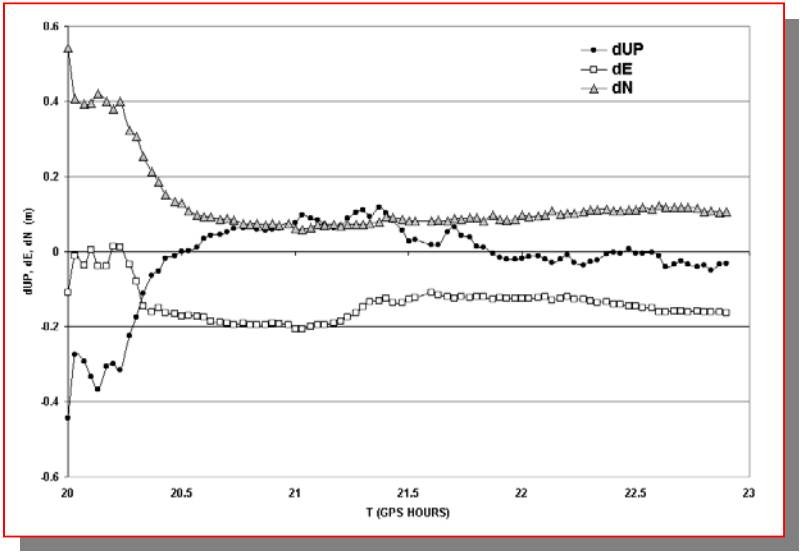


Differential positioning at 420 km from reference station

Broadcast orbits



Broadcast orbits (adjusted)



Differential Kinematic positioning with 429 km of baseline. Broadcast orbits (left) and adjusted broadcast orbits (right) used. Ambiguities "floated" (Lc biases estimated). Tropospheric refraction errors estimated. Triangles: dUP; black circles: dN, squares: dE; all meters.

Differential positioning with double differences using code and phase measurements (floating the ambiguities)

Phase measurements are modeled in similar way than code ones, but taking into account the phase **ambiguities** $\Delta\nabla B$, which must be estimated together with rover coordinates *(dx,dy,dz)*.

$$\begin{bmatrix} \Delta\nabla Prefit(P)^1 \\ \Delta\nabla Prefit(L)^1 \\ \cdots\cdots \\ \Delta\nabla Prefit(P)^n \\ \Delta\nabla Prefit(L)^n \end{bmatrix} = \begin{pmatrix} \nabla\left[\frac{x_{o,rov}-x^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^1_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^1_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \nabla\left[\frac{x_{o,rov}-x^1}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^1}{\rho^2_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^1}{\rho^2_{0,rov}}\right] & 0 & \cdots & \underset{(k)}{1} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \nabla\left[\frac{x_{o,rov}-x^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{y_{o,rov}-y^n}{\rho^n_{0,rov}}\right] & \nabla\left[\frac{z_{o,rov}-z^n}{\rho^n_{0,rov}}\right] & 0 & \cdots & 0 & \cdots & \underset{(l)}{1} & \cdots & 0 \end{pmatrix} \begin{bmatrix} dx_{rov} \\ dy_{rov} \\ dz_{rov} \\ \Delta\nabla B_1 \\ \vdots \\ \Delta\nabla B_k \\ \vdots \\ \Delta\nabla B_l \\ \vdots \\ \Delta\nabla B_s \end{bmatrix}$$

This system can be solved with a **Kalman filter**, assuming **ambiguities** $\Delta\nabla B$ constant along continuous carrier phase arcs, and white noise at cycle-slips.

FIXING versus FLOATING ambiguities

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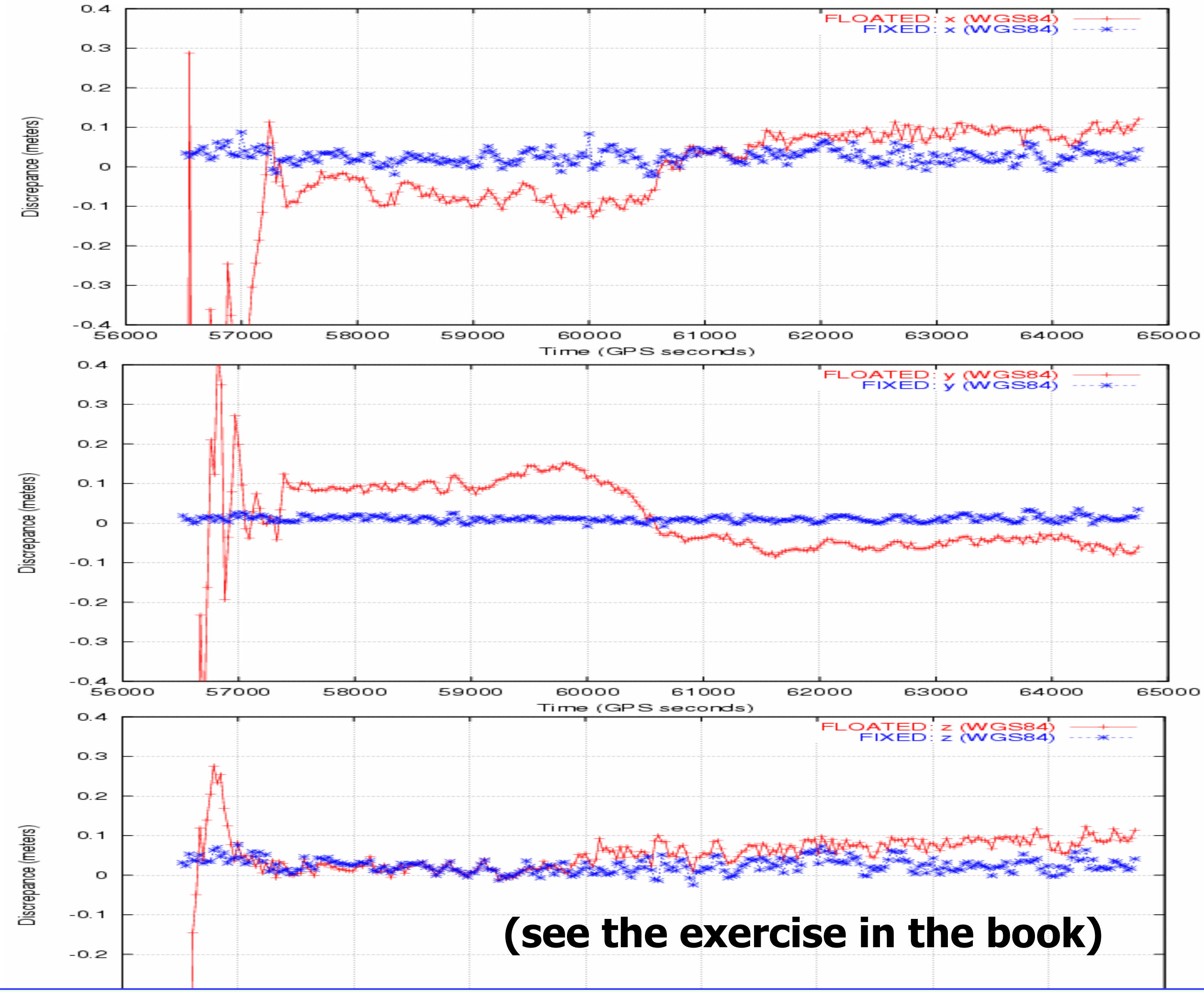


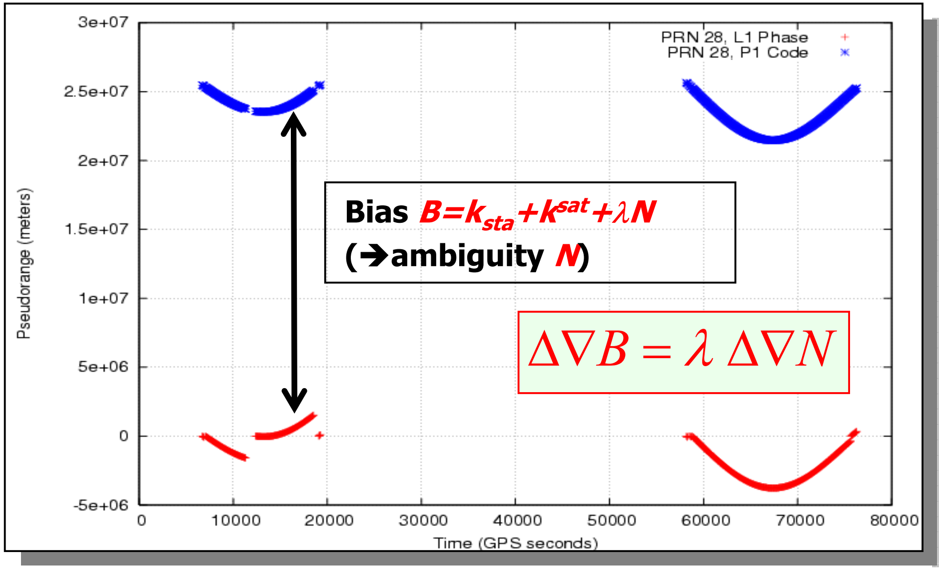
(see the exercise in the book)

All the ambiguities have been fixed in post-process.

After fixing the ambiguities the carrier phases are use for positioning as very accurate pseudoranges (with few millimeters of noise)

Resolving the Ambiguities:

- Ambiguities must be estimated with a Kalman filter, together with coordinates (and troposphere,…).



- The easier way to resolve the ambiguities is to treat them as **REAL numbers**, which are constant along continuous phase arcs and white noise when cycle-slips (this technique is called: **floating ambiguities**)

  ➔ The filter needs some time span to converge, and also there is estimation noise (due to the correlations with the other states of filter), which degrades the navigation solution at the level of few decimeters.

Notice: If the ambiguities were fixed, we would be positioning with measurements of few millimeters of noise.

On the other hand, the ionosphere can be removed using dual frequency measurements (with Lc, Pc combinations)!

Note:

- Klobuchar model only accounts for ~60%
- For long baselines the ionosphere does not cancel!

Differential with 2-freq and carrier phases (Lc,Pc)

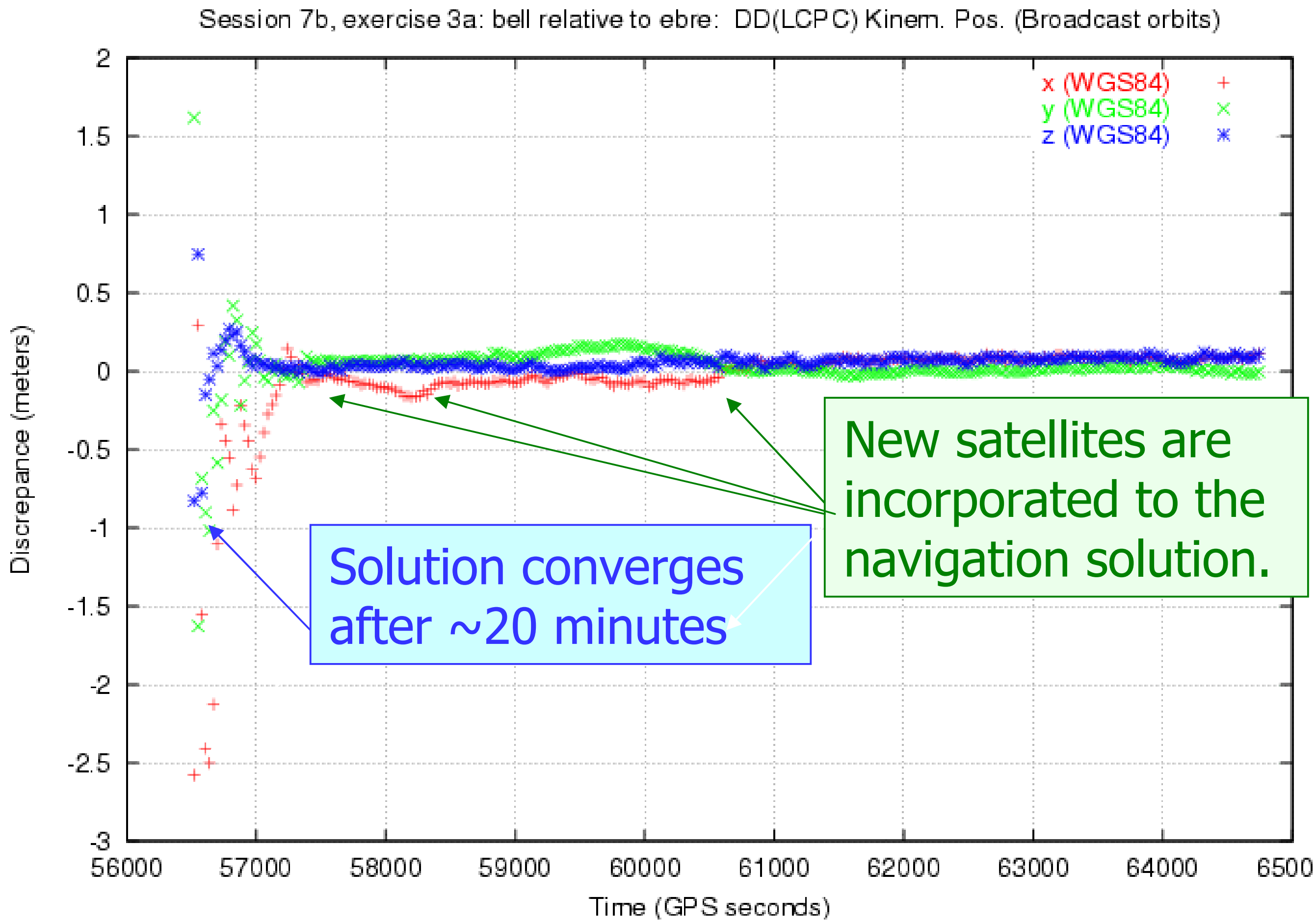
Ionosphere	**5-10 m**	**0.4m**	**~ mm**
Troposphere	**0.5-1.0 m**	**0.2 m**	**Estimate**
Receiver Noise	**0.5 m**	**0.5m**	**2 mm + Ambig.**
Multipath	**0.6 m**	**0.6m**	**5 mm**
SA on (SA=off)	**30 (0)m**	**0 m**	

Carrier phase ambiguities must be estimated/fixed with receiver coordinates:

The accuracy will depend on how they are solved.

