GPS data processing: code and phase
Algorithms, Techniques and Recipes

Research **g**roup of **A**stronomy and **GE**omatics (gAGE/UPC)

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To our families, they always solve our really important problems.

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Introduction

This volume contains a series of practical exercises about GPS data processing, addressed to all those professionals and students who would like to introduce themselves to the study of GPS signal and positioning algorithms with code and phase. The exercises are developed with a software package designed for this purpose and provided with no additional cost.

Its contents ranges from the analysis of basic observables (code and phase) to posing and solving navigation equations for absolute and differential positioning. Starting from RINEX files of observations and ephemeris, obtained from public domain data servers, the observables code and phase and their different combinations (ionospheric, free-ionosphere, wide-lane) are analyzed, making some of their aspects clearly and directly observable from graphs (phase *cycle-slips*, ionospheric refraction, multipath, etc.). Files collected under conditions of activated and deactivated Anti-Spoofing can be examined. From the navigation message, satellite coordinates and their clock synchronism errors are computed and, next, the different terms involved in modeling the pseudoranges (geometric distances, relativistic correction, atmosphere –ionosphere and troposphere–, instrumental delays, etc.) are calculated. The impact of Selective-Availability over modeled pseudoranges is studied, comparing the results with the ones obtained using files with precise clocks and orbits available through Internet. The navigation equation system is formulated and solved using estimation techniques such as least mean squares and Kalman filter. These techniques are introduced only from a conceptual point of view, with a view toward its implementation at an algorithmic level.

It is divided into 7 chapters, each of them containing a brief summary with theoretical foundations and a set of laboratory sessions with an approximate duration of two hours, to be carried out under a UNIX environment: Real data files and a specific software package containing different programs and routines designed for the implementation of the processing modules (GPS-Code-Analysis-Tool) will be used. Elementary routines are also provided for some specific functions: Computation of satellite coordinates (at reception and emission), Klobuchar model for ionospheric refraction, etc. The target is, from the start, *to give effectiveness* in the *instrumental use* of the concepts and techniques of GPS data processing.

The exercises are classified into different difficulty levels, which are indicated with "none", one, two or three asterisks. At the end of each session, a form is provided to record the answers from the sections we have considered more representative from an evaluation point of view.

Although a minimum knowledge of UNIX would be desirable, it is not essential to follow the book. Through different "guided" exercises[1], the reader is introduced, in a natural way and by immersion, in the syntax and possibilities of this environment. Our experience has shown us that students with no previous knowledge of UNIX do not find great difficulty in adapting themselves to this language –quite the reverse, they appreciate the fact that their training is taken place in the real context where these problems[2] are found–. Nevertheless, and taking into account that the fundamental purpose of this publication is GPS training, we have included (as appendices) some graphic results of the exercises, as well as different data files to develop most of the conceptual contents of these sessions without the need of executing the programs (some text files with the solutions of the exercises are also provided together with the software).

Its didactic outline is the result of more than fifteen years of university teaching experience. In the same way, the scientific/technological approach has been nourished by our experience in developing different research projects and contracts in Satellite Navigation.

[1] In the first session, some minimum computer elements are introduced (about UNIX, `gawk` and `gnuplot`), for all those who have never worked in this environment.

[2] Nowadays, it is possible to have a highly competitive *UNIX workstation* (LINUX) for little money, thanks to the LINUX operative system. It is a **freely distributable** software (*free-software*) that allows configurate a 486 PC with 4 Mb RAM memory and 200 Mb hard disk, or superior, as a UNIX machine with high performance. In the address *http://www.slackware.com* you could find information and software, in English, for LINUX installation.

Chapter 1
Basic concepts

The GPS system consists of a constellation of at least 24 satellites orbiting at an average height of $20200Km$ over Earth surface, continuously transmitting signals that enable users to determine their three-dimensional position. The positioning principle is based on solving an elemental geometric problem, starting with the distances to a minimum set of four GPS satellites (distances which are measured by the receiver using signals emitted by satellites) and known coordinates. One can determine the user coordinates with an accuracy of about ten meters.

An intuitive idea of GPS positioning

The basic observable of the GPS system is the time of propagation of the electromagnetic wave between the satellite (transmitter) and the receiver. This time, multiplied by the speed of light, gives us a measure of the distance (pseudorange) between them.

The following example[3] summarizes, for a two-dimensional case, the basic ideas about the GPS positioning:
Let's suppose that a lighthouse, with known coordinates within certain accuracy, is emitting acoustic signals at regular intervals of 1 minute (starting at 0h 0m 0s), and intense enough to be heard some kilometers away. Let's also assume a ship with a clock perfectly synchronized with the one in the lighthouse, receiving these signals at a moment of time not being an exact multiple of one minute, for example, 20 seconds later ($t = n * 1^m + 20^s$). These 20 seconds will correspond to the propagation time of sound from the lighthouse (transmitter) to the ship (receiver). Distance d between them will be obtained multiplying this value by the speed of sound $v \simeq 335m/s$: $d = 20s * 335m/s = 6.7Km$. Obviously, with a single lighthouse it is only possible to determine a measure of relative distance. So, the ship could be at any point over a circle of radius d (see figure 1).

[3]inspired in Kaplan (1996)

With a second lighthouse, the ship position will be given by the intersection of the two circumferences centered in the two lighthouses and radii determined by their relative distances to the ship (measured using the acoustic signals). In this case, the ship could be situated at any of the two points of intersection shown in figure 1. A third lighthouse will solve the previous ambiguity[4].

Although the preceding example corresponds to a two-dimensional case, the basic principle is the same in the GPS system:

- In the case of the lighthouses, one assumes that their coordinates are known. In the case of the GPS satellites, these are calculated from the transmitted ephemerides.

- In the GPS positioning, as well as in the example, distances between receiver and satellites are calculated using the propagation time of a signal (in this case, an electromagnetic wave) from the satellite to the receiver (see chapter 2).

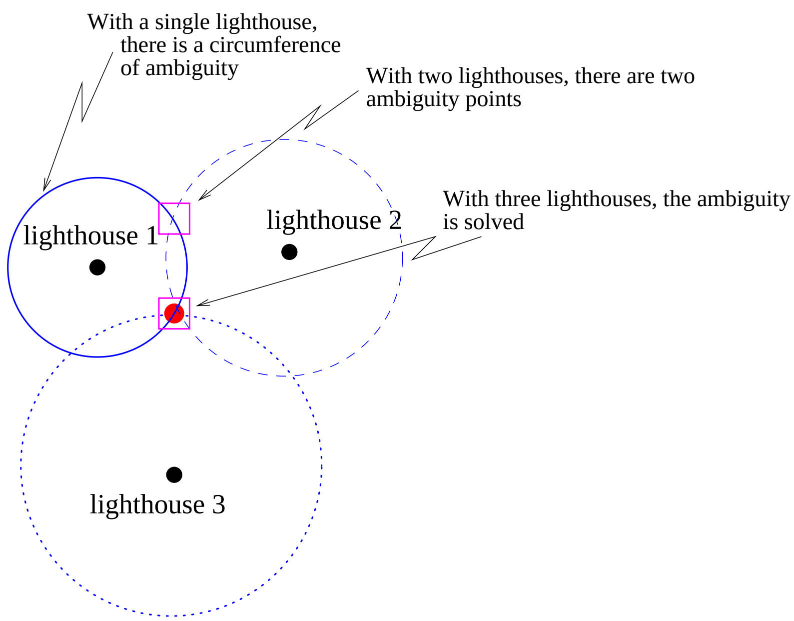


Fig. 1. 2D positioning

[4]In practice, a rough knowledge of the ship position may allow us to go without the third lighthouse. This is the case in the GPS positioning, where one starts from an *approximate* value of the receiver coordinates, being it iteratively refined (in the neighborhood of this point the problem is linearized in order to use minimum square techniques or Kalman filters –chapter 6–).

As shown in the example, an ideal situation with a perfect synchronism between lighthouses and ship clocks is supposed, but in practice this is very difficult to obtain.

A synchronism error between these clocks will produce a mistaken measure of signal propagation time, because it is something related to both clocks and in consequence, a mistaken value of the distance between them.

This situation is shown in figure 2, where the three circumferences do not intersect each other at one point, but in an uncertain region where the solution is placed.

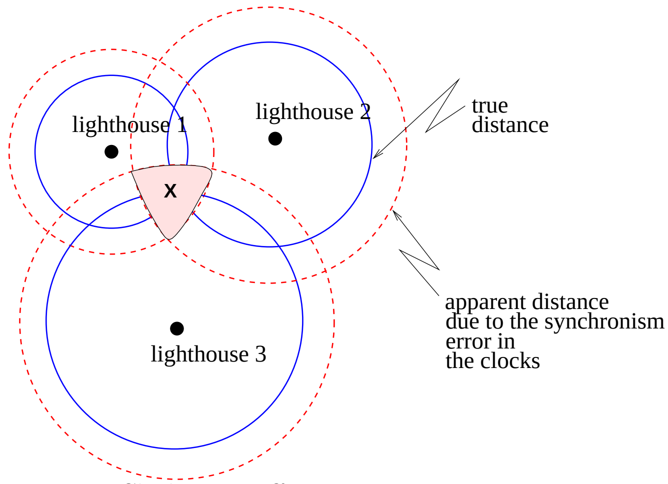


Fig. 2. Clock error effect in positioning

In order to assure the stability of clocks, GPS satellites are equipped with atomic oscillators with stabilities of about 10^{-13} (see chapter 2). In the case of commercial receivers, the used quartz clocks are much more economical but with a poorer stability. This inconvenient is overcome by estimating its synchronism error at the same time as the coordinates.

Finally, the geometry of the satellites depends on how the viewer sees them and how it affects the positioning error. This is illustrated in figure 3, where the size and shape of the region change depending on their relative position. This effect (Dilution Of Precision –DOP–) is studied in chapter 4.

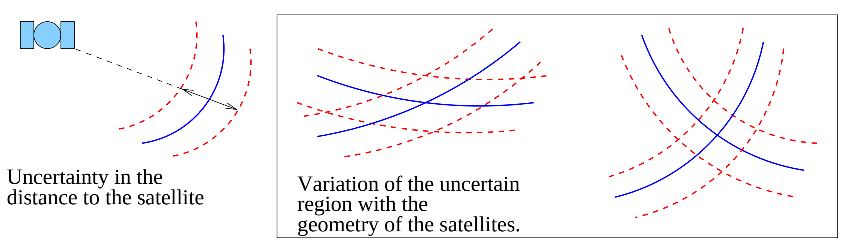


Fig. 3. DOP effect in positioning

Session 1
Computer tools

Objectives

To present a (very limited) set of UNIX instructions in order to run files and directories, as well as some basic elements of `gawk` programming and graphics environment `gnuplot`. The aim is not to teach UNIX or programming languages, but to provide some basic tools to develop these practical sessions.
NOTE: this session is very elementary and can be omitted if you already have some basic knowledge of UNIX, `gawk` *and* `gnuplot`.

Files to be used
`sxyz.eci`

Development

This session has been organized in a series of guided exercises thought to be done in the established order, introducing the main instructions to be used in following sessions.

1. [**First instructions**]

   (a) Show present working directory.
   Execute: `pwd`

   (b) See contents of present directory.
   Execute: `ls -lt`

   (c) Get into the personal directory or *home directory* ("~")[5]
   Execute: `cd` or `cd ~`

[5]If the installation has been properly done according to instructions in installation guide, one must find the following three directories: `files`, `programs` and `working` which will be hanging from the personal directory.

(d) Get into the directory `working` and see its contents.
Execute:
```
cd working
ls -lt
```

(e) Show a text line on the screen:
Execute:
```
echo "this is a test"
```

(f) Direct the contents to a file:
Execute:
```
echo "this is a test" > test
ls -lt
echo "this, too" >> test
```

(g) Show the contents of a file on the screen:
Execute:
```
cat test
```
Try and execute too: `echo test`. What happens?

(h) Edit a file
Execute:
```
textedit test
```

2. [**Directory management**]

(a) From any directory where you were working, place yourself in the directory `working` (which is hanging from the personal directory "~") and check if you are placed there. Create the directory `other` inside directory `working`. Access to it. Go back to the working directory (immediately above).
Execute:
```
cd ~/working
pwd
mkdir other
cd other
pwd
cd ..
pwd
```

3. **[File management]**

   (a) Place yourself in the directory `working`. Copy the file `test` in the personal directory (directory immediately above).
   Execute:
   ```
   cd ~/working
   cp test ../
   ls -lt
   ```
   (b) Copy file `test` to file `file1`[6]. Check the file contents of `file1`
   Execute:
   ```
   cp test file1
   ls -lt
   more file1
   ```
   (c) Create a "link"[7] from file `file2` to file `test`. Check the file contents. Check contents of `file2`.
   Execute:
   ```
   ln -s test file2
   ls -lt
   more file2
   ```
   (d) Using the program `textedit`, edit `file2` and change the word *test* for the word *wonderland*. Save changes and quit from `textedit`. Next, check the content of file `test` and its "link" `file2`. Has the content of the original file `test` been modified through its link `file2`?
   Execute:
   ```
   textedit file2
   more test
   more file1
   ```
   (e) Remove the file `file1` and its link `file2`. Check if they have been deleted. Remove the directory `other`.
   Execute:

[6] As file `file1` does not exist, a new file will be created with this name and same contents as file `test`.

[7] It differs from the previous case because `file2` is not a new file, but only a pointer to the file `test`. Thus, the "link" `file2` represents a minimum space expense, not depending on the file `test` size. Execute `man ln` to see the meaning and different types of links.

```
rm file1 file2
ls -lt
mkdir other
rm -r other
ls -lt
```

4. [**Programming environment** `gawk`][8]

   (a) Place yourself in the working directory and create a link from file `sxyz.eci` (which is placed in the directory `files`), to a file with the same name in the working directory.
   Execute:

   ```
   cd ~/working
   ln -s ~/files/sxyz.eci .
   ls -lt
   ```

   The file `sxyz.eci` has coordinates, with respect to the Earth mass center, of a set of satellites in different moments in time. It contains the following fields:

   ```
   SATELLITE time(sec) X(Km) Y(Km) Z(Km)
   ```

   (b) Execute the following instructions `cat`, `more` and `less` in order to display the file contents `sxyz.eci`. What differences are observed among these instructions[9]?
   Execute:

   ```
   cat sxyz.eci
   more sxyz.eci
   less sxyz.eci
   cat sxyz.eci | less
   ```

   (c) Using programming language `gawk`, print (on screen) the first and the third field of file `sxyz.eci`.
   Execute:

   ```
   gawk '{print $1,$3}' sxyz.eci |more
   o bien
   cat sxyz.eci |gawk '{print $1,$3}' |more
   ```

[8] `gawk` is an evolved version from `awk`

[9] the command "`|`" allows us to connect the output of a process with the input of another. For example, the output of `cat` can be sent to `more`.

(d) Now, print all the fields at the same time.
Execute:

```
cat sxyz.eci |gawk '{print $0}' |more
```

(e) The following instruction generates the file `prb1` which contains data from a single satellite. Which one is this satellite?
Execute:

```
cat sxyz.eci |gawk '{if ($1==5) print $0 }' > prb1
more prb1
```

(f) What is the meaning of the values in the second column of the file `prb2` generated with the next instruction?
Execute:

```
cat sxyz.eci|gawk '{if ($1==5)
    print $2,sqrt($3**2+$4**2+$5**2)}'> prb2
more prb2
```

(g) Discuss the structure of the following instruction that makes a "print" with format (Note: `%i`=integer, `%f`= float, `%s`= string)
Execute:

```
cat sxyz.eci |gawk '{printf
    "%2i %02i %11.3f %i %s \n",$1,$1,$3,$3, $1}'|more
```

(h) Access to the manual pages of `gawk`
Execute `man gawk`

5. [**Graphics environment** `gnuplot`]

(a) Enter the environment `gnuplot`. For the given file `prb1`, previously generated, plot the third field (x coordinate) as a function of the second one (time in seconds). Exit `gnuplot`
Execute:

```
gnuplot
plot "prb1" u 2:3
exit
```

(b) Redo the former graph for the interval [20000 : 30000] of x axis. Superpose a lattice (*grid*) in the figure. Then, repeat the graphic representation for interval $[-2e4 : 2e4]$ of y axis, and for any x value.
Execute:

```
gnuplot
set xrange[20000:30000]
set grid
plot "prb1" u 2:3
set auto x
set yrange [-2e4:2e4]
replot
exit
```

(c) For the same file used in the previous cases `prb1`, plot on the same graph the x coordinates (third field), y (fourth field) and z (fifth field) as a function of time (second field).
Execute:

```
gnuplot
plot "prb1" u 2:3,"prb1" u 2:4,"prb1" u 2:5
exit
```

(d) Visualize the different ways of graphic representations in the following instructions (with points "w p", with lines "w l", lines+points "w linespoints")
Execute:

```
gnuplot
set xrange[20000:25000]
plot "prb1" u 2:3
plot "prb1" u 2:3 w p 3
plot "prb1" u 2:3 w p 2
plot "prb1" u 2:3 w l
plot "prb1" u 2:3 w d
plot "prb1" u 2:3 w linespoints
exit
```

(e) In the following plots, different examples of `gawk` use inside `gnuplot` are shown.
Execute:

```
set xrange[0:90000]
set yrange [-3e4:3e4]
plot "< cat sxyz.eci |gawk '{if ($1==5) print $0}'" u 2:3
plot "< cat prb1 |gawk '{if ($2<20000||$2>50000) print $0}'" u 2:3
plot "< cat prb1 |gawk '{if ($2>30000 && $3>0) print $0}'" u 2:3
exit
```

Note:
"`if ($2<20000||$2>50000)`" means `$2<20000` <u>or else</u> `$2>50000`
"`if ($2<20000 && $2>50000)`" means `$2<20000` <u>and</u> `$2>50000`

(f) Consult "help" in `gnuplot`
Execute:

```
gnuplot
help
Help topic:  glossary
(try different options)
exit
```

Chapter 2
GPS system description

GPS system is formed by three great blocks: 1) Space segment, 2) Control segment and 3) User segment.

1. Space segment

Space segment main functions are, from instructions sent by the control segment, to provide an atomic time reference, generate RF pseudorandom signals, and store and retransmit the navigation message.

It consists of the following components:

Constellation

Space segment is formed by a satellite constellation of at least 24 satellites, arranged in 6 orbital planes, with an inclination of 55 degrees in relation to the equator. Orbits are nearly circular, with eccentricity less than 0.02, a semi-major axis of 26000 km and a period of 12 sidereal hours (11h 58min 2sec). The present configuration allows users to have a simultaneous observation of at least 4 satellites from any point on the Earth surface at any time, with an elevation above the visible horizon of the viewer greater than 15 degrees.

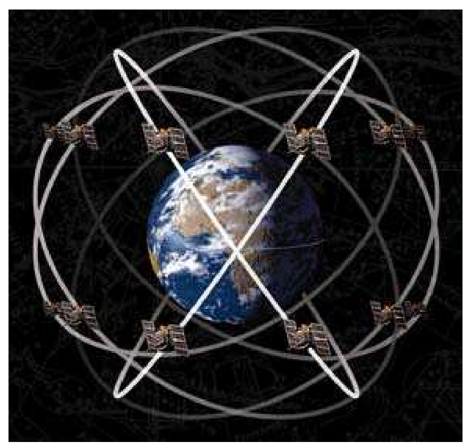


Fig 4. GPS satellite constellation

The satellites

Satellites have structures and mechanisms in order to keep themselves in orbit, communicate with the control segment, and emit signals to receivers. One of the critical aspects of the GPS system are satellite clocks. For this reason, satellites are equipped with atomic clocks (rubidium, cesium) which have a very high stability. (see in appendix I, the constellation configuration in the first third of year 2005).

The following group of satellites have been developed (A. Leick pag. 61):

- *Block I, Navigation Development Satellites.* Between 1978 and 1985, eleven satellites of this kind were launched. S/A was not implemented. They weighted about 845 Kg and had a planned averaged life of 4.5 years, although some of them lasted up to 10. They were capable of giving positioning service for 3 or 4 days without any contact with the control center.

- *Block II and IIA, Operational Satellites.* In present days, they are still operating. They consist of 28 satellites in total that were launched from 1989. They weight about 1500 Kg and have a planned average life of 7.5 years. From 1990, an improved version was used, block IIA (advanced), with capability of mutual communication. They are able to supply positioning service for 180 days with no contact with the control segment. However, under normal operating mode, they must communicate daily.

- *Block IIR, Replacement Operational Satellites.* From 1997, these satellites are being used as spare satellite of block II. It is formed by a set of 20 satellites, although it could be increased by 6 more. They weight about 2000 Kg and have a planned average lifespan of 10 years. These satellites will have the capability to autonomously determine their orbits and generate their own navigation message. They will be able to measure distances between them and transmit observations to other satellites or to the control segment. A satellite of this kind, completely developed, must be capable of operating about half a year without any support of control segment and no degradation in ephemeris accuracy. It is expected that some of them could be equipped with hydrogen masers.

- *Block IIF, Follow-on Operational Satellites.* Their launch is predicted as of 2001. Their theoretical averaged life is of about 10 years, and will have inertial navigation systems.

Clock types	Daily stability ($\Delta f/f$)	Time that takes to deviate a second
Quartz crystal	10^{-9}	30 years
Rubidium	10^{-12}	30 000 years
Cesium	10^{-13}	300 000 years
Hydrogen	10^{-15}	30 000 000 years

Table 1: Clock stabilities (source: A.Leick, pp.28)

GPS satellites are identified in different ways: by their position in the orbital plane [every satellite has a place (1, 2, 3, ...), inside the six orbits - A, B, C, D, E or F], their NASA reference number, their international identification number, PRN code (pseudorandom noise code), and their space vehicle number (SVN).

GPS signal

Every satellite transmits signals centered on two frequencies lying in the L band. These frequencies are derived from a fundamental frequency f_0 =10,23 MHz (with a relation $\frac{154}{120}$), generated by its atomic clocks with a stability about 10^{-13} (see table 1).

$$\boxed{\begin{array}{l} L1 = 154 \cdot 10.23\ MHz = 1575.42\ MHz \\ L2 = 120 \cdot 10.23\ MHz = 1227.60\ MHz \end{array}}$$

The fact that the satellites emit in two different frequencies allows the user to cancel one of the main error sources: ionospheric refraction. It is due to the ionosphere, which acts as a scattering media for GPS signals.

The following types of PRN codes and messages are modulated over the two carriers (see figure 5):

- *Coarse/Acquisition code [C/A(t)]*, it is also known as civilian code. This sequence is repeated every millisecond and at a velocity, or "chip-rate", of 1 Mbps, which is equivalent to a wavelength of 293.1 m. It is modulated only over $L1$.

- *Precision code [P(t)]*, it is reserved for military use and authorized civilian users. The sequence is repeated every 266 days (38 weeks) and a weekly portion of this code is assigned to every satellite, called PRN sequence. Its velocity or "chip-rate") is 10 Mbps, equivalent to a wavelength of 29.31 m and modulated over both carriers $L1$ and $L2$.

- *Navigation message [D(t)]*, it is modulated over both carriers at 50 bps, reporting information about ephemeris and satellite clock drifts, ionospheric model coefficients, constellation status, etc.

$$\boxed{\begin{array}{l} L1(t) = a_1 \cdot P(t) \cdot D(t) \cdot \sin(f_1 \cdot t + \phi_{P_1}) + a_1 \cdot C/A(t) \cdot D(t) \cdot \cos(f_1 \cdot t + \phi_c) \\ L2(t) = a_2 \cdot P(t) \cdot D(t) \cdot \sin(f_2 \cdot t + \phi_{P_2}) \end{array}}$$

The signal structure is summarized in the following figure:

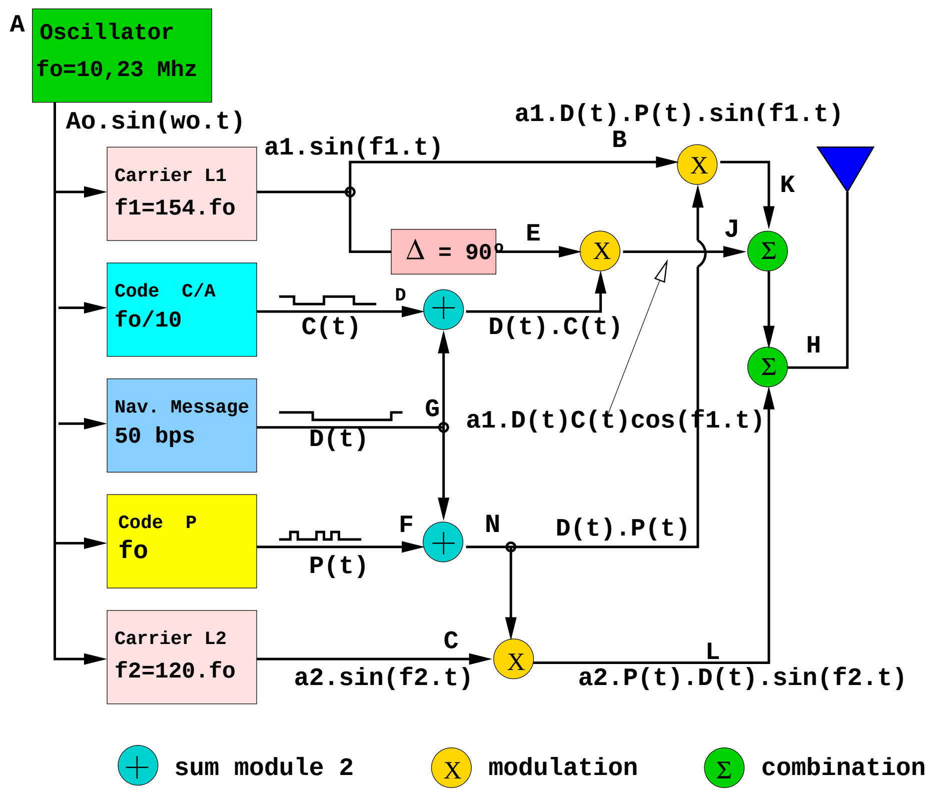


Fig. 5. GPS signal structure (source: G. Seeber, pp 218)

Atomic clock frequency	fo=10.23 MHz
Signal carrier L1	154 x fo
Frequency L1	1575.42 MHz
Wavelength L1	19.05 cm
Signal carrier L2	120 x fo
Frequency L2	1227.60 MHz
Wavelength L2	24.45 cm
P code frequency (chipping rate)	fo=10.23 MHz (Mbps)
P code wavelength P	29.31 m
P code period	266 days, 7 days/satellite
C/A code frequency (chipping rate)	fo/10=1.023 MHz
C/A code wavelength	293.1 m
C/A code period	1 millisecond
Navigation message frequency	50 bps
Frame length	30 seconds

Table 2. GPS signal structure (source: G. Seeber p 217)

In order to restrict civilian users access to full system accuracy, the next techniques have been developed:

- S/A or Selective Availability: intentional satellite clock degradation (process-δ) and ephemeris manipulation (process-ϵ). The effect on horizontal positioning implies going from about 10 m (S/A=off) to 100 m (S/A=on) (2σ-error). The process δ acts directly over satellite clock fundamental frequency, which has a direct impact on pseudoranges to be calculated by user's receivers. The process ϵ consists in truncating information related to the orbits.

- A/S or Anti-Spoofing: it consists in P code encryption by combining it with a secret W code, resulting the Y code, which is modulated over the two carriers $L1$ and $L2$. The purpose is to avoid the access of non authorized users to codes on both $P1$ and $P2$ frequencies, being solely C/A code (noisier) available over $L1$.

Control segment

The control segment is the responsible for GPS system functioning. Its basic functions are:

- To control and maintain the status and configuration of the satellite constellation.
- To predict ephemeris and satellite clock evolution.
- To keep GPS time scale (through atomic clocks).
- To update navigation messages of all satellite.

It is also responsible for Selective Availability, *S/A*, activation in signal transmission.

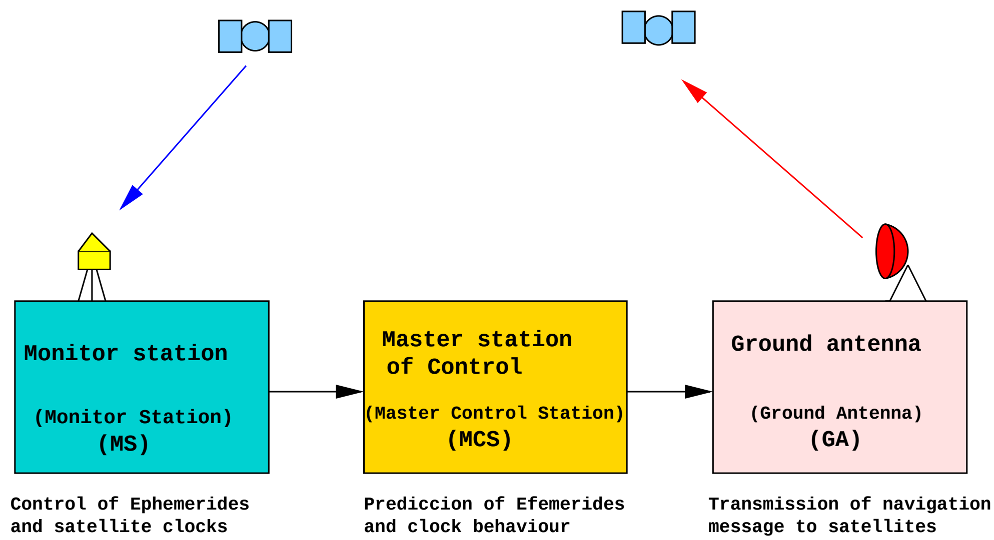


Fig. 6. Control segment diagram (source G. Seeber p. 215)

The control segment consists of five *Monitor Stations* located in Hawaii, Colorado Springs, Ascension Island in South Atlantic, Diego Garcia in Indian Ocean, and Kwajalein in North Pacific; a master control station located in Colorado Springs, and three ground antennas transmitting data to the satellites in Ascension, Diego Garcia, and Kwajalein.

Monitor stations continuously track the visible satellites from all these places. They are equipped with receivers that measure in both frequencies $L1$ and $L2$, all signals coming from satellites in view at local horizon. Tracking data are sent to the *Master Control Station*. Once there, they are processed to estimate satellite orbits (ephemeris) and clock errors, among other parameters. Orbits are affected by perturbations such as gravitational attraction from the Sun, Moon, and solar radiation pressure on the satellite, among others effects. For these reasons, correction calculations have to be made at some time intervals, which originates a new navigation message to be sent to the *Ground Control Stations* in order to be retransmitted to satellites. This is performed with ground antennas via S band radio links. Every satellite can be "uploaded" three times per day, i.e. every 8 hours; nevertheless, usually it is updated once a day.

User segment

The user segment is composed by GPS receivers. Their main function is to receive GPS signals, determine pseudoranges, and solve navigation equations in order to obtain their coordinates and provide a very accurate time.

Basic elements of a generic GPS receiver are an antenna with pre-amplification, a radio frequency section, a microprocessor, an intermediate-precision oscillator, a feeding source, a some memory for data storage, and an interface with the user. The calculated position is referred to antenna phase center.

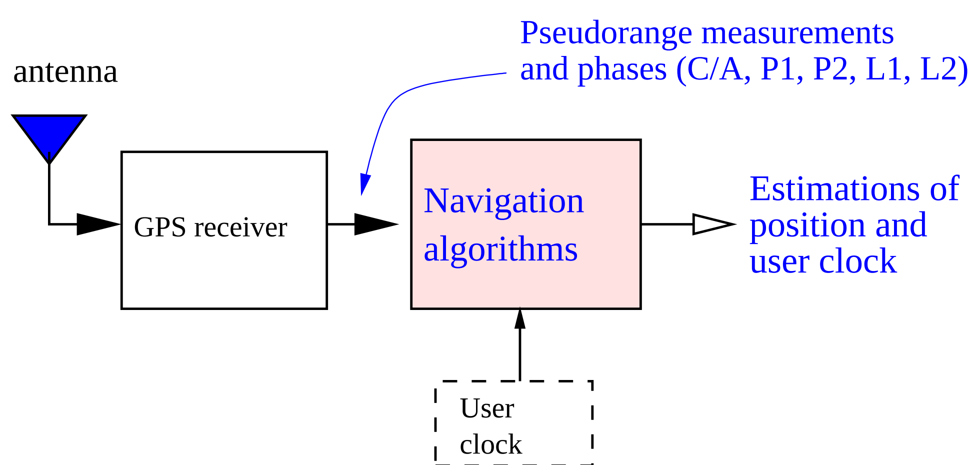


Fig 7. Basic scheme of a GPS receiver (source BW Parkinson, Vol I, p. 246)

Navigation message

Every satellite receives from the ground antennas, a message containing information about its orbital parameters, clock status, and other temporary data. This information is sent back to the user through the navigation message.

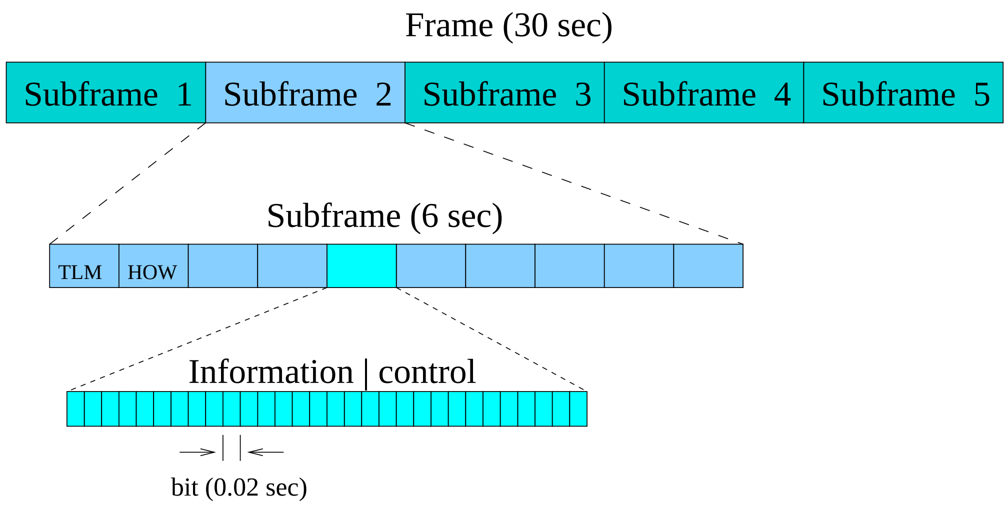


Fig. 8. Navigation message

Navigation message is modulated on both carriers at 50 bps. The whole message contains 25 pages or frames, forming the master frame that takes 12,5 minutes to be transmitted. Every frame is subdivided into 5 subframes of 6 seconds each; at the same time, every subframe consists of 10 words, with 30 bits per word. A frame takes 30 seconds to be sent.