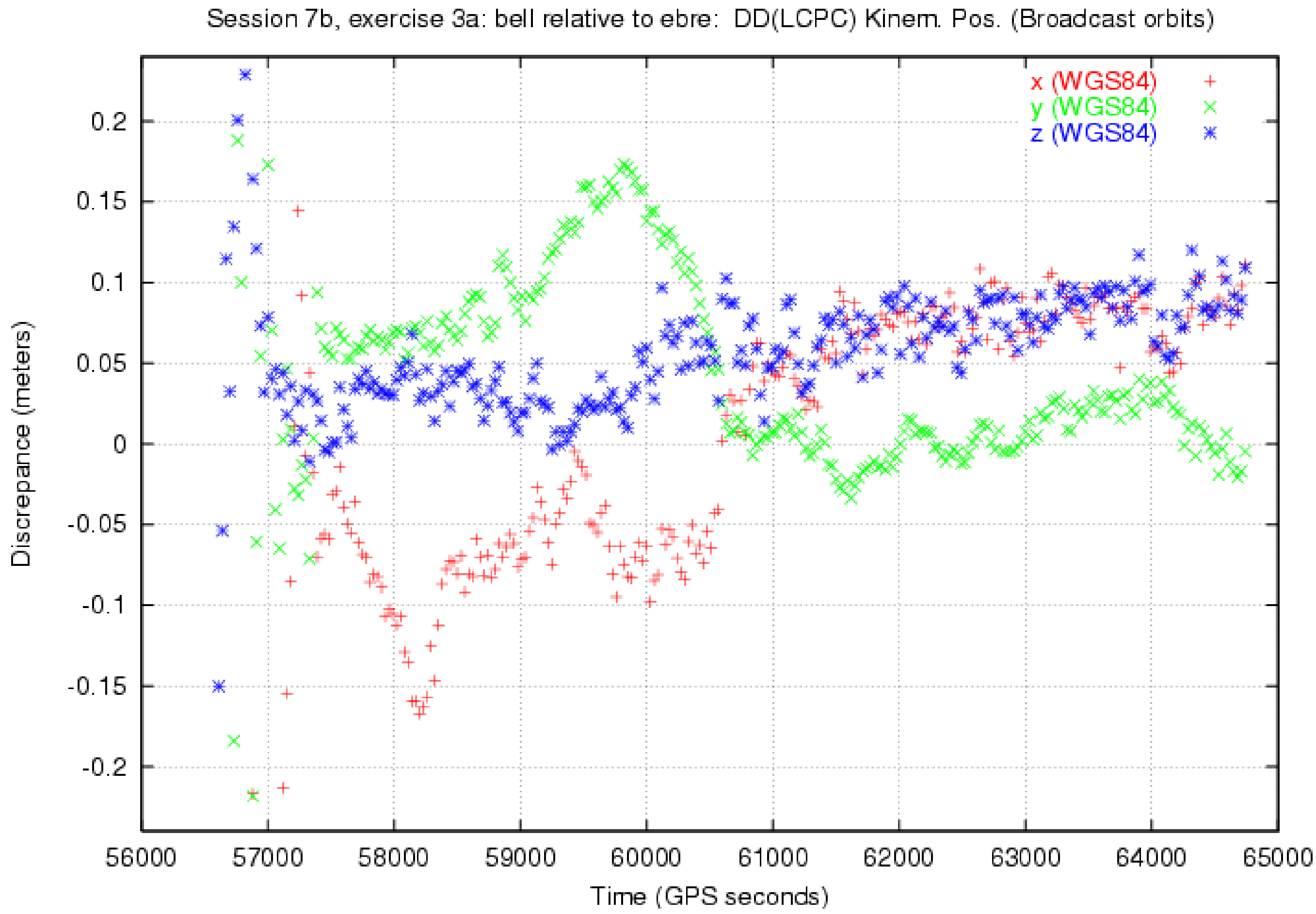
(also the tropospheric delay or orbits adjustments should be considered for high precision).

Differential



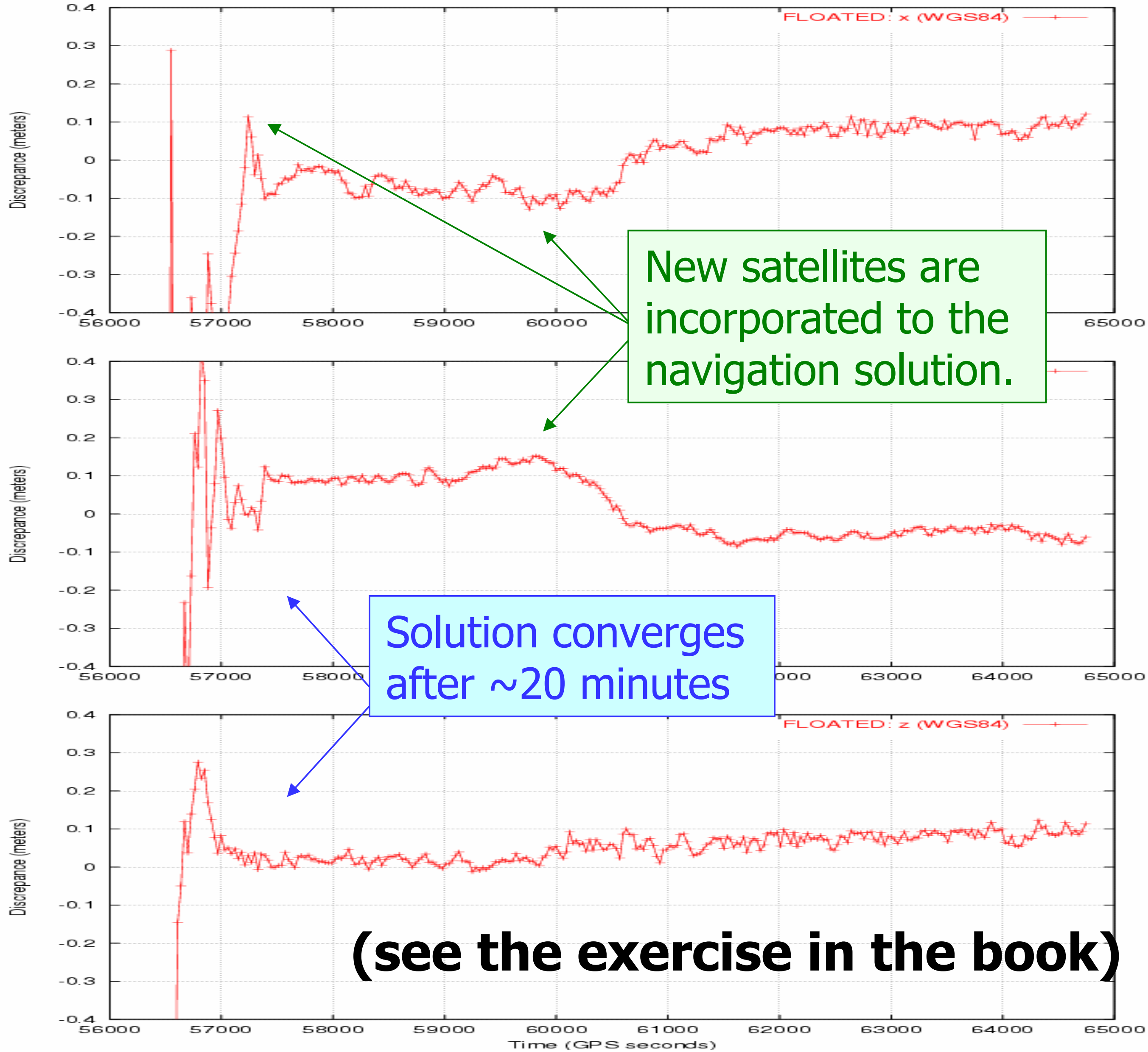
Differential Kinematic Positioning (floating ambiguities) using broadcast orbits and LC, PC measurements.

Baseline 100Km



Zoom of previous figure

Kinematic Posit. with Lc, Pc, FLOATING ambiguities
(base line 100Km)



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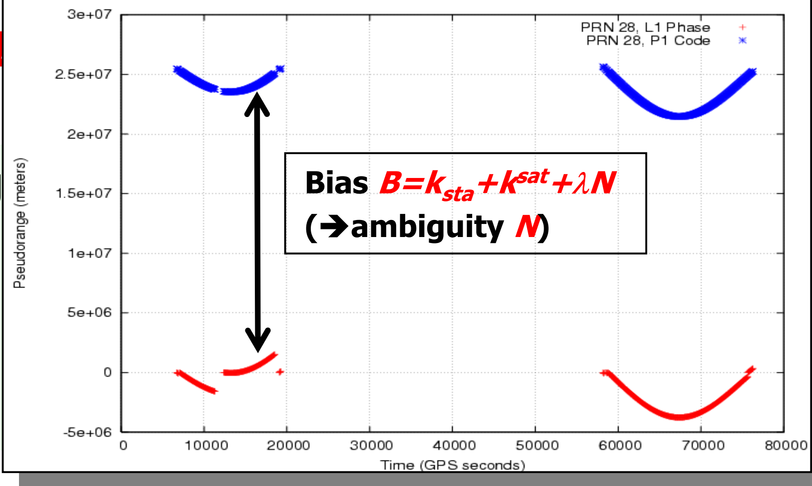
FIXING versus FLOA

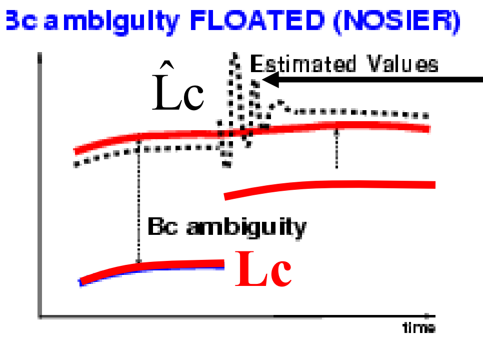
Fixing ambiguities will allow to g

- **More accuracy**: avoiding nosier it is fixed.
- **Faster convergence** of filter.



FLOATED:

The ambiguities have been estimated by filter as real numbers



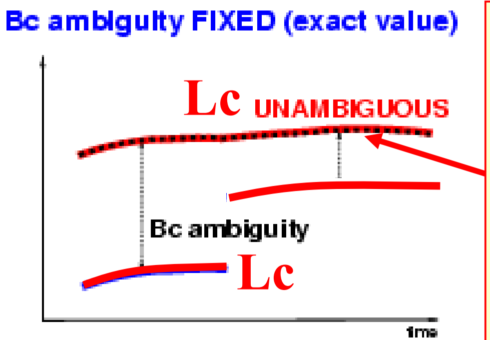
Carrier phase $\hat{L}c$ estimated floating the ambiguities.

The estimations need some time to converge, and do not converge to the true value.

t (GPS seconds)

FIXED:

The ambiguities have ben previously fixed (exact values)



The ambiguities are fixed to their "true" values. There is no estimation noise.

The carrier phase is fully "repaired" and provides a very accurate pseudorange (few millimeters of error)

Fixing the Ambiguities:

To obtain centimeter accuracy it is necessary to apply FIXING ambiguities techniques:

These techniques exploit the fact that the ambiguities in *f1* and *f2* are "**INTEGER NUMBERS**" of wavelengths, and if it is possible to build combinations of measurements with a level of noise under 1 wavelength, it will be possible to obtain the "**exact value" of the ambiguity by ROUNDING**.

Let's *B1* and *B2* be carrier phase bias in frequencies *f1* and *f2*. Thence:

$$B1_{rec}^{sat} = \lambda_1 N1_{rec}^{sat} + k_{1rec} + k_1^{sat}$$

$$B2_{rec}^{sat} = \lambda_2 N2_{rec}^{sat} + k_{2rec} + k_2^{sat}$$

N1,N1 are the **ambiguities (integers)**

k1, k2 are the **instrumental delays (real)**

The instrumental delays cancel out when making the double differences.

$$\Delta\nabla B1 = \lambda_1 \Delta\nabla N1$$

$$\Delta\nabla B2 = \lambda_2 \Delta\nabla N2$$

$$\Delta\nabla B_W = \lambda_W \Delta\nabla N_W$$

On the other hand ***Δ∇B1, Δ∇B2, Δ∇Bw*** are integer multiples of their wavelengths. But, the last is not the true for ***Δ∇Bc***

$$\Delta\nabla Bc = \lambda_C \left(\frac{\lambda_W \Delta\nabla N1}{\lambda_1} - \frac{\lambda_W \Delta\nabla N2}{\lambda_2} \right)$$

RTK: it is assumed that ionospheric refraction cancels out (base lines < 15-20 Km)

• Example of algorithm for fixing L1, L2 ambiguities

1. Using Lc, Pc, the Kalman filter estimates the ambiguity $\Delta\nabla\hat{B}_c$ together with the receiver coordinates (floating it such as in previous examples).