Every subframe always starts with the telemetry word (TLM), which is necessary for synchronizing. Next, the transference word (HOW) appears. Its mission is to allow a quick commutation from C/A code to P code.

The content of every subframe is as follow:

- *Subframe 1:* contains information about the parameters to be applied to satellite clock status for its correction. These values are some coefficients that allow to convert time on board to GPS time. It also has data about satellite health condition and information about message ambiguity.

- *Subframes 2 and 3:* these subframes contain satellite ephemeris.

- *Subframe 4:* in this part one can find ionospheric model parameters (in order to adjust ionospheric refraction), UTC information (Universal Coordinate Time), part of the almanac and indications whether the Anti-Spoofing, *A/S*, is activated or not (which transforms P code into the encrypted Y code).

- *Subframe 5:* contains data from the almanac and the constellation status. With this, one can quickly identify the satellite from which the signal comes. 25 frames are needed to complete the almanac.

SEGMENT	ENTRANCE	FUNCTION	PRODUCT
SPACE	Navigation message Commands	Provide a scale of atomic time Pseudocode signal generation Store and transmit the navigation message	Pseudorandom RF signals Navigation message Telemetry
CONTROL	Pseudorandom RF signals Telemetry UTC	Calibration of time scale, ephemeris prediction Keep active the spatial segment	Navigation message Commands
USER	Pseudorandom RF signals Navigation message	Solve navigation equations	Position Velocity Time

Table 3: Information flow between segments (source: A Leick, p. 60)

Time and reference frames

Time

There exist several time references based on different periodic processes associated with Earth rotation, celestial mechanics or transitions between energetic levels in atomic oscillators. The following table, based on Hofmann-Wellenhof et al. (1994), pag. 39, resumes the most important ones.

Periodic phenomenon	**Time**
Earth rotation	Universal Time (UT0, UT1, UT2) Sidereal time
Earth revolution	Terrestrial dynamic Time (TDT) Barycentric Dynamical Time (BDT)
Atomic Oscillators	International Atomic Time (IAT) Universal Coordinated TIME (UTC) GPS Time (GPST)

Table 4: Different types of time

Universal and Sidereal times are associated with Earth daily rotation. Universal time (solar time) uses the Sun as a reference. Sidereal time uses a direction out of the Solar system (Aries point). This leads to the fact that, in a year, both times differ in 24h (a lap), which supposes $3^m\,56.4^s$ per day.

$$1 \text{ mean sidereal day} = 1 \text{ mean solar day} - 3^m 56.4^s$$

Universal times UT0, UT1, UT2, in contrast of atomic times, are not completely uniform[10]. Due to this, Universal Coordinated Time (UTC) was introduced, which is an atomic time that keeps itself at least as close as 0.9s to UT1, by means of the systematic introduction[11] of certain number of seconds called *Leap Seconds*. This causes that, along time, the difference between UTC and IAT varies in integer leaps of 1 second.

GPS time is the reference time used in GPS applications. The origin epoch is 00:00 UTC (midnight) of 5th to 6th of January of 1980 ($6^d.0$). At that epoch,

[10]Earth rotation is not uniform. UT0 is a time based on instantaneous rotation of the Earth, UT1 is adjusted from periodic variation and UT2 is obtained correcting it from other additional irregularities.

[11]Due to the gradual decrease of the Earth rotation speed

the difference UTC−IAT was 19 seconds.

The following relations are met:

$$\text{IAT}=\text{GPST}+19^s.00$$
$$\text{IAT}=\text{TDT-}32^s.184$$
$$\text{IAT}=\text{UTC}+1^s * n$$

where n is the number of *Leap Seconds* introduced for a given epoch (..., 01-JAN-1996 $n = 30$, 01-JUL-1997 $n = 31$, 01-JAN-1999 $n = 32$,...)

In order to facilitate calculations in long time intervals[12], Julian date is used (invented by Julio Scaliger), having as a reference epoch the 1st of January of the year 4713 before our era, and starting from there, days are being counted in a correlative way. The Julian Day (JD) commences at 12^h of the corresponding civil day (f.e.: $6^d.0$ January 1980= JD 2,444,244.5). The current reference standard epoch for the scientific community is:

$$\boxed{\text{J2000.0} = 1^d.5 \text{ January 2000} = \text{JD } 2{,}451{,}545.0}$$

The Modified Julian Date (MJD) is also used, which is obtained subtracting 2,400,000.5 days from the Julian day.

The following relation allows us to calculate Julian Date (JD) from civil date[13] (YY MM DD UT):

$$\boxed{\begin{array}{l} JD = int[365.25 * y] + int[30.6001 * (m+1)] + DD + \frac{UT(hours)}{24} + 1720981.5 \\ \\ \text{where:} \quad \begin{array}{lcl} y = YY-1,\ m = MM+12 & , & MM \leq 2 \\ y = YY,\ m = MM & , & MM > 2 \end{array} \end{array}}$$

From Julian day, and taking into account that GPS reference data ($6^d.0$ January 1980) corresponds to the Julian day JD 2,444,244.5, one immediately obtains the GPS day and, from there and taking module, the GPS week[14] is obtained.

---

[12]The calendar has suffered important adjustments through history due to the fact that the length of a year is not exactly 365 days. For example, on Friday 5th of October of 1582, Pope Gregory XIII introduced a leap of 10 days –Gregorian reform–, becoming Friday 15th. As an anecdote, Santa Teresa de Jesus died on Thursday 4th and was buried on Friday 15th, the following day – `http://www.newadvent.org/cathen/14515b.htm` –).

[13]This expression is valid between March 1900 and February 2100 (year 2000 is a leap year).

[14]GPS week starts the night from Saturday to Sunday. For example, the day May 3rd, 1998 corresponded to GPS week 956.

Reference systems

Satellite coordinates and user receivers must be expressed in a well defined reference system. *Conventional Inertial System* and *Conventional Terrestrial System*[15] are introduced below.

- *Conventional Inertial System*[16] *(CIS)*
  It has its origin in the Earth mass center. X axis points in the direction of the mean equinox in J2000.0 epoch, Z axis is orthogonal to the plane defined by the mean equator in J2000.0 epoch (fundamental plane) and Y axis is orthogonal to the former ones, so the system is directly (right handed) oriented. The practical implementation is called Inertial Reference Frame (IRF) and it is determined from a fundamental set of stars. The mean equator and equinox J2000.0 were defined by International Astronomical Union (IAU) agreements in 1976, with 1980 nutation series (Seildelmann, 1982 and Kaplan, 1981), which are valid analytic expressions for long time intervals (the previous reference epoch was 1950.0).

- *Conventional Terrestrial System (CTS)*
  Also called Earth Centered Earth Fixed System (ECEF), it has its origin in the Earth mass center. Z axis is identical to the direction of the Earth rotation axis defined by CIO (Conventional International Origin), X axis is defined as the intersection of the orthogonal plane to Z axis (fundamental plane) and Greenwich mean meridian, and Y axis is orthogonal to both of them, making the system directly oriented. Examples of CTS systems are ITRS and WGS84 introduced, respectively, by IERS (International Earth Rotation Service) and DoD (Dept. of Defense, USA). Implementations of ITRS are ITRF, which are updated every year (ITRF98, ITRF99, ...). With respect to WGS84, except the initial one, its implementations are close to some of ITRS.

[15] One usually distinguishes between reference *System* and reference *Frame*. The first one is understood as "a theoretical definition", including models and standards for its implementation. The second one is its "practical implementation" through observations and a set of reference coordinates (set of stars or fiducial stations).

[16] It is not an inertial system in a strict way, because it is affected by the revolution movement of the Earth around the Sun.

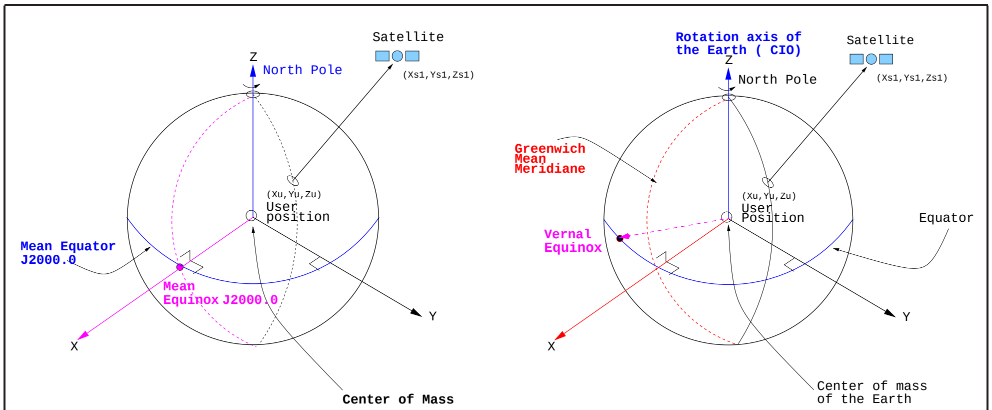


Fig. 9: Reference system CIS *Reference system CTS*

Coordinate transformations between CIS and CTS systems are performed by mean of rotations corresponding to (see, for example, transformations between these systems in Hofmann-Wellenhof et al. (1994)):

– *Precession and nutation* [forced rotation]: Earth rotation axis (and its equatorial plane) is not kept fixed in space, but it rotates about the pole of the ecliptic, as it is shown in figure 10. This movement is due to the effect of the gravitational attraction of the Moon and the Sun over the terrestrial ellipsoid. The total movement can be split into a secular component (*precession*, with a period of 26000 years) and a periodic component (*nutation*, with a period of 18.6 years).

– *Pole movement* [free rotation]: due to the structure of the Earth mass distribution and to its variation, the instantaneous pole shifts inside a square of about 20 meters in relation to a point with fixed coordinates on Earth. This movement has a period of about 430 sidereal days (Chandler period). On the other hand, Earth rotation velocity is not constant, but it variates in time, although in very small quantities[17], with a net decrease, which is the responsible for the necessity of introducing *leap-seconds* in order to maintain, with lesser than 0.9s, the difference between UTC (atomic time) and UT1 (time tied to the Earth rotation), defined in the previous section.

[17] Friction of water in shallow seas, atmosphere movements, abrupt displacements in the Earth interior (in 1955, the rotation suddenly delayed by $41^s \cdot 10^{-6}$), etc. Note that CTS system is tied to Greenwich meridian and therefore, rotates with the Earth.

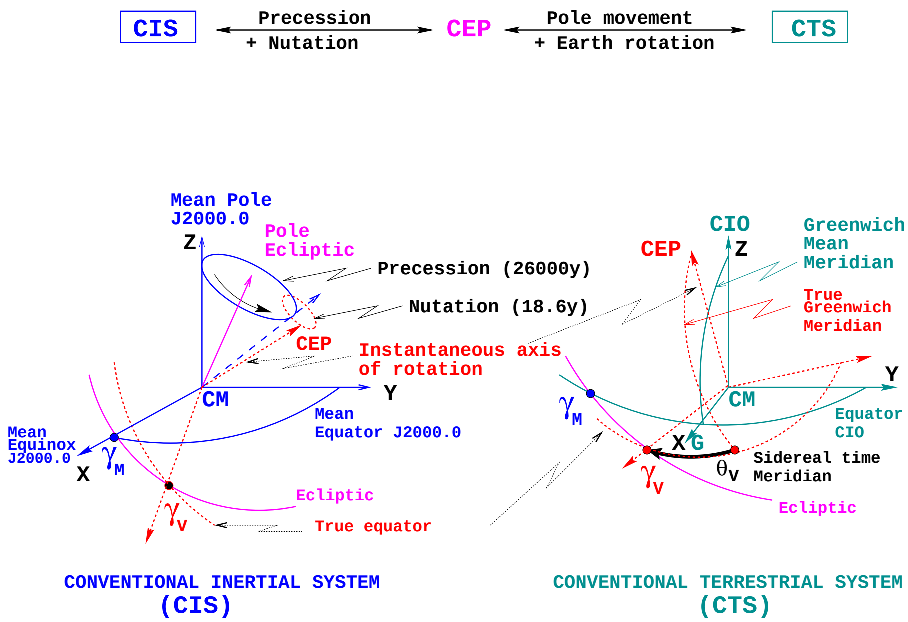


Fig. 10. Transformations between CIS and CTS systems

Figure 10 outlines the transformations needed to change from CIS system to CTS: through precession corrections and nutation one passes from mean equator and equinox J2000.0 to true equator and equinox at observation epoch. They define a reference system with the Z axis in the direction of the instantaneous rotation axis of Earth (Conventional Ephemeris Pole, CEP) and X axis pointing to true Aries point. Finally, using Earth rotation parameters and pole movement (Earth Orientation Parameters, EOP, Earth Rotation Parameters, ERP) one can pass from this system to CTS[18].

System WGS-84

From 1987, GPS uses the World Geodetic System WGS-84, developed by USA Defense Department, which is a unified terrestrial reference system for position and vector referencing[19].

[18]Opposite to *Precession* and *Nutation* series (defined for the mean equator and equinox J2000.0 in CIS system), which have valid analytical expressions for long time intervals, the rotation and orientation parameters of Earth can not be modeled theoretically and must be periodically updated using observations.

[19]The document "Modern Terrestrial Reference Systems PART 3: WGS 84 and ITRS" contains data and interesting references about WGS84 and ITRS

The original implementation is essentially equal to NAD83. Nevertheless, successive realizations approximate to (it is assumed that they are identical) some ITRS realizations. Thus, realizations WGS84(G730)[20] and WGS84(G873) correspond to ITRF92 and ITRF94, respectively.

WGS-84 system has the reference ellipsoid defined in the following table:

Semi-major axis of the ellipse	a	6 378.137 Km
Semi-minor axis of the ellipse	b	6 356.752 Km
Flattening factor	f	1/298.257223563
Earth angular velocity	ω_E	$7\ 292\ 115 \cdot 10^{-11}$ rad/s
Gravitational constant	μ	$3\ 986\ 005 \cdot 10^{8}\ \mathrm{m}^3/\mathrm{s}^2$

Table 5: Ellipsoidal parameters WGS-84

The routine `car2geo.f`, which is provided in appendix IV, makes the conversion from Cartesian coordinates (x,y,z) CTS to ellipsoidal (λ, ϕ, h), where λ and ϕ are respectively the longitude and latitude from the ellipsoid, and h the height above it.

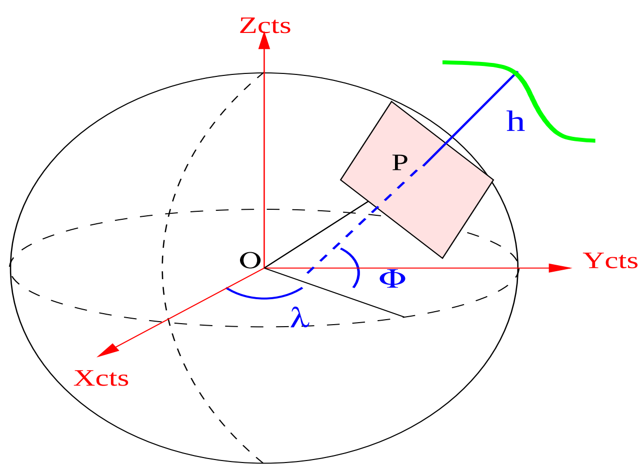


Fig. 11. Cartesian and ellipsoidal coordinates

(`http://www.ngs.noaa.gov/CORS/Articles/Reference-Systems-Part-3.pdf`)

[20]"G" indicates that it has exclusively been obtained with GPS observations and 730 indicates the GPS week.

Session 2
RINEX measurement and ephemeris files

Objectives

To know and manage RINEX format of observable files and GPS navigation, emphasizing in meaning of the data they contain.

Files to be used
`95oct18casa____r0.rnx`, `97jan09coco____r0.rnx`, `95oct18casa____r0.eph`, `Obsfile.html`, `Navfile.html`

Programs to use
`rnx2txt`, `eph2txt`

Foundations

RINEX means *Receiver INdependent EXchange*. The format consists of three types of files: 1) of observation (`95oct18casa____r0.rnx` or `lkhu0010.00o.gz`)[21], 2) of navigation (broadcast Ephemeris) (`95oct18casa____r0.eph` or `lkhu0010.00n.gz`) and 3) meteorological.

Development

1. Copy the corresponding files in the working directory.

2. The file `Obsfile.html` contains a tutorial of RINEX-2[22] format for observation files. One has to view the file `Obsfile.html` with a navigator and go over the different fields.

   Execute: `netscape Obsfile.html`

[21] They are different ways of naming files (under JPL/NASA, or under IGS).

[22] RINEX-2 is an extension of the initial RINEX format, that allows the incorporation of GLONASS (R) data, besides GPS (G). An extract of RINEX-2 format is available in appendix II. The whole document defining this format can be obtained in the address `http://www.ngs.noaa.gov/CORS/instructions2/`. In this same address, RINEX files are provided and also navigation (broadcast Ephemeris) and metereological files from year 1997. One can also find orbits and accurate clock files.

3. Repeat the previous exercise with navigation file `Navfile.html`.
   Execute: `netscape Navfile.html`

4. View rinex file `95oct18casa____r0.rnx` and answer the following questions:

   Execute: `more 95oct18casa____r0.rnx`

   (a) What kind of receiver and antenna do we have?

   (b) What are the station coordinates? (Indicate units and coordinate system.)

   (c) According to the file header, what is the time interval corresponding to registered observations?

   (d) How many satellites does the file contain? Are satellites PRN05 and PRN23 contained in the file? How many satellites are observed at instant $t = 0^h 0^m 30^s$?

   (e) What is the time interval between observations?

   (f) How many observations corresponding to satellite PRN25 have been registered for L1, L2, P1, P2?

   (g) What are the values of L1, L2, P1, P2 for the satellite PRN25 at the instant $t = 0^h 0^m 30^s$? What units are they expressed in? What is the signal/noise relation (SNR) corresponding to each of these data?

   (h) Why does some of these phase measurements (L1 or L2) have negative signs?

   (i) Give an estimation of the distance between the receiver and the satellite PRN25 at the instant $t = 0^h 0^m 30^s$.

   (j) (*)Is the *anti-spoofing* activated? What is the theoretical noise level of the pseudorange and phase observables from the file (see chapter 3)?

5. Considering now the file `97jan09coco____r0.rnx`:

   (a) Is the *anti-spoofing* activated?

   (b) Are the same observables (L1,P1, etc.) registered as in the previous file `95oct18casa____r0.rnx`? Why?

   (c) How could one explain that P2 is registered, while the *anti-spoofing* being activated?

6. The program `rnx2txt` applied over a file `*.rnx` generates a file `*.a` (more suitable for calculus), with L1, ..., P1,... data arranged in columns, with the following fields:

```
station day of year second satellite L1 L2 P1 P2 arc
(L1, L2, P1, P2 are expressed in meters)
```

   (a) Using program `rnx2txt`, generate the file `95oct18casa.a` from the file `95oct18casa____r0.rnx`.
   Execute:

```
rnx2txt 95oct18casa____r0.rnx
ls
textedit 95oct18casa.a
```

   (b) Starting from the file `95oct18casa.a`, generate another file containing only data from satellite PRN28.
   Execute:

```
cat 95oct18casa.a|awk '{if ($4==28) print $0 }'>a_PRN28
less a_PRN28
```

7. View the ephemeris files `95oct18casa____r0.eph` and answer the following questions (consulting appendix II):
   Execute:
```
more 95oct18casa____r0.eph
```

   (a) What are the clock parameters of satellite PRN04 at the instant $t = 2^h0^m0^s$?

   (b) What are the orbital parameters of satellite PRN04 at the instant $t = 2^h0^m0^s$?

   (c) (*)Make a drawing indicating the meaning of the orbital parameters (a, Ω, ω, λ, M, i) found in the file (see chapter 4).
   Note that orbital elements described in appendix I, `A1/2`$\equiv \sqrt{a}$, `omega`$\equiv \omega$, `io`$\equiv i$, `Mo`$\equiv M$ *correspond to satellite and observation epoch specified at the beginning of every data block, except from the element* `Omega` *(ascending node argument referring to Greenwich meridian), which refers to the beginning of the week, being* `TOE` *seconds passed through the week. Then, if* $\omega_e = 7.29210^{-5} rad/s$*, it is the Earth rotation velocity,* $\lambda =$`Omega`$-\omega_e$`TOE`*, it is the ascending node argument (referring to Greenwich meridian) for the epoch in question.*

8. The program `eph2txt` applied over a file `*.eph` generates files `*.b` and `*.clocks`, which contains ephemeris and parameters, respectively, for the calculation of satellite clock *offsets* (dt). The format is as follows:

File `95oct18.clocks`:
`satellite day of year` t`(in sec.)` a_0 a_1 a_2
[where $dt = a_0 + a_1(t - t_0) + a_2(t - t_0)^2$]

File `95oct18.b`:
`satellite day of year` t`(in sec.)` a e i λ ω M

   (a) Using program `eph2txt`, generate files `95oct18.b` and `95oct18.clocks` from the file `95oct18casa____r0.eph`.
   Execute:

```
cp 95oct18casa____r0.eph 95oct18.eph
eph2txt 95oct18.eph
more 95oct18.clocks
more 95oct18.b
```

   (b) What are the clock parameters of satellite PRN05 at the instant $t = 39104^s$?

   (c) What is the value of the semi-major axis (in km) of the orbit of satellite PRN05 at the instant $t = 39104^s$? Calculate length of semi-minor axis at that instant and evaluate different lengths.

Answers Session 2 RINEX data files and ephemerides
4.a
4.b
4.c
4.d
4.e
4.f
4.g
4.h
4.i
5.a
5.b
7.a
7.b
8.b
8.c

Chapter 3
GPS observables: L1,L2,P1,P2 and their combinations

GPS satellites emit signals in two different frequencies lying in the band L (L1= 1575.42 Mhz and L2= 1227.6 Mhz), which are multiples of a fundamental frequency 10.23 Mhz, with a relation of $\frac{154}{120}$. The following codes and messages are modulated over the carriers:

- the *Coarse/Acquisition code (C/A-code),* also known as *"Standard Positioning Service (SPS)*[23]*",* is available for civilian use.
- the *Precision Code (P-code)*, also known as *"Precise Positioning Service (PPS)",* is only available for military and authorized users.
- the *Navigation Message*, containing satellite orbits, clock corrections, and other system parameters.

From a generic point of view, one can say that the basic GPS observable is the delay, or time dT, that the signal takes to propagate from the phase center of the satellite antenna (at the emission time) to the phase center of the receiver (at the reception time). This value multiplied by the speed of light, gives us *apparent*[24] range $D = c\,dT$ between them. This propagation time dT can be obtained correlating the received code (P or C/A) from the satellite with a replica of itself generated in the receiver, so this replica is moved in time (Δt) until the maximum correlation is obtained (see figure 12).

[23]http://www.navcen.uscg.mil/pubs/gps/sigspec/default.htm

[24]It is called apparent to distinguish it from real range, since it includes different effects that makes it differ from it.

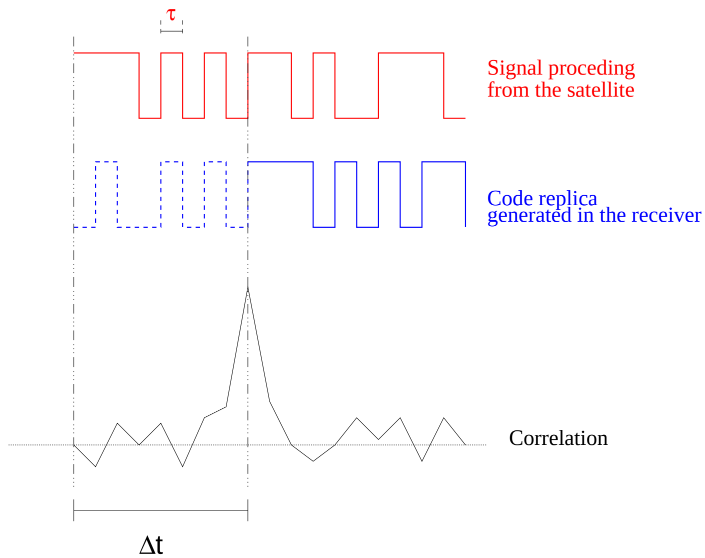


Fig. 12. Determination of the time of signal propagation

This shift Δt multiplied by the speed of light in a vacuum is what we know as *pseudorange or pseudodistance*. This observable is an "apparent range" between the satellite and the receiver that does not match with its geometric distance due to, among other factors, synchronism errors between receiver and satellite clocks. Taking into account, explicitly, possible synchronism errors between these clocks, the time passed between transmission and reception is obtained as a difference of times measured in two different scales: the satellite (t^j) and the receiver (t_i). Considering a reference time scale T, which we call GPS, one has that pseudorange for satellite i and receiver j may be expressed as:

$$P_i^j = c\,[t_i(T_2) - t^j(T_1)] \tag{1}$$

where:

- c is the speed of light in a vacuum.
- $t_i(T_2)$ is the time of signal reception, measured in the time scale given by the clock of receiver i.
- $t^j(T_1)$ is the time of signal transmission, measured in the time scale given by the satellite clock j.

Pseudorange P_i^j obtained using this procedure by the receiver includes, besides geometric range ρ_i^j between the receiver and the satellite, other non-geometric terms –apart from synchronism error between receiver and satellite clocks– due to signal propagation through the atmosphere (ionosphere and troposphere), relativistic effects, instrumental delays (from the satellite and the receiver), multipath interference, etc. (see figure 17 in page 77). Taking into account explicitly all these terms, the previous equation can be rewritten as follows, where P represents any of C/A, P1 or P2 codes:

$$P_i^j = \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j + \alpha_1 I_i^j + K1_i^j + M_{P,i}^j + \varepsilon_{P,i}^j$$

- ρ_i^j is the geometric distance between the satellite j antenna and receiver i antenna phase centers at emission and reception time, respectively: $\rho_i^j = \sqrt{(x^j - x_i)^2 + (y^j - y_i)^2 + (z^j - z_i)^2}$
- dt^j is the offset of the satellite j clock from the GPS time.
- dt_i is the offset of the receiver from GPS time.
- T_i^j is the tropospheric delay.
- I_i^j represents the ionospheric delay, which depends on signal frequency f ($\alpha_i = 40.3/f_i^2$).
- rel_i^j represents the relativistic effect.
- K_i^j represents the delays due to the instrumental constants of satellite and receiver, which are dependent on the frequency.
- $M_{P,i}^j$ represents the effect of multipath (*multipath*), also depending on the frequency.
- $\varepsilon_{P_1,i}^j$ is a noise term containing all non-modeled effects.

Apparent distance between satellite and receiver can also be measured from the carrier signal phase, obtaining in this case:

$$L_i^j = \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j - \alpha_1 I_i^j + B_i^j + w_L + m_{L,i}^j + \varepsilon_{L,i}^j$$

where, besides the former terms, one must take into account:

- w_L is a term due to signal polarization (*wind-up*)[25].

[25] a rotation of 360 degrees in the receiver antenna, keeping its position fixed, will introduce a variation of one wavelength in the measurement of apparent distance between the receiver and the satellite obtained from the phase.

- B is an ambiguity phase term owing to the signal acquisition, an ambiguity of an integer number of wavelengths $(N\,\lambda)$ appears to which one has to add instrumental constants k_i, k^j from satellite and receiver, respectively $(B_i^j = k_i + k^j + \lambda N_i^j)$.

Note that the ionospheric term has opposite sign for code and phase.

Next, a summary table points out the different terms taking part in modeling P codes and L phase observables for two given frequencies f_1 and f_2. Their order of magnitude is also shown..

GPS observables:

Codes (*pseudoranges*)

$$\mathrm{P1}_i^j = \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j + \alpha_1 I_i^j + K1_i^j + M_{P_1,i}^j + \varepsilon_{P_1,i}^j$$
$$\mathrm{P2}_i^j = \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j + \alpha_2 I_i^j + K2_i^j + M_{P_2,i}^j + \varepsilon_{P_2,i}^j$$

Phases (*carrier phases*)

$$\mathrm{L1}_i^j = \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j - \alpha_1 I_i^j + B1_i^j + w_{L1} + m_{L_1,i}^j + \varepsilon_{L_1,i}^j$$
$$\mathrm{L2}_i^j = \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j - \alpha_2 I_i^j + B2_i^j + w_{L2} + m_{L_2,i}^j + \varepsilon_{L_2,i}^j$$

ρ = geometric dist. ($\simeq$*20.000Km*)	dt = clock *offset* ($<$*300 Km*)	Where:
rel = relativistic effect ($\simeq$ *15 m*)	w = wind-up ($< \lambda$)	$K1_i^j = K_{1i} + TGD^j$
T = tropospheric delay $\simeq$*(2m*FO)*	I = ionosp. del. $\simeq$*([2-10m]*FO)*	$K2_i^j = K_{2i} + \frac{f_1^2}{f_2^2} TGD^j$
m, M = multipath ($m_L \simeq$ *0.1-1 cm*)	ε = noise ($\sigma_{\varepsilon_L} \simeq$*2mm*)	$B1_i^j = k_{1i} + k_1{}^j + \lambda_1 N1_i^j$
($M_P \simeq$ *0.1-1m*, $M_{CA} \simeq$ *0.5-5 m*)	($\sigma_{\varepsilon_P} \simeq$*0.1-0.3 m*, $\sigma_{\varepsilon_{CA}} \simeq$*0.5-3 m*)	$B2_i^j = k_{2i} + k_2{}^j + \lambda_2 N2_i^j$
B = phase ambiguity (*cm-Km*)	K = instrum. delay *(cm-m)*	
FO = Slant factor ($\simeq \frac{1}{sin(elev)}$)	$\gamma = (f_1/f_2)^2 = (77/60)^2$	N = integer ambiguity
$\alpha_i = 40.3/f_i^2$; $\lambda_i = c/f_i$; $\frac{1}{\lambda - 1} \simeq 1.546$	$\lambda_1 = 19.03$ cm, $\lambda_2 = 24.42$ cm	TGD, K, k = instrum. del.

Observable combinations

Starting from the basic observables previously described, one can define the following combinations (where P and L are expressed in meters):

- *Ionospheric free combination*: ionospheric effect depends on the square of the frequency ($\alpha_i = 40.3/f_i^2$). This allows its cancellation by this combinations:

$$\boxed{PC = \frac{f_1^2 P1 - f_2^2 P2}{f_1^2 - f_2^2} \quad ; \quad LC = \frac{f_1^2 L1 - f_2^2 L2}{f_1^2 - f_2^2}}$$

- *Narrow lane (PW) and wide-lane (LW) combinations*: *LW* combination gives an observable with a wavelength $\lambda_W = 86.2\,cm$, four times bigger than L1 or L2, which makes it very useful for (*cycle-slips*) detections. To do so, *Melbourne-Wübbena* combination is used ($W = LW - PW$).

$$\boxed{PW = \frac{f_1 P1 + f_2 P2}{f_1 + f_2} \quad ; \quad LW = \frac{f_1 L1 - f_2 L2}{f_1 - f_2}}$$

- *Ionospheric combination*: it cancels the geometric part of the measurement, leaving the ionospheric effect and the instrumental constants (besides multipath and observational noise). It is also used to detect cycle-slips in the phase. Note the change of factor order in LI and PI.

$$\boxed{PI = P2 - P1 \quad ; \quad LI = L1 - L2}$$

Replacing the expressions of P1, P2, L1 and L2 in the former definitions, one can obtain the following expressions and relations among ambiguities for PC, LC, PW, LW, PI and LI (demonstrations are left as an exercise):

Free ionospheric combination:

$PC = \rho + c(dt_i - dt^j) + rel + T + KC + M_{PC} + \varepsilon_{PC}$ $\qquad KC = \frac{f_1^2 K1 - f_2^2 K2}{f_1^2 - f_2^2} \equiv 0$

$LC = \rho + c(dt_i - dt^j) + rel + T + BC + m_{LC} + w_{LC} + \varepsilon_{LC}$ $\qquad BC = \frac{f_1^2 B1 - f_2^2 B2}{f_1^2 - f_2^2} = k_{ci} + k_c{}^j + \lambda_c Rc$

$Rc = \lambda_W (\frac{N_1}{\lambda_1} - \frac{N_2}{\lambda_2})$; $\lambda_C = \frac{c}{f_1 + f_2}$=10.7cm

Narrow-lane (PW) and wide-lane (LW) combinations:

$PW = \rho + c(dt_i - dt^j) + rel + T + \alpha_W I + KW + M_{PW} + \varepsilon_{PW}$ $\qquad KW = \frac{f_1 K1 + f_2 K2}{f_1 + f_2}$

$LW = \rho + c(dt_i - dt^j) + rel + T + \alpha_W I + BW + m_{L_W} + \varepsilon_{LW}$ $\qquad BW = \frac{f_1 B1 - f_2 B2}{f_1 - f_2} = k_{W\,i} + k_W{}^j + \lambda_W N_W$

$(\alpha_W = \frac{40.3}{f_1 f_2})$ $\qquad N_W = N_1 - N_2$; $\quad \lambda_W = \frac{c}{f_1 - f_2} = 86.2\,cm$

Ionospheric combination:

$PI = \alpha_I I + KI + M_{PI} + \varepsilon_{PI}$ $\qquad KI = K2 - K1$

$LI = \alpha_I I + BI + m_{LI} + w_{LI} + \varepsilon_{LI}$ $\qquad BI = B1 - B2 = k_{I\,i} + k_I{}^j + \lambda_1 N_W - \lambda_I N_2$

$(\alpha_I = \alpha_2 - \alpha_1 \simeq 1.05)$ $\qquad \lambda_I = \lambda_2 - \lambda_1 = 5.4\,cm$

Relations between ambiguities
(LW, LI, LC variations in terms of ΔN_1 and ΔN_2)

$$\Delta LW = \lambda_W \Delta N_W = \lambda_W\,(\Delta N_1 - \Delta N_2)$$
$$\Delta LI = \lambda_1 \Delta N_1 - \lambda_2 \Delta N_2 = \lambda_1 \Delta N_W - \lambda_I \Delta N_2 \qquad (N = \text{Integer ambig.})$$
$$\Delta LC = \lambda_C \left(\frac{\lambda_W}{\lambda_1} \Delta N_1 - \frac{\lambda_W}{\lambda_2} \Delta N_2 \right) =$$
$$= \lambda_C \Delta N_1 + \frac{f_2}{f_1 + f_2}\,\lambda_W\,\Delta N_W \simeq \lambda_C\,\Delta N_1 + \frac{1}{2}\,\lambda_W\,\Delta N_W$$

Session 3a
GPS Measurements: L1,L2,P1,P2 and their combinations

Objectives

To analyze graphically the GPS code and phase measurements and their combinations as well as to study their characteristics and properties: *cycle-slips*, ionospheric refraction, multipath, receiver noise; under Anti-Spoofing activated and deactivated conditions. To determine empirically the order of magnitude of these effects.