2. From the rough estimation of $\Delta\nabla\hat{B}_c$, the ambiguity $\boxed{\Delta\nabla N_w}$ is fixed:

$$\boxed{\Delta\nabla N_w = \mathrm{int}\left(\frac{\Delta\nabla\hat{B}_w}{\lambda_w}\right) = \mathrm{int}\left(\frac{\Delta\nabla L_w - \Delta\nabla L_c + \Delta\nabla\hat{B}_c}{\lambda_w}\right)}$$

3. Being fixed the ambiguity $\boxed{\Delta\nabla N_w}$, the ambiguities $\boxed{\Delta\nabla N_1}$ $\boxed{\Delta\nabla N_2}$ can be also fixed:

$$\boxed{\Delta\nabla N_1 = \mathrm{int}\left(\frac{\Delta\nabla\hat{B}_1}{\lambda_1}\right) = \mathrm{int}\left(\frac{\Delta\nabla L_1 - \Delta\nabla L_2 - \lambda_2\Delta\nabla N_w}{\lambda_1 - \lambda_2}\right)}$$

$$\boxed{\Delta\nabla N_2 = \Delta\nabla N_1 - \Delta\nabla N_w}$$

4. From the "exact" values of the ambiguities $\boxed{\Delta\nabla N_1}$ $\boxed{\Delta\nabla N_2}$, the "exact" value of the "NON-INTEGER ambiguity" in Lc can be fixed:

$$\boxed{\Delta\nabla B_c = \lambda_c\lambda_w\left(\frac{\Delta\nabla N_1}{\lambda_1} - \frac{\Delta\nabla N_2}{\lambda_2}\right)}$$

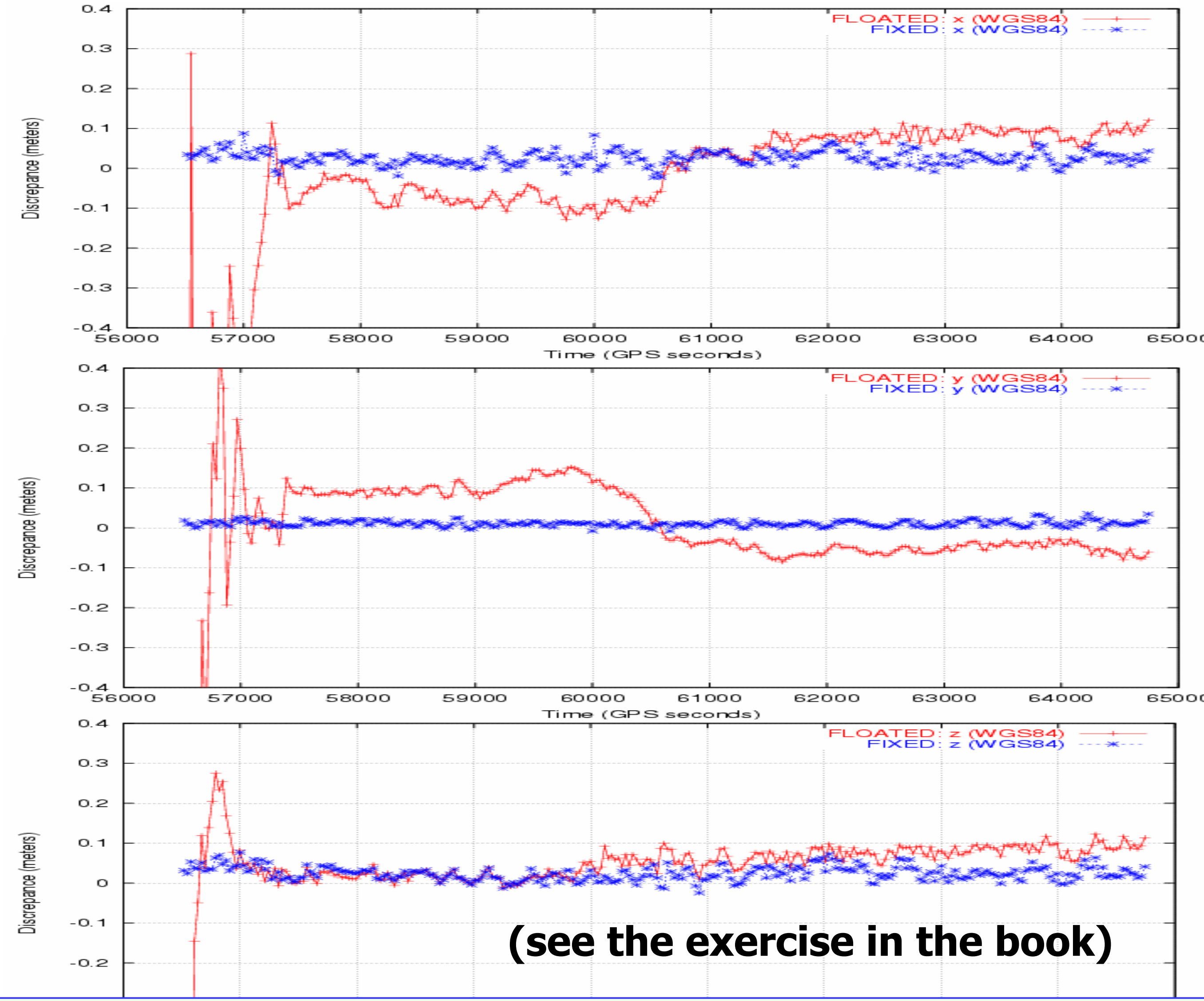
Being fixed the ambiguity in *Δ∇Bc*, then the measurement is accurate at the level of few millimeters!!!
(100 times more accurate than code).

Error budget	GPS Standalone	Differential CA code	Differential with 2-freq (Phases, L1,L2 [Lc])
Satellite clock	1.0 m	0 m	
Orbits	2.0 m	~0 m	
Ionosphere	5-10 m	0.4m	~mm
Troposphere	0.5-1.0 m	0.2 m	Estimate
Receiver Noise	0.5 m	0.5m	2 mm
Multipath	0.6 m	0.6m	5 mm
SA on (SA=off)	30 (0)m	0 m	

Typical	GPS	Differential	Differential
			3 cm
			4 cm
			5 cm

From the "conceptual point of view", when the ambiguity is fixed, the phase measurements are repaired and thence, the positioning is like if "very accurate codes (few millimeters of noise)" were used!!!

FIXING versus FLOATING ambiguities



(see the exercise in the book)

All the ambiguities have been fixed in post-process.

After fixing the ambiguities the carrier phases are use for positioning as very accurate pseudoranges (with few millimeters of noise)

Limitations:

➔The previous algorithm is based on assuming that ionospheric refraction is the same in the reference station and the rover, and cancels out when forming the double differences.

➔This hypothesis can only be assumed for short baselines (<15-20Km).

Wide Area Real Time Kinematics (WARTK)

➔For long baselines, the ionospheric refraction *$\Delta\nabla STEC$* must be taken into account in previous equations. Thence:

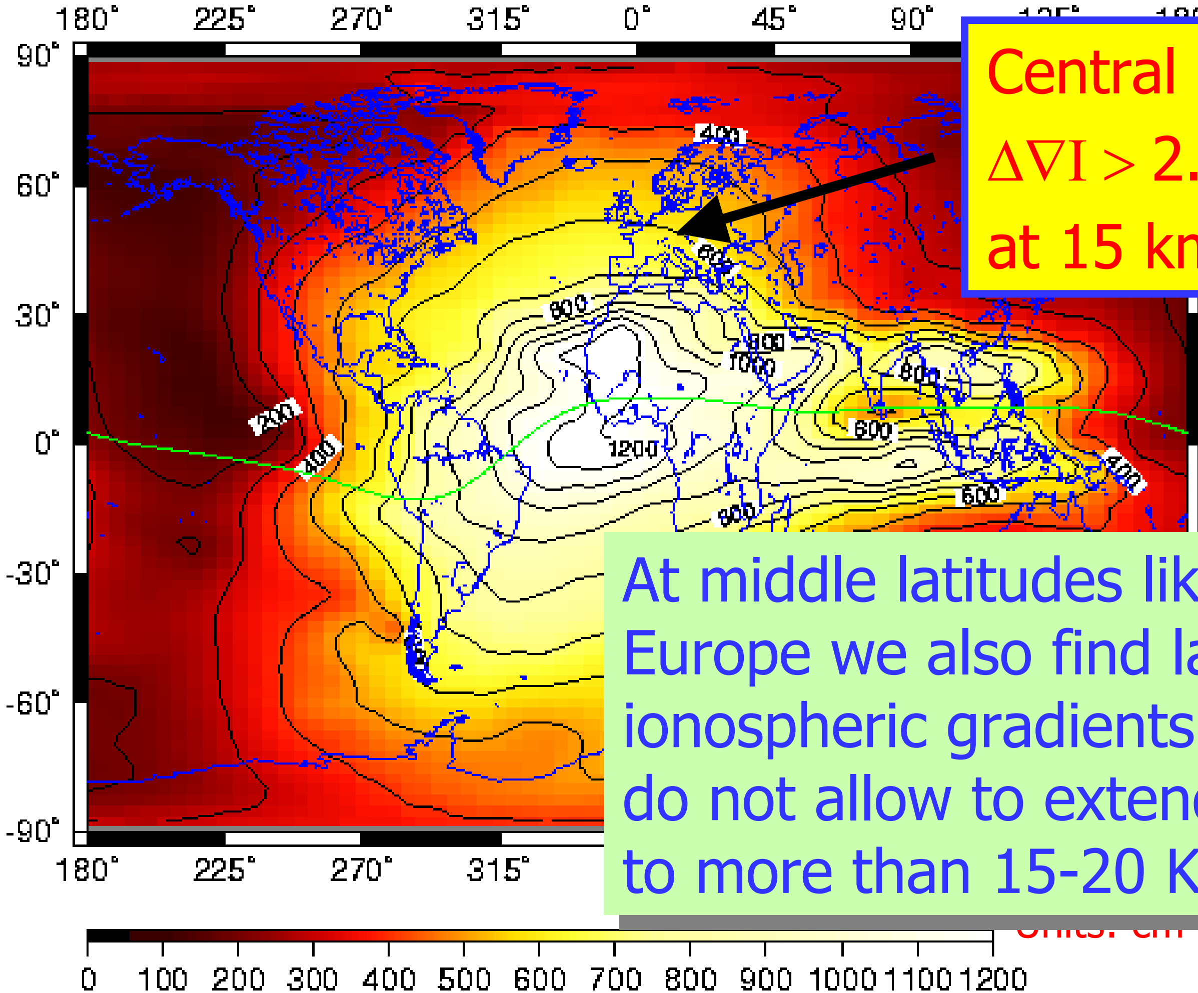
$$\Delta\nabla N_w = \mathrm{int}\left(\frac{\Delta\nabla L_w - \Delta\nabla L_c - \boxed{1.98\Delta\nabla STEC} + \Delta\nabla\hat{B}_c}{\lambda_w}\right)$$

$$\Delta\nabla N_1 = \mathrm{int}\left(\frac{\Delta\nabla L_1 - \Delta\nabla L_2 - \boxed{\Delta\nabla STEC} - \lambda_2\Delta\nabla N_w}{\lambda_1 - \lambda_2}\right)$$

Note: being *λ_2-λ_1 = 5.4 cm*, the accuracy in *$\Delta\nabla STEC$* must be better than 2.7 cm of L1-L2 delay.

Vertical Ionospheric Refraction

Experiment Day (077 2000, 15UT)



Central Europe:
$\Delta\nabla I > 2.7\,\mathrm{cm}_{L1-L2}$
at 15 km or more

At middle latitudes like in Europe we also find large ionospheric gradients which do not allow to extend RTK up to more than 15-20 Km.

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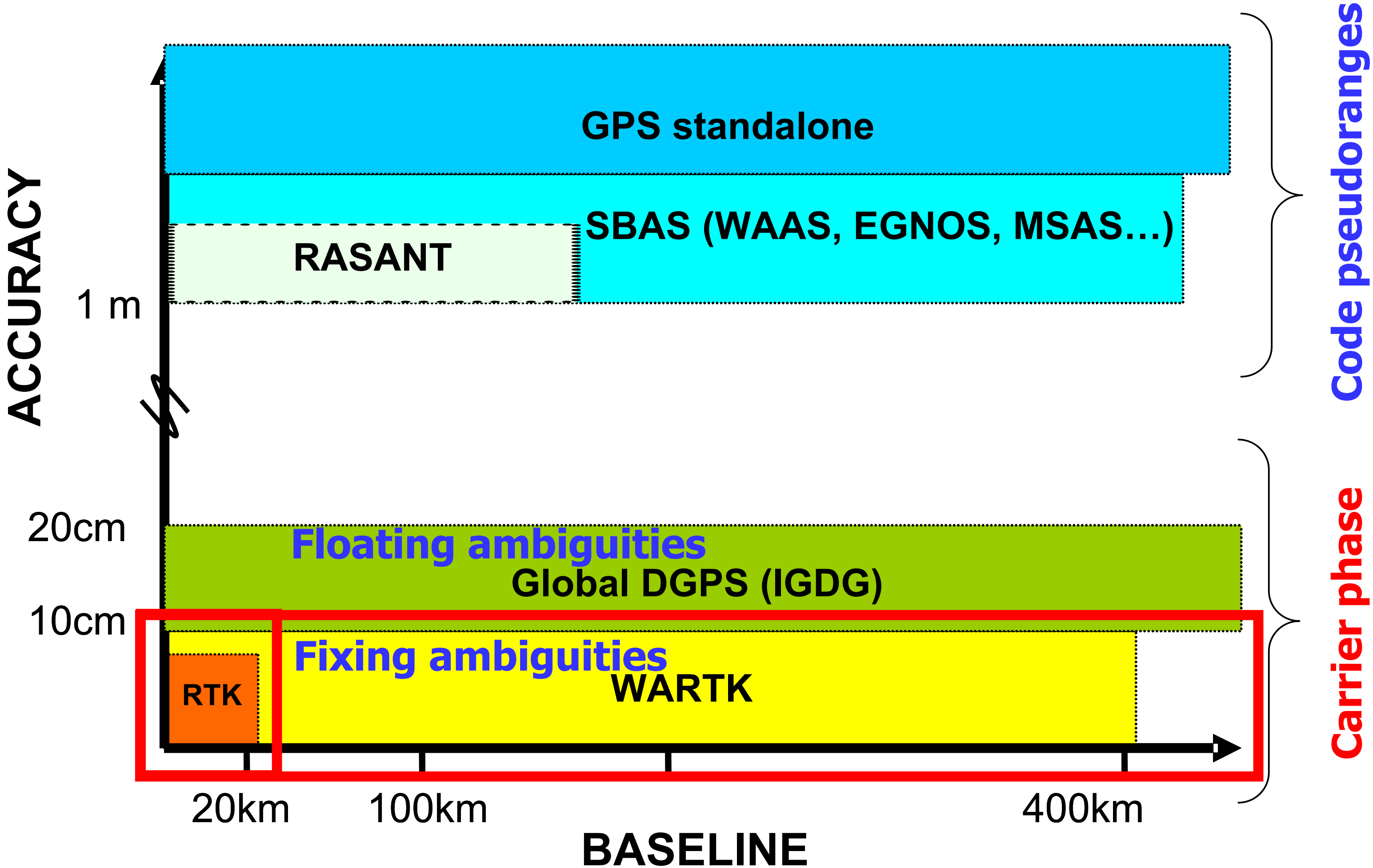
If ACCURATE ENOUGHT ionospheric corrections could be provided to the user, centimetic navigation could be achieved for LONG-BASELINES (hundred Km far from reference station)!!!! → WARTK

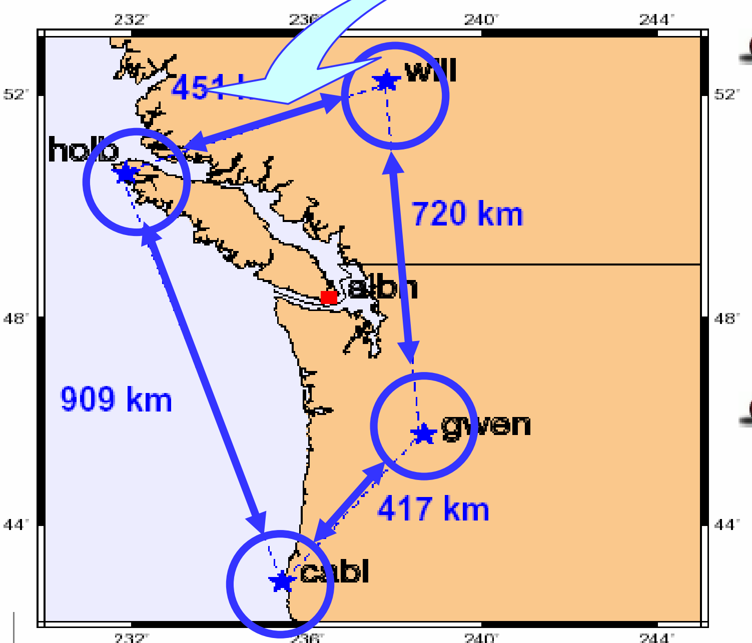


But, how to achieve *error ($\Delta\nabla$ STEC)* < 2.7 cm L1-L2 at the user location?

Working with two frequency receivers (reference stations and Rover), the precise ionospheric determinations are used to help the ambiguity resolution.

The following scheme is applied:

$$\Delta\nabla LI = \Delta\nabla STEC + \lambda_1 \Delta\nabla N_1 - \lambda_2 \Delta\nabla N_2 + \Delta\nabla w_I + \varepsilon$$



- Resolving the ambiguities in the reference stations $\Rightarrow \nabla\Delta STEC$ **"unambiguous"** ($\sigma_{\nabla\Delta STEC} \simeq mm$)

  Tomographic model

  σ_{STEC} <10 cm in L1-L2

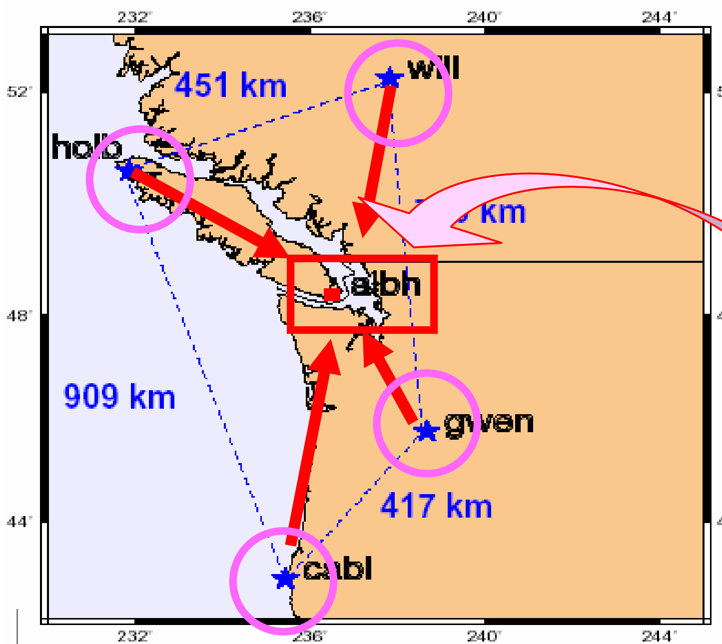
- Interpolating the unambiguous $STEC$ to rover position $\nabla\Delta STEC_{ro}$

(error $<$ 2.7 cm $\simeq 1/4$ TECU)

But, how to achieve *error ($\Delta\nabla$ STEC)* < 2.7 cm L1-L2 at the user location?

Working with two frequency receivers (reference stations and Rover), the precise ionospheric determinations are used to help the ambiguity resolution.

The following scheme is applied:



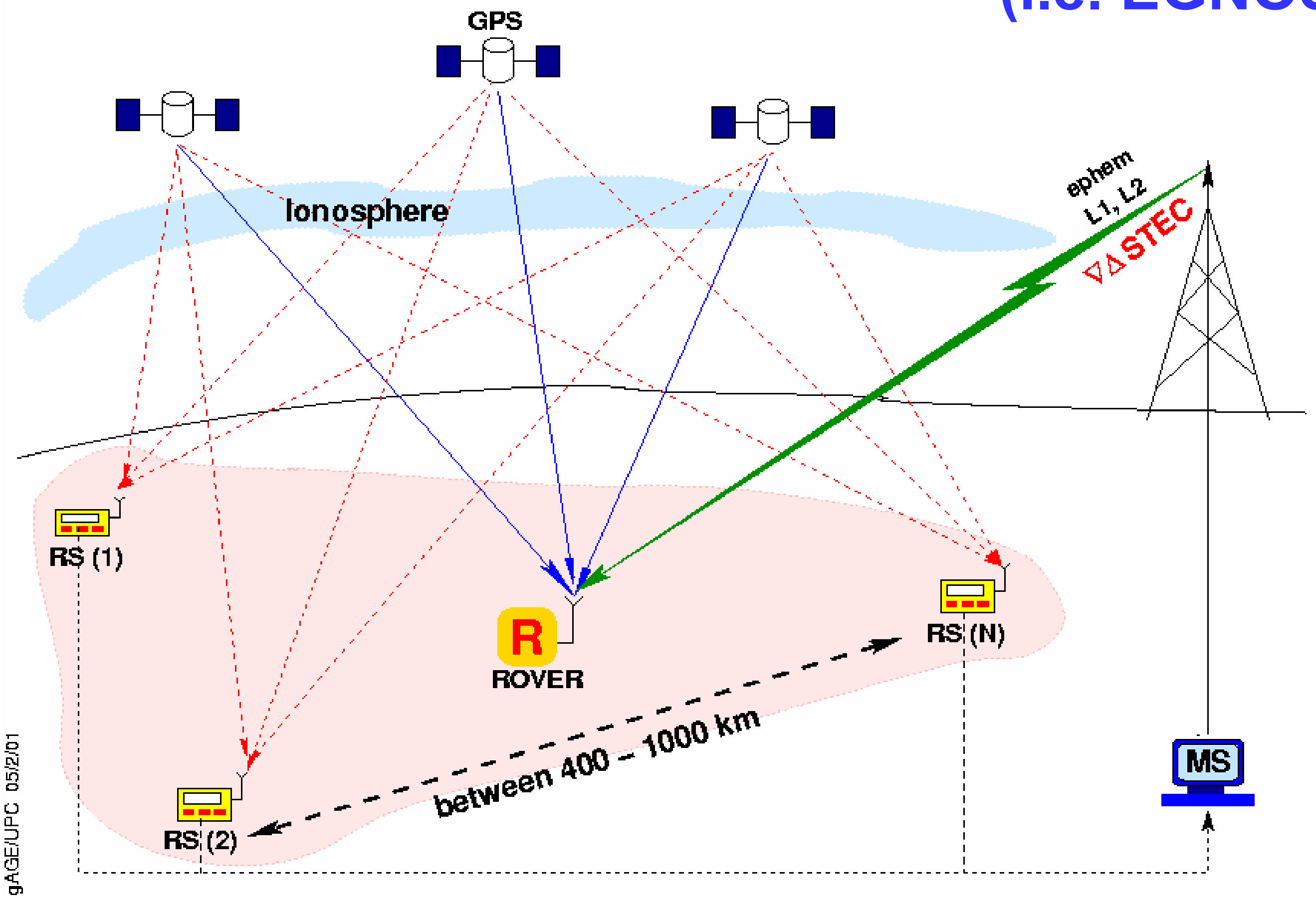
- Resolving the ambiguities in the reference stations $\Rightarrow \nabla\Delta STEC$ **"unambiguous"** ($\sigma_{\nabla\Delta STEC} \simeq mm$)

  Tomographic model
  σ_{STEC} <10 cm in L1-L2

- Interpolating the unambiguous $STEC$ to rover position $\nabla\Delta STEC_{ro}$

  (error $<$ 2.7 cm $\simeq 1/4$ TECU)

WARTK: Can be based on Wide Area GPS Networks (i.e. EGNOS)



An ***ΔVSTEC*** **with error less than 2.7 cm of L1-L2 delay must achieved at the rover location!!**

[gAGE/UPC patent,2000]

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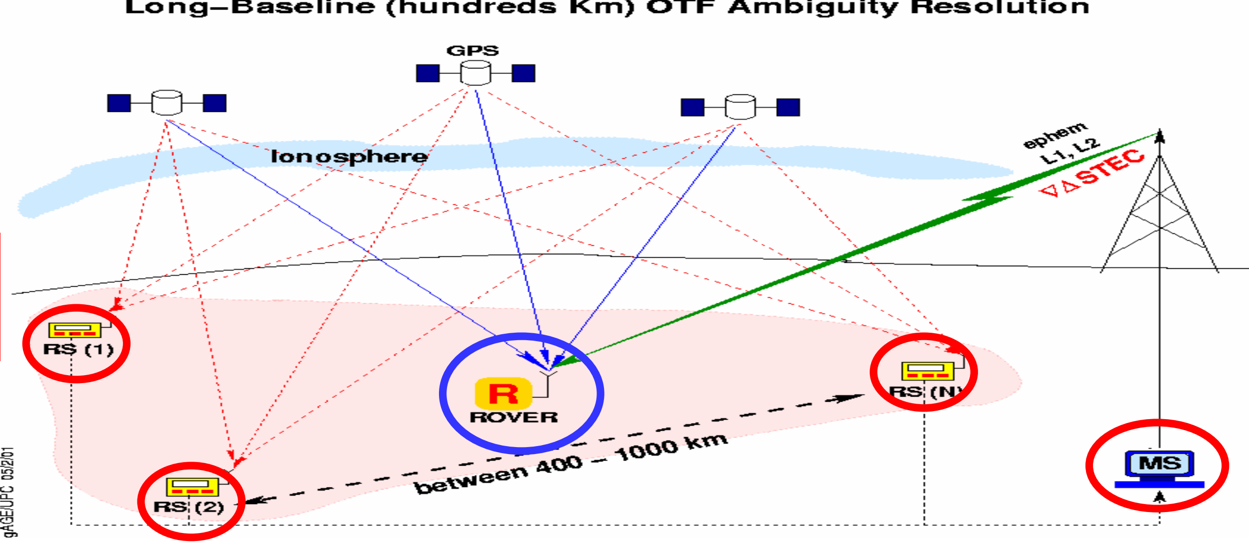
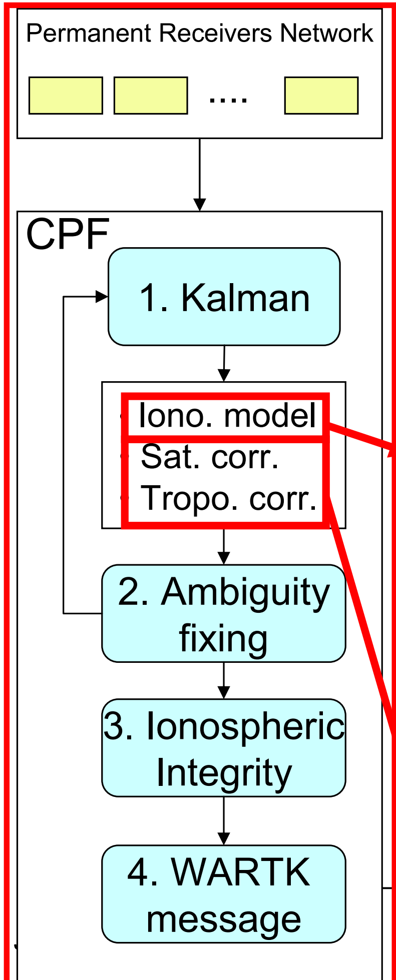
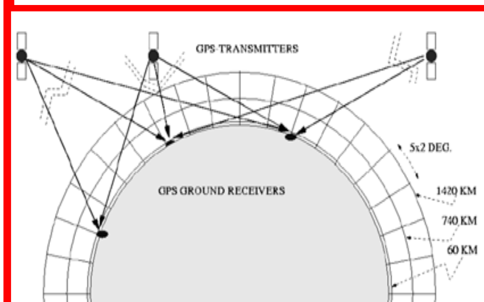
WARTK layout