Files to be used
`95oct18casa____r0.rnx`, `97jan09coco____r0.rnx`,
`gage2710.98o.a`,`gage2720.98o.a`,`gage2730.98o.a`,
`upci00178.tec0.anim.gif`

Programs to be used
`rnx2txt`

Development

1. Copy the programs and files of this lesson in the working directory.

2. [**Reading the RINEX file**] Using the program `rnx2txt`, it generates the file `95oct18casa.a` from `95oct18casa____r0.rnx` (notice that this file was collected under *A/S=off conditions*).

   Execute:
   ```
   rnx2txt 95oct18casa____r0.rnx
   ls
   textedit 95oct18casa.a
   ```

   The obtained file `95oct18casa.a` contains the following fields:
   ```
   receiver_name day_of_year second satellite L1 L2 P1 P2 arc
   (L1, L2, P1, P2 are given in meters)
   ```

(a) Plot the phase L1 in function of time for the satellite PRN28 and identify the epochs in which the *cycle-slips* happen. Does it make sense to have negative values in phase measurements?
Execute:

```
cat 95oct18casa.a| gawk '{if ($4==28) print $3,
       $5,$6,$7,$8}' >casa.a_28
gnuplot
set grid
plot "casa.a_28" u 1:2
```

(b) Plot phase L1 and code P1 in the same graph.
Execute:

```
cat 95oct18casa.a| gawk '{if ($4==28) print $3, $5}' >L1.a
cat 95oct18casa.a| gawk '{if ($4==28) print $3, $7}' >P1.a
gnuplot
plot "L1.a" u 1:2,"P1.a" u 1:2
```

(c) Repeat the same for L2 and P2 (optional).

3. **[Ionospheric Refraction]** Plot the *ionospheric combination* L1-L2 for the satellite PRN28. What is the physical meaning of this combination? Execute, for instance:

```
plot "<cat 95oct18casa.a|gawk '{if($4==28)print $3,$5-$6}'"
```

(a) The same for P1-P2. Why does this combination show different sign from the previous one? Does it make sense that the graphic cuts the x-axis? (*) From which factors may the ionospheric refraction value depend? (geometry, local time, ...)[26] ?

(b) Superpose the combinations L1-L2 and P2-P1 (notice that the last one is P2-P1 to avoid the problem with the sign) for the satellite PRN28 in the same graph.
Execute for instance:

[26] Executing `netscape upci00178.tec0.anim.gif` can be seen as a "movie" showing the ionospheric delays (TEC) at a planetary scale. The "slant" delay (STEC) (in the direction of satellite-user ray) could be obtained by multiplying the TEC by the slant factor $FO \simeq 1/sen(elev)$, where $elev$ is the elevation of the satellite referred to the user horizon. (`IONEX Daily ionospheric TEC: ftp://cddis.gsfc.nasa.gov/gps/products/ionex/`)

```
plot "<cat 95oct18casa.a|gawk '{if($4==28)print $3,$5-$6}'",
  "<cat 95oct18casa.a|gawk '{if ($4==28) print $3, $8-$7}'"
```

Taking into account the previous plots, which combination shows greater level of noise, L1-L2 or P1-P2?

(c) Plot the combination (L1-L2)-(P2-P1) in the same graph. Why does the dispersion increase at the beginning and the end of the arcs? Why is the noise not centered at zero?

4. (*) [**Measurement noise**] Using the following values for the code and phase measurement noise $\sigma_{\varepsilon_{L1}} \simeq \sigma_{\varepsilon_{L2}} \simeq 2mm$, $\sigma_{\varepsilon_{P1}} \simeq \sigma_{\varepsilon_{P2}} \simeq 30cm$ (see table in theoretical basis), compute the theoretical values of noise[27] $\sigma_{\varepsilon_{LI}}, \sigma_{\varepsilon_{PI}}$ for the combinations LI, PI. Are the obtained values in agreement with the previous plots[28]? What is the effect of the multipath?

5. [**Measurement noise: antispoofing**] Generate the file `97jan09coco.a` from `97jan09coco____r0.rnx` (repeat the same steps as at the beginning of exercise 2) –file collected under *anti-spoofing activated*–

   (a) Does it make sense that the file contains records of P1 and P2, being the antispoofing activated?

   (b) Plot L1-L2 and P2-P1 for the satellite PRN15 in the same graph. Explain the structure of the dispersion of the points in the figure[29].

   (c) Represent L1-L2 and P2-P1 for the satellite PRN01 (for instance) in the same plot. Compare the dispersion of the values obtained for P1-P2 with that of the previous exercise (for 18 of October of 1995, with *anti-spoofing* off). Are there significants differences?

   (d) (*) Repeat the calculation of the previous exercise, using the following values $\sigma_{L1} \simeq \sigma_{L2} \simeq 2mm, \;\; \sigma_{P1} \simeq \sigma_{P2} \simeq \sigma_{C/A} \simeq 3m.$

[27] Apply the following result: given two independent random variables X, Y and two constants a, b, then, $\sigma_{aX+bY} = \sqrt{a^2\,\sigma_X^2 + b^2\,\sigma_Y^2}$.

[28] In the case of zero mean Gaussian noise, the 68.27% of the cases are inside of the interval $[-\sigma, +\sigma]$.

[29] Note: the code corresponds to C/A measurements and P2 is a synthetic code obtained from C/A and the cross correlation of the encrypted P1 and P2 (Y1 and Y2).

6. [**Multipath**] The code multipath can be observed plotting the combination P1-L1 (with a sampling rate of 1Hz, it is also possible to follow its evolution). Due to their geometric nature, the effect of multipath repeats when the geometry receiver-satellite repeats. The files `gage2710.98o.a`, `gage2720.98o.a`, and `gage2730.98o.a` contain observations at 1Hz collected at the same period of time for three consecutive days[30]. Plot the combination P1-L1 and identify the effect of multipath[31].

   Execute:
   ```
   gnuplot
   set grid
   plot "< cat gage2710.98o.a|gawk '{if ($4==14) print $3,$7-$5-23690187}'",
    "< cat gage2720.98o.a|gawk '{if ($4==14) print $3,$7-$5-22202591}'",
    "< cat gage2730.98o.a|gawk '{if ($4==14) print $3,$7-$5-22800909}'"
   exit
   ```

   (a) Compare the plots corresponding to the three days. Are they similar?

   (b) Repeat the same plots, but shifting $3^m56^s = 236^s$ the plot of the second day, and $2*(3^m56^s) = 472^s$ the plot of the third day.

   Execute:
   ```
   gnuplot
   set grid
   plot "< cat gage2710.98o.a|gawk '{if ($4==14) print $3,$7-$5-23690187}'",
    "< cat gage2720.98o.a|gawk '{if ($4==14) print $3+236,$7-$5-22202591}'",
    "< cat gage2730.98o.a|gawk '{if ($4==14) print $3+472,$7-$5-22800909}'"
   set xrange[41500:41985]
   replot
   exit
   ```

   What is the reason of the observed displacement of 3^m56^s between the graphs of two consecutive days? Why can we affirm that we are basically observing the effect of the code multipath?

   (c) What is the origin of the observed drift in the plots?

   (d) Is it also possible to detect the code multipath using the PC-LC combination (*)? What advantages or disadvantages does the P1-L1 combination have?

[30] The format is the same as in files `*.a`. It has been collected with the one frequency receiver Lassen-SK8 (Trimble). This low cost receiver (about 180 ECUs) provides codes and "truncate" phases at the frequency $f1$.

[31] Superpose the graphics and shift them along the y-axis to make the comparison easier.

Answers

Session 3a
GPS observables and their combinations

2.a
3.a
3.b
3.c
5.a
5.b
5.c
6.a
6.b
6.c
6.d

Session 3b
Cycle-slips detection

Objectives

To study the combinations of GPS observables and their applications to the detection of cycle-slips of phase measurements. To study the relationship between the ambiguities for the different observables and their combinations.

Files to be used
`95oct18casa____r0.rnx`

Programs to be used
`rnx2txt, P3b_2.scr, P3b_3.scr, P3b_5.scr, plots_P3b.gnu`

Development

1. Copy the programs and files of this lesson in the working directory.

2. Following the same steps as in exercise 2 of session 3a, generate the file `95oct18casa.a` from `95oct18casa____r0.rnx` using the programs `rnx2txt`. Select the fields `[sec, L1, L2, P1, P2]` for the satellite PRN18 of file `95oct18casa.a`. Rename as `"s18.org"` the obtained file. Execute:

```
cat 95oct18casa.a | gawk '{if ($4==18)
                                  print $3,$5,$6,$7,$8}' >s18.org
```

   Insert a cycle-slip of 1 cycle (0.19m) in L1 at time $t = 5000s$. Rename as `"s18.cl"` the resulting file.
   Execute:

```
cat s18.org | gawk '{if ($1>=5000) $2=$2+0.19;
      printf "%s %f %f %f %f \n", $1,$2,$3,$4,$5}' > s18.cl
```

   Make a graphical study of how the introduced cycle-slip in L1 can be detected by plotting the following data: a) L1, b) L1-P1, c) LC-PC, d) LW-PW e) LI-PI, f) LI
   Note: Make the graphs in cycles of the corresponding magnitude (i.e., $\lambda_1 = 0.19m$, $\lambda_C = 0.107m$, $\lambda_w = 0.862m$, $\lambda_I = 0.054m$).

a) $\boxed{\texttt{L1}}$. Execute:

```
cat s18.org| gawk '{printf "%f %f \n", $1,$2/0.19}' > l1.org
cat s18.cl | gawk '{printf "%f %f \n", $1,$2/0.19}' > l1.cl
gnuplot
set grid
set xrange[3000:8000]
plot "l1.org","l1.cl"
exit
```

From this graph, the time in which the cycle-slip happens could be detected. How many cycles change L1 (approximately) into two consecutive observations (replot, for instance, the previous graph in the interval [4900:5100] (`set xrange[4900:5100]`))? How many cycles are involved in the cycle-slip that we try to detect?

b) $\boxed{\texttt{L1-P1}}$. Execute:

```
cat s18.org|gawk '{print $1,($2-$4+24027475.6)/0.19}' >lp1.org
cat s18.cl |gawk '{print $1,($2-$4+24027475.6)/0.19}' >lp1.cl
gnuplot
set grid
set xrange[3000:8000]
plot "lp1.org","lp1.cl"
plot "lp1.cl"
exit
```

Note: Observe that both graphs have been shifted 24027475.6 units along the y-axis for a better visualization.

From previous graph, is it possible to detect, in a confident way, the "time" in which the cycle-slip happens? Is the difference between the code and phase measurements constant?. Why? (justify theoretically the answer from the expressions of these observable). How large, in cycles (approximately), is the noise observed in the graph? How large is, in cycles, the jump observed in the graph at the time in which the cycle-slip happens ($t = 5000s$) [see, for instance, the previous graphic in the interval [4900:5100] (`set xrange[4900:5100]`))?

c) LC-PC. Execute:

```
cat s18.org| gawk 'BEGIN{f1=1575.42;f2=1227.6}{print $1,
 ((f1^2*$2-f2^2*$3)/(f1^2-f2^2) - (f1^2*$4-f2^2*$5)/(f1^2-f2^2)
  +24027475.6)/0.107}' > lc.org
cat s18.cl | gawk 'BEGIN{f1=1575.42;f2=1227.6}{print $1,
 ((f1^2*$2-f2^2*$3)/(f1^2-f2^2) - (f1^2*$4-f2^2*$5)/(f1^2-f2^2)
  +24027475.6)/0.107}' > lc.cl
gnuplot
set grid
set xrange[3000:8000]
set yrange[10:60]
plot "lc.org","lc.cl"
plot "lc.cl"
exit
```

Answer to the same questions as in previous section. (*) Taking into account the relations between ambiguities (see page 41) theoretically justify the observed number of cycles of λ_C when the cycle-slip happens. (*) Using the values $\sigma_{L1} = \sigma_{L2} = 2mm$, $\sigma_{P1} = \sigma_{P2} = 30cm$, theoretically calculate the noise that should be expected for this combinations of observable LC-PC (give the result in centimeters and in cycles of LC ($\lambda_C = 10.7cm$))

d) LW-PW. Execute:

```
cat s18.org| gawk 'BEGIN{f1=1575.42;f2=1227.6}{print $1,
 ((f1*$2-f2*$3)/(f1-f2) - (f1*$4+f2*$5)/(f1+f2)
  +24027475.6)/0.862}' > lw.org
cat s18.cl| gawk 'BEGIN{f1=1575.42;f2=1227.6}{print $1,
 ((f1*$2-f2*$3)/(f1-f2) - (f1*$4+f2*$5)/(f1+f2)
  +24027475.6)/0.862}' > lw.cl
gnuplot
set grid
set xrange[3000:8000]
set yrange[-4:4]
plot "lw.org","lw.cl"
plot "lw.cl"
exit
```

Answer to the same questions as in previous case ($\lambda_W = 86.2cm$).

e) $\boxed{\texttt{LI-PI}}$. Execute:

```
cat s18.org|gawk '{print $1,(($2-$3)-($5-$4))/0.054}' >lpi.org
cat s18.cl |gawk '{print $1,(($2-$3)-($5-$4))/0.054}' >lpi.cl
gnuplot
set grid
set xrange[3000:8000]
set yrange[-60:0]
plot "lpi.org","lpi.cl"
plot "lpi.cl"
exit
```

Answer to the same questions as in previous case ($\lambda_I = 5.4cm$).

f) $\boxed{\texttt{LI}}$. Execute:

```
cat s18.org| gawk '{print $1,($2-$3)/0.054}' > li.org
cat s18.cl | gawk '{print $1,($2-$3)/0.054}' > li.cl
gnuplot
set xrange[3000:8000]
set yrange[-60:0]
plot "li.org","li.cl"
plot "li.cl"
exit
```

From previous graph, is it possible to detect in a confident way the "time" in which the cycle-slip happens? How many cycles does the el cycle-slip in LI has[32]? How large is the variation (approximately) of the ionospheric refraction between two observations 30 seconds apart (visualize, for instance, the previous graphic in the interval[4900:5100] (`set xrange[4900:5100]`). Give the result in cycles of LI and in centimeters. What does this value depend on?

[32]Note: the term *cycle of LI* is used in a wide sense, as a multiple of λ_I, because the ionospheric combination does not define a physic wave: in absence of ionosphere and without instrumental delays, this combination will be null.

Additional Exercises

3. If in the previous exercise one cycle were added to both carriers (L1 and L2), would a cycle-slip in LW occur? And in LI? How many cycles? Justify theoretically the answer taking into account the relations between the ambiguities from the Theoretical Basis given in section (page 41).

   Note: executing the script `P3b_3.scr`, the files `s18.cl, l1.org, l1.cl, lp1.org , lp1.cl , lc.org, lc.cl, lw.org` and `lw.cl` corresponding to the same questions as in the previous section, can be generated. The corresponding plots can be visualized executing `gnuplot plots_P3b.gnu`

   Execute:

   ```
   P3b_3.scr
   gnuplot plots_P3b.gnu
   ```

   Note: Executing `textedit P3b_3.scr` or `textedit plots_P3b.gnu`, the content of these scripts can be visualized (in order to check that they implement the same instructions as in the previous exercise).

4. (*) [**wind-up**] Consider a user that, **without changing the antenna location** rotates the antenna 360 degrees around its symmetry axis. Will this operation affect the code P1 or P2 measurements, and the phase measurements L1 and L2? How much? The same questions for the combinations LW, LC, LI. Justify theoretically the answers.

5. Is it possible to have a cycle-slip in LW but not in LI? In which way? (*) Justify theoretically the answer. Execute:

   ```
   P3b_5.scr
   gnuplot plots_P3b.gnu
   ```

6. (*) [**Demonstrations**] From the expressions P1, P2, L1 y L2 given in the Theoretical Basis section, proof the expressions of PC, PW and PI, and the relations between the ambiguities given in that section.

Answers

Session 3b

Cycle-slips detection

2.a
2.b
2.c
2.d
2.e
2.f

Chapter 4
Orbits and clocks of GPS satellites

The knowledge of the orbits and clocks of the satellites is fundamental to have a right positioning. An error in the GPS satellite coordinates or satellite clock will lead to a positioning error. Information about orbital parameters and clocks is transmitted in the navigation message.

Next, orbital elements are defined, navigation message is introduced and the algorithm for the coordinate calculation will be deduced from it.

Keplerian elements (two-body problem)

If one only considers the attractive force between two masses, the motion of mass m_2 relative to another mass m_1 is defined by the differential equation:

$$\frac{d^2\underline{r}}{dt^2} + \frac{\mu}{r^3}\underline{r} = \underline{0}$$

where $\underline{r}$ is their relative position vector, $\mu = G(m_1 + m_2)$ and G the universal gravitational constant. In the case of motion of an artificial earth satellite, its mass can be neglected with respect to the mass of the Earth.

The integration of this equation leads us to the Keplerian orbit of the satellite[33] as:

$$\underline{r}(t) = \underline{r}(t; a, e, i, \Omega, \omega, \tau)$$

defined by the following six orbital parameters (see figures 13, 14, 15 and 16):

- [Ω] *Right ascension of ascending node:* is the geocentric angle between the ascending node direction and the Aries point. The node line is the intersection with the equatorial plane and the orbital plane. Its intersection with the unit sphere defines two points: the ascending node, through which the satellite crosses to the region of positive Z, and the descending one.

[33] we restrict ourselves to elliptic orbits.

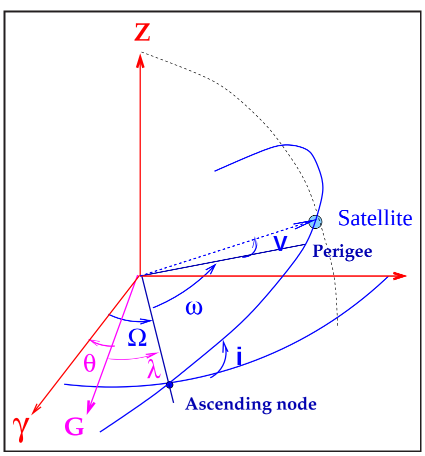
- $[i]$ *Inclination of orbital plane:* is the angle between the orbital plane and equator.

- $[\omega]$ *Argument of perigee:* It is the angle between node directions and perigee, measured in the orbital plane. The perigee is the point of closest approach of the satellite with respect to the center of mass of the Earth. The most distant position is the apogee. Both are in the semi-major axis direction.

- $[a]$ *Semi-major axis of orbital ellipse*: It is the semi-major axis of the ellipse defining the orbit.

- $[e]$ *Numerical eccentricity of the orbit*: It is the eccentricity of the orbital ellipse.

- $[T_0]$ *Perigee passing time*: It is the time of the satellite passage through the closest approach with the Earth (perigee). Satellite orbital position can be obtained at a moment t using $\tau(t) = t - T_0$ or any of the three following anomalies:
  - $[v(t)]$ *True anomaly:* It is the geocentric angle between perigee direction and satellite direction.
  - $[E(t)]$ *Eccentric anomaly:* It is the angle, measured from the center of the orbit, between the perigee and the direction of the intersection point of the normal line to the major axis passing through the satellite with the circle of radius a (see figure 14).
  - $[M(t)]$ *Mean anomaly:* It is a mathematical abstraction.



a = orbital semi-major axis
e = eccentricity
i = inclination
ω = argument of perigee
Ω = arg. ascending node (Aries)
λ = arg. ascending node (Greenwich)
T = perigee passing time
M = mean anomaly
V = true anomaly
θ = sidereal time

Fig. 13. Orbital parameters

The three anomalies are related by the formulas:

$$M(t) = n(t - T_0)$$

$$E(t) = M(t) + e \sin E(t)$$

$$V(t) = 2 \arctan \left[\sqrt{\frac{1+e}{1-e}} \tan \frac{E(t)}{2} \right]$$

$$n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

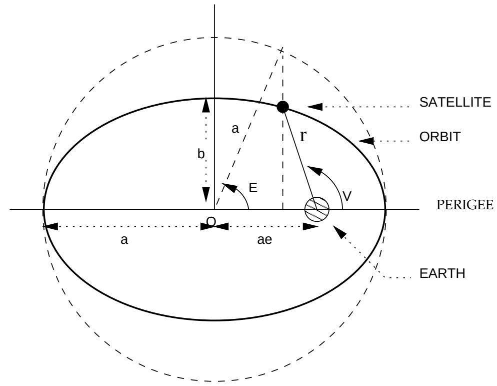


Fig. 14. Elliptic orbital representation.

where n denotes the mean angular velocity of the satellite, or mean motion, with revolution period P. Replacing $a = 26560km$ (nominal value for GPS satellites) in the last of the above equations, an orbital period of 12 sidereal hours is obtained[34].

Perturbed motion

The two-body problem considered in the previous section is only a first approximation to the real case. In practice, an additional set of accelerations $\underline{k}$ or disturbing terms must be added to the equation, therefore, our previous differential equation becomes:

$$\ddot{\underline{r}} = -\frac{\mu}{r^3}\underline{r} + \underline{k}$$

These perturbations are mainly due to:

1. Non-sphericity of the Earth and non-homogeneous mass distribution[35].
2. The presence of other celestial bodies, foremost, the Sun and the Moon.
3. Tidal effect.
4. Solar radiation pressure.

[34] a sidereal day is $3^m 56^s$ shorter than a solar day (see chapter 2).

[35] Spherical harmonic expansion is considered. Term n=0 corresponds to the central body, C_{20} coefficient explains the effect due to the oblateness of the Earth. Its magnitude is nearly 1000 times higher than the rest of coefficient.

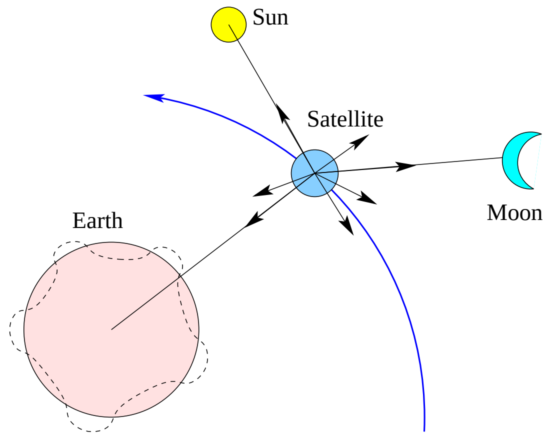


Fig. 15. Perturbations over the satellite orbit (source G. Seeber p. 73)

Perturbation	Acceleration	Orbital effect	
	m/s^2	in 3 hours	in 3 days
Central force (as a reference)	0.56		
C_{20}	5.10^{-5}	2 km	14 km
rest of the harmonics	3.10^{-7}	50-80 m	100-1500 m
Solar + Moon grav.	5.10^{-6}	5-150 m	1000-3000 m
Tidal effects	1.10^{-9}	-	0.5-1.0 m
Solar rad. pressure	1.10^{-7}	5-10 m	100-800 m

Table 6: Different perturbation magnitudes and their effect over the GPS orbit.

A way of taking into account the effect of all these perturbations is to consider *osculating*[36] orbital elements changing with time, thus:

$$\underline{r}(t) = \underline{r}(t; a(t), e(t), i(t), \Omega(t), \omega(t), \tau)$$

In the navigation message, all the parameters needed for the calculation of these orbital elements are transmitted at every observation epoch. The parameters obtained in the navigation message are renewed every two hours and must not be used out of the prescribed

[36]From the Latin verb *osculor* (to kiss). It is used in the way that the perturbed orbit and the nominal orbit are tangent to each other at every moment in time.

time (about four hours), because the extrapolating error grows exponentially beyond this period.

Parameter	**Explanation**
$IODE$	Series number of ephemerides data
t_{oe}	Ephemerides reference epoch
$\sqrt{a}$	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω	Ascending node's right ascension
Δn	Mean motion difference
$\dot{i}$	rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
c_{uc}, c_{us}	Latitude argument correction
c_{rc}, c_{rs}	Orbital radius correction
c_{ic}, c_{is}	Inclination correction

Table 7: Ephemerides in the navigation message

In order to compute WGS84 satellite coordinates from navigation message, the following algorithm must be used [GPS/SPS-SS, table 2-15] (see subroutine FORTRAN `orbit.f`, appendix IV):

Satellite coordinate calculation using navigation message

The following steps are needed:

- Calculation of the time t_k from the ephemerides reference epoch t_{oe} (t and t_{oe} are expressed in seconds in the GPS week):

  $$t_k = t - t_{oe}$$

  If $t_k > 302400$ sec, subtract 604800 sec from t_k. If $t_k < -302400$ sec, add 604800 sec

- Calculation for the mean anomaly for t_k,

  $$M_k = M_o + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n \right) t_k$$

- Solving (iteratively) Kepler equation for the eccentricity anomaly E_k:

$$M_k = E_k - e \sin E_k$$

- Calculation of real anomaly v_k:

$$v_k = \arctan\left(\frac{\sqrt{1-e^2}\sin E_k}{\cos E_k - e}\right)$$

- Calculation of the argument of latitude u_k from the argument of perigee ω, real anomaly v_k and corrections c_{uc} and c_{us}:

$$u_k = \omega + v_k + c_{uc}\cos 2\left(\omega + v_k\right) + c_{us}\sin 2\left(\omega + v_k\right)$$

- Calculation of the radial distance r_k, considering corrections c_{rc} and c_{rs}:

$$r_k = a\left(1 - e\cos E_k\right) + c_{rc}\cos 2\left(\omega + v_k\right) + c_{rs}\sin 2\left(\omega + v_k\right)$$

- Calculation of inclination i_k of the orbital plane from the inclination i_o at the reference time t_{oe}, and corrections c_{ic} and c_{is}:

$$i_k = i_o + \dot{i}\, t_k + c_{ic}\cos 2\left(\omega + v_k\right) + c_{is}\sin 2\left(\omega + v_k\right)$$

- Calculation of the longitude of the ascending node λ_k (referring to Greenwich), using its right ascension Ω_o at the beginning of the current week, corrected from the apparent sidereal time variation in Greenwich between the beginning of the week and reference time $t_k = t - toe$, and the change in longitude of the ascending node from the reference time toe.

$$\lambda_k = \Omega_o + \left(\dot{\Omega} - \omega_E\right) t_k - \omega_E t_{oe}$$

- Calculation of coordinates in CTS frame, by applying three rotations (about u_k, i_k, λ_k):

$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = \mathbf{R}_3\left(-\lambda_k\right)\mathbf{R}_1\left(-i_k\right)\mathbf{R}_3\left(-u_k\right)\begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$

Now, a scheme is provided with the necessary calculations to obtain the osculating orbital elements starting from the position and velocity of the satellite and vice-versa:

Calculation of the orbital elements starting from the position and velocity

$$\boxed{(x,\, y,\, z,\, v_x,\, v_y,\, v_z) \Longrightarrow (a,\, e,\, i,\, \Omega,\, \omega,\, T)}$$

$\vec{c} = \vec{r} \times \vec{v} \Longrightarrow p = \frac{c^2}{\mu} \Longrightarrow p$
$v^2 = \mu(2/r - 1/a) \Longrightarrow \boxed{a}$
$p = a(1 - e^2) \Longrightarrow \boxed{e}$

$\vec{c} = c\vec{S} \Longrightarrow \Omega = \arctan(-c_x/c_y);\ i = \text{arcs}(c_z/c) \Longrightarrow \boxed{\Omega}, \boxed{i}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cos(\omega+V) - \sin\Omega\sin(\omega+V)\cos i \\ \sin\Omega\cos(\omega+V) + \cos\Omega\sin(\omega+V)\cos i \\ \sin(\omega+V)\sin i \end{pmatrix}$$
$\Longrightarrow \omega + V$

$r = \frac{p}{1+e\cos(V)} \Longrightarrow \boxed{\omega}, V$

$\tan(E/2) = (\frac{1-e}{1+e})^{1/2}\tan(V/2) \Longrightarrow E$

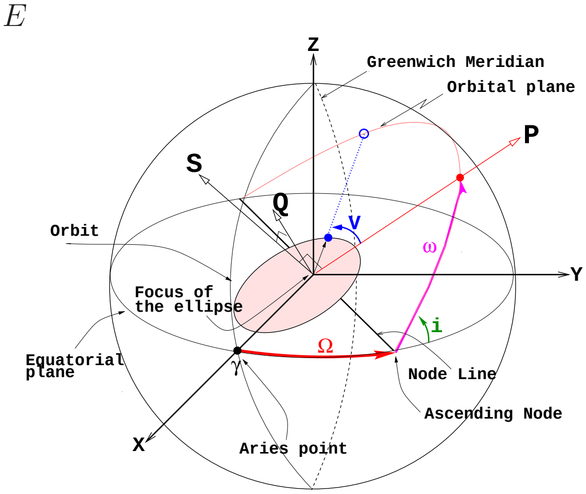


Fig. 16. Orbit in space.

Calculation of the position and velocity from the orbital elements

$$\boxed{(a,\, e,\, i,\, \Omega,\, \omega,\, \underbrace{T; t}_{V}\,) \Longrightarrow (x,\, y,\, z,\, v_x,\, v_y,\, v_z)}$$

$$\boxed{\begin{array}{ccccccc} \text{t} & \Longrightarrow & M & \Longrightarrow & E & \Longrightarrow & (r, V) \\ & M = n(t-T) & & M = E - e\sin E & & r = a(1 - e\cos E) & \\ & & & & & \tan(V/2) = (\frac{1+e}{1-e})^{1/2} tan(E/2) & \end{array}}$$

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos V \\ r\sin V \\ 0 \end{pmatrix} \; ; \; \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{na^2}{r}\{\vec{Q}(1-e^2)^{1/2}\cos E - \vec{P}\sin E\}}$$

where

$$\begin{aligned} R &= R_3(-\Omega)R_1(-i)R_3(-\omega) = \\ &= \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} P_x & Q_x & S_x \\ P_y & Q_y & S_y \\ P_z & Q_z & S_z \end{pmatrix} = [\vec{P}\ \vec{Q}\ \vec{S}] \end{aligned}$$

$$\boxed{n^2a^3 = \mu; \quad \mu = G(M+m) = 3.986005\,10^{14}\, m^3 s^{-2}} \qquad \boxed{c = \sqrt{a^2 - b^2}}$$

$$\boxed{n = \frac{2\pi}{P} = 1.46\,10^{-4}\, rad\, s^{-1}} \qquad \boxed{e = c/a}$$

Session 4a
Orbital elements and reference frames

Objectives

To get familiar with orbital elements and reference frames, as well as working with different coordinate frames. To visualize variations of the orbital elements due to different perturbations.

Files to use
`95oct18casa____r0.rnx, 95oct18casa____r0.eph, 1995-10-18.eci`

Programs to use
`eph2txt, orb2xyz, rv2ele_orb, eq2wgs_ts,cart2esf`

Development

1. Copy the programs and files of this session in the working directory.

2. [**Satellite coordinates**] The program `orb2xyz` allows us to calculate the satellite positions in a reference frame **tied to the Earth**[37] (with its origin in the center of the mass of the Earth, x axis with the direction of the Greenwich meridian, z axis parallel to the rotation axis of the Earth and y axis forming a direct frame with them) starting from the orbital elements of the `*.b` files. In these files, the longitude of the ascending node is related to the Greenwich meridian.

   (a) Generate the file `95oct18.b` from `95oct18casa____r0.eph` and, using it, calculate satellite positions for the day 18th of October of 1995, referring to the reference frame tied to the Earth.
   Execute:

```
cp 95oct18casa____r0.eph 95oct18.eph
eph2txt 95oct18.eph
cat 95oct18.b| orb2xyz >pos.b
more pos.b
```

[37] Basically, it is CTS system (see page 26).

(b) (*) Edit the program `orb2xyz.f`[38] and compare it with the algorithm described on page 59. Describe different steps made by the program `orb2xyz.f` for the calculation of these coordinates.

(c) Calculate the geocentric distance (in km) of the satellite PRN15 in the moments in time registered in the file `95oct18.b`.
Execute:

```
cat pos.b | awk '{if ($1==15) print $3 ,sqrt($4^2+$5^2
        +$6^2)/1000}' > dist.b
gnuplot
plot "dist.b"
exit
```

Which is the variation range observed? Calculate relative variation $(r_{max} - r_{min})/r_{min} \times 100$.
(*)Calculate the orbital period from the semi-major axis value (see equations in the section of theoretical basis). The satellite TOPEX/POSEIDON orbits at a height of 1.400km over the Earth surface. Which is its orbital period? (Earth radius $\simeq$ 6.400km)

3. [**Orbital elements variation**] The program `rv2ele_orb` allows us to calculate *osculating* orbital elements of a satellite, starting from its position and velocity in an inertial referential frame (in fact pseudoinertial), "non tied" to the daily rotation of the Earth. This system will be called **equatorial system**: its origin is the center of the mass of the Earth, x axis is in the direction of point Aries, z is parallel to the Earth rotation axis and y axis forms a direct frame with them.
The exit of the program `rv2ele_orb` presents the following fields:

$$\boxed{\texttt{a, e, i, }\Omega\texttt{, }\omega\texttt{, M}}$$

Files `*.eci` contain precise orbits and clocks, processed by the *Jet Propulsion Laboratory (JPL)* available in some days time. These orbits are expressed in the referential frame *Conventional Inertial System (CIS)*[39] that, obviating precession corrections, nutation, etc., and for the purpose of these practical sessions, we will considered as the equatorial system just defined.

[38]This program basically implements the first algorithm of the satellite coordinates calculation starting from the orbital elements. It does not consider perturbative terms parameters of the navigation message. The subroutine `orbit.f` implements the whole algorithm in accordance with the document (GPS/SPS-SS).
[39]See page26.

Data contained in these files are organized in the following fields:

```
sat year month day hh mm ss.ss x y z vx vy vz flag
```

where coordinates and velocities are expressed in km and km/s respectively.