WARTK Central Processing Facility



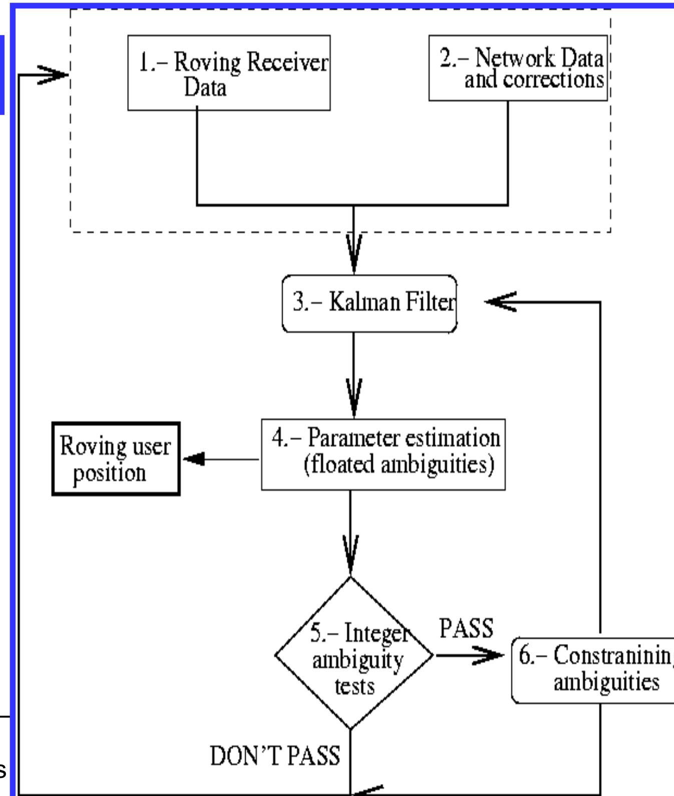
Real-time Iono. model



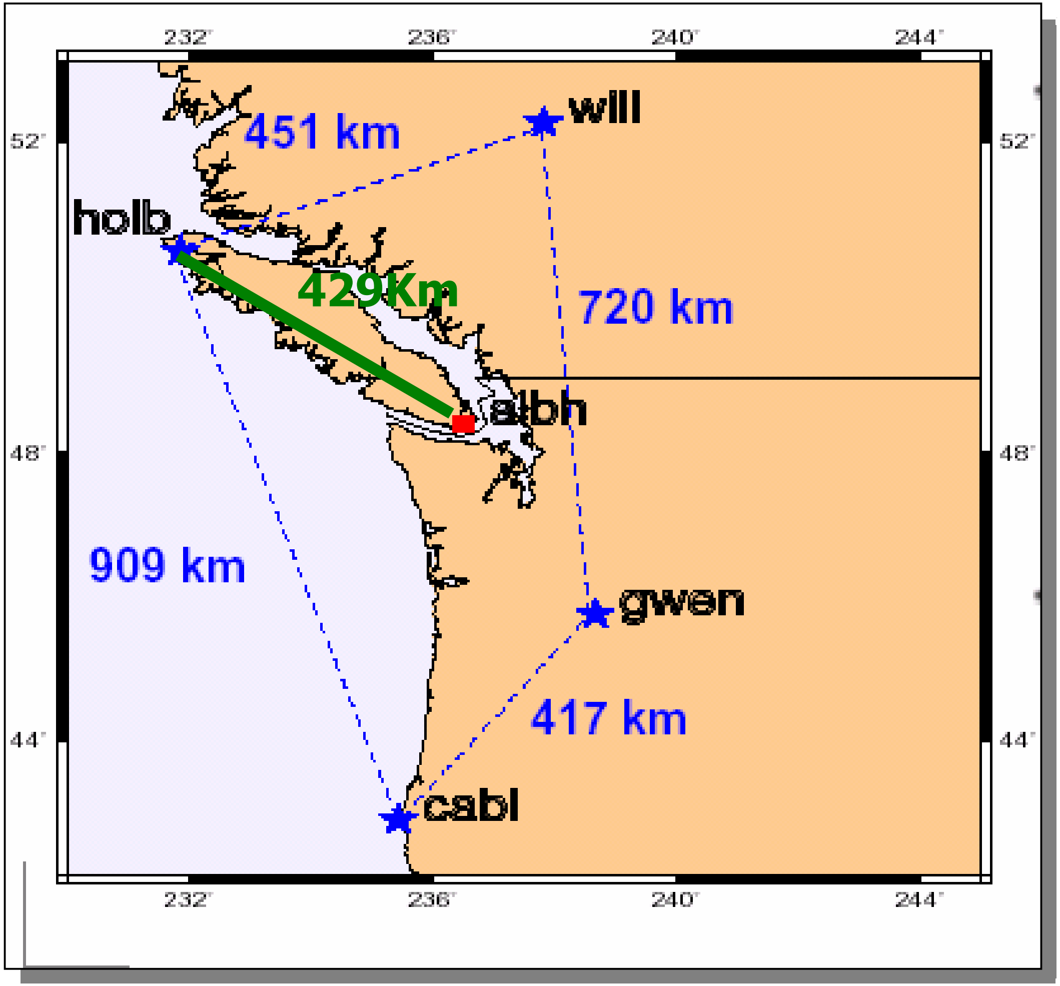
$$L_I = STEC + B_I = \int_{REC}^{SAT} N_e\,dl + B_I = \sum_i \sum_j \sum_k (N_e)_{i,j,k}\,\Delta s_{i,j,k} + B_I$$

Real-time geodetic model

WARTK User



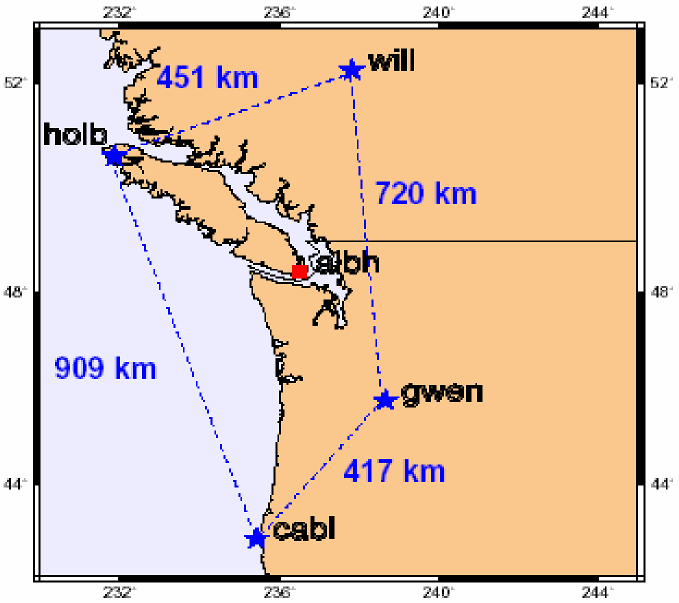
The Pacific Northwest Test: 1998 March 3rd (Kp>4)



Ref.: Colombo O., Hernández-Pajares M., J.M. Juan, J. Sanz. Wide-Area, carrier-phase ambiguity resolution using a tomographic model of the Ionosphere. Navigation 49(1), pp. 61- 69, 2002.

RESULTS: The Pacific Northwest Test (Kp>4)

1998 May 3rd



- All observations were dual frequency carrier-phase and pseudorange, collected at typical IGS rate of 30 sec.
- Data sets from four of them (cabl, wen, holb, will) were used as REFERENCE STATIONS.
- The station albh was treated as the ROVER.

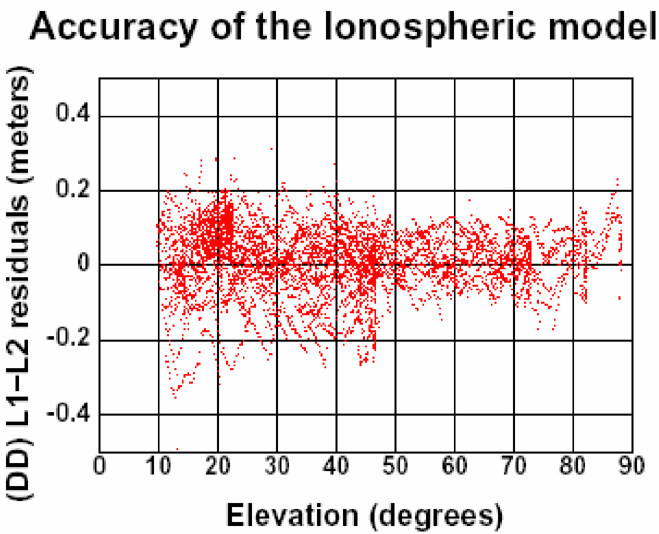
While all data had been collected before we carried out the calculations, we took care to process them as they would be processed in a truly real-time application

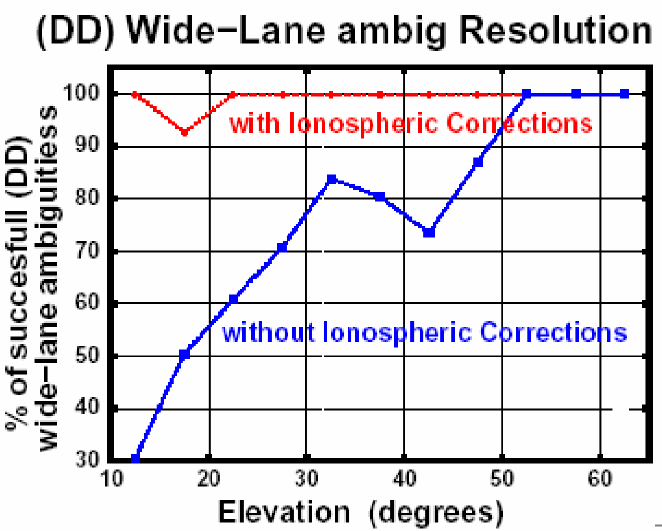
Ref.: Colombo O., Hernández-Pajares M., J.M. Juan, J. Sanz. Wide-Area, carrier-phase ambiguity resolution using a tomographic model of the Ionosphere. Navigation 49(1), pp. 61- 69, 2002.

Reference Stations

Checking the Ionospheric Correction at the Refrence stations [Kp>4, 450-1300 km, (05/03/98)]

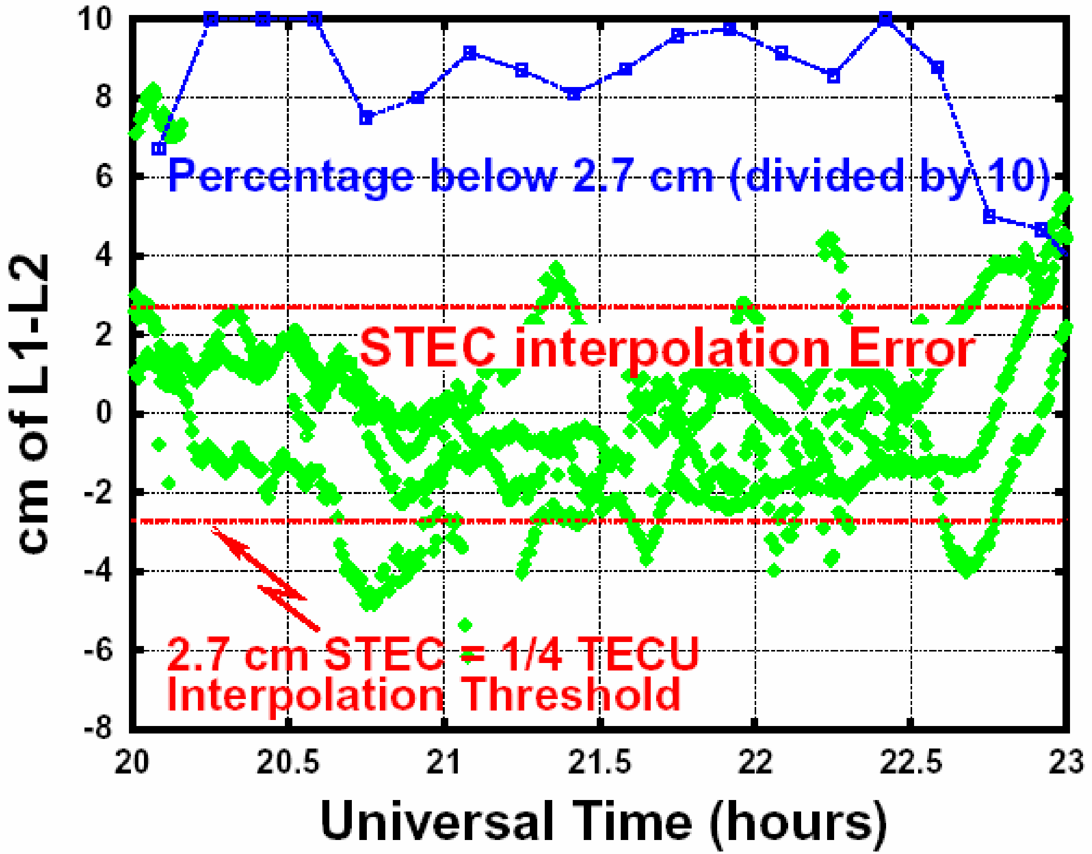
- Ionospheric model: provides RMS=0.9 TECU for double difference ionospheric corrections.
- Thence, most of the time, the (DD) widelane ambiguities are successfully Fixed.