(a) From the file `1995-10-18.eci` and using the previous program, calculate orbital elements of the satellite PRN15 for the day 18th of October of 1995.
Execute:

```
cat 1995-10-18.eci|awk '{if($1==15&&$4==18)print $8,$9,
        $10,$11,$12,$13}'| rv2ele_orb > eleorb
cat 1995-10-18.eci| awk '{if ($1==15 && $4==18)
print $5*3600+$6*60+$7}' > time
paste time eleorb >orb.jpl
more orb.jpl
```

(b) (*)Make a scheme of all the steps needed to calculate the orbital elements from the satellite position and velocity in an inertial frame.

(c) Study graphically the variation of the orbital elements of the satellite PRN15 as a function of time.
Execute, for example:

```
gnuplot
plot "orb.jpl" u 1:2
exit
```

i. Indicate the variables plot in every case (x axis and y axis). Indicate their unities.
ii. Indicate the magnitude order of the observed variations for every orbital element.

4. [**Comparison of broadcast and accurate orbital elements**] Compare the values of the orbital elements obtained from the file `*.eci` with the ones in the ephemerides file `*.eph`. What is the order of magnitude of the observed differences?
   Note that in files `*.eci` or in `*.b` coordinates are expressed in km, and velocity components in km/s. On the other hand, in files `*.eph`, the ascending node argument is expressed referring to Greenwich meridian and, in some cases, it can have an accumulate offset of several cycles.

   Execute, for example:

   ```
   sed 's/D/E/g' 95oct18.b >nada
   cat nada|awk '{if ($1==15 && $2==291) print $3,$4/1000,
   $5,$6,$7+6*3.1416,$8,$9}' > orb.b
   gnuplot
   plot "orb.b" u 1:3,"orb.jpl" u 1:3
   exit
   ```

   Discuss differences. What are they due to?

5. [**Coordinate frames**] Plot a graph (in spherical coordinates) of the satellite PRN15 positions for the day 18th of October of 1995, referring to the equatorial frame (non tied to the daily rotation of the Earth).

   Execute:

   ```
   cat 1995-10-18.eci|awk '{if($1==15&&$4==18)print $8,$9,
                $10 }'|cart2esf >pos_eq
   gnuplot
   plot "pos_eq" u 2:3
   exit
   ```

   (a) Using *script* `eq2wgs_ts`, transform equatorial coordinates of file `*.eci` to terrestrial coordinates (for the satellite PRN15).
   Execute:

   ```
   cat 1995-10-18.eci|awk '{if($1==15 && $4==18)print $2,$3,
   $4,$5+$6/60+$7/3600,$8,$9,$10}'|eq2wgs_ts|cart2esf >pos_ter
   ```

   (b) Make a scheme of the transformation.

   (c) Plot satellite PRN15 positions referring to the terrestrial surface for the day 18th of October of 1995.

Execute:

```
gnuplot
plot "pos_ter" u 2:3
exit
```

(d) In one of the former figures, one can appreciate strokes formed by two contiguous points. What are they due to?

6. [**Miscellaneous**]

   (a) How long does a signal take to travel from the satellite to the receiver approximately? (take an approximate value of 20.000Km for the satellite-receiver distance)

   (b) How much has the satellite moved its position (approximately) during this time? (take an approximate value for the satellite velocity: for example the file `1995-10-18.eci` with velocities (in Km/s) referred to a (quasi)-inertial system).

   (c) How much has the ground receiver moved its position (approximately), due to the Earth rotation?

Answers Session 4a Orbital elements and reference systems
2.c
3.c.i
3.c.ii
4
5.d
6.a
6.b
6.c

Session 4b
Errors in orbits and clocks. S/A effect.

Objectives

It is to study and quantify errors in orbits and clocks of satellites (broadcast and accurate). Analyze the effect of the Selective availability from files with S/A=on and S/A=off.

Files to use
`eph.on, sp3.on, eph.off, sp3.off, eph_5.on, eci_5.on, eph_5.off, eci_5.off`

Development

1. Copy programs and files for this session in the working directory.

2. [**Errors in broadcast satellite orbits and clocks: S/A=on**] In the file `eph.on`, coordinates[40] (x,y,z) are provided in the frame tied to the Earth WGS-84 and satellite clock offsets, calculated from the navigation message from the 23rd of March of 1999 (`99mar23.eph`). The file `sp3.on` contains precise coordinates and clocks, for the same day. It has been obtained from the file `igp10022.sp3` provided from the server IGS[41]. These values will be used as a reference (their errors are less than 10cm). These files contain the following fields: `[PRN seconds X Y Z dT]`, where coordinates and clock are expressed in meters.

   (a) Calculate the differences between broadcast coordinates and clocks `eph.on` and the accurate ones `sp3.on`. Execute:

```
paste eph.on sp3.on |
  gawk '{print $1,$2,$3-$9,$4-$10,$5-$11,$6-$12}'
                                          > dif_xyzt.on
```

[40] Calculated using subroutine `orbit.f`
[41] ftp://igscb.jpl.nasa.gov/igscb/product/

(b) Represent graphically the values obtained and evaluate the errors in the orbits and clocks.
Execute:

```
gnuplot
set grid
plot "dif_xyzt.on" u 2:3
plot "dif_xyzt.on" u 2:4
plot "dif_xyzt.on" u 2:5
plot "dif_xyzt.on" u 2:6
exit
```

(c) Rationalize what must be taking into account in order to evaluate positioning error: the whole error vector or only its projection over satellite-receiver direction?

(d) Calculate error in satellite-receiver direction for a user standing in Barcelona city (coordinates WGS84: `[4789048, 176682, 4194989]`). Plot the obtained results.
Execute:

```
cat eph.on|gawk 'BEGIN{x0=4789048;y0=176682;z0=4194989}
  {printf "%02d %6d %14.4f \n",
  $1,$2,sqrt(($3-x0)**2+($4-y0)**2+($5-z0)**2)}' >eph.rho

cat sp3.on|gawk 'BEGIN{x0=4789048;y0=176682;z0=4194989}
  {printf "%02d %6d %14.4f \n",
  $1,$2,sqrt(($3-x0)**2+($4-y0)**2+($5-z0)**2)}' >sp3.rho

paste eph.rho sp3.rho |
  gawk '{printf "%02d %6d %9.3f \n",$1,$2,$3-$6}'
                                              > dif_rho.on

gnuplot
set grid
plot "dif_rho.on" u 2:3
exit
```

Limit errors in broadcast orbits and clocks (eph) with S/A=on.

(e) In the global error computation of orbits and clocks about pseudorange, what proportion (%) (approximately) belongs to each one?

(f) Compare the satellite clock errors of PRN15 and PRN19. Why are they so different?

Execute:

```
cat dif_xyzt.on | gawk '{if ($1==15) print $2,$6}' > reloj15.on
cat dif_xyzt.on | gawk '{if ($1==19) print $2,$6}' > reloj19.on
gnuplot
set grid
set yrange [-100:100]
plot "reloj15.on","reloj19.on"
exit
```

3. [**Errors in broadcast orbits and satellite clocks: S/A=off**]

   (a) Repeat same calculations as in previous exercise for the files `eph.off`, `sp3.off`. Limit errors in broadcast orbits and clocks (eph) for S/A=off.

   (b) At the time in which the file of the previous exercise was collected (1999-03-23), was the S/A equally affecting orbits and clocks?

   (c) Which error has to affect positioning more: Selective Availability (S/A) or Antispoofing (A/S)?

4. [**Selective Availability**] Files `eph_5.off`, `eci_5.off`, `eph_5.on` and `eci_5.on` contain the same sort of data as the previous ones, but every 5 minutes. For the day 5th of May of 2000, compare graphically the clock values of the satellite PRN06 provided by the navigation message (file `eph_5.off`), and by the file of accurate orbits and clocks (`eci_5.off`).

   Execute:

```
gnuplot
set grid
plot "< cat eph_5.off|gawk '{if ($1==6) print $2,$6}'",
     "< cat eci_5.off|gawk '{if ($1==6) print $2,$6}'"
set xrange[20000:60000]
replot
exit
```

   (a) What is the clock mean drift (through a day) of the satellite PRN06? Extrapolate that value for 1 year? (give the result in meters and ns (c$\simeq 3\,10^8 m/s$)

   (b) Was S/A activated when these data were collected?

(c) (*)Why is the oscillation observed in the figure?

(d) Repeat the previous plot for the satellites PRN10 and PRN17. What drifts are found?

Repeat the same study for the day 23rd of March of 1999.

(files eph_5.on and eci_5.on).

(e) What is the extend of the observed oscillations for the satellite PRN06? Was S/A activated when these data were collected?

(f) According to the obtained plots, Does S/A imply an alteration in the clock values in the navigation message, or an alteration of its own clock (from the oscillator) of the satellites?

(g) From the figure, could an upper limit be given for the correlation time of S/A?
Execute:

```
gnuplot
set grid
set xrange[20000:60000]
plot "< cat eci_5.on|gawk '{if ($1==6) print $2, $6}'"
     w linespoints

plot "< cat eci_5.on|gawk '{if ($1==10) print $2, $6}'"
     w linespoints

plot "< cat eci_5.on|gawk '{if ($1==17) print $2, $6}'"
     w linespoints
exit
```

5. (*)Accurate orbits and clocks files eci_5.off and sp3.off have been generated by independent centers. The first one originated in JPL[42] has been implemented and the second one is an average of different centers estimations (IGS). Compare accurate clock estimations contained in these files for the satellite PRN10:

[42] Files eci_5.off and eci_5.on have been obtained from 1999-03-23.eci.Z, 1999-03-23.tdpc.Z, 2000-05-15.eci.Z and 2000-05-15.tdpc.Z from the server ftp://sideshow.jpl.nasa.gov of JPL. The coordinates of these files have been transformed from the system CIS to CTS, taking into account precession and nutation terms and using the Earth rotation parameters 1999-03-23tpeo.nml.Z, and 2000-05-15tpeo.nml.Z, available in the same server.

Execute:

```
gnuplot
set grid
plot "< cat eph_5.off|gawk '{if ($1==10) print $2,$6}'",
     "< cat eci_5.off|gawk '{if ($1==10) print $2,$6}'",
     "< cat sp3.off|gawk '{if ($1==10) print $2,$6}'"
exit
```

(a) What is the difference (accumulated through all the day) between both estimations? Repeat this comparison for other satellites (for example, PRN06, PRN01, PRN09,...)

Calculate differences between accurate orbits and clocks of both files. To do so, first generate a file with common observations (notice the peculiar structure of the next instruction –that must be executed in one single line–):

Execute:

```
cat sp3.off eci_5.off| gawk '{i=$1*1" "$2*1;
    if (length(X[i])!=0) {
       printf "%02d %6d %8.4f %8.4f %8.4f %8.4f \n",
                 $1,$2,$3-X[i],$4-Y[i],$5-Z[i],$6-T[i]
                              }
          else {X[i]=$3;Y[i]=$4;Z[i]=$5;T[i]=$6}
                                 }' > eci_sp3.xyzt

gnuplot
set grid
plot "eci_sp3.xyzt" u 2:3,"eci_sp3.xyzt" u 2:4,
     "eci_sp3.xyzt" u 2:5
exit
```

(b) What order are the obtained differences between accurate coordinates of the satellites?

Plot a graph with the difference between estimations of accurate clocks.

```
gnuplot
set grid
plot "eci_sp3.xyzt" u 2:6
exit
```

(c) How could one interpret the drift detected in the figure?

(d) (**)Would this drift affect (common to all satellites) the accurate positioning (if it was done with one or the other files of the accurate clocks)?

(e) (**)Which one will affect more, the (common) drift or value dispersion? Why?

6. **(*)[Program orbits]** Edit subroutine FORTRAN `orbit.f`, and identify the different parts of the algorithm described in the last section of the theoretical fundamentals. Also identify different orbital parameters originated in the navigation message (see RINEX format).

Answers

Session 4b
Errors in orbits and clocks

2.c
2.d
2.e
2.f
3.b
3.c
4.a
4.b
4.d
4.e
5.a
5.b
5.c

Chapter 5
Pseudorange modeling (code)

Pseudorange or apparent distance between the satellite and the receiver, obtained through the correlation of the modulated code in the received signal from the satellite with the replica generated in the receiver, $P = c\,\Delta T$, is affected by a series of terms which are added to the geometric distance. In figure 17, a scheme with the different contributions is shown:

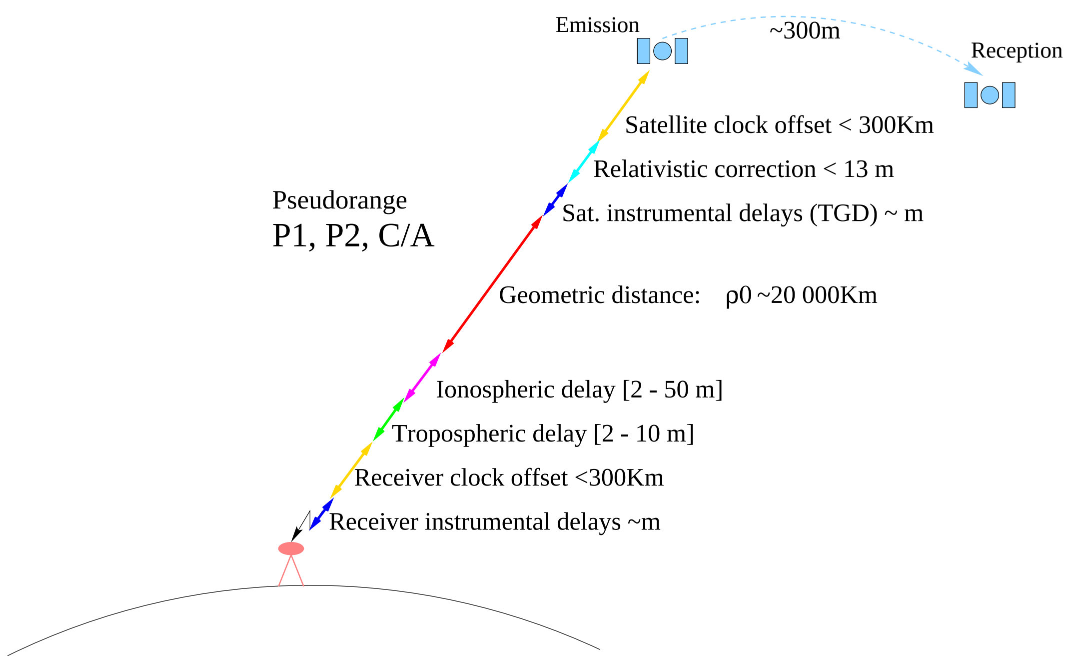


Fig. 17. Pseudorange components

The modeling of pseudorange measures P1 (or C/A) and P2, between a receiver i and a satellite j, must take into account the following terms [43] (ICD-GPS-200, 1992):

[43]These components of the model are implemented in **GCAT** (see page 90), a software module used in practical sessions from this chapter on.

$$
\begin{aligned}
\mathrm{P1}_i^j &= \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j + \alpha_1 I_i^j + K1_i^j + M_{P_1,i}^j + \varepsilon_{P_1,i}^j \\
\mathrm{P2}_i^j &= \rho_i^j + c(dt_i - dt^j) + rel_i^j + T_i^j + \alpha_2 I_i^j + K2_i^j + M_{P_2,i}^j + \varepsilon_{P_2,i}^j
\end{aligned}
$$

where:

- **Geometric distance** (ρ_i^j)
  It corresponds to the Euclidean distance between satellite position at *emission* epoch and the receiver[44] at the moment of the signal *reception*:

$$\rho_i^j = \sqrt{(x_{i,rec} - x_{ems}^j)^2 + (y_{i,rec} - y_{ems}^j)^2 + (z_{i,rec} - z_{ems}^j)^2}$$

  See on page 85 the algorithm for the coordinate calculation at the emission epoch starting from the observation epoch, and the receiver approximate position (implemented in subroutines `coord_ems_P.f` and `rec2ems.f`).

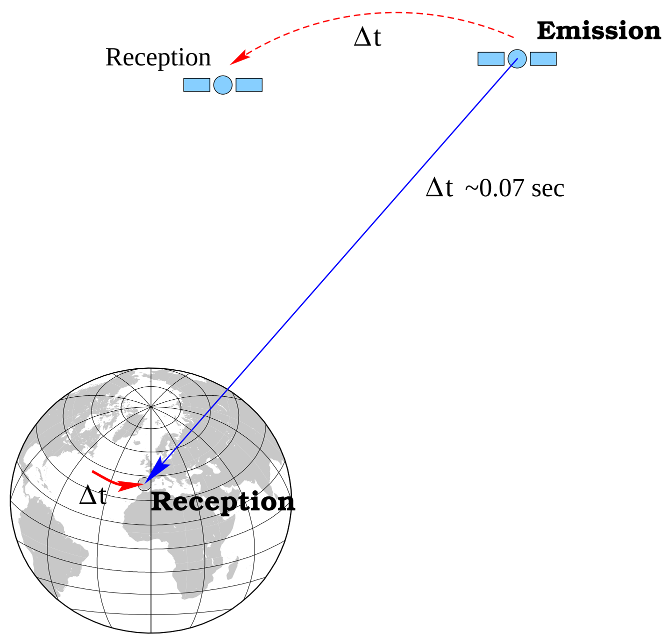


Fig. 18. Coordinates at emission and reception

[44]As the receiver coordinates are not known with accuracy (they actually are the unknowns to determine), a nominal value is taken *"a priori"* in the navigation equations $(x0_i, y0_i, z0_i)$ and ρ is linearized in the neighborhood of this point: $\rho_i^j = \rho 0_i^j + \nabla \rho_i^j \cdot (dx_i, dy_i, dz_i)$, being the deviations from this nominal value $dx_i = x_i - x0_i$, $dy_i = y_i - y0_i$, $dz_i = z_i - z0_i$ unknowns to estimate as well as the receiver clock offset dt_i (see chapter 6).

- **Offsets of receiver clock (dt_i) and satellite clock (dt^j)**
  They correspond to the clock synchronism errors referring to GPS time scale.
  – The offset of the receiver clock (dt_i) is estimated at the same time as its coordinates.
  – The offset of the satellite clocks (dt^j) can be calculated from values a_0, a_1, a_2 and $t0$ which are transmitted in the **navigation message**, according to the following expression:

$$dt^j = a_0 + a_1(t - t0) + a_2(t - t0)^2$$

  being: a_1= clock drift, a_2= clock drift rate, $t0$= time of clock (see RINEX Format in appendix II)
  NOTE: This correction is implemented by default in **GCAT** for broadcasted orbits. It may be activated or not for precise orbits, using option `Satellite clock interpolation` (see page 90).

- **Relativistic correction (rel_i^j)**
  The rate of advance of two identical clocks, placed one in the satellite and the other on the terrestrial surface, will differ due to the difference of the gravitational potential (general relativity) and to the relative speed between them (special relativity). This difference can be broken into (Hofmann-Wellenhof):

  – A constant component that only depends on the nominal value of the semi-major axis of the satellite orbit, which is adjusted modifying the clock oscillating frequency of the satellite[45]:

$$\frac{f_0' - f_0}{f_0} = \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{\triangle U}{c^2} \simeq -4.464 \cdot 10^{-10}$$

  – A periodical component due to the orbit eccentricity (that must be adjusted by the user receiver):

$$rel = 2\frac{\sqrt{\mu a}}{c}\, e\, sin(E) = 2\,\frac{\mathbf{r} \cdot \mathbf{v}}{c} \quad (in\ meters)$$

[45] Being $f_0 = 10.23MHz$, one has $\Delta f_0 = 4.464 \cdot 10^{-10}\, f_0 = 4.57 \cdot 10^{-3}Hz$ thus the satellite must use $f_0' = 10.22999999543\, Mhz$. Note that f_0' is the frequency "emitted" by the satellite and f_0 is the one "received" on the terrestrial surface, i.e., an apparent increase of the frequency is of $4.57 \cdot 10^{-3}Hz$, which is corrected decreasing in this quantity the oscillating frequency of the satellite.

being $\mu = 3.986005 \cdot 10^{14}$ (m^3/s^2) the universal gravitational constant, $c = 299792458$ (m/s) the speed of light in a vacuum, a the semi-major axis of the orbit, e its eccentricity, E the eccentric anomaly of the satellite in the orbit, and r and v satellite geometric position and velocity in an inertial system.

NOTE: This correction is implemented in **GCAT** as `Relativistic Correction` option (see page 90).

- **Tropospheric delay** (T_i^j)

  At the frequency which the GPS signal is emitted, the troposphere[46] behaves like a non dispersive media, being its effect independent of the frequency. The tropospheric delay can be modeled in an approximate way (nearly a 90%) using the following expression:

$$T_i^j = (d_{dry} + d_{wet}) \cdot m(elev)$$

  where d_{dry} corresponds to the vertical delay due to the dry component of the troposphere (basically formed by oxygen and nitrogen in hydrostatic equilibrium) and d_{wet} corresponding to the vertical delay associated with the wet component (due to the water vapor of the atmosphere), being[47]

$$\begin{array}{l} d_{dry} = 2.3\, exp\left(-0.116 \cdot 10^{-3} \cdot H\right) \quad (m) \\ d_{wet} = 0.1 \quad (m) \qquad \text{(H: height over the see level, in meters)} \end{array}$$

  Finally, $m(elev)$ is the slant factor in order to project the vertical delay in the direction of the satellite observation.

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + sin^2(elev)}}$$

  where $elev$ is the elevation referring to the local horizon of the receiver.

  NOTE: Model is implemented in **GCAT** as `Tropospheric Correction` option.

- **Ionospheric delay** (αI_i^j)

  The ionosphere is the zone of the terrestrial atmosphere that extends itself from about 60km until more than 2000km high. Due to the interaction with free electrons, electromagnetic signals that go through it suffer a delay/advancement in relation to the propagation in a vacuum that is expressed by:

$$\delta_{ion} = \int (n-1)\; ds$$

[46] Region of the atmosphere that extends itself about 60km high.

[47] More complete models can be found, for example in Hofmann-Wellenhof, p. 109.

where the integral extends itself through the ray trajectory and $n = \frac{c}{v}$ is the refraction index. As ionosphere is a dispersive media, its refraction index depends on the frequency and affects, in a different way, phase and code. This dependence on the signal frequency allows us to adjust its effect using two different frequencies[48]. For receivers with only one frequency, a model of ionospheric prediction can be used. The model defined in the (GPS/SPSS-SS) is the one of Klobuchar, its parameters are transmitted in the navigation message. Apart from being a very simple ionospheric model in which one assumes that all electrons are concentrated in a thin layer placed at 350Km high over the surface (see figure 19), one is able to reduce the ionospheric effect between a 50% and a 60%. See its implementation (GPS/SPSS-SS) in routine `klob.f` (appendix IV).

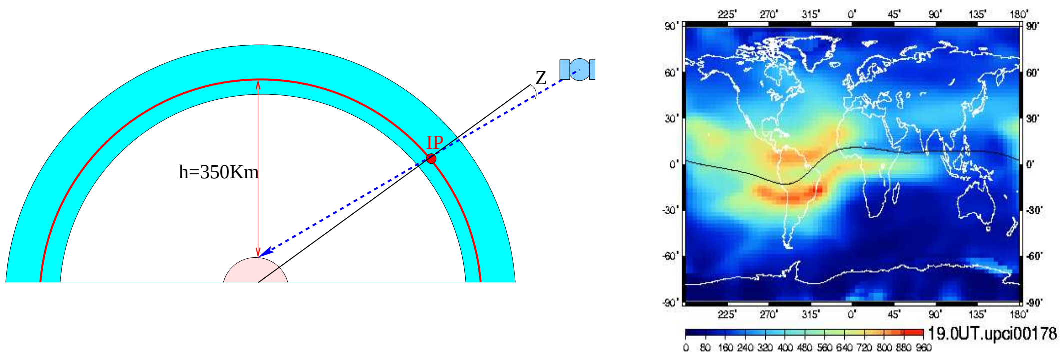


Fig. 19. Klobuchar model of a thin layer (left). Vertical delay distribution (TEC in unities of 0.1 TECUs≃ 1.6cm of delay in L1) at 19UT of the 26th of June of 2000 (right). Geomagnetic equator is also indicated.

NOTE: Implementation of Klobuchar model in **GCAT** corresponds to `Ionospheric Correction` option (see page 90).

The refraction indexes of the ionosphere for phase velocity v_f, and group velocity v_g, of the GPS signal are given, in the first order of approximation, by:

$$n_f \simeq 1 - \alpha_f \cdot N$$
$$n_g \simeq 1 + \alpha_f \cdot N$$

[48]Through free combination of ionosphere PC or LC, the ionospheric effect can be canceled up to 99.9%.

where:

- N is the electron density of the ionosphere (e^-/m^3).
- $\alpha_f = \frac{40.3}{f^2}$ (m^2/e^-).
  In particular:
  $\alpha_{f_1} = 1.6237 \cdot 10^{-17}$
  $\alpha_{f_2} = 2.6742 \cdot 10^{-17}$
  $\alpha_I = \alpha_{f_2} - \alpha_{f_1} = 1.0505 \cdot 10^{-17}$
- f is the signal frequency (Hz).

With all this, the ionospheric delay (in meters), in the first approximation, is given by:

$$\boxed{\delta_{ion} = \alpha_f \cdot I}$$

being I the electron number per area unity in the direction of the observation or STEC (*Slant Total Electron Content*)[49]:

$$I = \int N_e \, ds$$

The ionospheric delay corresponding to **phase** measurements is $-\delta_{ion}$ and the corresponding to **pseudorange** measurements is $+\delta_{ion}$, that is to say, phase measurements suffer an advancement when crossing ionosphere and pseudorange measurements suffer a delay[50].

- **Instrumental delays (K_i^j)**

  Possible sources of these delays are antennas, cables, as well as different filters used in receivers and satellites.

  They break into a delay corresponding to the satellite and another to the receiver, which depend on the frequency:

  $$K1_i^j = R1_i - T_{GD}^j \qquad ; \qquad K2_i^j = R2_i - \frac{f_1^2}{f_2^2} T_{GD}^j$$

  where

  - $R1_i$ can be assumed to be zero (including it in the receiver clock offset)
  - T_{GD}^j **is transmitted in the navigation message** (*Total Group Delay*) from the satellite.

[49] $1\,TECU = 10^{16}\,e^-/m^2 = 0.105\,m_{LI} = 0.162\,m_{L1} = 0.267\,m_{L2}$
$1\,m_{LI} = \frac{1}{\gamma - 1}\,m_{L1} = 1.54573\,m_{L1} \quad ; \quad \gamma = (\frac{77}{60})^2$

[50] Note that although the phase travels faster than the speed of light, no information is carried so the relativity principle is not violated.

According to ICD GPS-2000, the control segment tracks the satellite *timing* in order to cancel T_{GD} completely when making the ionosphere free combination. This is the reason why one has $\alpha_2/\alpha_1 T_{GD}^j$ for frequency f_2.

- **Multipath** ($M_{P,i}^j$)

The interference by multipath is generated when a signal arrives, by different ways, at the antenna (see figure 20). Its principal cause is the antenna closeness to the reflecting structures, and it is important when the signal comes from the satellite with low elevation. This error is different for different frequencies. It affects the phase measurements, as well as the code measurements. In the case of the code, it can reach a theoretical value of 1.5 times the wavelength ("chip"). This means that, for C/A code, it reaches up to 450 m although upper values more than 15m are difficult to observe. Typically, it is less than 2 or 3 meters. In the case of the phase, its theoretical maximum value is a quarter of the wavelength. This means about 5 cm for L1 or L2.

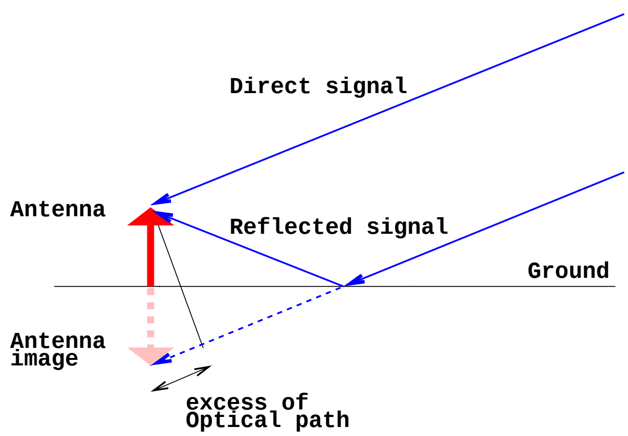


Fig. 20. Difference of the optical path between the direct signal and the reflected signal

This error can be minimized improving the antenna quality, that is to say, making them reject signal coming from certain directions, and moving the antenna away from reflecting objects.

- **Noise** ($\varepsilon^j_{P,i}$)

  In this term, the measurement noise of pseudorange is included and all non previously modeled effects.

  The accuracy of pseudorange measurements is higher than 1% of the wavelength ("chip"). This means a noise with a maximum value of 3 m for the civil C/A code and about 30 cm for the protected P codes. However, current receivers, using the smoothing of the code with the phase, can provide C/A codes with noise about 50 cm.

Annex 5.1: Algorithm for coordinate calculation at the emission epoch

- **Emission epoch calculation**

The following algorithms allow us to calculate the emission epoch of the signal coming from the satellite starting from its reception epoch:

A) Algorithm using pseudorange

The emission epoch may be directly obtained from the reception epoch, taking into account that pseudorange P is a direct measurement of the time difference between both epochs, each one of them measured in the corresponding clock (t^{sat} or t_{sta}): $P = c\ (t_{sta}[reception] - t^{sat}[emission])$

So, the signal emission epoch, measured with satellite clock (t^{sat}), is given by:

$$t^{sat}[emission] = t_{sta}[reception] - \Delta t$$

where,

$$\Delta t = P/c$$

In order to calculate satellite coordinates, it must be used emission epoch "measured in GPS time scale" $T[emision]$ (i.e., the time scale defined by Control Segment clocks). This time scale may be obtained correcting t^{sat} value with satellite clock offset $dt^{sat} = t^{sat} - T$, that may be acquired from navigation message. Thence we finally get:

$$T[emission] = t^{sat}[emission] - dt^{sat} = t_{sta}[reception] - P/c - dt^{sat}$$

Note that the former expression relates the emission epoch $T[emission]$ in the GPS time scale with observation epochs (t_{sta}) recorded by the receiver, referring to receiver internal clock.

The former algorithm has the advantage of providing the signal emission epoch directly, without iterative calculation, although it does need pseudorange measurements in order to relate both instants. The accuracy in determination of $T[emision]$ is very high, and essentially depends on dt^{sat} error figure: less than 10 or 100 nanoseconds with S/A=off and S/A=on, respectively. This allows to calculate satellite coordinates with errors below one tenth of millimeter in both cases (GPS satellite speeds is in the order of few km/s). This algorithm is implemented by default in **GCAT** software, under "`Satellite coordinates at emission:Using the PR`" option (see page 90).

B) Purely geometric algorithm

The former algorithm provides signal emission epoch tied to satellite clock (t^{sat}). On the other hand, the following algorithm ties this epoch to receiver clock (t_{sta}):

$$t_{sta}[emission] = t_{sta}[reception] - \Delta t$$

where Δt is now calculated by iteration (assuming some approximate receptor coordinates $r_{0_{sta}}$ are known) according to this procedure (it converges very fast):

1. Calculate the position r^{sat} of the satellite at signal reception epoch t_{sta}.
2. Calculate the geometric distance between satellite coordinates obtained previously and receiver position[51], and from it, calculate the signal propagation time between both points:
$$\Delta t = \frac{\|r^{sat} - r_{0_{sta}}\|}{c}$$
3. Calculate satellite position at the instant: $t = t_{sta} - \Delta t \Longrightarrow r_{sat}$.
4. Compare the new position r^{sat} with the former position. If they differ more than certain threshold value, reiterate the process starting from step 2.

Finally, emission epoch at GPS time scale will be given by[52]:

$$T[emission] = t_{sta}[emission] - dt_{sta}$$

where dt_{sta} is receiver clock offset referred to GPS time, that may be obtained from navigation solution (although "a posteriori").

[51] At this point one must pay special attention to make sure that satellite and receiver coordinates are expressed in the same reference system, because when satellite-receiver ray is generated, a common reference system must be considered

[52] Rigorously,

$$T[emission] = f(T[reception]) = f(t_{sta}[reception] - dt_{sta}) \simeq t_{sta}[emission] - dt_{sta}$$

where function $f(\cdot)$ represents geometric algorithm.

Comment:

A similar algorithm for satellite coordinate calculations at the reception epoch is used in JPL's GIPSY OASIS-II software, allowing a larger modularity because accurate pseudorange measurements are not needed to calculate emission epoch.

If receiver clock offset is small [53] this term may be neglected, and besides, it will not be known until navigation solution has been calculated (it could also be extrapolated from previous estimations). If dt_{sta} is big (in the order of 1 millisecond), it may introduce errors of about one meter in satellite coordinates, and this must be taken in account when building the navigation model[54]; or more the point, in the partial derivative respect to receiver clock in the design matrix.

It also must be taken into account the possible error due to utilization of an approximate value for receiver coordinates[55] $r_{0_{sta}}$. In this way, if receiver coordinates are not known with certain degree of accuracy, that error must be considered when calculating partial derivatives regarding receiver coordinates, and they may end up more complicated than those corresponding to pseudorange method described in the former section (see annex II in the following chapter).

This algorithm is implemented in **GCAT** with the option "`Satellite coordinates at emission:  Geometric`" (see page 90).

[53]Some modern receivers adjust their clocks epoch-by-epoch, providing offsets of about 10 nanoseconds. However, a big amount of the other receivers wait until gathering an offset of 1 millisecond.

[54]in the "design matrix" or Jacobian matrix, obtained when linearizing the model with respect to coordinates and receiver clock errors –see following chapter–).

[55]Although its impact is very small for errors of a few meters.

- **Satellite coordinate calculation**

Once the emission epoch of the signal is known, satellite coordinates may be calculated at that instant, and in order to do so it may be used an inertial system or a system tied to the Earth.

If satellite coordinate calculations are made in a system tied to the Earth (for example, using routine `orb.f`), one must use (logically) the same reference system for the receiver and satellite coordinates, because when forming the satellite-receiver ray both must be expressed in a common reference system.

If a system tied to the Earth is adopted "at the signal reception instant"[56], you should use the following algorithm:

1. Calculate satellite coordinates at the emission epoch, and in the system tied to the Earth at the same instant.
   Using, for example, routine `orb.f`, you would do:

$$T[emission] \Longrightarrow [\texttt{orb.f}] \Longrightarrow r^{sat}$$

2. Transform satellite coordinates from the system tied to the Earth at "emission epoch" to the system tied to the Earth at "reception epoch". In order to do so, one must consider the Earth rotation during the time interval Δt that the signal takes to propagate from the satellite to the receiver:

$$\tilde{r}^{sat} = R_3\left(\omega_E \Delta t\right) \cdot r^{sat}$$

   where,

$$\Delta t = \frac{\sqrt{(x_{0_{sta}} - x^{sat})^2 + (y_{0_{sta}} - y^{sat})^2 + (z_{0_{sta}} - z^{sat})^2}}{c}$$

   Note: It is advisable to calculate Δt using the former expression, even when pseudorange method is used to figure out signal propagation time. The reason for this is that "P/c" includes other delays (clock offsets, ...) besides just the purely geometric part ρ/c. In other words, P/c establish a very precise link between receiver clock (at reception side) and satellite (at emission). Nevertheless, as a geometric distance it is biased, thence the name "pseudorange".

[56]In that case, the coordinates of a fixed receiver must always be the same for different observation epochs.

Session 5a

Pseudorange modeling (code): Propagation and effects depending on satellites

Objectives

It is to study the different components of the pseudorange modeling for the code. In particular, the ionospheric and tropospheric delay, instrumental constants and satellite clock offsets

Files to use
`13oct98.a, 13oct98.eph, sta.pos`

Programs to use
`GCAT`

Development

Files used in this session have been collected with the plaque Lassen-SK8 (Trimble). It is a low cost receiver (about 40.000 pts in year 1998) which provides the code[57] (by means of the protocol TSIP) for the frequency f_1. The file `13oct98.a` was registered under A/S=on conditions.

1. Copy programs and files of this session in the working directory.

2. Through the application of `GCAT`, generate a file with the different terms of the pseudorange modeling for the file `13oct98.a` data, using broadcast orbits `13oct98.eph`. To do so, follow these steps:

   - Execute **GCAT &**.
     The panel shown immediately after will be introduced (figure 21 on the left):

[57] it also provides the truncated phase.

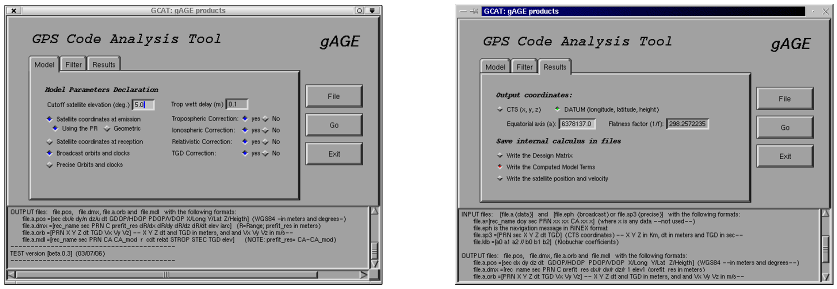


Fig. 21. Main panel and file Results of GCAT application

- Press **File** and select file `13oct98.a` (default the file `13oct98.eph` will also be selected for broadcast orbits)
- In the file **Results** (figure 21 right), select the option: `Write the Computed Model terms`. Leave default values of the remaining parameters in all the files.
- Execute **Go** (it takes some seconds in processing the file)

Once the process is finished, the file `13oct98.a.mdl` will have been generated containing the following fields:

rec_name sec PRN CA CA_model ρ cdt rel $STROP$ $STEC$ T_{GD} $elev$

where:

"`rec_name`" is the name of the receiver, "`sec`" are seconds in the day, `PRN` indicates the satellite, "CA" corresponds to the pseudorange value –in meters– measured by the receiver (code CA), "`CA_model`" is the modeled pseudorange value (in meters), "ρ" is the geometric Euclidean distance (in meters) satellite receiver, "cdt" is the satellite clock offset –in meters–, "rel" is the relativistic correction –in meters– due to the orbit eccentricity, "$STROP$" and "$STEC$" corresponds to modeled tropospheric and ionospheric slant delays –in meters–, "T_{GD}" is the instrumental delay of the satellite–in meters– and "$elev$" is the satellite elevation referring to the local horizon of the observer –in degrees–.

3. [**Tropospheric delay**] Represent graphically the modeled tropospheric slant delay "`STROP`" as a function of time for the satellite PRN14. The same as a function of the elevation. Repeat for other satellites.

Execute:

```
gnuplot
set grid
set yrange[0:20]
plot "<cat 13oct98.a.mdl|gawk '{if ($3==14) print $2,$9}'"

plot "<cat 13oct98.a.mdl|gawk '{print $2,$9}'"

plot "<cat 13oct98.a.mdl|gawk '{print $12,$9}'"
exit
```

(a) With the obtained results in sight, set bounds to the value of tropospheric slant delay.

(b) Why are the plots of the different satellites superimposed when making the representation as a function of the elevation?

(c) Represent graphically $STROP * sin(elev)$ as a function of time. The same as a function of the elevation.
Execute:

```
gnuplot
set grid
set yrange[0:5]
plot "<cat 13oct98.a.mdl|
                  gawk '{print $2,$9*sin(3.14/180*$12)}'"

plot "<cat 13oct98.a.mdl|
                 gawk '{print $12,$9*sin(3.14/180*$12)}'"
exit
```

What does the observed value correspond to in the plot (approximately)?

(d) Is the tropospheric slant delay (as it has been modeled in the program GCAT) a quantity depending on the hour of the day? And on elevation?

(e) Give an approximate value for the tropospheric zenith (vertical) delay.

(f) (**) Propose a plain model for the tropospheric delay.

(g) (**) What percentage of the real tropospheric delay could be adjusted using the proposed model (approximately)?

(h) Instead of the C/A code, the free ionospheric combination (LC) was used. Could one use the same model for the tropospheric delay? What about using a code in frequency f_2?

4. **[Ionospheric delay]** Graphically represent the modeled ionospheric slant delay "STEC" as a function of time, and as a function of the elevation for the satellite PRN14. Repeat for other satellites.
Execute:

```
gnuplot
set grid
set yrange[0:20]
plot "<cat 13oct98.a.mdl|gawk '{if ($3==14) print $2,$10}'"

plot "<cat 13oct98.a.mdl|gawk '{print $2,$10}'"

plot "<cat 13oct98.a.mdl|gawk '{print $12,$10}'"
exit
```

(a) With the figure in sight, set bound to the ionospheric slant delay.

(b) Graphically represent $STEC * sin(elev)$ as a function of time and as a function of the elevation
Execute:

```
gnuplot
set grid
set yrange[0:5]
plot "<cat 13oct98.a.mdl|
                  gawk '{print $2,$10*sin(3.14/180*$12)}'"

plot "<cat 13oct98.a.mdl|
                 gawk '{print $12,$10*sin(3.14/180*$12)}'"
exit
```

Why, in contrast to what happened with troposphere, are the different satellite curves not superimposing when making graph as a function of the elevation?

(c) (*) What variables does the ionospheric delay STEC depend on according to the Klobuchar model (see subroutine `Klob.f`)? Which ones are provided by the navigation message?

(d) (**) Edit subroutine `klob.f` and identify the implementation of the Klobuchar algorithm defined in the document GPS/SPS-SS.

(e) (**) What percentage of the real ionospheric delay can be adjusted using Klobuchar model (approximately)?

(f) What value must be considered for the ionospheric delay when the ionospheric free combination LC is used (if one has to consider any). (*) And what would happen if a code in frequency L2 was used?

5. [**Satellite instrumental constants (TGD)**] Graphically represent the instrumental constants ("Total Group Delay" or "interfrequency bias") for the satellite PRN14 as a function of time. Repeat the graph for all the satellites at the same time.

Execute:

```
gnuplot
set grid
plot "<cat 13oct98.a.mdl|awk '{if ($3==14) print $2,$11}'"

plot "<cat 13oct98.a.mdl|awk '{print $2,$11}'"
exit
```

(a) What is the range where one finds TGD values?

(b) These values, are they directly obtained from the navigation message or must they be calculated by the positioning program?

(c) (**) If T_{GD} were common to all the satellites, should they be taken into account in the pseudorange modeling for positioning? Why?

(d) (**) What values must be considered for the instrumental delays when using the ionosphere free combination (LC)? And if a code in frequency L2 was used? Why?

6. [**Satellite clock offset**] Represent, graphically, the clock offset of the satellite PRN14 as a function of time. Repeat the graph for all the satellites at the same time.
Execute:

```
gnuplot
set grid
plot "<cat 13oct98.a.mdl|awk '{if ($3==14) print $2,$7}'"

plot "<cat 13oct98.a.mdl|awk '{print $2,$7}'"
exit
```

(a) What is the range of values where one finds the satellite clock offsets?

(b) These values, are they directly obtained from the navigation message, or must they be calculated by the positioning program?

(c) What accuracy can the satellite clocks be given with through the navigation message when S/A is activated? And when is it disabled?

(d) (**) If the satellite clock offsets *cdt* were common to all satellites (though variable in time), should they be taken into account when pseudorange modeling for positioning? Why?

(e) (**) Must the same values for satellite clock offsets be considered when using ionosphere free combination (LC)? Why?

Answers session 5a Pseudorange model. Propagation and effects depending on satellites
3.a
3.b
3.c
3.d
3.e
4.a
4.b
4.f
5.a
5.b
6.a
6.b
6.c

Session 5b

Pseudorange modeling (code): Relativistic correction, geometric distance and modeled pseudorange.

Objectives

Study the different components of pseudorange modeling for the code. In particular, the effect on the geometric distance receiver-satellite of considering satellite coordinates at the emission or reception epoch, relativistic effects and the comparison of pseudorange measured by the receiver and the modeled one, before solving the navigation equations (prefit-residual). Study its impact on the positioning error.

Files to use
`13oct98.a, 13oct98.eph, sta.pos`

Programs to use
`GCAT`

Development

1. Copy programs and files of the session in the working directory.

2. Using application `GCAT` generate a file with the different terms of pseudorange modeling for the data file `13oct98.a`, using broadcast orbits `13oct98.eph`. To do so, follow the steps:

   - Execute **GCAT**.

   - Press **File** and select file `13oct98.a` (default, the file `13oct98.eph` is also selected for broadcast orbits)

- In file **Results**, select options: `Write the Computed Model terms` and `Write the satellite position and velocity`. Leave default values of the remaining parameters in all files.
- Execute **Go** (it takes some seconds to process the file)

Once the process is finished, the files `13oct98.a.mdl` and `13oct98.a.orb` will be generated that contain the following fields:

– File `13oct98.a.mdl`:

`rec_name sec PRN CA CA_model` ρ cdt rel $STROP$ $STEC$ T_{GD} $elev$

where:
`CA CA_model` ρ cdt rel $STROP$ $STEC$ TGD are expressed in meters and $elev$ in degrees.

– File `13oct98.a.orb`:

`PRN sec X Y Z dt TGD Vx Vy Vz`

where:
`X,Y,Z,dt,TGD` are expressed in meters and `Vx,Vy,Vz` in m/s.

3. [**Relativistic correction**] Represent, graphically, the relativistic correction for the different satellites as a function of time.

```
gnuplot
set grid
set yrange[-5:5]
plot "<cat 13oct98.a.mdl|gawk '{print $2,$8}'"
exit
```

(a) Which is the variation range?

(b) (*) Theoretically justify the obtained result in the former section.

(c) What would this correction be if the orbit was perfectly circular?

(d) (**) How much must the clock oscillation frequency be modified in order to compensate the mean value of the relativistic effects due to 1) gravitational potential difference between the satellite and the receiver positions (general relativity), and 2) satellite velocity (special relativity)?

4. [**Euclidean distance: emission-reception coordinates**] In previous calculations, one has considered satellite coordinates at the emission epoch of the signal (calculated using pseudorange method, see page 85 –option `Using PR` in folder `Model`–)). Repeat the process, but taking satellite coordinates at the reception epoch instead of emission. To do so, it will be enough to follow the same steps as in section 2, but selecting the option `Satellite coordinates at reception`.
   Note: before executing **GCAT**, rename as `13oct98.a.orb_em` the file previously obtained. Name the new file as `13oct98.a.orb_rc`.

   (a) Compare satellite coordinates between emission and reception epochs. Make a graph with the difference between both of them for different epochs registered in the files. Represent, separately, the coordinates x, y, z and the module of the difference vector. Which is the variation range of the obtained values?

   Execute:

```
gnuplot
Coordinate x
plot "< paste 13oct98.a.orb_em 13oct98.a.orb_rc
      |awk '{print $2,($3-$13)}'"
Coordinate y
plot "< paste 13oct98.a.orb_em 13oct98.a.orb_rc
      |awk '{print $2,($4-$14)}'"
Coordinate z
plot "< paste 13oct98.a.orb_em 13oct98.a.orb_rc
      |awk '{print $2,($5-$15)}'"
Module of difference vector:
plot "< paste 13oct98.a.orb_em 13oct98.a.orb_rc
      |awk '{print $2,sqrt(($3-$13)**2+($4-$14)**2+($5-$15)**2)}'"
exit
```

   (b) (*) Taking into account the distance satellite-receiver ($\simeq$ 20.000km) justify the satellite shift obtained in the previous section (take $v \simeq 4km/s$) theoretically. Should Earth rotation be considered?

   (c) Calculate the modeled pseudorange error due to considering coordinates at the emission or reception epoch for the different satellites. Which is the variation range of the obtained values?
   Note: receiver coordinates at the moment of data capturing were $[4789031, 176612, 4195008]$ (Barcelona).

Execute:

```
gnuplot
plot "< paste 13oct98.a.orb_em 13oct98.a.orb_rc|
   gawk 'BEGIN{x0=4789031;y0=176612;z0=4195008}
       { print $2,(($3-$13)*($3-x0)+($4-$14)*($4-y0)+
                     ($5-$15)*($5-z0))/sqrt(($3-x0)**2+
                                ($4-y0)**2+($5-z0)**2)}'"
exit
```

(d) (**) Design an algorithm that allows us to determine the signal emission epoch starting from the reception epoch and receiver and satellite coordinates in a reference system tied to the Earth (an example of such an algorithm may be found in FORTRAN subroutine `rec2ems.f`).

5. [**Modeled pseudorange**] Compare modeled pseudorange (CA_mod) with the observed one (CA) (the one measured by the receiver –P1–).

(a) Represent, in the same graph and as a function of time, the observed and modeled pseudorange for the satellite PRN14. Repeat the graph for the satellite PRN19.
Execute:

```
gnuplot
set grid
plot "<cat 13oct98.a.mdl|awk '{if ($3==14) print $2,$4-$5}'"

plot "<cat 13oct98.a.mdl|awk '{if ($3==19) print $2,$4-$5}'"

plot "<cat 13oct98.a.mdl|awk '{print $2,$4-$5}'"
exit
```

(b) What can the observed saw tooth be due to in the figures?

(c) (*)Make a graph (as a function of time) between the observed pseudorange differences for the satellites PRN16 and PRN19 (i.e. simple differences: $\nabla^{16,19}CA = CA^{16} - CA^{19}$)
Execute (in one single line)

```
cat 13oct98.a.mdl|
  gawk '{
           if ($3==16) {R[$2]=$4}
             else {if ($3==19 && length(R[$2])!=0)
                                   print $2,$4-R[$2]}
         }' > CA_16_19
```

Execute:

```
gnuplot
plot "CA_16_19"
exit
```

Why has the saw tooth disappeared?

(d) (*) Give the mathematical expression of the simple differences between two satellites observed from the same receiver. Is any term canceled?

(e) (*)Make a graph (as a function of time) between the modeled pseudorange differences for the satellites PRN16 and PRN19 (i.e. $\nabla^{16,19}CA\_mod = CA^{16} - CA^{19}$)

Execute (in one single line)

```
cat 13oct98.a.mdl|
  gawk '{
          if ($3==16) {R[$2]=$5}
            else {if ($3==19 && length(R[$2])!=0)
                                    print $2,$5-R[$2]}
        }' > CAm_16_19
gnuplot
plot "CAm_16_19"
exit
```

(f) (*)Represent, as a function of time, the differences $\nabla^{16,19}CA - \nabla^{16,19}CA\_mod$.
Execute

```
gnuplot
plot "< paste CA_16_19 CAm_16_19 |awk '{print $1,$2-$4}'"
exit
```

What can be visualized in the graph (SA, multipath, noise...)?

6. (**) [**Modeled pseudorange calculation**] Using registered values in files `13oct98.rnx` and `13oct98.eph`[58], calculate *"by hand"* the modeled pseudorange for the satellite PRN14 corresponding to the instant t=38230sec. To do so, the next steps must be followed.

   (a) *[Orbital elements selection]* From the file `13oct98.eph`, select the block of orbital elements closer to the time instant t=38230sec.

   (b) *[Geometric distance satellite(emission)--receiver(reception)]*

[58]These files have been collected by a static receiver at the coordinate point WGS'84 (4789031, 176612, 4195008) –in meters–

i. *[Coordinates at emission]* Applying the algorithm for pseudorange (see page 85), calculate PRN14 satellite coordinates at signal emission time.
Note: use program `coord_ems_P.f` and subroutine `orbit.f`. If you want to calculate coordinates at emission time with geometric algorith, subroutine `rec2ems.f` may be used (see details at code heading).

ii. *[Coordinates at emission]* Applying the algorithm defined in routine `rec2ems.f`, calculate coordinates at the emission instant (this point requires the reiterative application of the former point).

iii. Assuming statical receiver at the coordinate point (4789052, 176614, 4195020), calculate the geometric distance between the receiver and the satellite at the signal emission epoch.

(c) *[Satellite clock offset]*. From the coefficients a_0, a_1, a_2 in the navigation message for the instant t_0 corresponding to the block of selected orbits, calculate the satellite clock offset:
$cdt = a_0 + a_1\,(t - t_0) + a_2\,(t - t_0)^2$

(d) *[Satellite instrumental delay]*. Select the TGD value from the navigation message corresponding to the instant t_0 of the previous section.

(e) *[Relativistic effect]* Applying any of the following expression $rel = 2\,\frac{r\,v}{c} = 2\frac{\sqrt{\mu\,a}}{c} e\,sinE$, calculate the relativistic correction due to the orbit eccentricity.

(f) *[Ionospheric delay]* Applying the algorithm defined for the calculation of ionospheric delay from the Klobuchar model, calculate ionospheric correction (see subroutine `klob.f`).

(g) *[Tropospheric delay]* Adopting a value of $tr_{dry} = 2.3m$ for the dry component[59] of the troposphere and $tr_{wet} = 10cm$ for the wet component and adopting the slant factor $m(elev) = \frac{1.001}{\sqrt{0.002001 + sin^2(elev)}}$, calculate the tropospheric delay according to the below expression:
$trop = m(elev) \cdot\ (tr_{dry} + tr_{wet})$

(h) Calculate the value of modeled pseudorange:
$CA_{mod} = \rho + rel + T + I - cdt + TGD$

[59]In the application **GCAT**, the following model is used for the calculation of the nominal value of the dry troposphere: $trop_{dry} = 2.3e^{-0.116\ 10^{-3} * h}$, where h is the height over the ellipsoid (GIPSY OASIS-II).

7. (***) Design a program that implements the former steps.

8. From the results obtained in the previous exercises, complete the following summarizing table of errors in the different components of the model (*absolute error*) and its impact over pseudorange.

Component	**Before adjusting**	**After adjusting**	**Model**
	Absolute error	*Absolute error*	*Parameters*
	Pseudorange er.	*Pseudorange er.*	*of the model*
Error due to the delay ionospheric	[2-10m]*FO	[1-5m]*FO	Klob. model a0,a1,a2,a3 b0,b1,b2,b3 nav. message
	[2-10m]*FO	[1-5m]*FO	
Error due to the tropospheric delay			
Error due to relativistic correction (eccentricity of the orbit)		mm	$2rv/c = \frac{\sqrt{\mu\, a}}{c} sinE$
		mm	
Error due to instrumental delay of the satellites (TGD)			
		mm	
Error due to the clock offset of satellites (S/A=off)			$a0, a1, a2$ nav. message
Error due to the clock offset of satellites (S/A=on)			
Error in distance (ρ) due to error in satellite coordinates (S/A=off)	xxxxxxxxxxxxxxxx		
	xxxxxxxxxxxxxxxx		
Error in distance (ρ) due to error in satellite coordinates (S/A=on)	xxxxxxxxxxxxxxxx		
	xxxxxxxxxxxxxxxx		
Error in coordinates are taking at the reception epoch instead of emission			

Answers Session 5b Pseudorange modeling. Relativistic effects. Geometric distance and modeled pseudodistance
3.a
3.c
4.a
4.c
5.a
5.b

Note: Complete table on previous page.

Chapter 6
Solving navigation equations (with code)

It is to determine the position $\vec{r}$ and the *offset* dt of the clock of a receiver from the pseudoranges P^j, with at least 4 satellites, and positions $\vec{r^j}$ and *offsets* dt^j of the clocks of these satellites (see Hofmann-Wellenhof p. 179).

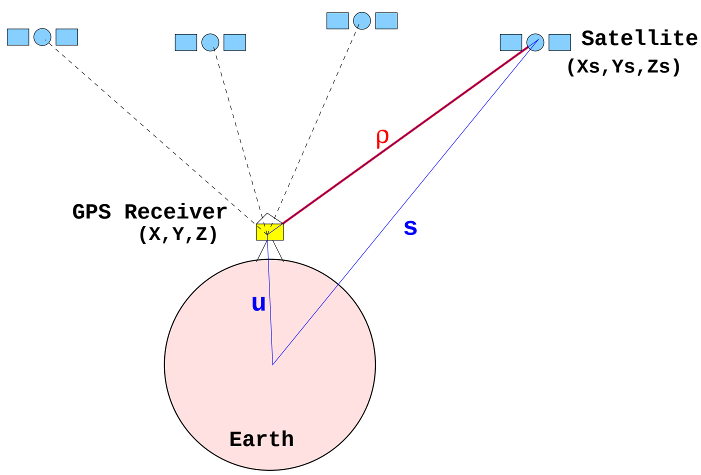


Fig. 22. Positioning GPS

Data
– *Pseudoranges* (receiver-satellite *j-th*): P^j
– The *navigation message*. In particular:
  * satellite positions when emitting the signal: $\vec{r^j} = (x^j, y^j, z^j)$
  * *offsets* of satellite clocks: dt^j
  (j=1,2,...,n) (n≥4)

Unknowns
receiver position: $\vec{r} = (x, y, z)$
offset of receiver clock: dt

From the pseudoranges between satellite and receiver:

$$P^j = \rho^j + c(dt - dt^j) + rel^j + T^j + \alpha_1 I^j + TGD^j + M^j_{P_1} + \varepsilon^j_{P_1}$$

one has an equation system with four unknowns (x, y, z, dt) with the form:

$$\boxed{\begin{array}{l} P^j + cdt^j - \delta^j \simeq \sqrt{(x-x^j)^2 + (y-y^j)^2 + (z-z^j)^2} + cdt \\ j = 1, 2, ..., n \qquad (n \geq 4) \end{array}}$$

(where the multipath terms have been rejected and noise in general, and it has been called $\delta = rel^j + T^j + \alpha_1 I^j + TGD^j$).

It is a *non linear* system, and in general overdimensioned, its usual resolution technique consists of linearizing the distance ρ in the neighborhood of a point (x_0, y_0, z_0) corresponding to an approximate position of the receiver[60].

Then, linearizing $\rho^j(x, y, z) = \sqrt{(x-x^j)^2 + (y-y^j)^2 + (z-z^j)^2}$ in the point $\vec{r}_0 = (x_0, y_0, z_0)$, one obtains: $\rho^j = \rho^j_0 + \frac{x_0 - x^j}{\rho^j_0} dx + \frac{y_0 - y^j}{\rho^j_0} dy + \frac{z_0 - z^j}{\rho^j_0} dz$

with:

$$dx = x - x_0;\ dy = y - y_0;\ dz = z - z_0$$

resulting into the linear equation system:

$$\boxed{\begin{array}{l} P^j = \rho^j_0 + \frac{x_0 - x^j}{\rho^j_0} dx + \frac{y_0 - y^j}{\rho^j_0} dy + \frac{z_0 - z^j}{\rho^j_0} dz + c(dt - dt^j) + \delta^j \\ \\ j = 1, 2, ..., 4 \qquad (n \geq 4) \end{array}}$$

The system of navigation equations[61] may be expressed as a matrix:

$$\boxed{\left[\begin{array}{c} P^1 - \rho^1_0 + cdt^1 - \delta^1 \\ \vdots \\ P^n - \rho^n_0 + cdt^n - \delta^n \end{array}\right] = \left(\begin{array}{cccc} \frac{x_0 - x^1}{\rho^1_0} & \frac{y_0 - y^1}{\rho^1_0} & \frac{z_0 - z^1}{\rho^1_0} & 1 \\ \vdots & \vdots & \vdots & \\ \frac{x_0 - x^n}{\rho^n_0} & \frac{y_0 - y^n}{\rho^n_0} & \frac{z_0 - z^n}{\rho^n_0} & 1 \end{array}\right) \left[\begin{array}{c} dx \\ dy \\ dz \\ c\,dt \end{array}\right]}$$

In general, overdimensioned systems will be obtained (for $n > 4$) and will have to be solved using the least mean square technique or Kalman filter.

Note that the differences (dx, dy, dz) are being estimated between the true position (x, y, z) and the approximate one(x_0, y_0, z_0) where a linearization has been made. This value can be refined, iterating with successive corrections obtained for the same epoch, to the point of reducing the error underneath a threshold.

[60]that can be obtained, for example, using the Bancroft method (see page 111).

[61]Strictly speaking, this system corresponds to the case where satellite coordinates at emission epoch have been calculated using pseudorange algorithm described in page 85. In case you use purely geometric algorithm, the elements of the associated matrix (design matrix or Jacobian) will vary slightly (see details in annex II, page 119).

- *Solving navigation equation:*
  - **Least mean square solution**
    One must solve the linear overdimensioned system $Y = AX$
    The least mean square solution[62] is:
    $$\boxed{\hat{X} = (A^t A)^{-1} A^t Y}$$
  - **Least mean square with weights solution**
    If W is a weight matrix for the observations vector Y, then the least mean square solution with weight matrix W is:
    $$\boxed{\hat{X} = (A^t\, W\, A)^{-1} A^t\, W\, Y}$$
    The weight matrix W is usually expressed as below:
    $$W = \begin{pmatrix} 1/\sigma^2_{y_1} & & \\ & \ddots & \\ & & 1/\sigma^2_{y_n} \end{pmatrix}$$
    where $\sigma^2_{y_i}$ is the noise variance of the observations $Y = (y_1, ..., y_n)^t$.
    If P_Y is the covariance matrix of the observation vector Y, for $W = P_Y^{-1}$ the least variance solution is obtained for X, being:
    $$\boxed{P_{\hat{X}} = (A^t\, W\, A)^{-1}}$$
  - **Kalman filter**
    If $\widehat{X}(n-1)$ is the obtained estimation for the n-th epoch, a prediction of the vector $X(n)$ will be done for the following epoch $\widehat{X}^-(n)$, according to the model[63]
    $$\boxed{\begin{array}{l} \widehat{X}^-(n) = \Phi(n-1)\ \widehat{X}(n-1) \\ P^-_{\widehat{X}(n)} = \Phi(n-1)\ P_{\widehat{X}(n-1)}\ \Phi(n-1)^T + Q(n-1) \end{array}}$$
    With these predictions $\widehat{X}^-(n)$, one can extend the observation equation $Y(n) = A(n)\, X(n)$, as if they were new observations, obtaining the system,
    $$\begin{bmatrix} Y(n) \\ \widehat{X}^-(n) \end{bmatrix} = \begin{pmatrix} A(n) \\ I \end{pmatrix} X(n) \ ; \qquad W = \begin{pmatrix} P_{Y(n)} & \\ & P^-_{\widehat{X}(n)} \end{pmatrix}^{-1}$$

[62] Naming $\hat{Y} = A\,\hat{X}$, this solution minimizes the remainder $\|Y - \hat{Y}\|^2 = \sum (y_i - \hat{y}_i)^2$, or else $\|Y - \hat{Y}\|^2_W = \sum w_i\,(y_i - \hat{y}_i)^2$ for the least mean square case with weights.

[63] It is a first order model of Gauss-Markov. The dynamical character is established through the state transition matrix Φ and the noise matrix of the process Q.

which is solved in the usual way by the least mean squares with weight matrix W:

$$\boxed{\begin{aligned}\widehat{X}(n) &= \left(A(n)^T P_{Y(n)}^{-1}\, A(n) + \left(P_{\widehat{X}(n)}^{-}\right)^{-1}\right)^{-1} \left(A(n)^T P_{Y(n)}^{-1}\, Y(n) + \left(P_{\widehat{X}(n)}^{-}\right)^{-1} \widehat{X}^{-}(n)\right)\\ P_{\widehat{X}(n)} &= \left(A^T P_{Y(n)}^{-1}\, A + \left(P_{\widehat{X}(n)}^{-}\right)^{-1}\right)^{-1}\end{aligned}}$$

The algorithm can be summarized in the following scheme[64]:

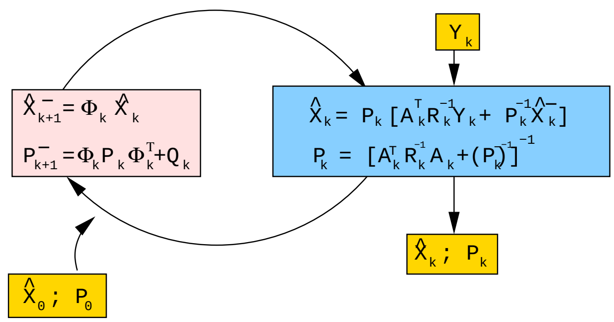


Fig. 23. Kalman filter diagram. Notation: $R_k = P_{Y(k)}$, $P_k = P_{\widehat{X}(k)}$.

Note: The formulation here presented is algebraically equivalent to the classical formulation defined in the following scheme:

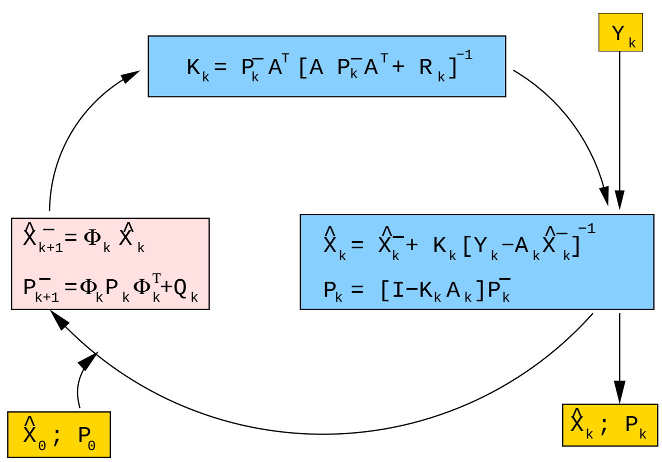


Fig. 24. Classical formulation of Kalman filter.

[64]If one desires to go deeply into the theme, an excellent book by G. J. Bierman (1977) is recommended. Special chapters relating to U-D covariance filter and SRIF.

Some easy examples of matrix definitions Φ and Q

a) Static positioning
The state vector to determine $\widehat{X}$ is given by $\widehat{X} = (x_{rec}, y_{rec}, z_{rec}, dt_{rec})$ where the coordinates $(x_{rec}, y_{rec}, z_{rec})$ are considered as constants (because the receiver is kept fixed) and the clock offset dt_{rec} as white noise with mean zero. Under these conditions matrix Φ and Q have the same shape:

$$\Phi(n) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \qquad Q(n) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & \sigma_{dt}^2 \end{pmatrix}$$

being σ_{dt} the noise of the process associated with the clock offset (in a way, the uncertainty in the clock value).

b) Kinematic positioning

1. If it is a vehicle running at a high velocity, the coordinates[65] will be modeled as a white noise with mean zero (*white noise*) the same as the clock offset:

$$\Phi(n) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \qquad Q(n) = \begin{pmatrix} \sigma_x^2 & & & \\ & \sigma_y^2 & & \\ & & \sigma_z^2 & \\ & & & \sigma_{dt}^2 \end{pmatrix}$$

2. If it is a vehicle running at a low velocity, the coordinates can be modeled as a random path (*random walk*) with the process spectral density $Q' = \frac{d\sigma^2}{dt}$:

$$\Phi(n) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \qquad Q(n) = \begin{pmatrix} Q'_x\delta t & & & \\ & Q'_y\delta t & & \\ & & Q'_z\delta t & \\ & & & \sigma_{dt}^2 \end{pmatrix}$$

[65]we are referring to deviations from nominal values (dx, dy, dz), that is what it is estimated from the navigation equations.

- *Dilution of precision (DOP):*

  Let A be a matrix associated with the equation system $Y = AX$ defined previously (having as many rows as satellites being observed at a given time).

  Then, having the matrix:

  $$Q_{xyzt} = (A^t A)^{-1} = \begin{pmatrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{xy} & q_{yy} & q_{yz} & q_{yt} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{pmatrix}$$

  - *Geometric Dilution of Precision:* GDOP=$\sqrt{q_{xx} + q_{yy} + q_{zz} + q_{tt}}$

  - *Position Dilution of Precision:* PDOP=$\sqrt{q_{xx} + q_{yy} + q_{zz}}$

  - *Time Dilution of Precision:* TDOP=$\sqrt{q_{tt}}$

  If the rotation matrix $R = [\vec{e}, \vec{n}, \vec{u}]^T$ has the directions $\{\vec{e}, \vec{n}, \vec{u}\}$ of the axis of the local coordinate system (east,north,vertical) as columns, and $Q_{enu} = R\, Q_{xyz}\, R^T$ is given, where Q_{xyz} is the submatrix of Q containing solely the geometric components, then:

  - *Horizontal Dilution of Precision:* HDOP=$\sqrt{q_{ee} + q_{nn}}$

  - *Vertical Dilution of Precision:* VDOP=$\sqrt{q_{uu}}$

  Basically, DOP represents an approximate ratio factor between the precision in positioning and precision in measurements (σ_0) in the navigation equations:

$GDOP\,\sigma_0$	...	geometric precision in position and time
$PDOP\,\sigma_0$	...	precision in position
$TDOP\,\sigma_0$	...	precision in time
$HDOP\,\sigma_0$	...	precision in horizontal positioning
$VDOP\,\sigma_0$	...	precision in vertical positioning

  Note that the precision in navigation solutions depend on two factors: 1) the precision in the measurement (σ_0), and 2) the geometry of the visible satellites (DOP).