Rover

Checking the Ionospheric Corr. at the ROVER



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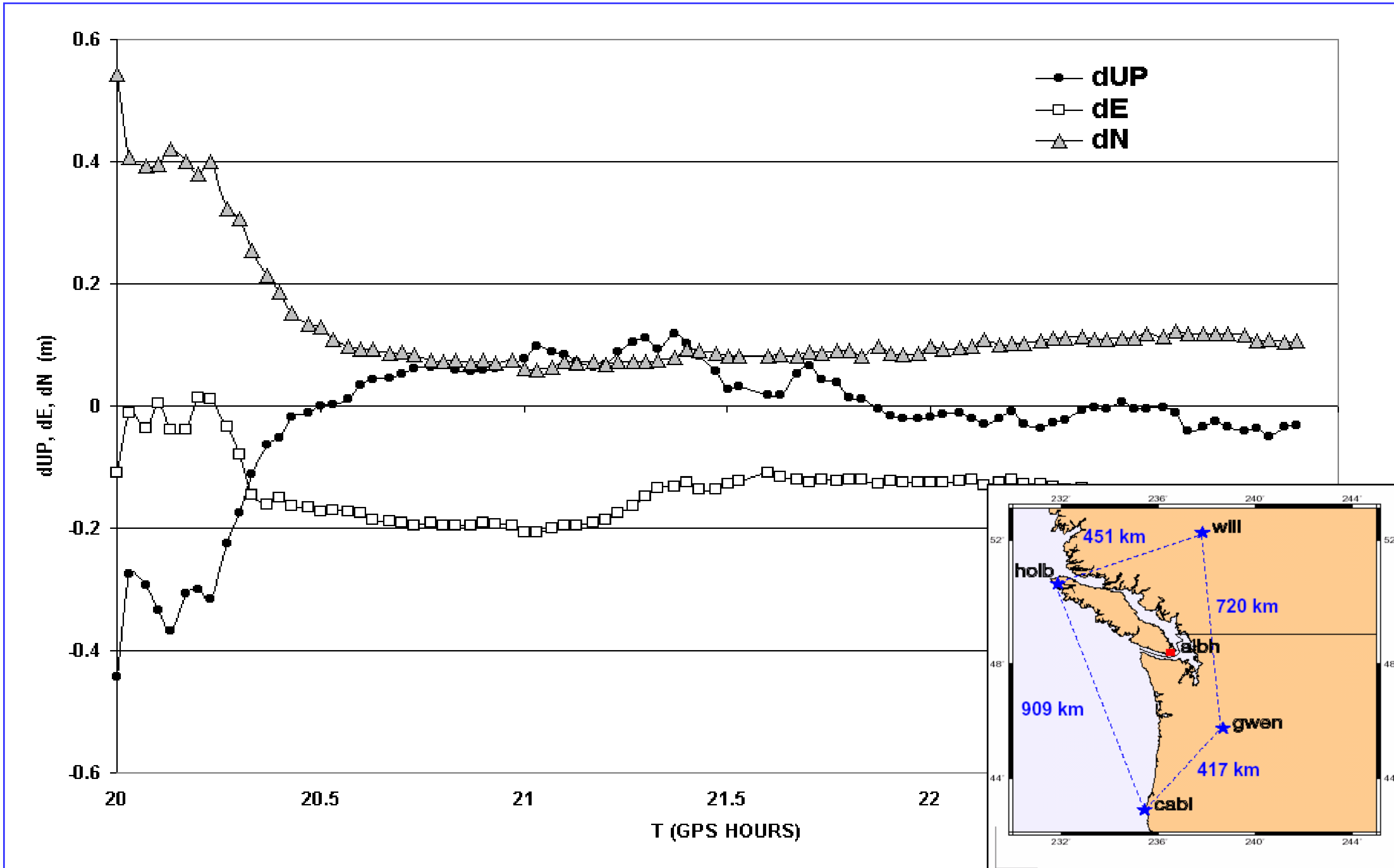
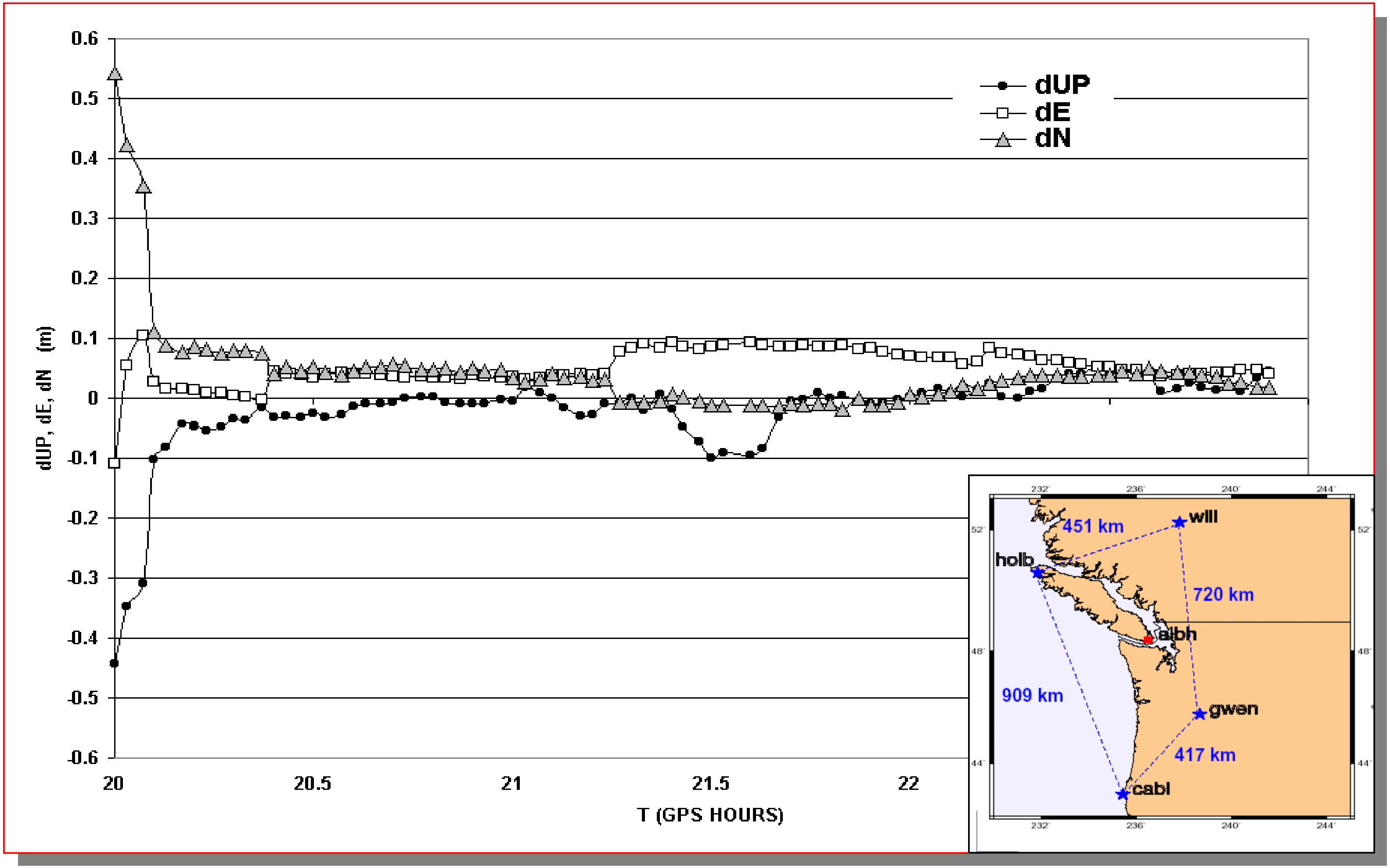
Barcelona, Spain

Floating ambiguities: Broadcast orbits adjusted



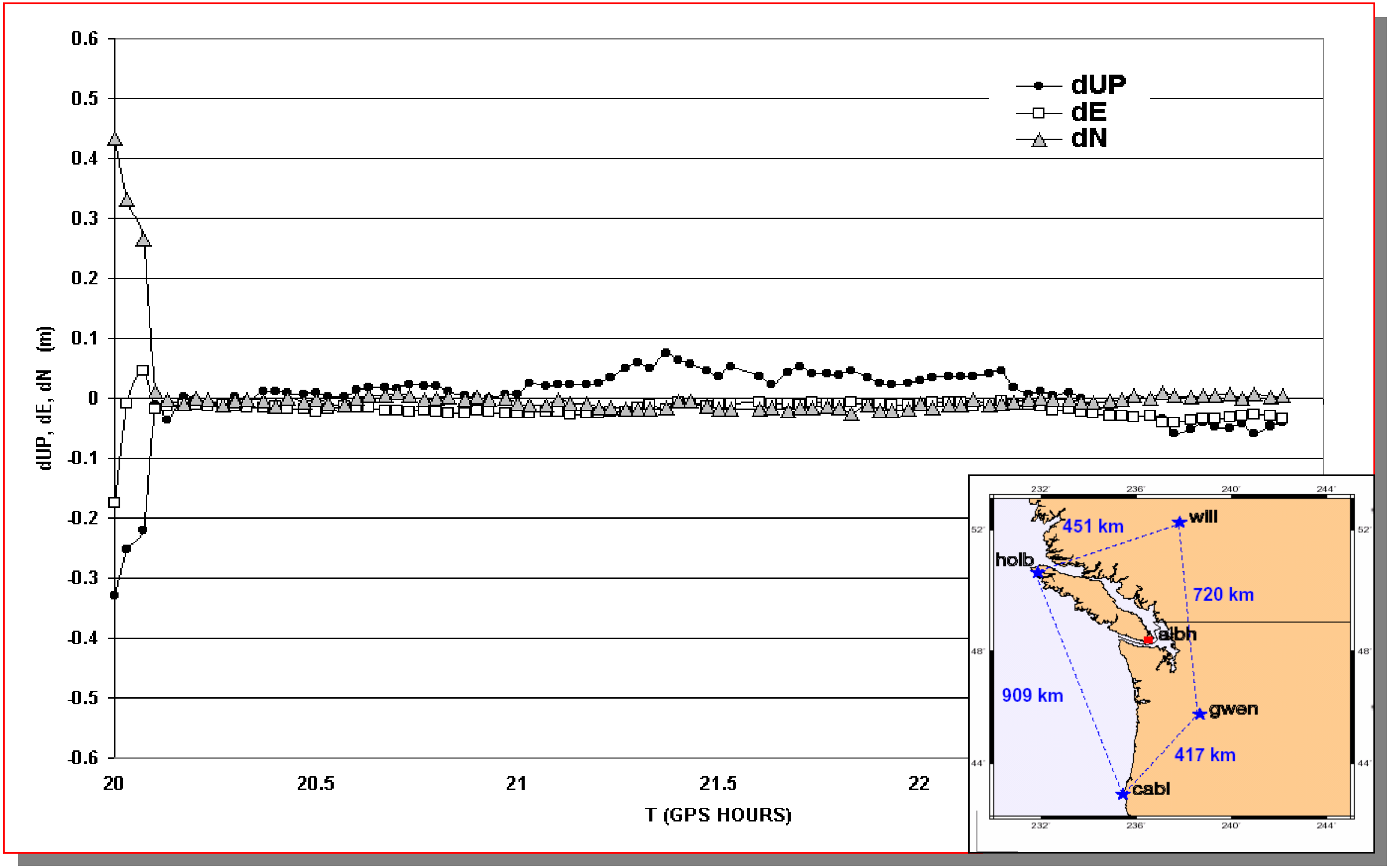
Kinematic versus true position of ALBH, with HOLB as base station (a 429 km baseline). Adjusted broadcast orbits used. Ambiguities "floated" (Lc biases estimated). Tropospheric refraction errors estimated. Triangles: dUP; black circles: dN, squares: dE; all meters.

Fixing ambiguities: Broadcast orbits adjusted



Kinematic versus true position of ALBH, with HOLB as base station (a 429 km baseline). Adjusted broadcast orbits used. Ambiguities "fixed" (Lc biases fixed). Tropospheric refraction errors estimated. Triangles: dUP; black circles: dN, squares: dE; all meters.

Fixing ambiguities: Precise SP3 orbits

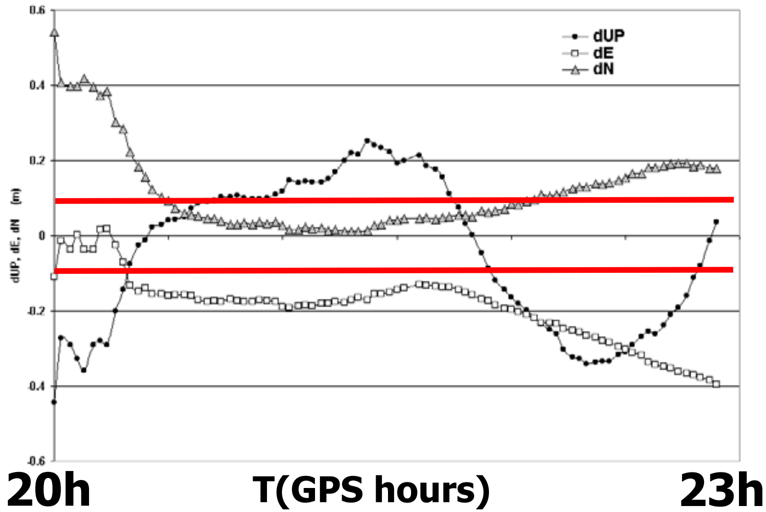


Kinematic versus true position of ALBH, with HOLB as base station (a 429 km baseline). Precise SP3 orbits used. Ambiguities "fixed" (Lc biases fixed). Tropospheric refraction errors estimated. Triangles: dUP; black circles: dN, squares: dE; all meters.

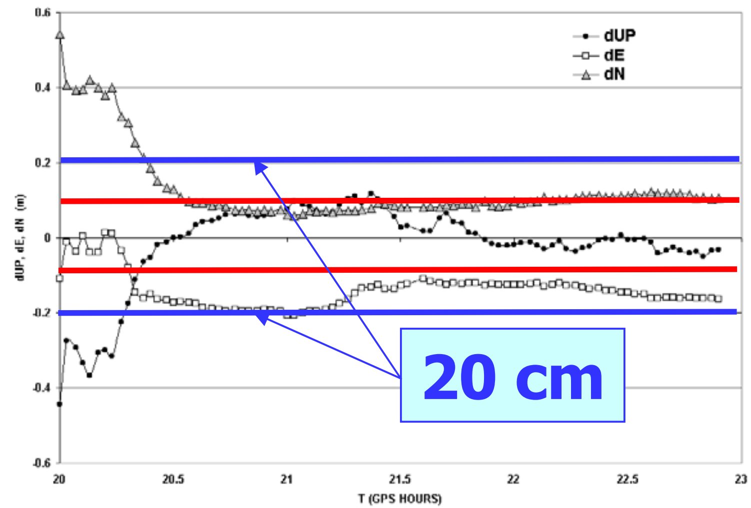
Differential positioning at 420 km from reference station

Floating

Broadcast orbits

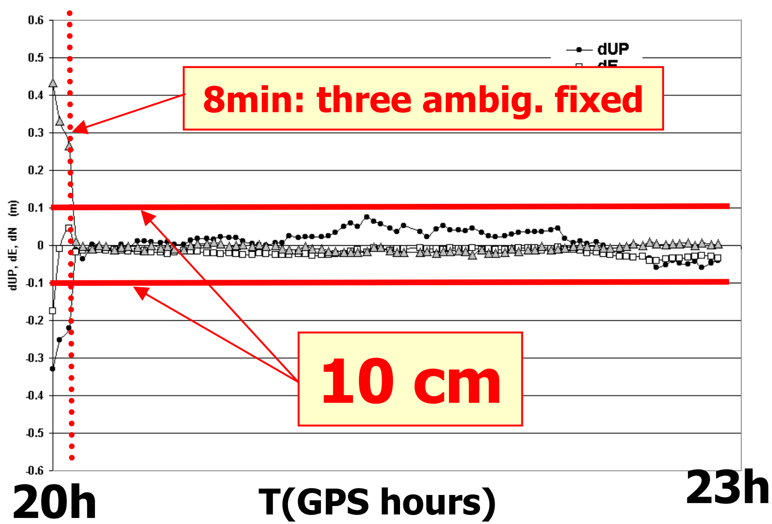


Broadcast orbits Adjusted

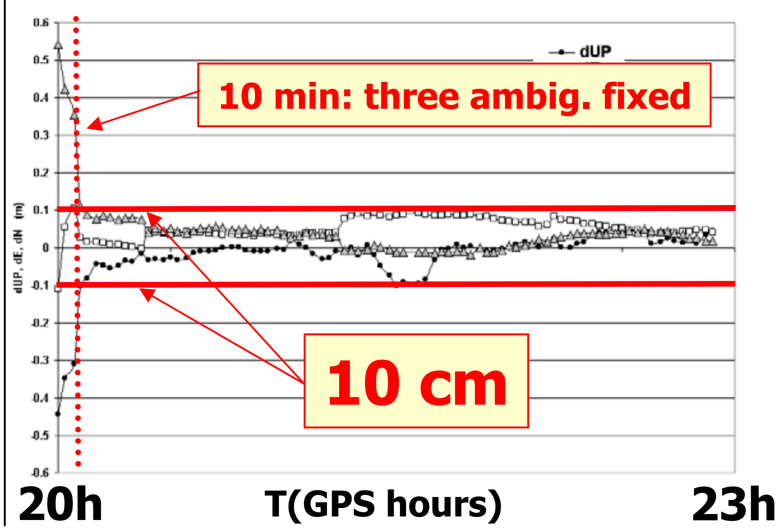


Fixing

Precise SP3 orbits



Broadcast orbits Adjusted



Fixing ambig. with three frequencies: Galileo

- Techniques proposed to improve the instantaneous positioning by using three-frequency systems share a similar basic approach: the double differenced integer ambiguities are successively solved from the longest to the shortest beat-wavelength, including "extra-widelane" and "widelane" combinations of carrier phases.
- In particular TCAR is a straightforward approach that tries to instantaneously solve (single-epoch) the full set of ambiguities. But TCAR (and ITCAR) is strongly affected by the ionos. refraction decorrelation with the distance.
- Ionosphere is a problem (for both 2 & 3 freq.) when :

$$\boxed{\nabla\Delta I > 1/4 TECU \approx 2.7 cm\ of\ \ L1 - L2\ delay}$$

Classical TCAR Technique (1)