Annex 6.1

Bancroft method for the direct calculation of the receiver position and the satellite offset

The Bancroft method allows us to obtain a direct solution of the receiver position and the clock offset, without requesting any knowledge kind "a priori" for the receiver. Thus, this method can provide an initial value (x_0, y_0, z_0) for the navigation equations seen before.

Raising and resolution:

Developing equation $P^j = \sqrt{(x-x^j)^2+(y-y^j)^2+(z-z^j)^2} + cdt$, one obtains:

$$\left[{x^j}^2+{y^j}^2+{z^j}^2-{P^j}^2\right]-2\left[x^j x+y^j y+z^j z-P^j\, cdt\right]+\left[x^2+y^2+z^2-(cdt)^2\right]=0$$

then, calling $\mathbf{r}=[x,y,z]^t$ and considering the inner product of Lorentz[66] can be expressed in a more compact way as:

$$\frac{1}{2}\left\langle \begin{bmatrix}\mathbf{r}^j\\ P^j\end{bmatrix},\begin{bmatrix}\mathbf{r}^j\\ P^j\end{bmatrix}\right\rangle - \left\langle \begin{bmatrix}\mathbf{r}^j\\ P^j\end{bmatrix},\begin{bmatrix}\mathbf{r}\\ cdt\end{bmatrix}\right\rangle + \frac{1}{2}\left\langle \begin{bmatrix}\mathbf{r}\\ cdt\end{bmatrix},\begin{bmatrix}\mathbf{r}\\ cdt\end{bmatrix}\right\rangle = 0$$

The former equation can be raised for every satellite (or measurement P^j).

Let us suppose that one disposes of four measurements P^j, and let us consider the following matrix which contains the available information of satellite coordinates and pseudoranges (every row corresponds to a satellite):

$$B=\begin{pmatrix} x^1 & y^1 & z^1 & P^1\\ x^2 & y^2 & z^2 & P^2\\ x^3 & y^3 & z^3 & P^3\\ x^4 & y^4 & z^4 & P^4\end{pmatrix}$$

Then, calling:

$$\Lambda=\frac{1}{2}\left\langle \begin{bmatrix}\mathbf{r}\\ cdt\end{bmatrix},\begin{bmatrix}\mathbf{r}\\ cdt\end{bmatrix}\right\rangle \;,\; \mathbf{1}=\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \;,\; \mathbf{a}=\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} \text{ being } a_j=\frac{1}{2}\left\langle \begin{bmatrix}\mathbf{r}^j\\ P^j\end{bmatrix},\begin{bmatrix}\mathbf{r}^j\\ P^j\end{bmatrix}\right\rangle$$

---

[66] $$\langle \mathbf{a},\mathbf{b}\rangle = \mathbf{a}^t\,\mathbf{M}\,\mathbf{b} = \begin{bmatrix} a_1, a_2, a_3, a_4\end{bmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$$

The four equations for pseudorange can be expressed as:

$$\mathbf{a} - B\,M \begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix} + \Lambda\,\mathbf{1} = 0 \ , \quad \text{being} \quad M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

from where:

$$\begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix} = MB^{-1}(\Lambda\,\mathbf{1} + \mathbf{a})$$

Then, taking into account that the equality is carried out $\langle M\mathbf{g}, M\mathbf{h}\rangle = \langle \mathbf{g}, \mathbf{h}\rangle$, and that $\Lambda = \frac{1}{2}\left\langle \begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix}, \begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix} \right\rangle$, from the former expression, one obtains:

$$\left\langle B^{-1}\mathbf{1}, B^{-1}\mathbf{1}\right\rangle \Lambda^2 + 2\left[\left\langle B^{-1}\mathbf{1}, B^{-1}\mathbf{a}\right\rangle - 1\right]\Lambda + \left\langle B^{-1}\mathbf{a}, B^{-1}\mathbf{a}\right\rangle = 0$$

The previous expression is a quadratic equation in Λ (note that matrix B and the vector $\mathbf{a}$ are also known) and provides two solutions, one of them is the searched solution $\begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix}$.

Generalization to the case of n-observations:

If more than four observations are done, the matrix B is not square. However, multiplying by B^t, one obtains (least mean square solution):

$$B^t\mathbf{a} - B^t B\,M \begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix} + \Lambda\,B^t\mathbf{1} = 0$$

where:

$$\begin{bmatrix} \mathbf{r} \\ cdt \end{bmatrix} = M(B^tB)^{-1}B^t(\Lambda\,\mathbf{1} + \mathbf{a})$$

and then:

$$\left\langle (B^tB)^{-1}B^t\mathbf{1}, (B^tB)^{-1}B^t\mathbf{1}\right\rangle \Lambda^2 + 2\left[\left\langle (B^tB)^{-1}B^t\mathbf{1}, (B^tB)^{-1}B^t\mathbf{a}\right\rangle - 1\right]\Lambda + \left\langle (B^tB)^{-1}B^t\mathbf{a}, (B^tB)^{-1}B^t\mathbf{a}\right\rangle = 0$$

Annex 6.2

Calculating partial derivatives of design matrix

As was seen at the beginning of this chapter (page 106), receiver-satellite pseudorange may be expressed as:

$$P = \rho + c(dt_{sta} - dt^{sat}) + rel + T + \alpha_1 I + TGD^{sat} + M_{P_1} + \varepsilon_{P_1}$$

where ρ is the geometric distance between satellite coordinates at emission time $\vec{r}^{sat}$ and receiver (or station) coordinates at reception time $\vec{r}_{sta}$. Both epochs ($t^{emission}$ and $t_{reception}$) are referred to GPS time scale, as set by control segment clocks.

Since both receiver coordinates and signal emission epoch $t_{reception}$ are unknown[67], you may get an initial approach to distance ρ using a first-order Taylor series:

$$\rho = \left[\frac{\partial \rho}{\partial x}\right]_{\rho_0} \Delta x + \left[\frac{\partial \rho}{\partial y}\right]_{\rho_0} \Delta y + \left[\frac{\partial \rho}{\partial x}\right]_{\rho_0} \Delta z + \left[\frac{\partial \rho}{\partial t}\right]_{\rho_0} \Delta t$$

where $\Delta x = x - x_0$, $\Delta y = y - y_0$, $\Delta z = z - z_0$ are the corrections to be applied to nominal value $\vec{\rho}_0 = (x_0, y_0, z_0)$ in order to get the precise position of receiver $\vec{r}_{sta}$ and Δt is a clock offset.

Calculation of the former partial derivatives depends on how you get ρ_0. At the former chapter, two algorithm were developed to calculate signal emission epoch and thence, to calculate distance ρ and satellite coordinates at emission epoch.

The following lines will develop expressions to determine such partial derivatives for each of the aforementioned algorithms:

1. Computation of derivative $\frac{\partial \rho}{\partial t}$

As mentioned in former chapter, pages 85 and 86, the aforementioned algorithms relate emission epoch $t^{emission}$ whether with satellite clock $\iota^{emission}$ or with receiver clock $\tau^{emission}$. That is:

Pseudorange algorithm:

$$t^{emission} = \tau_{reception} - P/c - d\iota = \iota^{emission} - d\iota$$

Geometric algorithm:

$$t^{emission} = f(\tau_{reception} - d\tau) \simeq \tau^{emission} - d\tau$$

---

[67] The reception epoch is known, but according to receiver clock $\tau_{reception}$. On the other hand, determination of $\vec{r}_{sta}$ coordinates is the objective of positioning.

where $f(\tau_{reception})$ means emission epoch computed from geometric algorithm, being a function of reception epoch τ (according to receiver clock) and that may be estimated with emission epoch at receiver clock $\tau^{emission}$ plus offset $d\tau$.

1 A. Case of pseudorange-based algorithm

If only ρ variation regarding time is considered, for this case you get:

$$\rho(t) \simeq \rho(\iota) + \frac{\partial\rho}{\partial t}(t-\iota) = \rho(\iota) - \dot{\rho}(\iota)d\iota$$

where $d\iota = \iota - t$.

In the former lineal approximation the error incurred when geometric distance ρ is calculated using emission epoch (according to satellite clock ι), instead of GPS time scale t, is proportional to receiver-satellite distance variation rate[68] and to the synchronism error between both time scales, $d\iota$.

In practice, offset $d\iota$ may be calculated from navigation message with a precision of about 10 to 100 nanoseconds, depending on S/A=on or A/S=off. In this case, and taking in account that $\dot{\rho} < 1km/s$, the error in ρ calculation is lower than 1 millimeter and may be ignored.

1 B. Case of purely geometric algorithm

As in the former case, linearizing ρ around τ, and considering only variation regarding time, results[69]:

$$\rho(t) \simeq \rho(\tau) + \frac{\partial\rho}{\partial t}(t-\tau) = \rho(\tau) - \dot{\rho}(\tau)d\tau$$

where $d\tau = \tau - t$.

In this case, receiver clock offset is the unknown, and it will be computed along receiver coordinates in navigation solution.[70].

[68]Rigorously it must be considered $\dot{\rho} = \frac{\partial\rho}{\partial t^{emission}}$, because observable P directly provides emission epoch $\iota^{emission}$ very accurately (at P noise level, few nanoseconds), but according to receiver error. Thence, the error incurred when calculating ρ will be produced by errors determining $t^{emission} = \iota^{emission} - d\iota$, due to synchronism error between satellite clock and GPS time scale.

[69]In the other hand, in this case the direct measure available is reception epoch $\tau_{reception}$ (according to receiver clock), computing emission epoch $t^{emission}$ as a function of it $t^{emission} = f(t_{reception}) = f(\tau_{reception} - d\tau)$. Therefore, it must be considered: $\dot{\rho} = \frac{\partial\rho}{\partial t_{reception}}$ (see calculation of this derivative in page 118).

[70]It may also be extrapolated from previous estimations, although it is not necessary.

Although some modern receivers update their clocks epoch-by-epoch, so offset $d\tau$ is kept between few tens of nanoseconds, lots of receivers don't update their clocks until offset reaches 1 millisecond. In this case, and taking into account that $\dot{\rho} < 1km/s$, the error in ρ calculation may be over several decimeters.

If, as usual, $d\tau$ offset is determined in navigation solution, then correction $-\dot{\rho}(\tau)d\tau$ must be taken into account in receiver clock coefficient when navigation equations are built (i.e., in design matrix or Jacobian, see page 106). Therefore, coefficient 1 of $cd\tau$ must be substituted by $1 - \frac{\dot{\rho}}{c}$. Indeed:

$$
\begin{aligned}
P = c(\tau_{reception} - \iota^{emission}) = & \quad c(t_{reception} - t^{emission}) + c(d\tau - d\iota) = \\
= & \quad \rho(t) + c(d\tau - d\iota) \simeq \\
\simeq & \quad \rho(\tau) - \dot{\rho}(\tau)d\tau + c(d\tau - d\iota) = \\
= & \quad \rho(\tau) + \left[1 - \tfrac{\dot{\rho}(\tau)}{c}\right] cd\tau - cd\iota
\end{aligned}
$$

2. Computation of derivatives $[\frac{\partial\rho}{\partial x}, \frac{\partial\rho}{\partial y}, \frac{\partial\rho}{\partial z}]$

As in computation of partial derivative $\frac{\partial\rho}{\partial t}$, it must be distinguished between the case where emission epoch is calculated using pseudorange algorithm and the case where purely geometric algorithm is used, because implicit relationships among involved variables are different in each case:

2 A. Case of pseudorange-based algorithm

In this case, the nominal value chosen for receiver position no affects, in any way, to computation of signal emission epoch, nor to satellites coordinates at that instant. In other words, receiver coordinates $\vec{r}_{sta} = (x, y, z)$ and satellite coordinates $\vec{r}^{sat} = (x^{sat}, y^{sat}, z^{sat})$ are "independent variables".

In consequence, as stated in page 106 when building navigation equations:

$$
\frac{\partial\rho}{\partial x} = \frac{x - x^{sat}}{\rho} \quad , \quad \frac{\partial\rho}{\partial y} = \frac{y - y^{sat}}{\rho} \quad , \quad \frac{\partial\rho}{\partial z} = \frac{z - z^{sat}}{\rho},
$$

or equivalently:

$$
\left[\frac{\partial\rho}{\partial x}, \frac{\partial\rho}{\partial y}, \frac{\partial\rho}{\partial z}\right] = \frac{\vec{\rho}}{\rho}
$$

2 B. Case of purely geometric algorithm

In the following development these items must be considered:

- **Effect of receiver clock synchronism error** regarding GPS time scale (dt_{rec}):
  Given a reception epoch, geometric algorithm calculates emission epoch (using an iterative procedure) considering only receiver-satellite geometry. Indeed, it computes signal propagation time supposing that it has been received in a given epoch.

  Then, if reception epoch is expressed in GPS time scale, it will also be for obtained emission epoch. If on the contrary, it is given by receiver clock, synchronism error between receiver clock and GPS time will introduce an error in satellite coordinates (because they are not calculated exactly "at GPS emission epoch") and, as a consequence, an error in geometric range ρ.

- **Effect of errors on nominal value** of receiver coordinates $r_0 = (x_0, y_0, z_0)$:
  In order to compute receiver-satellite geometric distance, algorithm uses a nominal value of receiver coordinates $r_0 = (x_0, y_0, z_0)$. In consequence, any error in these coordinates will affect result, and thence, geometric range ρ.

Taking into account the former considerations, computation of partial derivatives yields to reiterative use of chain rule:

Given geometric distance

$$\rho = c\left(t_{reception} - t^{emission}\right) = \sqrt{\vec{\rho}^{t} \cdot \vec{\rho}} = \left\|\vec{r}_{sta} - \vec{r}^{sat}\right\|$$

results:

$$\frac{\partial \rho}{\partial x} = \frac{1}{\rho}\vec{\rho}^{t} \cdot \frac{\partial \vec{\rho}}{\partial x}$$

On the other hand,

$$\frac{\partial \vec{\rho}}{\partial x} = \frac{\partial\left(\vec{r}_{sta} - \vec{r}^{sat}\right)}{\partial x} = \frac{\partial \vec{r}_{sta}}{\partial x} - \frac{\partial \vec{r}^{sat}}{\partial \vec{r}_{sta}} \cdot \frac{\partial \vec{r}_{sta}}{\partial x}$$

Taking into account that satellite coordinates at emission epoch $\vec{r}^{sat}$ depend on emission epoch $t^{emission}$ obtained by geometric algorithm, and this depends on receiver coordinates $\vec{r_{sta}}$ used to calculate receiver-satellite geometric distance, it results:

$$\frac{\partial \vec{r}^{sat}}{\partial \vec{r}_{sta}} = \frac{\partial \vec{r}^{sat}}{\partial t^{emission}} \cdot \frac{\partial t^{emission}}{\partial \vec{r}_{sta}} = \dot{\vec{r}}^{sat} \cdot \frac{\partial t^{emission}}{\partial \vec{r}_{sta}}$$

Derivative $\frac{\partial t^{emission}}{\partial \vec{r}_{sta}}$ may be obtained implicitly differencing the equation

$$\rho^2 = c^2 \left(t_{reception} - t^{emission}\right)^2 = \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \left(\vec{r}_{sta} - \vec{r}^{sat}\right)$$

leading to:

$$c^2 \left(t_{reception} - t^{emission}\right) \left(-\frac{\partial t^{emission}}{\partial \vec{r}_{sta}}\right) = \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \left(\frac{\partial \vec{r}^{sta}}{\partial \vec{r}_{sta}} - \frac{\partial \vec{r}^{sat}}{\partial \vec{r}_{sta}}\right)$$

Then, taking into account in the former expression that $\frac{\partial \vec{r}^{sat}}{\partial \vec{r}_{sta}} = \dot{\vec{r}}^{sat} \cdot \frac{\partial t^{emission}}{\partial \vec{r}_{sta}}$ y $\frac{\partial \vec{r}^{sta}}{\partial \vec{r}_{sta}} = I_3$, we get:

$$\frac{\partial t^{emission}}{\partial \vec{r}_{sta}} = \frac{-\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t}{-c^2\left(t_{reception} - t^{emission}\right) + \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \dot{\vec{r}}^{sat}} = \frac{-\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t}{c\rho\left(1 - \frac{\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}\right)}$$

Finally, substituting in the equation of $\frac{\partial \rho}{\partial x}$:

$$\frac{\partial \rho}{\partial x} = \frac{1}{\rho} \vec{\rho}^t \cdot \frac{\partial \vec{\rho}}{\partial x} = \frac{1}{\rho} \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \left[I_3 + \frac{\dot{\vec{r}}^{sat} \cdot \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t}{c\rho\left(1 - \frac{\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}\right)}\right] \frac{\partial \vec{r}_{sta}}{\partial x}$$

where $\frac{\partial \vec{r}_{sta}}{\partial x} = (1, 0, 0)^t$.

In general, considering that $\vec{\rho} = \vec{r}_{sta} - \vec{r}^{sat}$, then:

$$\left[\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, \frac{\partial \rho}{\partial z}\right] = \frac{\vec{\rho}^t}{\rho} \cdot \left[I_3 + \frac{\frac{\dot{\vec{r}}^{sat}}{c} \cdot \frac{\vec{\rho}^t}{\rho}}{1 - \frac{\vec{\rho}^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}}\right]$$

Complement: Computation of range derivative $\dot{\rho} = \frac{\partial \rho}{\partial t_{reception}}$

Calculating partial derivative $\frac{\partial}{\partial t_{reception}}$ of equation $\rho = c\left(t_{reception} - t^{emission}\right)$, it results:

$$\dot{\rho} = c\left(1 - \frac{\partial t^{emission}}{\partial t_{reception}}\right)$$

Equally, from equation $\rho^2 = c^2\left(t_{reception} - t^{emission}\right)^2 = \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \left(\vec{r}_{sta} - \vec{r}^{sat}\right)$, we get:

$$2c\left(t_{reception} - t^{emission}\right)\left(1 - \frac{\partial t^{emission}}{\partial t_{reception}}\right) = 2\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \left(\dot{\vec{r}}_{sta} - \dot{\vec{r}}^{sat}\frac{\partial t^{emission}}{\partial t_{reception}}\right)$$

from there, solving $\frac{\partial t^{emission}}{\partial t_{reception}}$, it results:

$$\frac{\partial t^{emission}}{\partial t_{reception}} = \frac{c\rho - \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \dot{\vec{r}}_{sta}}{c\rho - \left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \dot{\vec{r}}^{sat}}$$

and, therefore:

$$\dot{\rho} = c\left(1 - \frac{\partial t^{emission}}{\partial t_{reception}}\right) = \frac{\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t \cdot \left(\dot{\vec{r}}_{sta} - \dot{\vec{r}}^{sat}\right)}{c\rho\left(1 - \frac{\left(\vec{r}_{sta} - \vec{r}^{sat}\right)^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}\right)}$$

Or, likewise:

$$\dot{\rho} = \frac{\partial \rho}{\partial t_{reception}} = \frac{\frac{\vec{\rho}^t}{\rho} \cdot \left(\frac{\dot{\vec{r}}_{sta}}{c} - \frac{\dot{\vec{r}}^{sat}}{c}\right)}{1 - \frac{\vec{\rho}^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}}$$

3. Design matrix

Taking into account former results, the matrix associated to navigation equations system $Y = \mathbf{A} \cdot x$, or design matrix $\mathbf{A}$ (see page 106), is given by:

3 A. Case of pseudorange-based algorithm

$$A = \begin{pmatrix} \frac{x_0 - x^1}{\rho_0^1} & \frac{y_0 - y^1}{\rho_0^1} & \frac{z_0 - z^1}{\rho_0^1} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_0 - x^n}{\rho_0^n} & \frac{y_0 - y^n}{\rho_0^n} & \frac{z_0 - z^n}{\rho_0^n} & 1 \end{pmatrix}$$

3 B. Case of purely geometric algorithm

$$A = \begin{pmatrix} \frac{\partial \rho}{\partial x}\Big|_{\rho_0^1} & \frac{\partial \rho}{\partial y}\Big|_{\rho_0^1} & \frac{\partial \rho}{\partial z}\Big|_{\rho_0^1} & 1 - \frac{\dot{\rho}_0^1}{c} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \rho}{\partial x}\Big|_{\rho_0^n} & \frac{\partial \rho}{\partial y}\Big|_{\rho_0^n} & \frac{\partial \rho}{\partial z}\Big|_{\rho_0^n} & 1 - \frac{\dot{\rho}_0^n}{c} \end{pmatrix}$$

Being:

$$\left[\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, \frac{\partial \rho}{\partial z}\right] = \frac{\vec{\rho}^t}{\rho} \cdot \left[I_3 + \frac{\frac{\dot{\vec{r}}^{sat}}{c} \cdot \frac{\vec{\rho}^t}{\rho}}{1 - \frac{\vec{\rho}^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}} \right] \quad ; \quad \dot{\rho} = \frac{\frac{\vec{\rho}^t}{\rho} \cdot \left(\frac{\dot{\vec{r}}_{sta}}{c} - \frac{\dot{\vec{r}}^{sat}}{c} \right)}{1 - \frac{\vec{\rho}^t}{\rho} \cdot \frac{\dot{\vec{r}}^{sat}}{c}}$$

In order to get more details, consult document Observation Model and Parameter Partials from the JPL Geodetic GPS Modeling Software "GPSOMC". O.J. Sovers and J.S. Border. JPL/NASA, June 15, 1990.

Session 6a

Solving navigation equations: Positioning and S/A effect

Objectives

It is to solve the navigation equations, positioning with broadcast orbits and, positioning qith accurate orbits and clocks. Studying the S/A effect on positioning. Studying the implementation of the Kalman filter for static and kinetic positioning (white-noise, random walk).

Files to use

`13oct98.a, 13oct98.eph, 13oct98.sp3, 13oct98.a.klb`
`30may00.a, 30may00.eph, sta.pos`

Programs to use

`GCAT`

Development

The file **Filter** of the application **GCAT** contains the following options:

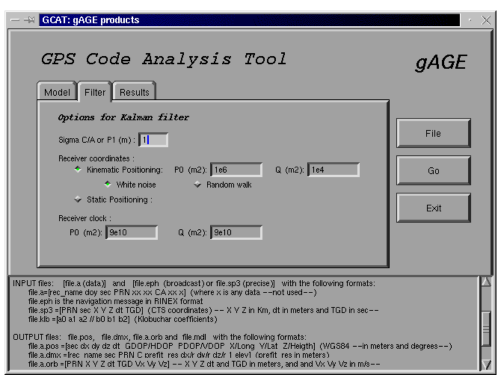


Fig. 25. File Filter of application GCAT

- **Receiver coordinates**: it allows defining the type of positioning to be done, as well as its associated parameters:
  - `Static Positioning`: coordinates are considered constants in the filter (this is the default option). The initial covariance $\sigma^2_{x_0} = \sigma^2_{y_0} = \sigma^2_{z_0} = P0\ (m^2)$ is configurable.
  - `Kinematic positioning`: there are two available options:
    - `White noise`: the correction referring to the nominal for the coordinates is considered as a white noise with mean zero and variance $\sigma^2 = Q$, that is to say, no dynamics is assumed any dynamics in the filter. The initial covariance $\sigma^2_{x_0} = \sigma^2_{y_0} = \sigma^2_{z_0} = P0\ (m^2)$ and the noise of the process $Q\ (m^2)$ are configurable parameters.
    - `Random walk`: the correction referring to the nominal value for the coordinates is considered a random walk (its uncertainty grows in time $\sigma^2 = Q'\delta t$). The initial covariance $\sigma^2_{x_0} = \sigma^2_{y_0} = \sigma^2_{z_0} = P0\ (m^2)$ and $Q'\ (m^2/sec)$ are configurable parameters.

- **Receiver Clock**: the receiver clock offset is considered as white noise with mean zero and variance $\sigma^2 = Q$. The initial covariance $\sigma^2_{dt_0} = P0\ (m^2)$ and the noise of the process $Q\ (m^2)$ are configurable parameters.

1. Copy programs and files for the session in the working directory.

2. [**Positioning with broadcast orbits and S/A=on**] Using the application **GCAT**, calculate coordinates (x,y,z) WGS'84 of the receiver, processing files `13oct98.a` and `13oct98.eph` in static mode[71]. To do so, one must follow the below steps:

   - Execute $\boxed{\textbf{GCAT \&}}$.
     The panel of next figure 26 left will appear on the screen:

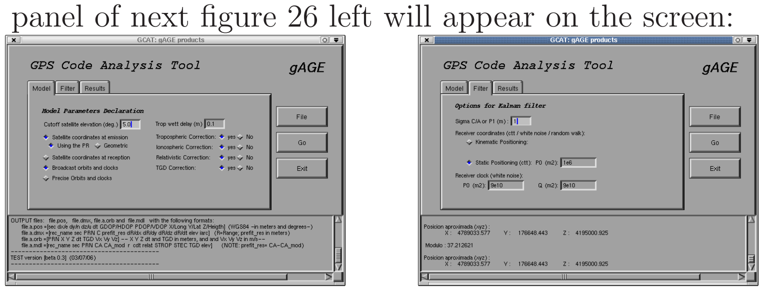


Fig. 26. Main panel and Filter of the application GCAT

[71]Data were registered maintaining the receiver stationary. By the time these data were collected, the S/A was turned on.

- Press **File** and select file `13oct98.a` (by default, the file `13oct98.eph` is also selected for broadcast orbits)
- Maintain all the default established options in tabs **Model**, **Filter** and **Results** (all the activated buttons must be in blue –see figure 26–).
- Execute **Go** (it takes some seconds to process the file)

Once the process is finished, the file `13oct98.a.pos` will be generated containing the following fields:

sec dx/e dy/n dz/u dT GDOP/HDOP PDOP/VDOP x/λ, y/ϕ, z/h

where:
- (dx,dy,dz) or (de,dn,du) are the deviations estimated by the filter referring to the nominal value (x_0, y_0, z_0) established in the file `sta.pos`. All of them are expressed in meters in the WGS'84 system.
- dT is the estimation of the receiver clock offset (in meters).
- GDOP and PDOP are provided by the option underlineCTS in the file **Results**, and HDOP and VDOP with the option DATUM.
- x/λ, y/ϕ, z/h are Cartesian coordinates CTS [WGS84 (x, y, z) (in meters)], or else, ellipsoidal ones DATUM [longitude (grad), latitude (grad), height over the ellipsoid (meters)] (see file **Results**).

3. [**Static positioning**] Graphically represent deviations (dx,dy,dz) referring to the nominal value[72] (x_0, y_0, z_0).

   (a) Graphically represent values (dx,dy,dz) and interpret the obtained results.
   Execute:

```
gnuplot
set grid
set yrange[-200:200]
plot "13oct98.a.pos" u 1:2 w d,"13oct98.a.pos" u 1:3 w d,
     "13oct98.a.pos" u 1:4 w d
exit
```

   (b) Represent estimated values for the receiver clock offset dt.
   Execute:

[72]The nominal value (x_0, y_0, z_0) adopted (see `sta.pos`) corresponds to the true value of the receiver coordinates (x, y, z) , thus (dx,dy,dz) are in fact the error in the determination of the receiver coordinates (which was fixed).

```
gnuplot
set grid
set auto
plot "13oct98.a.pos" u 1:5
exit
```

What is the saw tooth shown in the graph due to ?

(c) Which is the initialized value of (dx,dy,dz, dt)? Which is the accuracy ($\sigma_x, \sigma_y, \sigma_z$) assumed for the known coordinates? And the clock offset?

(d) Which is the value taken for the process noise `Q` of the clock? Repeat the data processing taking `Q=0.0001` for the clock. Why is the solution so corrupted? Which stochastical character is given to the clock taking `Q`$\simeq$ `0`?

4. [**Kinematic positioning: white-noise**] Activate the options Kinematic Positioning and white noise in the file `Filter` and repeat the data processing (taking the default values for the initial covariance `P0` and process noise `Q` of the coordinates and the clock).

 (a) Graphically represent deviations referring to the nominal value (dx,dy,dz). Which is the range of positioning error? Taking into account that the file was collected under S/A=on conditions, is it consistent with the expected error?
 Execute:

```
gnuplot
set grid
set yrange[-200:200]
plot "13oct98.a.pos" u 1:2 w d,"13oct98.a.pos" u 1:3 w d,
     "13oct98.a.pos" u 1:4 w d
exit
```

 (b) Graphically represent the estimated values for the receiver clock offset dt.
 Execute:

```
gnuplot
set grid
set auto
plot "13oct98.a.pos" u 1:5
exit
```

Is there any difference referring to the obtained estimations in the case of static positioning? (*)Should there be any?

(c) `[Clock synchronization]` Using the obtained estimations to adjust the clock drift dt, what accuracy can the GPS time be determined with?

5. [**Kinematic positioning: random-walk**] Activate options `Kinematic Positioning` and `random walk` in the file `Filter` and repeat the data processing (taking the default values for the initial covariance `P0` and process noise `Q` of the coordinates and the clock).

   (a) Represent deviations referring to the nominal value (dx, dy,dz) as a function of time and interpret the obtained graphs.
   Execute:

```
gnuplot
set grid
set yrange[-200:200]
plot "13oct98.a.pos" u 1:2 w d,"13oct98.a.pos" u 1:3 w d,
     "13oct98.a.pos" u 1:4 w d
exit
```

   (b) Graphically represent the estimated values for the receiver clock offset dt.
   Execute:

```
gnuplot
set grid
set auto
plot "13oct98.a.pos" u 1:5
exit
```

   Is there any difference referring to the obtained estimations in the case of static positioning? (*)Should there be any?

   (c) Repeat the data processing taking $Q' = 0$. Do the same for $Q' = 9999$. Compare these results with the ones obtained in static and kinetic positioning (`white-noise`). (*) Theoretically justify why the case $Q' = 0$ corresponds to static positioning and the case $Q' = 9999$ with the kinematic `white-noise`.

6. [**Precise orbit and clocks processing (S/A=on)**] Repeat the file data processing `13oct98.a`, but using precise orbits and clocks of the file `13oct98.sp3` *(make the data processing in kinematic mode with the white noise option)*. To do so, select in the file `Model`:

   - <u>`Precise orbits and clocks`</u>: `Orbit interpolation polynomial degree: 10`
   - <u>`Satellite clock interpol.`</u>: `No`

   (a) Compare the estimations (dx, dy, dz, dt) with the ones obtained using broadcast orbits.

   (b) (*)Give an approximate value for the range error due to broadcast orbits and clocks. The same for precise orbits and clocks (see sessions 5a and 5b).

   (c) Repeat the data processing interpolating satellite clocks using a polynomial of degree 1.
   To do so, select in the file `Model`:

   - <u>`Satellite clock interpol.`</u>: `yes`

     `Clock interpolation`
     `Polynomial degree` 1

   Why are the results so corrupted?

7. [**Broadcast orbit positioning (S/A=off)**] Calculate receiver coordinates (x, y, z) WGS84 processing the file `30may00.a` in kinematic mode (with white-noise option). Using the default options in the file `Model` (all buttons must be colored in blue), as well as broadcast orbits `30may00.eph`. Note: the file `30may00.a` has been collected maintaining the receiver position as fixed.

   (a) Graphically represent the values[73] of (dx,dy,dz) as a function of time.
   Execute:

[73] The nominal value for the coordinates corresponds to the true position (see `sta.pos`), thus the deviations referring to this nominal value directly give the positioning error.

```
gnuplot
set grid
set yrange[-200:200]
plot "30may00.a.pos" u 1:2 w d,"30may00.a.pos" u 1:3 w d,
     "30may00.a.pos" u 1:4 w d
set yrange[-40:40]
replot
exit
```

(b) Which is the error order? Was S/A on?

8. [**DOP**] The former data processing has been made with the option <u>`CTS`</u> of the file **Results** activated. Thus, the file `30may00.a.pos` contains the values of the `GDOP` and `PDOP` in the fields 6 and 7, respectively (see page 123).

   (a) Graphically represent the obtained values of `GDOP` and `PDOP`, as a function of time.
   Execute:

```
gnuplot
set grid
set yrange[0:4]
plot "30may00.a.pos" u 1:6,"30may00.a.pos" u 1:7
exit
```

   (b) Activate the option <u>`DATUM`</u> of the file **Results** and repeat the process in order to calculate `HDOP` and `VDOP`. Graphically represent the obtained values as a function of time.
   Execute:

```
gnuplot
set grid
set yrange[0:4]
plot "30may00.a.pos" u 1:6,"30may00.a.pos" u 1:7
exit
```

   (*) Think of a physical reason why `VDOP` is always greater than `HDOP`.

Complementary exercises

9. (*)In the tables shown immediately after, the different error components are summarized (1-sigma)[74] for the cases: 1) Standard Positioning Service (SPS) with S/A=off, 2) Standard Positioning Service (SPS) with S/A=on and 3) Precise Positioning Service (PPS). Every error component is described through a *bias* (that persists for some minutes) and a *random* which corresponds to a white noise.

SPS error model with SA=off	One-sigma error, m		
Error Source	Bias	Rand.	Total
Ephemeris data	2.1	0.0	2.1
Satellite clock	2.0	0.7	2.1
Ionosphere	4.0	0.5	4.0
Troposphere	0.5	0.5	0.7
Multipath	1.0	1.0	1.4
Receiver Measurement	0.5	0.2	0.5

SPS error model with SA=on	One-sigma error, m		
Error Source	Bias	Rand.	Total
Ephemeris data	2.1	0.0	2.1
Satellite clock	20.0	0.7	20.0
Ionosphere	4.0	0.5	4.0
Troposphere	0.5	0.5	0.7
Multipath	1.0	1.0	1.4
Receiver Measurement	0.5	0.2	0.5

PPS error model, P/Y code dual frequency	One-sigma error, m		
Error Source	Bias	Rand.	Total
Ephemeris data	2.1	0.0	2.1
Satellite clock	2.0	0.7	2.1
Ionosphere	1.0	0.7	1.2
Troposphere	0.5	0.5	0.7
Multipath	1.0	1.0	1.4
Receiver Measurement	0.5	0.2	0.5

(a) Assuming that the different error components are non correlated, calculate the UERE (User Equivalent Range Error)[75]

[74] Source: BW Parkinson Vol. I, pag 481-483.

[75] UERE (*rms*): statistical error in (1-sigma) that represents the total contribution of the different error sources over pseudorange.

(b) There is an average sample of 16 taken by the receivers, calculate the new UERE.

(c) Assuming non correlated[76] observations of the different satellites, and adopting values HDOP=2.0 and VDOP=2.5, give an estimation for the horizontal and vertical positioning errors.

[76]Naturally, this assumption is not followed systematically. Thus, for example, an error in the vertical ionospheric delay value is transmitted proportionally (through the slant factor) to the pseudorange measurements of the different observed satellites. Due to this, a part of this error can be absorbed by the receiver clock error, which is common to all satellites and is estimated together with coordinates.

Answers Session 6a Solving navigation equations: positioning and S/A effect
3.a
3.b
3.c
3.d
4.a
4.c
6.a
6.c
7.b

Session 6b

Solving navigation equations: Model components analysis and their impact over positioning

Objectives

To solve navigation equations. Analyze their effect on the positioning of the different components of the modeled pseudorange: relativistic effects, signal propagation, clocks, number of satellites, etc. Study the correlation effect between parameters to be estimated.

Files to use
`13oct98.a, 13oct98.eph, 13oct98.sp3, 13oct98.a.klb`
`30may00.a, 30may00.eph, sta.pos, kalman.nml_e6b`

Programs to use
`GCAT, kalman0`

1. Copy programs and files of this session in the working directory.

2. **Model components analysis**] In this exercise it will be studied the effect of the different model components of the pseudorange on positioning with code[77].
   Using the default options in tab `Model` (all buttons must be colored blue), and activating the option `DATUM` in tab `Results`, process kinematically (with the white noise option in tab `Filter`) the file `30may00.a` with broadcast orbits `30may00.eph`.
   Rename as `30may00.a.pos.org`.

[77]In the case of phase positioning, one should add the *wind-up* (see chapter 3) due to the GPS signal polarization and to the satellite rotation in a motion related to the observer.

Execute:

```
cp 30may00.a.pos 30may00.a.pos.org
```

(a) Disable the option `Tropospheric Correction` in tab `Model` and repeat the data processing (maintaining the default values of the remaining options –colored blue–). Represent in the same graph as a function of time, the height estimations over the ellipsoid h of the obtained file (`30may00.a.pos`) and of the original file (`30may00.a.pos.org`).

Execute:

```
gnuplot
set grid
set auto
plot "30may00.a.pos.org" u 1:10, "30may00.a.pos" u 1:10
exit
```

What is the order of resulting differences? Which is its incidence over pseudorange (see chapter 5)?

(b) The same for the option `Ionospheric Correction`[78].

(c) The same for the option `Relativistic Correction`.

(d) The same for the option `TGD Correction`.

(e) The same taking satellite coordinates at the reception instant instead of emission (option `Satellite coordinates at reception: Using PR`).

3. [**Effect of the number of satellites**] Represent graphically the observed satellites as a function of time for the file `30may00.a`.

Execute:

```
gnuplot
set grid
set auto
plot "30may00.a" u 3:4
exit
```

(a) Eliminate the satellite PRN21 from the file `30may00.a` and run the process again using the same options as the ones that generated the file `30may00.a.pos.org`.

[78] With all the remaining options activated.

Execute:

```
mv 30may00.a 30may00.a.org
cat 30may00.a.org|gawk '{if ($4!=21) print $0}' > 30may00.a
```

Process again `30may00.a` with **GCAT**

(b) Represent in the same graph the height values over the ellipsoid h of the obtained file and of the file `30may00.a.pos.org`.
Execute:

```
gnuplot
set grid
set auto
plot "30may00.a.pos.org" u 1:10, "30may00.a.pos" u 1:10
exit
```

How much is the height variation?.

(c) Now eliminate satellites PRN21 and PRN03 of the file `30may00.a.org` and get a position solution again using the same options as when the file was generated `30may00.a.pos.org`.
Execute:

```
cat 30may00.a.org|
   gawk '{if ($4!=21 && $4!=3) print $0}' > 30may00.a
```

Process again `30may00.a` with **GCAT**

Which is the height variation? Try with other satellites.

4. [**Filtrating satellites by elevation**] In tab `Model`, select a minimum elevation of 15 degrees for the satellites (`Cutoff satellite elevation`) and process it again with the rest of the default options –colored blue–.

(a) Compare the obtained VDOP with the file `30may00.a.pos.org`
Execute:

```
gnuplot
set grid
plot "30may00.a.pos" u 1:7, "30may00.a.pos.org" u 1:7
exit
```

(b) Compare the obtained height estimations over the ellipsoid with the ones in the file `30may00.a.pos.org`.
Execute:

```
gnuplot
set grid
plot "30may00.a.pos" u 1:10, "30may00.a.pos.org" u 1:10
exit
```

Why does the height estimation improves in spite of a worse VDOP?.

5. (*)[**Correlation between parameters**] Disable the option `Tropospheric Correction` in tab `Model` and repeat the data processing (keeping the default remaining options –colored blue–).

   Execute:

   ```
   paste 30may00.a.pos 30may00.a.pos.org|gawk '{print$1,$5-$15}'> dclk
   paste 30may00.a.pos 30may00.a.pos.org|gawk '{print$1,$10-$20}'> dh
   gnuplot
   set grid
   set xrange [31900:32350]
   plot "dclk","dh"
   exit
   ```

   (a) What should happen with height over the ellipsoid when deactivating tropospheric correlation? What actually happens?.

   (b) Compare the obtained height estimations over the ellipsoid h with those in file `13oct98.a.pos.org` How much is the clock variation? And what happens to the vertical coordinates? How much you supose is the vertical tropospheric delay (approximately)? How are these values related?.

   (c) Explain why the tropospheric error has been transferred in such a way to h and to clocks.

Complement

6. [**Kalman filter and effect of Jacobian matrix on navigation solution**]
   Process file `30may00.a` using **GCAT** software, according to the following scheme:

   - Apply pseudorange algorithm to calculate emission epoch: In `Model` tab, option `Satellite coordinates at emission: Using PR` should

be activated.
In `Filter` tab, please activate `Kinematic Positioning: White noise` option.
In `Results` tab, the `Write the design matrix` option should also be activated.
Leave default values for the rest.

Rename resulting files as:

Execute:

```
cp 30may00.a.pos 30may00.a.posP
cp 30may00.a.dmx 30may00.a.dmxP
```

- Apply geometric algorithm to calculate emission epoch: In `Model` tab should be activated the option:
  `Satellite coordinates at emission: Geometric`.
  In `Filter` tab, please activate `Kinematic Positioning: White noise` option.
  In `Results` tab, the `Write the design matrix` option should also be activated.
  Leave default values for the other options.

  Rename resulting files as:

  Execute:

  ```
  cp 30may00.a.pos 30may00.a.posG
  cp 30may00.a.dmx 30may00.a.dmxG
  ```

(a) Represent graphically the resulting solutions and compare results. Do you get the same navigation solution in both cases?

Execute:

```
gnuplot
set grid
plot "30may00.a.posP" u 1:2, "30may00.a.posG" u 1:2
plot "30may00.a.posP" u 1:3, "30may00.a.posG" u 1:3
plot "30may00.a.posP" u 1:4, "30may00.a.posG" u 1:4
exit
```

(b) Compute the difference between navigation solutions `30may00.a.posP` and `30may00.a.posG`, and represent results in a graph.

Execute:

```
paste 30may00.a.posP 30may00.a.posG|
  gawk '{if ($1==$11) print $1,$2-$12,$3-$13,$4-$14}'
                                          > diff.dat
gnuplot
set grid
plot "diff.dat" u 1:2 t "x","diff.dat" u 1:3 t "y",
     "diff.dat" u 1:4 t "z"
exit
```

What is the order of the differences? To what causes may they be ascribed?

(c) Taking **GCAT**-generated files `30may00.a.dmxP` and `30may00.a.dmxG` as starting point, calculate navigation solution using `kalman0` routine[79], and compare its results with those of **GCAT**.

Take the following values for `kalman.nml` namelist[80]:

```
$parameters
Pxx=1.d+6    fi_x=0.d0  Qxx=1.d+4
Pyy=1.d+6    fi_y=0.d0  Qyy=1.d+4
Pzz=1.d+6    fi_z=0.d0  Qzz=1.d+4
Ptt=9.d+10   fi_t=0.d0  Qtt=9.d+10
$end
```

(also, you may execute: `cp kalman.nml_e6b kalman.nml`).

Execute:

```
Adapt format of 30may00.a.dmxP file to kalman0 format:
 [obs_type sec PRN Prefit_res σ_prefit_res ∂R/∂x ∂R/∂y ∂R/∂z ∂R/∂t]
cat 30may00.a.dmxP|
    gawk 'print "C1",$2,$3,$5,"1",$6,$7,$8,$9'> fileP.dat

Process using kalman0:
cat fileP.dat |kalman0 > posP.dat

Compare these results with GCAT results:
gnuplot
set grid
plot "30may00.a.posP" u 1:2,"posP.dat" u 2:3 w p 3
plot "30may00.a.posP" u 1:3,"posP.dat" u 2:4 w p 3
plot "30may00.a.posP" u 1:4,"posP.dat" u 2:5 w p 3
exit

Repeat the same procedure with 30may00.a.posG file
```

[79]This routine implements Kalman filter. Filter parameters are established using a namelist `kalman.nml` (see details at `kalma0.f` code heading).

[80]These are the same values applied by **GCAT**.

Do you get the same results running **GCAT** and `kalman0`?

(d) In this last item we intend to analyze the effect of using the Jacobian matrix corresponding to the case when signal emission epoch is calculated using pseudorange algorithm

$$\left(\text{i.e., } \frac{\partial R}{\partial x} = \frac{x_0 - x^{sat}}{\rho},\ \frac{\partial R}{\partial y} = \frac{y_0 - y^{sat}}{\rho},\ \frac{\partial R}{\partial z} = \frac{z_0 - z^{sat}}{\rho},\ \frac{\partial R}{\partial t} = 1\right),$$

when emission epoch is computed using geometric algorithm (see from page 85 on, and annex II of this chapter).

- Substitute columns corresponding to partial derivatives `∂R/∂x ∂R/∂y ∂R/∂z ∂R/∂t]` in file `30may00.a.dmxG` with those of file `30may00.a.dmxP`[81].

  Execute:

```
cat 30may00.a.dmxP 30may00.a.dmxG |
 gawk '{if (length(r[$2" "$3])!=0){$5=r[$2" "$3]; print $0}
                       else{r[$2" "$3]=$5}}' >30may00.a.dmxN
```

- Reprocess, again, with `kalman0` routine:

  Execute:

```
cat 30may00.a.dmxN|
    gawk 'print "C1",$2,$3,$5,"1",$6,$7,$8,$9'> fileN.dat

cat fileN.dat |kalman0 > posN.dat
```

- Compute the difference regarding the solution obtained with `fileP.dat` and represent graphically the result. Include receiver clock offset in the plot (supplied, for example, by posP.dat file).

  Execute:

```
paste posP.dat posN.dat|
 awk '{if ($2==$8) print $2,$3-$9,$4-$10,$5-$11}'>diff.dat

gnuplot
set grid
plot "diff.dat" u 1:2 t "x","diff.dat" u 1:3 t "y",
     "diff.dat" u 1:4 t "z",
     "posP.dat" u 2:($6/100000) t "receiver clock"
```

What is the order of differences between `posP.dat` and `posN.dat` navigation solutions? Is there any relationship between receiver clock offset and resulting differences?

[81] substitute only the part related to partial derivatives. Prefit-residuals are kept with their original values. Note that the effect of using coordinates supplied by geometric algorithm, instead of pseudorange algorithm, will mainly manifest in prefit-residuals.

7. [**Navigation equation system**] Process the file `13oct98.a` again with broadcast orbits `13oct98.eph`, activating the following options of the file `Results`:

   - Write the design matrix
   - Write the Computed Model terms
   - Write the Satellite Position and velocity

   The files `13oct98.a.dmx`, `13oct98.a.mdl` and `13oct98.a.orb` will be generated.
   These files are organized in the following fields:

   file.a.dmx:
   [rec_name sec PRN C prefit_res $\partial R/\partial x$ $\partial R/\partial y$ $\partial R/\partial z$ $\partial R/\partial t$ elev]
   (prefit_res in meters)

   ```
   file.a.orb:
   [PRN sec X Y Z dt TGD Vx Vy Vz]
   (X Y Z dt and TGD in meters and V in m/s)

   file.a.mod:
   [rec_name sec PRN CA CA_mod  r  cdt relat STROP STEC TGD elev]
   (NOTA: prefit_res= CA-CA_mod)
   ```

   Build the navigation equation system (see section of foundations) for the instant $t = 38230sec$. To do so, one can proceed in the following way:
   - From the file `13oct98.a.dmx`, obtain the value $CA - CA_{mod}$ (prefit residual). Compare it with the obtained value in exercise 6 of session 5b for the satellite PRN14.
   - From the file `13oct98.a.orb`, obtain coordinates at the emission epoch.
   - From the file `sta.pos`, obtain the nominal value for the coordinates (x_0, y_0, z_0) of receiver `gage`.
   Compare the obtained values with the ones in the file `13oct98.a.dmx`[82]

[82] There can be small variations due to the fact that **GCAT** implements some more accurate formulas for the calculation of partial derivatives.

8. (*) `[Solving with LMS (least mean squares)]` Calculate the former equation system solution applying the least mean square solving technique.

   (a) Which case of Kalman filtering would correspond to this calculation?.

   (b) Would the result vary if this equation system was solved assigning the same weight $\omega = 1/\sigma^2$ to all the observations?

9. [**DOP**] Calculate the GDOP and the PDOP for this epoch.

10. (*)[**Solving with Kalman filter**] Consider three blocks of observations corresponding to three consecutive epochs $t = 38230,\ t = 38231,\ t = 38232$ from the file `13oct98.a.mdl`.

    (a) Write the resolution algorithm using Kalman filter, indicating the equations to use in every case, to solve this system. Consider coordinates as constants with `P=`$10^6\ m^2$ and the clock as white-noise with `P0=`$9\,10^{10}\ m^2$`,` `Q=`$9\,10^{10}\ m^2$. Take $\sigma_y^2 = 1m^2$ for the noise of the observations.

    (b) Calculate, numerically, the solution after these three iterations.

Answers

Session 6b
Solving navigation equations: analysis of the model components and their impact on positioning

2.a
2.b
2.c
2.d
2.e
3.b
3.c

Chapter 7
Differential positioning with code and phase

The idea is to compute receiver coordinates with respect to another which acts as a reference with known coordinates. This allows us to reduce the positioning error in an important way, due to the cancellation of common range errors.

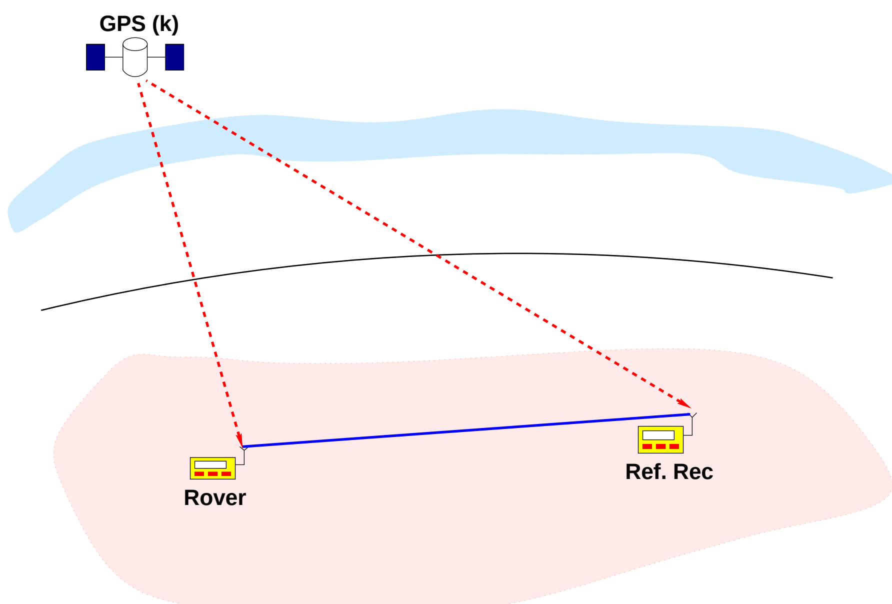


Fig. 27. Differential positioning with simple differences. Two receivers observe the same satellite at the same epoch.

Basically, the reference station (whose coordinates are fixed and known) provides range corrections for several different visible satellites that are used by the receiver (”rover”) to compute its position, canceling the part of the non-modeled effects which is common to both receivers (S/A, orbits and clocks errors, ionosphere, troposphere, ...).

1. Positioning with simple differences (with code)

If P^j_{rov} and P^j_{ref} are the code measurements of the rover (rov) and of the reference station (ref), respectively, for the satellite j-th, you get :

$$P^j_{rov} - \rho^j_{rov,0} + cdt^j - \delta^j_{rov} = \frac{x_{rov,0}-x^j}{\rho^j_{rov,0}}\, dx + \frac{y_{rov,0}-y^j}{\rho^j_{rov,0}}\, dy + \frac{z_{rov,0}-z^j}{\rho^j_{rov,0}}\, dz + cdt_{rov} + \varepsilon^j_{rov}$$

$$P^j_{ref} - \rho^j_{ref} + cdt^j - \delta^j_{ref} = cdt_{ref} + \varepsilon^j_{ref} \qquad j = 1, 2, ..., 4 \quad (n \geq 4)$$

where the left member corresponds to "prefit-residuals" $(prefit^j_\bullet = P^j_\bullet - \rho^j_\bullet + cdt^j - \delta^j_\bullet)$ and contains the modeled part of the range errors[83]. The term ε contains the non modeled error part[84]. Note that ρ has been linearized only for the rover, because the reference station coordinates are known.

Introducing notation $\Delta\diamondsuit^j \equiv \diamondsuit^j_{rover} - \diamondsuit^j_{ref}$ (simple differences between stations), you obtain:

$$\Delta prefit^j = \frac{x_{rov,0} - x^j}{\rho^j_{rov,0}}\, dx + \frac{y_{rov,0} - y^j}{\rho^j_{rov,0}}\, dy + \frac{z_{rov,0} - z^j}{\rho^j_{rov,0}}\, dz + \Delta(cdt) + \Delta\varepsilon^j$$

where[85], being a great part of the errors common to both receivers, "the noise term" $\Delta\varepsilon^j$ would have reduced considerably.

Thus, a linear equation system of the same kind for the absolute positioning is obtained, but the clock offset to be estimated is relative to reference receiver clock $\Delta(cdt) = cdt_{rov} - cdt_{ref}$.

$$\boxed{\begin{bmatrix} \Delta prefit^1 \\ \vdots \\ \Delta prefit^n \end{bmatrix} = \begin{pmatrix} \frac{x_0-x^1}{\rho^1_0} & \frac{y_0-y^1}{\rho^1_0} & \frac{z_0-z^1}{\rho^1_0} & 1 \\ \vdots & \vdots & \vdots & \\ \frac{x_0-x^n}{\rho^n_0} & \frac{y_0-y^n}{\rho^n_0} & \frac{z_0-z^n}{\rho^n_0} & 1 \end{pmatrix} \begin{bmatrix} dx \\ dy \\ dz \\ \Delta\,(c\,dt) \end{bmatrix}}$$

To solve this system, the same techniques as in the case of the absolute positioning with code (LMS, WMS, Kalman filter,...) are applied.

---

[83] See chapter 6, page 106.

[84] The ionospheric and tropospheric modeled error, satellite orbits and clocks, and especially S/A. To all this, one must add the multipath and the code noise.

[85] In practice, a range correction is calculated from reference station for each satellite $PRC^j = P^j_{ref} - \rho^j_{ref} - \delta^j_{ref}$, which is transmitted to the user in order to cancel/mitigate common differential errors. In this way, and similarly to the former equation, we get: $\Delta prefit^j = P^j_{rov} - \rho^j_{rov,0} - \delta^j_{rov} - PRC^j$ (note that satellites clocks have canceled out when computing the difference).

In the following table, the values(1σ) are shown for the different error types in the absolute and differential positioning which indicate the degradation to the geographic decorrelation. Note that the error cancellation, due to the orbits and especially to the ionosphere and troposphere, degrades when increasing the distance (base line). On the other hand, errors due to the receiver noise and multipath do not cancel.

Type of Error	Without correction DGPS		Base line and Null latency		Geographic Decorrel.
	Bias (m)	Random (m)	Bias (m)	Random (m)	(m/100Km)
Receiver noise	0.5	0.2	0.5	0.3	0.0
Multipath	0.3 - 3	0.2 - 1	0.4 - 3	0.2 - 1	0.0
Sat clock(SA=on)	21	0.1	0.0	0.0	0.0
Sat clock (SA=off)	3	0.0	0.0	0.0	0.0
Orbits (**)	< 5	0.0	0.0	0.0	< 0.05
Ionosphere (*)	(1 - 10)*FO	< 0.1*FO	0.0	0.0	< 0.2
Troposphere (*)	0.3 *FO	< 0.1*FO	0.0	0.0	< 0.2

Table 8. Scheme of absolute and differential errors. (*) Non modeled effects [FO= Slant factor]. (**) Assuming S/A is not applied to the ephemerides.

2. Positioning with double differences (with code)

This technique has its main application in solving phase ambiguities (see following section). However, for the sake of continuity in concept presentation, we will introduce first the results for positioning with code, and later on for the phase.

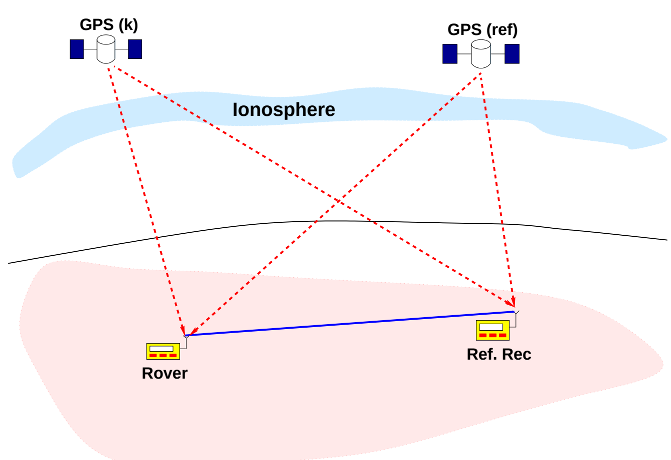


Fig. 28. Positioning with double dif.. Two receivers observing two satellites at the same epoch.

Taking a station and a satellite as a reference, double differences can be formed. Introducing notations:

$$\begin{aligned}
&\Delta\diamondsuit^{\bullet} \equiv \diamondsuit^{\bullet}_{rov} - \diamondsuit^{\bullet}_{ref}\\
&\nabla\diamondsuit_{\bullet} \equiv \diamondsuit^{j}_{\bullet} \quad - \diamondsuit^{k}_{\bullet}\\
&\Delta\nabla\diamondsuit \equiv \Delta\diamondsuit^{j} - \Delta\diamondsuit^{R} = \nabla\diamondsuit_{rov} - \nabla\diamondsuit_{ref}
\end{aligned}$$

It results in:

$$\Delta\nabla\diamondsuit = [\diamondsuit^{j}_{rov} - \diamondsuit^{j}_{ref}] - [\diamondsuit^{R}_{rov} - \diamondsuit^{R}_{ref}] = [\diamondsuit^{j}_{rov} - \diamondsuit^{R}_{rov}] - [\diamondsuit^{j}_{ref} - \diamondsuit^{R}_{ref}]$$

With single differences between receivers (Δ), the common terms associated with the satellite cancel out (clock, ephemerides, atmospheric propagation, ...). In the same way, single differences between satellites (∇) cancel common errors associated to the receiver (i.e, $\nabla(cdt) = 0$) . In consequence[86]:

$$\Delta\nabla(cdt) = \Delta\left[\nabla(cdt)\right] = \Delta 0 = 0$$

Similar to the previous section, applying these double differences to the code equations, it results[87] in:

$$\Delta\nabla prefit = \nabla\left[\frac{x_{rov,0} - x^{j}}{\rho^{j}_{rov,0}}\right] dx + \nabla\left[\frac{y_{rov,0} - y^{j}}{\rho^{j}_{rov,0}}\right] dy + \nabla\left[\frac{z_{rov,0} - z^{j}}{\rho^{j}_{rov,0}}\right] dz + \Delta\nabla\varepsilon^{j}$$

where the terms $\nabla\left[\frac{x_{rov,0}-x^{j}}{\rho^{j}_{rov,0}}\right]$ indicate the single differences between the satellite j-th and the one you took as reference[88].

Therefore, a linear equation system is obtained in which the receiver clock offset has disappeared as a parameter to be estimated.

$$\boxed{\begin{bmatrix} \Delta\nabla prefit^{1}\\ \vdots \\ \Delta\nabla prefit^{n}\end{bmatrix} = \begin{pmatrix} \nabla\left[\frac{x_{rov,0}-x^{1}}{\rho^{1}_{rov,0}}\right] & \nabla\left[\frac{y_{rov,0}-y^{1}}{\rho^{1}_{rov,0}}\right] & \nabla\left[\frac{z_{rov,0}-z^{1}}{\rho^{1}_{rov,0}}\right]\\ \vdots & \vdots & \vdots \\ \nabla\left[\frac{x_{rov,0}-x^{n}}{\rho^{n}_{rov,0}}\right] & \nabla\left[\frac{y_{rov,0}-y^{n}}{\rho^{n}_{rov,0}}\right] & \nabla\left[\frac{z_{rov,0}-z^{n}}{\rho^{n}_{rov,0}}\right] \end{pmatrix} \begin{bmatrix} dx\\ dy\\ dz\end{bmatrix}}$$

[86]See comments at the end of this section.

[87]Note that the term corresponding to the receiver clock has been cancelled out: $\Delta\nabla(cdt) = \Delta\left[\nabla(cdt)\right] = \Delta 0 = 0$

[88]Notation: $\nabla\left[\frac{x_{rov,0}-x^{j}}{\rho^{j}_{rov,0}}\right] \equiv \frac{x_{rov,0}-x^{j}}{\rho^{j}_{rov,0}} - \frac{x_{rov,0}-x^{R}}{\rho^{R}_{rov,0}}$

To solve this system, the same techniques are applied as in the case of the absolute positioning with code (least mean squares, Kalman filter,...).

Comments:

The result $\Delta\nabla(cdt) = 0$ has been based on assuming that the term corresponding to receiver clock "cdt" in P code modeling is the same for all observations, and therefore it cancels out when forming single differences between satellites (i.e., $\nabla(cdt) = 0$). This property will be fulfilled when signal emission epoch for satellite coordinates calculation has been determined using Pseudorange Algorithm described in page 85. *However, this can not be ascertained if Geometric Algorithm of page 86 is used.* Indeed:
As shown in Annex II, page 113, receiver clock coefficient is no longer "1" when geometric method is used, being affected by a correction factor depending of satellite:

$$1 - \frac{\dot{\rho}^j}{c}$$

and thence, it does not cancels out completely when satellite differences are computed.

The impact of this effect on navigation solution arises on exercises 4 and 5 in practice 7a (positioning with code), and exercises 3 and 4 in practice 7b (positioning with code and phase).

3. Positioning with double differences with code and phase (Floating)

If, besides code observations P, phase observations L are also available, the previous equation system could be extended with those new measurements.

Phase observations will be modeled in a similar way as code[89], although taking the ambiguity terms of the phase into account (see chapter 3), which are unknowns quantities that should be estimated together with the rover position.

Then, adding the phase measurements to the previous equation system in double differences and introducing the bias of the double differentiated arcs[90] as additional parameters to estimate, the following equation system is obtained:

$$\boxed{\begin{bmatrix} \Delta\nabla prefit(P)^1 \\ \Delta\nabla prefit(L)^1 \\ \\ \vdots \\ \\ \Delta\nabla prefit(P)^n \\ \Delta\nabla prefit(L)^n \end{bmatrix} = \begin{pmatrix} \nabla\left[\frac{x_{rov,0}-x^1}{\rho^1_{rov,0}}\right] & \nabla\left[\frac{y_{rov,0}-y^1}{\rho^1_{rov,0}}\right] & \nabla\left[\frac{z_{rov,0}-z^1}{\rho^1_{rov,0}}\right] & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ \nabla\left[\frac{x_{rov,0}-x^1}{\rho^1_{rov,0}}\right] & \nabla\left[\frac{y_{rov,0}-y^1}{\rho^1_{rov,0}}\right] & \nabla\left[\frac{z_{rov,0}-z^1}{\rho^1_{rov,0}}\right] & 0 & \dots & \underbrace{1}_{k} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & & & & & & \\ \nabla\left[\frac{x_{rov,0}-x^n}{\rho^n_{rov,0}}\right] & \nabla\left[\frac{y_{rov,0}-y^n}{\rho^n_{rov,0}}\right] & \nabla\left[\frac{z_{rov,0}-z^n}{\rho^n_{rov,0}}\right] & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ \nabla\left[\frac{x_{rov,0}-x^n}{\rho^n_{rov,0}}\right] & \nabla\left[\frac{y_{rov,0}-y^n}{\rho^n_{rov,0}}\right] & \nabla\left[\frac{z_{rov,0}-z^n}{\rho^n_{rov,0}}\right] & 0 & \dots & 0 & \dots & \underbrace{1}_{l} & \dots & 0 \end{pmatrix} \begin{bmatrix} dx \\ dy \\ dz \\ \Delta\nabla B^1 \\ \vdots \\ \Delta\nabla B^k \\ \vdots \\ \Delta\nabla B^l \\ \vdots \\ \Delta\nabla B^s \end{bmatrix}}$$

Equally, a linear equation system is obtained that can be solved using Kalman filter, considering arc bias $\Delta\nabla B^i$ as "constants" along continuous phase arcs, and "white-noise" at the instants when cycle-slips are produced.

This solving procedure is called *floating* ambiguities. Float in the sense that they are estimated by the filter "as real numbers". The bias estimations $\Delta\nabla B^i$ will converge into a solution after overcoming a transition, and transition length depends on the observation geometry, model quality and data noise. In general, one must expect errors in the order of decimeters in pure kinematic positioning (i.e., coordinates (x, y, z) modeled as white-noise).

[89] In phase modeling, one should take into account the *wind-up* effect as well, due to signal polarization (see page 39).

[90] That is to say, double differences of the phase arc ambiguities (B_i^j) for every pair satellite–receiver: $\Delta\nabla B^j \equiv B^j_{rov} - B^R_{rov} - (B^j_{ref} - B^R_{ref})$.

4. Solving phase ambiguities: Fixing versus Floating

The solution that one obtains floating the ambiguities does not provides the "exact" values of the bias $\Delta\nabla Bi$, due to the estimation noise. The mean quadratic error (ECM) of these estimations will depend on the quality of the model, the geometry of receivers-satellites (i.e., correlations between parameters) and the observations noise. Likewise, errors in these bias will be transfered to coordinates (which are estimated together), degrading the kinetic solution in real time (x,y,z) and showing, in general, errors exceeding the decimeter, even after the arcs stabilization.

If one desires to obtain subdecimeter precision, it requires applying techniques based on solving ambiguities (to its exact value). These consist of exploding the fact that ambiguities in both frequencies are integer multiples of wavelength[91] and, therefore, if one manages to get observable combinations with noise lower than one wavelength, its exact value could be obtained rounding it [92].

Like an example, the following set of equations (based on Colombo et al., 1999) is introduced that allow us to fix ambiguities $\Delta\nabla N_1$ and $\Delta\nabla N_2$ starting from the phase measurements in two frequencies for two close receivers[93]. To be exact, they allow us to obtain the "exact value" of the bias $\Delta\nabla Bc$ from an estimation $\Delta\nabla\widehat{Bc}$ provided by the filter by "floating" this ambiguity as a real number:

Ambiguity resolution $\Delta\nabla N_W$ for the phase measurements in two frequencies and the estimations $\Delta\nabla\widehat{Bc}$:

$$\Delta\nabla N_W = nint\left[\frac{\Delta\nabla L_W - \Delta\nabla Lc + \Delta\nabla\widehat{Bc}}{\lambda_W}\right]$$

Being the measurements $\Delta\nabla L_W$ and $\Delta\nabla Lc$ accurate to the level of some millimeters, the limiting factor is the estimation error of $\Delta\nabla\widehat{Bc}$ that must be lower than $\lambda_W/2 \simeq 40cm$ in order to round it up to the correct value.

[91]Note that, if $B_{1}{}_i^j = k_{1i} + k_1{}^j + N_{1}{}_i^j$ and $B_{2}{}_i^j = k_{2i} + k_2{}^j + N_{2}{}_i^j$ are phase ambiguities L_1 and L_2, respectively (b are instrumental delays —real values— and N integer number of cycles, see chapter 3), when forming double differences, the instrumental constants will be canceled, having $\Delta\nabla B_1 = \lambda_1\Delta\nabla N_1$, $\Delta\nabla B_2 = \lambda_2\Delta\nabla N_2$.
The ambiguity in the wide-lane combination is also an integer: $\Delta\nabla B_W = \lambda_W\Delta\nabla N_W$. The same does not happen for the ambiguity in the ionosphere-free combination $\Delta\nabla Bc$, although it can be expressed as a function of $\Delta\nabla N_1$ and $\Delta\nabla N_2$: $\Delta\nabla Bc = \lambda_c\lambda_W\left(\frac{\Delta\nabla N_1}{\lambda_1} - \frac{\Delta\nabla N_2}{\lambda_2}\right)$.

[92]Or by means other search procedures. See for example Leick (1994).

[93]Thus, one can assume that ionospheric refraction is canceled (i.e., $\Delta\nabla STEC \simeq 0$). This will be valid, in general, for baselines lower than 10-20 Km.

Ambiguity resolution $\Delta\nabla N_1$ and $\Delta\nabla N_2$ starting from the ambiguity $\Delta\nabla N_W$ solved previously, and also using phase measurements $\Delta\nabla L_1$ and $\Delta\nabla L_2$:

$$\Delta\nabla N_1 = nint\left[\frac{\Delta\nabla L_1 - \Delta\nabla L_2 - \lambda_2\Delta\nabla N_W}{\lambda_1 - \lambda_2}\right]$$

$$\Delta\nabla N_2 = \Delta\nabla N_1 - \Delta\nabla N_W$$

Obtaining the "exact" value of the bias $\Delta\nabla Bc$, once the ambiguities $\Delta\nabla N_1$ and $\Delta\nabla N_2$ are solved:

$$\Delta\nabla Bc = \lambda_W\left(\frac{\Delta\nabla N_1}{\lambda_1} - \frac{\Delta\nabla N_2}{\lambda_2}\right)$$

From the former equations, one could define the following algorithm to solve ambiguities in real time:

- The filter starts estimating ambiguities $\Delta\nabla\widehat{Bc}$ (as real numbers—floating them—) together with coordinates (x,y,z)[94].
- From the estimations of $\Delta\nabla\widehat{Bc}$, the "exact" values of $\Delta\nabla Bc$ are calculated using the previous equations. The ambiguities $\Delta\nabla Bc$ will not be considered as solved until they overcome a statistical test of null hypothesis.
- The solved ambiguities $\Delta\nabla Bc$ will be assimilated in the Kalman filter, FIXING their values until a new cycle-slip is produced.
- Every solved ambiguity (to its exact value) is assimilated for the filter (i.e., fixed) assuming a less parameter to estimate, therefore it decreases correlations and gives more strength to the navigation solution.