TCAR

1. To solve the extra-wide lane ambiguity by adding a pseudo-range combination

2. The wide lane combination ambiguity is estimated from the unambiguous extra-wide lane carrier phase, obtained in step 1

3. The L1 phase ambiguity is derived from the difference between L1 and the unambiguous wide lane obtained previously

Code Multipath Mitigation

$$P=\frac{1}{3}\left(P_1+P_2+P_3\right)$$

$$\frac{1}{\lambda_{ew}}\nabla\Delta\left(L_{ew}-P_{ew}\right)=\nabla\Delta N_{ew}+\frac{1}{\lambda_{ew}}\left(\nabla\Delta M_{ew}+\nabla\Delta\varepsilon_{ew}\right)+\ldots\Rightarrow\nabla\Delta N_{ew}$$

Classical TCAR Technique (2)

TCAR

1. To solve the extra-wide lane ambiguity by adding a pseudo-range combination

2. The wide lane combination ambiguity is estimated from the unambiguous extra-wide lane carrier phase, obtained in step 1

3. The L1 phase ambiguity is derived from the difference between L1 and the unambiguous wide lane obtained previously

Phase multipath

0.058 cycles Nw/TECU

$$\frac{1}{\lambda_w}\left(\nabla\Delta L_w - \nabla\Delta L_{ew} + \lambda_{ew}\nabla\Delta N_{ew}\right) = \nabla\Delta N_w - \frac{1}{\lambda_w}\nabla\Delta(\varepsilon_{ew} + \boxed{m_{ew}} - m_w) + \frac{1}{\lambda_w}(\alpha_w - \alpha_{ew})\nabla\Delta I + \ldots \Rightarrow \nabla\Delta N_w$$

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Classical TCAR (3)

TCAR

1. To solve the extra-wide lane ambiguity by adding a pseudo-range combination

2. The wide lane combination ambiguity is estimated from the unambiguous extra-wide lane carrier phase, obtaine

$$error(\nabla\Delta I) < 1/4\, TECU = 2.7 cm_{L1-L2}$$

3. The L1 phase ambiguity is derived from the difference between L1 and the unambiguous wide lane obtained previou

-1.95 cycles N1/TECU

$$\frac{1}{\lambda_1}\nabla\Delta\left(L_1 - L_w + \lambda_w N_w\right) = \nabla\Delta N_1 - \frac{1}{\lambda_1}\nabla\Delta(\varepsilon_w + m_w - m_1) + \frac{1}{\lambda_1}(\alpha_1 - \alpha_w)\nabla\Delta I \Rightarrow \nabla\Delta N_1$$

Comments about TCAR

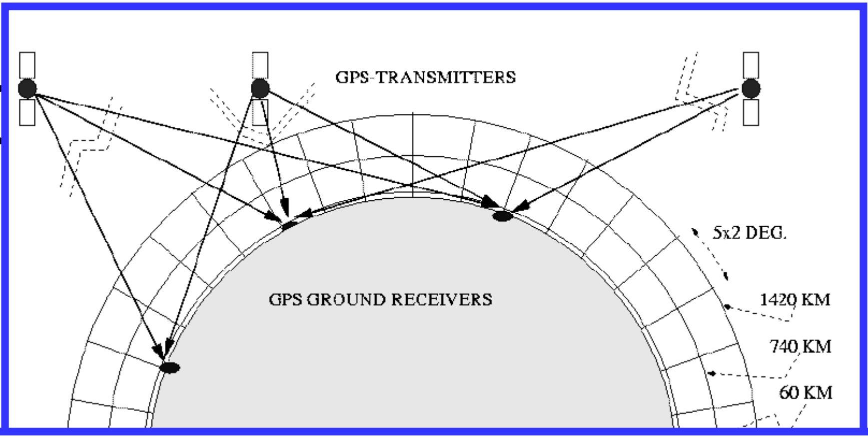
- TCAR is a straightforward approach that tries to instantaneously solve (single-epoch) the full set of ambiguities in three-frequency systems. But TCAR (and ITCAR) is strongly affected by the ionos. refraction decorrelation with the distance.

- The Ionosphere is a problem (for both 2 & 3 freq.) when :

$$\boxed{\nabla \Delta I > 1/4 TECU \approx 2.7 cm \ of \ L1 - L2 \ delay}$$

C iono.

WARTK-3



$$L_I = STEC + B_I = \int_{REC}^{SAT} N_e\, dl + B_I = \sum_i \sum_j \sum_k (N_e)_{i,j,k}\, \Delta s_{i,j,k} + B_I$$

2. The wide lane combination ambiguity is estimated from the unambiguous extra-wide lane carrier phase, obtained in step 1

3. The L1 phase ambiguity is derived from the difference between L1 and the unambiguous wide lane obtained previously

$$\nabla\Delta I < 1/4\; TECU = 2.7\, cm_{L1-L2}$$

-1.95 cycles N1/TECU

$$\nabla\Delta\hat{N}_1 = \frac{1}{\lambda_1}\nabla\Delta\left(L_1 - L_w + \lambda_w N_w\right) = \nabla\Delta N_1 - \frac{1}{\lambda_1}\nabla\Delta(\varepsilon_w + m_w - m_1) + \frac{1}{\lambda_1}(\alpha_1 - \alpha_w)\nabla\Delta I$$

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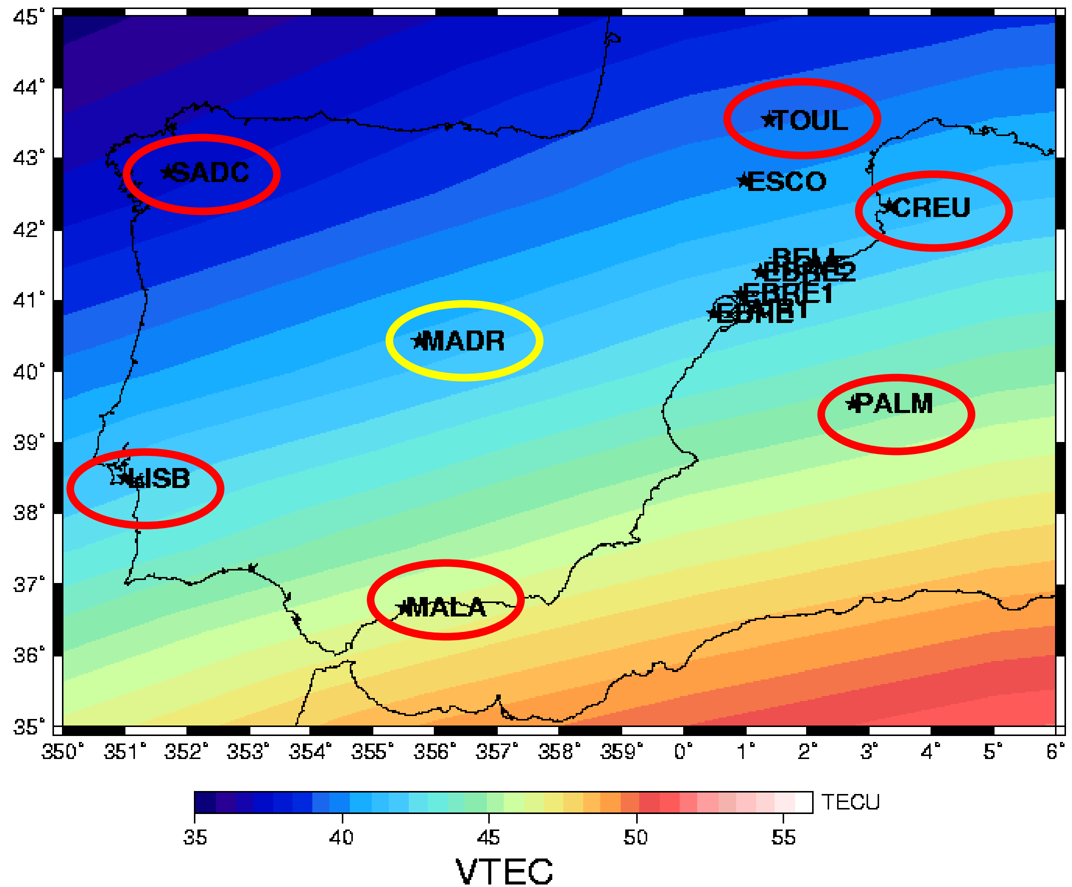
GALILEO: WARTK3 [gAGE-ESA patent,2002]

Centimetric navigation at hundreds of km from ref. stations

	ADVANTAGES	DISADVANTAGES
TCAR (3 frequency)	Low computational load.	Seriously limited by iono. refraction
ITCAR (3 frequency)	Improved results by integrating TCAR in a navigation filter	The iono. delay still limits the 3rd ambiguity fixing.
WARTK-2 (2 frequency)	Accurate RT ionos. modelling, allows precise navigation at hundreds of km far from the nearest site	In spite of speeding up the navigation Kalman filter, a significant convergence time can be still needed (5-15 min).
WARTK-3 (3 frequency)	Uses the extra-widelane, and an accurate iono. model to provide single-epoch navigation capabilities at hundreds of km far, and greatly speeding up the convergence of the Nav. Filter to just a few epochs.	

WARTK-3 allows INSTANTANEOUS (i.e., single epoch) ambiguity fixing at hundreds of Km far from the nearest ref. station

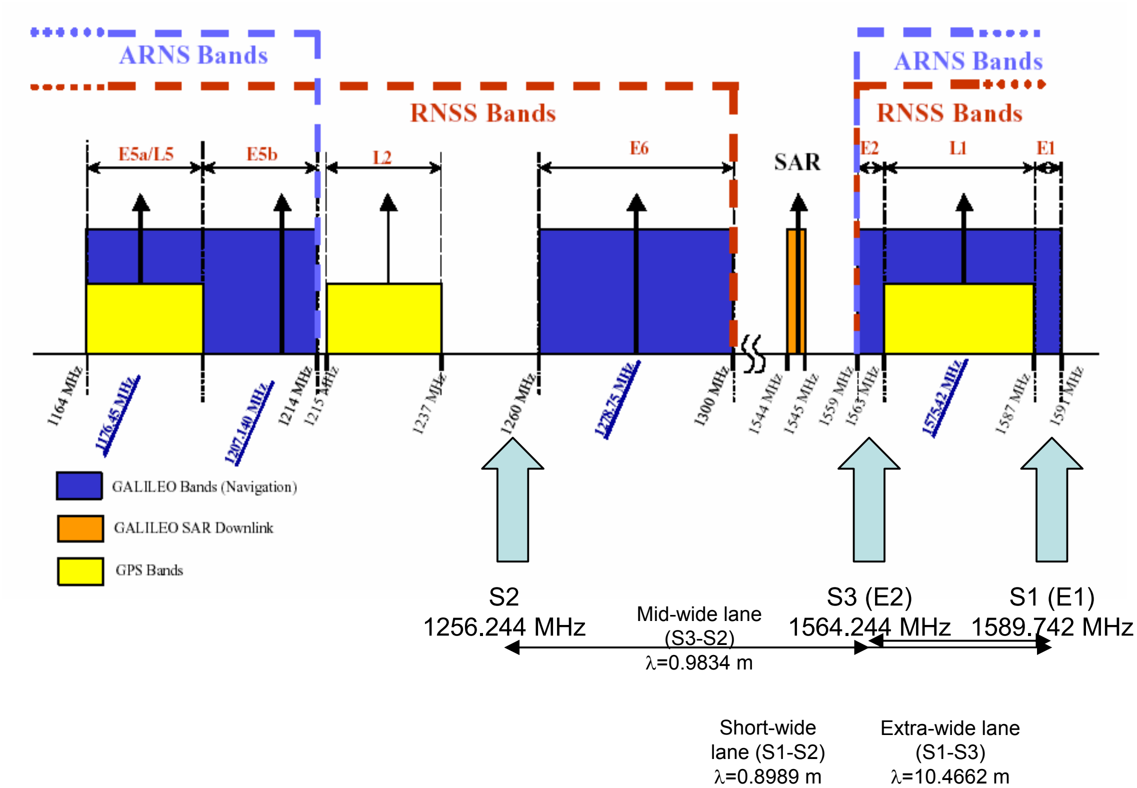
Experiment : MADR at 404km from the nearest station (EBRE) at Solar Maximum Conditions



FIXED STATION TREATED AS ROVER (MADR)

FIXED STATIONS (NETWORK)

WARTK3 Lab.Test camp.: Frequencies

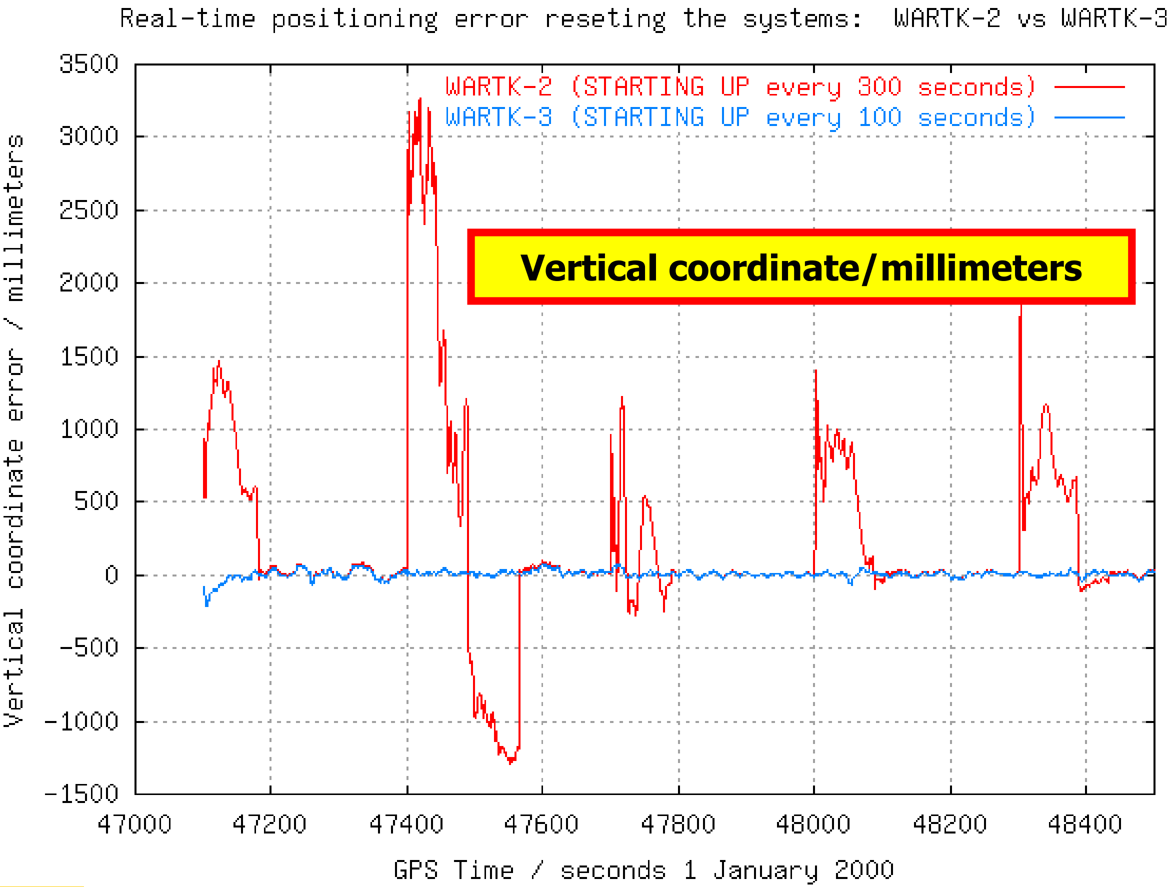


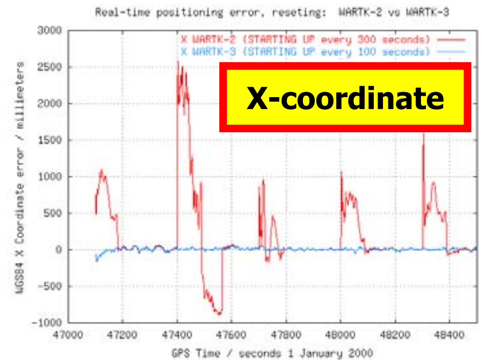
Extra-wide lane (S1-S3): λ=10.4662 m

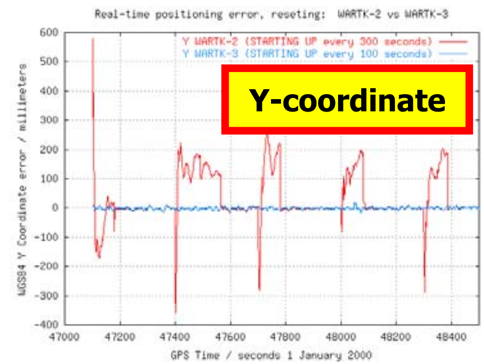
Wide lane (S1-S2): λ= 0.8989 m

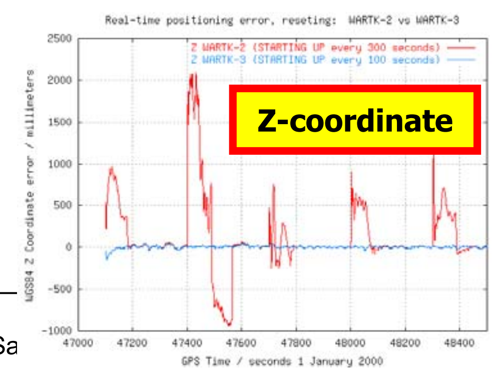
S1: λ= 0.1886 m

ROVE: WARTK3 vs WARTK2 (cool-start each 100/300s, including tropo.). Baseline > 400Km



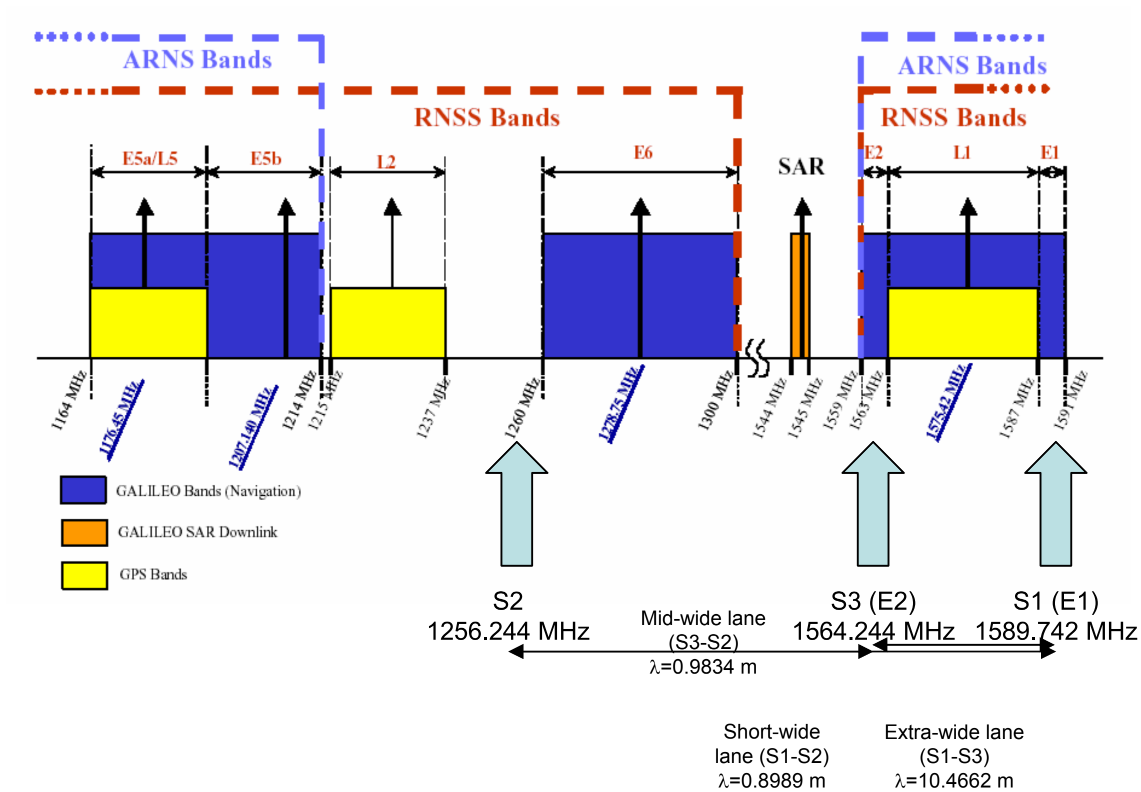






Starting-up everything (cool start): WARTK-2 (i.e. with GPS data) provides equivalent results to WARTK-3 (RMS of 2 cm and 100% amb. fixed), but after a convergence time of ~100 sec. (instead of instantaneously).

WARTK3 Lab.Test camp.: Frequencies



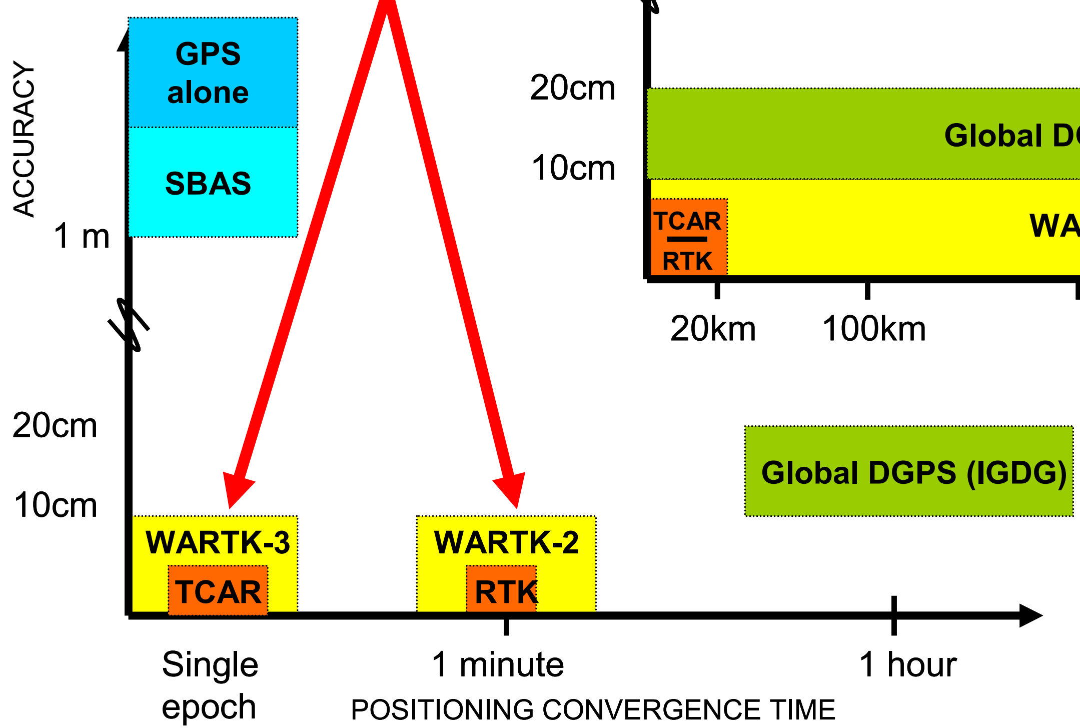
Extra-wide lane (S1-S3): λ=10.4662 m

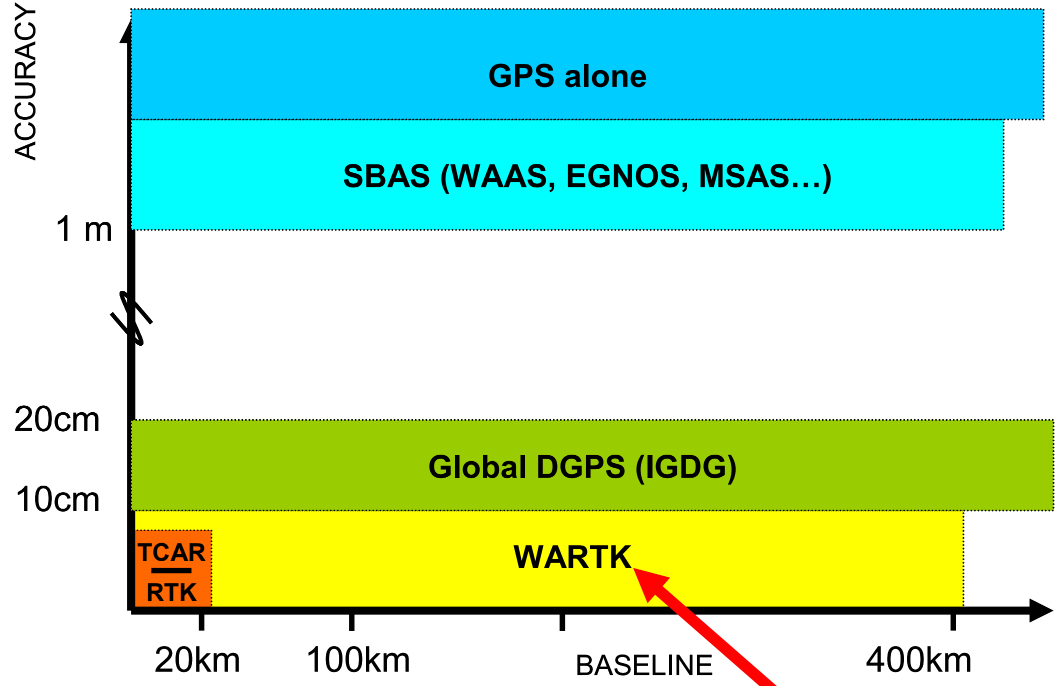
Wide lane (S1-S2): λ= 0.8989 m

S1: λ= 0.1886 m

WARTK: subdecimeter-error navigation hundreds of kilometers away, and in single-epoch with 3-frequency systems

After general cycle-slips: Instantaneous recovery with 3-freq. systems (Galileo, modernized GPS), and about 1 minute with GPS.





Potential use of WARTK on SBAS service areas

WARTK*: experiments & results

Experiment	Shortest baseline/km Rover/Fixed	Ionospheric Activity	Fixed Rec. Ambiguity success %	Roving Rec. Ambiguity success %	Kind of rover	Region	Dates	Reported in
BellKin99	116 / 286	Mid.Solar Cycle & Quite	97	80-100	4x4 Car	Catalonia, NE Spain	23-03-99	Colombo et al. 99 (ION)
NWPacific1	400/900	Mid Solar Cycle & Active high lat. (Kp=6)	90-100	80	IGS Site	NWCanada-USA	03-05-98	Hernández et al. 00a, Colombo et al. 00 (GRL, PLANS)
NWPacific2	162/900	Mid-Low Solar Cycle & Irreg.	95-100	80-90	IGS Site	NWCanada-USA	28-04 to 01-05-98	Hernández et al. 00b (ION)
SolarMax1	130/500	Solar Maximum	85-95	80	IGS Site	Central Europe	19 to 22-04-00	Hernández et al. 01 (GRL)
SolarMax2	130/500	Solar Max. & Supestorm	50-95	80	IGS Site	Central Europe	12 to 15-07-00	Hernández et al. 00b (ION)
Baltic99	144/285	Travelling Iono. Disturb. (TIDs)	97	83	IGS Site	North Europe	25-08-99	Hernández et al. 01b (ION)
Equator01	1000-3000/.	Solar Max. & Equator & Very Active (Kp to 9)	90	---	IGS Sites	Central Asia to Oceania	06-03 to 02-04-01	Hernández et al. 02 (JGR)
TCARdata (simulated)	130/300	Solar Max.	100	92 (single-epoch)	Sim. car	Central Europe	17-03-00 (noon)	Hernández et al. 03a-b (IEEE TGARS, Navigation)
UNBAR01	70-115/100	Solar Max. & Strong TIDs	100	~80 (with integrity)	Car	Barcelona, NE Spain	11-06-01	ION-GPS2004
WARTK3 Lab.Test 1-2	178-238 /250-600	Solar Max.	100	100	1:Car 2:Air.	Iberian Peninsula	31-03-90 (ionos.)	ION-GPS 2004
WARTK3 Lab. Test 3	416/250-600	Solar Max.	100	99	Fixed Site	Iberian Peninsula	31-03-90 (ionos.)	ION-GPS 2004

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Some papers about WARTK and TCAR

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• Colombo O.L., Hernández-Pajares M., Juan J.M. and Sanz J., Ionospheric Tomography Helps Resolve GPS Ambiguities On-the-Fly At Distances of Hundreds of Kilometers During High Geomagnetic Activity, Position Location and Navigation Symposium (PLANS 2000 IEEE conference), San Diego (USA), March 2000.

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That's all,

Thank you for your attention!