Besides the reduction of the navigation error, another advantage of the methods used to solve ambiguity resolution in real time is the convergence velocity of the solution. In a few minutes, several ambiguities can be assimilated thereby decreasing the solution error in an important way.

[94] Orbital parameters could be adjusted too, estimating the tropospheric delay, ...

Solving ambiguities in a hundred kilometers scale

For distances superior than 10-20 Km, the hypothesis that the ionospheric refraction is canceled when forming double differences is not valid any more, the term $\Delta\nabla STEC$ must be added in the previous two first equations:

$$\Delta\nabla N_W = nint\left[\frac{\Delta\nabla L_W - \Delta\nabla Lc - 1.98\Delta\nabla \widetilde{STEC} + \lambda_c \Delta\nabla \widehat{Bc}}{\lambda_W}\right]$$

$$\Delta\nabla N_1 = nint\left[\frac{\Delta\nabla L_1 - \Delta\nabla L_2 - \Delta\nabla \widetilde{STEC} - \lambda_2 \Delta\nabla N_W}{\lambda_1 - \lambda_2}\right]$$

Solving ambiguities On-The-Fly (OTF) for long baselines is a subject of recent investigation. In Colombo et al. (2000) the proof of subdecimetric navigation concept is presented with hundreds of kilometers from the closer reference station and under high geomagnetic activity conditions.

One of the keys is being able to provide to the rover with very accurate ionospheric corrections so that it can predict the $\Delta\nabla STEC$ with a lower error[95], a $1/4\,TECU \simeq \lambda_I/2$. These corrections can be calculated through an elaborated process having a tomographic model in its core of the ionosphere in real time that processes the observations collected (continuously) by a net of permanent stations (Hernández-Pajares et al., 1999). In an intuitive way, one could say that the GPS satellite constellation and the Earth receivers are used as a huge scanner of planetarium scale. This technique is nowadays in a validation phase. It has been applied successfully in WADGPS scale (some hundreds of kilometers) and high ionospheric variability (Hernández-Pajares et al., 2000, 2001).

Comments:
Why is the ionospheric refraction $\Delta\nabla STEC$ needed, when working with ionospheric-free combination $\Delta\nabla Lc$?

The combination $\Delta\nabla Lc$ allow us to cancel ionospheric refraction p to 99.9%, remaining range measurements free of this perturbation. So, for positioning, "floating" the ambiguities $\Delta\nabla Bc$ no ionospheric model is needed.

In the case of the solving ambiguities, no $\Delta\nabla STEC$ is required to correct the range LC (because it is free from it), but to solve integer ambiguities $\Delta\nabla N_1$ and $\Delta\nabla N_2$ and, from them on, the real ambiguity $\Delta\nabla Bc$ is "exactly" solved.

[95]Note that the phase measurements $\Delta\nabla L_1$ and $\Delta\nabla L_2$ have a noise of some millimeters, thus, being $\Delta\nabla N_W$ an exact value, the limiting factor is the error in the ionospheric term $\Delta\nabla STEC$, that must be lower than $\lambda_I/2 = (\lambda_2 - \lambda_1)/2 \simeq 2.7cm$.

Session 7a
Differential positioning with code.

Objectives

It is to study differential positioning with code in single and double differences. Let us emphasize that the "manual" calculation procedure is made in order to attempt the strategy for session 7b in which code and phase will be used.

Files to use
`99mar23bell_ebre.s.gz, kalman.nml_D_WN, kalman.nml_DD_WN, sta.pos, 99mar23bell.eph, 99mar23ebre.eph,99mar23bell.a.pos, 99mar23ebre.a.pos`

Programs to use
`GCAT, kalman0, Dbell_ebre.scr, DDbell_ebre21.scr`

Development

1. Copy programs and files of the session in the working directory.

2. [**Differential positioning : Single Differences**].
The file `99mar23bell_ebre.s.gz` contains observations every 30 seconds, registered by two receivers (bell, ebre)[96] located at about 100Km of distance. Data are displayed according to the following fields[97]:

   `[sta doy sec PRN LC LI PC PI arc]`

   Use the program **GCAT** to generate the design matrix[98] (with broadcast orbits) for every receiver. Next, calculate single differences of the "prefit-residuals" of the station `bell` referring to `ebre`. Finally, follow the scheme developed in chapter 7 of theory, pose and solve navigation equations with this single differences.

   To do so, the below steps are proposed to be followed:

[96]With a view to form single or double differences between observations of both stations, the only ones selected have been the ones corresponding to the satellites registered by both receivers at the same time.

[97]Note that, in contrast to the files used in the previous sessions in the fifth and seventh fields of the file `99mar23bell_ebre.s.gz`, the observables LC and PC are provided, (i.e, ionospheric-free combinations), instead of L1 and P1.

[98]That is to say, generate the files "`*.dmx`" with the prefit-residuals and partial derivatives (see page 119) for each receiver: $\boxed{\texttt{[sta sec PRN "C" prefit } \partial R/\partial x\ \partial R/\partial y\ \partial R/\partial z\ \partial R/\partial t \texttt{ elev iarc]}}$.

(a) From the file `99mar23bell_ebre.s.gz`, generate the data files `99mar23bell.a` and `99mar23ebre.a`, selecting the corresponding observations of every station separately (`bell` and `ebre`). These files, together with the ones of the orbits[99] (`99mar23bell.eph`, `99mar23ebre.eph`), would constitute the INPUT of the program **GCAT**.

Execute:

```
Select observations for every receiver
zgrep bell 99mar23bell_ebre.s.gz > 99mar23bell.a
zgrep ebre 99mar23bell_ebre.s.gz > 99mar23ebre.a
```

(b) Using the program **GCAT**, calculate the design matrix for each station separately *(we will also make use of this to determine the kinematic position of every receiver)*.
To do so, the following options must be selected (leaving the rest as default values):

- File `[MODEL]`
  `[Ionospheric refraction = NO]`, since we work with ionospheric-free combination PC.
  `[Satellite coordinates at emission:  Using the PR]`, because pseudorange-based algorithm is used.
- File `[FILTER]`
  `[Kinematic Positioning] [White noise]`
- File `[RESULTS]`
  `[Write Design matrix]`: to write the design matrix (with the prefit residuals and partial derivatives for the processed station).

Finally, press **File**, with the options previously indicated, select the file `99mar23bell.a` and press **Go**[100]. Repeat the same for the file `99mar23ebre.a`.
With all this, the files `99mar23bell.a.dmx` and `99mar23ebre.a.dmx` will be generated, containing the below data:

[99] They are the broadcast orbits provided by the navigation message. Both files are identical to the `auto0820.99n` obtained from the server `ftp://lox.ucsd.edu/pub/rinex/99data/082/auto0820.99n.Z`.

[100] Note that **GCAT** processes the observations corresponding to the seventh column of the data file `*.a` and does not use the values in the fifth, sixth or eighth columns . According to the format previously indicated the files `99mar23bell.a` or `99mar23ebre.a` are contained in their seventh column code measurements `PC`, and therefore, these will be the ones to process the default.

```
sta sec PRN "C" prefit ∂R/∂x ∂R/∂y ∂R/∂z ∂R/∂t elev iarc
```

Note: `iarc` indicates the number of phase arc. It is useful to identify the epochs in which the cycle-slips in the phase are produced. As we are working with code measurements, we can ignore this field. "The character C" is fixed.

The files have also been generated: `99mar23bell.a.pos` and `99mar23ebre.a.pos`, with coordinate estimations of every receiver in kinematic mode (remember that the options `[Kinematic Positioning] [White noise]` have been selected).

(c) Calculate single differences of `bell` station prefit-residuals respect to `ebre` ($[\Delta\texttt{prefit}]_{bell,ebre} = \texttt{prefit}_{bell} - \texttt{prefit}_{ebre}$) and generate a file with the following fields (INPUT of the program `kalman0`):

```
type sec PRN [Δprefit]_bell,ebre 10 [∂R/∂x]_bell [∂R/∂y]_bell [∂R/∂z]_bell [∂R/∂t]_bell iarc
```

where partial derivatives $[\partial R/\partial x]_{bell}$, $[\partial R/\partial y]_{bell}$, $[\partial R/\partial z]_{bell}$ and $[\partial R/\partial t]_{bell}$ are from station `bell`, and the value "10 (meters)" is the noise of the observations (σ_{obs}) adopted for the code "PC".

Execute[101]:

```
cat 99mar23ebre.a.dmx 99mar23bell.a.dmx |
   gawk '{if ($1=="ebre") {r[$2 $3]=$5}
    else {if (length(r[$2 $3])!=0) printf "%s %6i %02i %14.6f
    %6.3f %14.9f %14.9f %14.9f %14.9f %3i \n",
    "PC",$2,$3,$5-r[$2 $3],10,$6,$7,$8,$9,$11}}'>Dbell_ebre.mod
```

(d) According to the scheme defined in section 7.1 of the theory (page 142), write the navigation equation system for this problem in single differences.

(e) Calculate the navigation solution using Kalman filter implemented in the program `kalman0`[102]. Model receiver coordinates and clock as "white-noise" (pure kinematic positioning), establishing the next parameters in the name list `kalman.nml`[103]:

[101]This instructions are in the script `Dbell_ebre.scr`. Therefore, it is enough to execute: `Dbell_ebre.scr`.

[102]See description of `kalman0` in the header of the code `textedit kalman0.f`.

[103]These values have been saved in the file `kalman.nml_D_WN`.

```
Pxx=1.d+8 m2      fi_x=0.d0     Qxx=1.d+8 m2
Pyy=1.d+8 m2      fi_y=0.d0     Qyy=1.d+8 m2
Pzz=1.d+8 m2      fi_z=0.d0     Qzz=1.d+8 m2
Ptt=9.d+16m2      fi_t=0.d0     Qtt=9.d+16m2
```

Execute:

```
cp kalman.nml_D_WN kalman.nml
cat Dbell_ebre.mod | kalman0 > Dbell_ebre.pos
```

The obtained file `Dbell_ebre.pos` contains the following fields (see the header of the program `kalman0`):

```
sec x y z t
```

where `x, y, z` are the deviation of the estimations referring to the nominal value adopted (a priori corresponding to the file `sta.pos`[104]) (WGS'84) and `t` is the receiver clock offset of the station `bell` relative to the one in the station `ebre` (note that we are using the observable differences between `bell` and `ebre` as data).

(f) Graphically represent the obtained values for `x,y,z`, when kinematically positioning the station `bell` relative to the station `ebre` (its coordinates have been assumed to be known and fixed).

Execute:

```
gnuplot
set yrange[-20:20]
plot "Dbell_ebre.pos" u 2:3, "Dbell_ebre.pos" u 2:4,
     "Dbell_ebre.pos" u 2:5
set yrange[-200:200]
replot
exit
```

(g) Compare the differential estimations obtained in the former section (`Dbell_ebre.pos`), with the absolute estimations of the files `99mar23bell.a.pos` and `99mar23ebre.a.pos`, obtained in the section (b) when processing every station separately with **GCAT**.

[104] As in previous sessions, values contained in the file `sta.pos` define the true receiver coordinates. Therefore, values `x,y,z` are directly the positioning error (or discrepancy referring to the true value).

Execute:

```
gnuplot
set yrange[-200:200]
plot "99mar23bell.a.pos" u 2:3,"99mar23bell.a.pos" u 2:4,
     "99mar23bell.a.pos" u 2:5
plot "99mar23ebre.a.pos" u 2:3,"99mar23ebre.a.pos" u 2:4,
     "99mar23ebre.a.pos" u 2:5
exit
```

i. Was the S/A on at the epoch these observations were registered (23rd of May of 1999)?
ii. What error could one expect in the absolute positioning of every receiver?
iii. Why has the positioning error in differential mode been reduced so notably?

(h) Calculate the difference of individual estimations contained in the files `99mar23bell.a.pos` and `99mar23ebre.a.pos` and compare them with the values in the file `Dbell_ebre.pos`.
Execute:

```
cat 99mar23ebre.a.pos 99mar23bell.a.pos|
    gawk '{if (length(x[$1])!=0)
    {print $1,$2-x[$1],$3-y[$1],$4-z[$1],$5-t[$1]}
    else {x[$1]=$2;y[$1]=$3;z[$1]=$4;t[$1]=$5}}' > dif.pos

gnuplot
set yrange[-20:20]
plot "Dbell_ebre.pos" u 2:3,"dif.pos" u 2:3
plot "Dbell_ebre.pos" u 2:4,"dif.pos" u 2:4
plot "Dbell_ebre.pos" u 2:5,"dif.pos" u 2:5
plot "Dbell_ebre.pos" u 2:6,"dif.pos" u 2:6
exit
```

i. With this result in sight, justify "intuitively" why the error decreases when positioning in differential mode.
ii. Should the values of the files `dif.pos` and `Dbell_ebre.pos` match "exactly"?

3. [**Differential positioning: Double Differences**].

   The same as in the previous exercise, use the program **GCAT** to calculate the design matrix (with broadcast orbits) for every receiver. Next, calculate double differences of the "prefit-residuals" and partial derivatives, with the aim of pose and solve navigation equations (in double differenced mode). Take `"ebre"` as reference station and `PRN21` as reference satellite.

   To do so, one can take the following steps:

   (a) From the file, `99mar23bell_ebre.s.gz`, and selecting corresponding observation to the stations `bell` and `ebre`, generate the files `99mar23bell.a` and `99mar23ebre.a` for them to be processed by **GCAT**.

   ```
   zgrep bell 99mar23bell_ebre.s.gz > 99mar23bell.a
   zgrep ebre 99mar23bell_ebre.s.gz > 99mar23ebre.a
   ```

   (b) Using the program **GCAT**, calculate the design matrix for every station separately.

   To do so, the same options as in the former exercise have to be selected:

   - File `[MODEL]`
     `[Ionospheric refraction = NO]`, because we are working with ionospheric-free combination PC.
     `[Satellite coordinates at emission: Using the PR]`, because pseudorange-based algorithm is used.
   - File `[FILTER]`
     `[Kinematic Positioning] [White noise]`
   - File `[RESULTS]`
     `[Write Design matrix]`: in order to write the design matrix containing the prefit-residuals and partial derivatives for the processed station.

   Finally, press **File**, with the options previously indicated, select the file `99mar23bell.a` and press **Go**. Repeat the same for the file `99mar23ebre.a`.
   The following files will have been generated: `99mar23bell.a.dmx` and `99mar23ebre.a.dmx`

(c) Taking as a reference the station `ebre` and the satellite `PRN21`, calculate double differences of the next fields [`prefit` $\partial R/\partial x$ $\partial R/\partial y$ $\partial R/\partial z$] between both files and generate a new file with the corresponding format for the input of the program `kalman0`. To do so, use the script `DDbell_ebre21.scr`[105]:

Execute:

```
DDbell_ebre21.scr PC 99mar23bell.a.dmx 99mar23ebre.a.dmx
```

As a result, the file `DDbell_ebre21_PC.mod` will be generated containing the following fields[106]:

type sec PRN $\nabla\Delta$prefit 10 $\nabla\partial R/\partial x$ $\nabla\partial R/\partial y$ $\nabla\partial R/\partial z$ $\nabla\partial R/\partial t$ iarc

(d) According to the scheme defined in the section 7.2 of the theory (page 143), write the navigation equation system for this problem in double differences.

(e) Calculate the navigation solution through the Kalman filter implemented in the program `kalman0`. Modelate coordinates as "white-noise" (pure kinematic positioning) and "fix the receiver clock" according to the next parameters[107]:

```
Pxx=1.d+8 m2      fi_x=0.d0     Qxx=1.d+8 m2
Pyy=1.d+8 m2      fi_y=0.d0     Qyy=1.d+8 m2
Pzz=1.d+8 m2      fi_z=0.d0     Qzz=1.d+8 m2
Ptt=9.d-16m2      fi_t=0.d0     Qtt=9.d-16m2
```

Why must the receiver clock be fixed in the `kalman0`?

Execute:

```
cp kalman.nml_DD_WN kalman.nml
cat DDbell_ebre21_PC.mod | kalman0 > DDbell_ebre21.pos0
```

The obtained file `DDbell_ebre21.pos0` contains the following fields (see header of program `kalman0`) `sec x y z t`, where `x,y,z` are the deviations of the estimations referring to the nominal value adopted (a priori corresponding to the file `sta.pos`) (WGS'84) and `t` is the receiver clock offset (that has been fixed to zero).

[105] For further details about this double differences calculation, edit and examine this script `textedit DDbell_ebre21.scr`.

[106] Notation: $\nabla\Delta\diamondsuit = (\diamondsuit^j_{bell} - \diamondsuit^{21}_{bell}) - (\diamondsuit^j_{ebre} - \diamondsuit^{21}_{ebre})$.

[107] These values have been saved in the file `Kalman.nml_DD_WN`.

(f) Graphically represent the obtained values for `x,y,z` when kinematically positioning (in double differenced mode) the station `bell` relative to `ebre`.
Execute:

```
gnuplot
set yrange[-20:20]
plot "DDbell_ebre21.pos0" u 2:3
plot "DDbell_ebre21.pos0" u 2:4
plot "DDbell_ebre21.pos0" u 2:5
exit
```

(g) Compare the obtained estimations `DDbell_ebre21.pos0` with the ones in the former exercise `Dbell_ebre.pos`.
Execute:

```
gnuplot
set yrange[-20:20]
plot "DDbell_ebre21.pos0" u 2:3,"Dbell_ebre.pos" u 2:3
plot "DDbell_ebre21.pos0" u 2:4,"Dbell_ebre.pos" u 2:4
plot "DDbell_ebre21.pos0" u 2:5,"Dbell_ebre.pos" u 2:5
exit
```

Should these estimations match?

(h) Graphically represent the receiver clock estimations of `bell` and `ebre` contained in files `99mar23bell.a.pos` and `99mar23ebre.a.pos`, obtained previously when processing every station separately with **GCAT** in section (b).

Execute:

```
gnuplot
set auto
plot "99mar23bell.a.pos" u 1:5,
     "99mar23ebre.a.pos" u 1:5
exit
```

May the fact that receiver clocks of `bell` and `ebre` are not synchronized referring to the GPS time scale (with an offset up to 1 millisecond in the case of `ebre`) affect positioning when working in double differences? (see exercise 2 of session 7b). In which way?

4. Repeat the former exercise, but using geometric algorithm (i.e., activate option `[Satellite coordinates at emission: Geometric]` in GCAT) for satellite coordinate computation at emission epoch (note that an equation system on double differences is assumed, as in theory section 7.2, page 143.). Rename the resulting file as `DDbell_ebre21.pos1`.

5. Use the offset estimations of receiver clocks `bell` and `ebre`, obtained in the previous exercise, to correct time marks[108] of the files `99mar23bell.a` and `99mar23ebre.a`.

   To do so, the following procedure is proposed:

   i. Generate the original files again:

```
zgrep bell 99mar23bell_ebre.s.gz > 99mar23bell.a
zgrep ebre 99mar23bell_ebre.s.gz > 99mar23ebre.a
```

   ii. Generate files with the clock offset values of the receivers `bell` and `ebre`. To do so, the clock estimations of the files `99mar23bell.a.pos` and `99mar23ebre.a.pos` can be used which were obtained in the former exercise[109].

```
cat 99mar23bell.pos|gawk '{print $1,$5}' > bell_clock
cat 99mar23ebre.pos|gawk '{print $1,$5}' > ebre_clock
```

   iii. Adjust time marks of the registered epochs in the observation files `99mar23bell.a` and `99mar23ebre.a` to express them in GPS time scale.

```
cat bell_clock 99mar23bell.a | gawk '{if (NF==2){s[$1*1]=$2}
  else {if (length(s[$3*1])!=0) {a=$3-s[$3*1]/3e8;
  printf "%s %s %14.8f %s %14.10f %14.10f %14.10f %14.10f %s \n",
  $1,$2,a,$4,$5,$6,$7,$8,$9}}}' > nada
mv nada 99mar23bell.a

cat ebre_clock 99mar23ebre.a | gawk '{if (NF==2){s[$1*1]=$2}
  else {if (length(s[$3*1])!=0) {a=$3-s[$3*1]/3e8;
  printf "%s %s %14.8f %s %14.10f %14.10f %14.10f %14.10f %s \n",
  $1,$2,a,$4,$5,$6,$7,$8,$9}}}' > nada
mv nada 99mar23ebre.a
```

[108] Note that every receiver registers observation epochs (time marks) according to its internal clock, which can present an important offset (up to a 1 millisecond) referred to GPS time scale.

[109] These files are also available in the file directory.

iv. Repeat the same steps as in the previous exercise (4) in order to determine `bell` station position relative to `ebre` in double differenced mode, using the new files `99mar23bell.a` and `99mar23bell.a` whose observation epochs (time marks) have already been adjusted to GPS time scale. Use option `[Satellite coordinates at emission: Geometric]` in GCAT.

Answer the following questions:

(a) Graphically represent deviations `x, y, z` and compare them with the ones in the previous exercise (4.) (`DDbell_ebre21.pos0`) (remember that in this exercise geometric method was used to compute signal emission epoch).
Execute:

```
gnuplot
set yrange[-20:20]
plot "DDbell_ebre21.pos" u 2:3,"DDbell_ebre21.pos0" u 2:3
plot "DDbell_ebre21.pos" u 2:4,"DDbell_ebre21.pos0" u 2:4
plot "DDbell_ebre21.pos" u 2:5,"DDbell_ebre21.pos0" u 2:5
exit
```

Do these results match with those of exercise 4?

i. At which time interval is the maximum time discrepancy observed between estimations contained in the files `DDbell_ebre21.pos` and `DDbell_ebre21.pos0`? What is the order of these discrepancies?

ii. Why do the estimations of `DDbell_ebre21.pos` and `DDbell_ebre21.pos0` match again starting from time $t \simeq 63000$?

iii. At which instant the readjustment of 1 millisecond is done in the receiver clock `ebre`?

iv. Would this problem affect positioning with single differences?

(b) Represent graphically deviations `x, y, z` and compare with exercise (3.) (`DDbell_ebre21.pos0`) (remember that in that exercise pseudorange method was used to compute signal emission epoch).
Execute:

```
gnuplot
set yrange[-20:20]
plot "DDbell_ebre21.pos" u 2:3,"DDbell_ebre21.pos0" u 2:3
plot "DDbell_ebre21.pos" u 2:4,"DDbell_ebre21.pos0" u 2:4
plot "DDbell_ebre21.pos" u 2:5,"DDbell_ebre21.pos0" u 2:5
exit
```

Do these results match those of exercise 3? Why?

Answers Session 7a Differential positioning
2.d
2.g
2.h
3.e
3.g
3.h
4.g
4.h
5.a
5.b

Session 7b
Differential positioning with code and phase.

Objectives

It is to study differential positioning with code and phase solving an equation system with the double differentiated observables. Study the exact resolution of the ambiguities and compare the solutions obtained by "floating" and "fixing" ambiguities.

Files to use
`99mar23bell_ebre.s.gz`, `DDbell_ebre21.ion`, `DDbell_ebre21.bc`, `sta.pos`, `99mar23bell.eph`, `99mar23ebre.eph`, `99mar23bell.sp3`, `99mar23ebre.sp3`, `DDbell_ebre21_eci.mod`, `DDbell_ebre21_eph.mod`, `99mar23bell.a_PC.pos`, `99mar23ebre.a_PC.pos`, `kalman.nml_DD_WN`

Programs to use
`GCAT`, `kalman`, `ambisolv`, `DDbell_ebre21.scr`, `DDobs.scr`, `add.scr`

Development

1. Copy the programs and files for the session in the working directory.

2. [**Differential positioning with LC PC in double differences**]
As we have already indicated in the previous session, `99mar23bell_ebre.s.gz` contains observations every 30 seconds registered by two receivers (bell, ebre) located at a distance about 100Km. Data are displayed in the following fields: `[sta doy sec PRN LC LI PC PI arc]`.

   Use the program **GCAT** to generate the design matrix (with broadcast orbits) for the PC code measurements of every receiver. Repeat the same procedure for LC phase measurements. Next, calculate double differences of the "prefit-residuals" and partial derivatives with the aim of posing and solving navigation equations in double differenced mode for code and phase. Take `"ebre"` as reference station and `PRN21` as reference satellite.

   To do so, the next steps are proposed to be as follow:

◇ *Generation of the design matrix for PC code measurements:*
We will proceed identically as in exercise 2 of session 7a.

(a) From the file, `99mar23bell_ebre.s.gz`, and selecting the observations corresponding to stations `bell` and `ebre`, generate files `99mar23bell.a` and `99mar23ebre.a` for them to be processed by **GCAT**.

```
zgrep bell 99mar23bell_ebre.s.gz > 99mar23bell.a
zgrep ebre 99mar23bell_ebre.s.gz > 99mar23ebre.a
```

(b) Using the program **GCAT** calculate the design matrix for every station separately.

To do so, the same options as in session 7a must be selected:

- File `[MODEL]`
  `[Ionospheric refraction = NO]`, since we are working with ionosphere-free combination PC.
  `[Satellite coordinates at emission: Using the PR]`, because pseudorange-based algorithm is used.
- File `[FILTER]`
  `[Kinematic Positioning] [White noise]`
- File `[RESULTS]`
  `[Write Design matrix]`: to write the design matrix containing the prefit-residuals and partial derivatives for the processed station.

Finally, press **File**, with the options previously indicated, select the file `99mar23bell.a` and press **Go**[110]. Repeat the same for the file `99mar23ebre.a`.
The following files[111] will be generated:
`99mar23bell.a.dmx` and `99mar23ebre.a.dmx`, that will be renamed as:

[110]Note that **GCAT** processes the observations corresponding to the seventh column of the data file `*.a` and does not use the values of fifth, sixth or eighth columns. According to the format previously indicated the files `99mar23bell.a` and `99mar23ebre.a` contain in their seventh column code measurements `PC`, and therefore, these would be the ones that process default.

[111]Their format is: `sta sec PRN "C" prefit` $\partial R/\partial x$ $\partial R/\partial y$ $\partial R/\partial z$ $\partial R/\partial t$ `elev iarc`.
Note: `iarc` indicates the number of phase arc. It is used for identifying the instants when cycle-slips in phase are produced. If we are working solely with code measurements, we could omit from this field. "The character C" is fixed.

```
mv 99mar23bell.a.dmx 99mar23bell.a_PC.dmx
mv 99mar23ebre.a.dmx 99mar23ebre.a_PC.dmx
```

These files are also generated: `99mar23bell.a.pos` and `99mar23ebre.a.pos`, that are stored with the following names for latter exercises:

Execute:

```
mv 99mar23bell.a.pos 99mar23bell.a_PC.pos
mv 99mar23ebre.a.pos 99mar23ebre.a_PC.pos
```

◇ *Design matrix generation for code measurements LC*[112]:

As we have already quoted, the program **GCAT** processes default the observations corresponding to the seventh column of the data file `*.a`, and does not use values in the fifth, sixth or eighth columns. In order to process the phase measurements "LC" (which are placed in the fifth column of the files `99mar23bell.a`, `99mar23 ebre.a`), the contents of the seventh column (PC) should be swapped by the fifth (LC) and repeat the same steps as in the former section (b). That is to say, execute:

```
cat 99mar23bell.a |gawk '{$7=$5; print $0}'> nada
mv nada 99mar23bell.a
cat 99mar23ebre.a |gawk '{$7=$5; print $0}'> nada
mv nada 99mar23ebre.a
```

Next repeat the same operations as in
section (b), over this new files.
Once **GCAT** has been executed, rename the files:

```
mv 99mar23bell.a.dmx 99mar23bell.a_LC.dmx
mv 99mar23ebre.a.dmx 99mar23ebre.a_LC.dmx
```

(c) Taking as reference the station `ebre` and the satellite `PRN21`, calculate double differences of the following fields `[prefit` $\partial R/\partial x$ $\partial R/\partial y$ $\partial R/\partial z$`]` between both files and generate a new file with the corresponding format for the input of the program `kalman`[113]. To do so, use the script `DDbell_ebre21.scr`:

[112] The program **GCAT** has been designed to only process the code measurements and therefore, does not incorporate the correction due to the *wind-up* (which affects only the phase). Nevertheless, as they are receivers at 100Km, most part of this correction is canceled when forming differences between them.

[113] `kalman` is similar to `kalman0` except for the fact that it is prepared to estimate the bias of

Execute:

```
DDbell_ebre21.scr PC 99mar23bell.a_PC.dmx 99mar23ebre.a_PC.dmx
DDbell_ebre21.scr LC 99mar23bell.a_LC.dmx 99mar23ebre.a_LC.dmx
```

The following two files `DDbell_ebre21_PC.mod`, `DDbell_ebre21_LC.mod`, will be generated, containing the following fields[114]:

type sec PRN $\nabla\Delta$prefit σ_{obs} $\nabla\partial R/\partial x$ $\nabla\partial R/\partial y$ $\nabla\partial R/\partial z$ $\nabla\partial R/\partial t$ iarc

(Note: The value of `iarc` indicates continuous arcs of phase, changing every time a cycle-slip[115] is produced. The adopted value for σ_{obs} is 10m for the "PC" code measurements and 0.01m for the "LC" phase)

(d) Join the two former files in one single file and put them in the order of time.
Execute:

```
cat DDbell_ebre21_LC.mod DDbell_ebre21_PC.mod|
                          sort -n +1 +2 > DDbell_ebre21.mod
```

Rename the file `DDbell_ebre21.mod` obtained in order to using it late:

```
cp DDbell_ebre21.mod DDbell_ebre21_eph.mod
```

(e) According to the scheme defined in section 7.3 of the theory (page 143), write the navigation equation system, with code and phase, for the problem in double differences.

(f) Calculate the navigation solution with LC PC, using the Kalman filter implemented in the program `kalman`. Modelate the coordinates as "white-noise" (pure kinematic positioning) and "fix the receiver clock" according to the following parameters[116]:

```
Pxx=1.d+8 m2     fi_x=0.d0     Qxx=1.d+8 m2
Pyy=1.d+8 m2     fi_y=0.d0     Qyy=1.d+8 m2
Pzz=1.d+8 m2     fi_z=0.d0     Qzz=1.d+8 m2
Ptt=9.d-16m2     fi_t=0.d0     Qtt=9.d-16m2
```

the phase arcs (see the header of code: textedit `kalman.f`). These are modeled as constants along continuous arcs and white-noise (with $\sigma^2 = 9\ 10^{16} m^2$) at the instants when the cycle-slips are produced. The namelist is the same as in `kalman0`.

[114] Notation: $\nabla\Delta\diamondsuit = (\diamondsuit^j_{bell} - \diamondsuit^{21}_{bell}) - (\diamondsuit^j_{ebre} - \diamondsuit^{21}_{ebre})$.

[115] For `iarc` values have been added, in stead of forming double differences.

[116] These values have been saved in the file `kalman.nml_DD_WN`.

Execute:

```
cp kalman.nml_DD_WN kalman.nml
cat DDbell_ebre21.mod | kalman | grep X > DDbell_ebre21.pos
```

The obtained file `DDbell_ebre21.pos` contains the following fields (see the header of the program `kalman`): `sec x y z t`
where `x,y,z` are the deviations of the estimations referring to the adopted nominal values (a priori corresponding to the file `sta.pos`) (WGS'84) and `t` is the receiver clock offset (in this case, it has been fixed to zero, since it does not have to be estimated).

(g) Graphically represent the obtained values for `x,y,z` when kinematically positioning, with code and phase, and in double differenced mode, the estimation `bell` relative to `ebre`.
Execute:

```
gnuplot
plot "DDbell_ebre21.pos" u 2:3, "DDbell_ebre21.pos" u 2:4,
     "DDbell_ebre21.pos" u 2:5
exit
```

i. Are the obtained results reasonable?
ii. Do you observe a jump in $(\Delta x, \Delta y, \Delta z)$ estimations at the end of analized data? Did you observed it when only-with-code positioning was done?

(h) Graphically represent estimations of receivers clocks belonging to `bell` and `ebre`, contained in the already created files `99mar23bell.a_PC.pos` and `99mar23ebre.a_PC.pos` obtained before, when processing every station individually with **GCAT**.

Execute:

```
gnuplot
plot "99mar23bell.a_PC.pos" u 1:5, "99mar23ebre.a_PC.pos" u 1:5
exit
```

Answer only if you observe any leap in $(\Delta x, \Delta y, \Delta z)$ estimations toward the end of analyzed data set:

i. Does any relation exist between the leap observed in the navigation solution and the clock offsets of `bell` and `ebre`?
ii. How could one explain the fact that the receiver clocks of `bell` and `ebre` "keep on showing" in spite of having "been canceled" when forming double differences?
iii. How could the positioning result be improved?

3. Repeat the former exercise, but using geometric algorithm to compute satellite coordinates at emission epoch (note that the navigation equations system on double differences shown in theory section 7.3, page 146, is assumed.)

4. Following the same procedure as in the previous exercise 5 of session 7a, use the estimations of the receiver clocks offsets `bell` and `ebre` obtained in the former exercise to correct time marks of the files `99mar23bell.a` and `99mar23ebre.a`.

   To do so, the following procedure is proposed:

   i. Generate the original files again:

```
zgrep bell 99mar23bell_ebre.s.gz > 99mar23bell.a
zgrep ebre 99mar23bell_ebre.s.gz > 99mar23ebre.a
```

   ii. Generate some files with the values of the receiver clock offsets `bell` and `ebre`. To do that, use the clock estimations of the files `99mar23bell.a_PC.pos` and `99mar23ebre.a_PC.pos` obtained in the previous exercise[117].

```
cat 99mar23bell.a_PC.pos|gawk '{print $1,$5}' > bell_clock
cat 99mar23ebre.a_PC.pos|gawk '{print $1,$5}' > ebre_clock
```

   iii. Correct time marks of the epochs registered in the files to make them be expressed in GPS time scale. For such purpose, use option `[Satellite coordinates at emission: Geometric]` in **GCAT**.

```
cat bell_clock 99mar23bell.a | gawk '{if (NF==2){s[$1*1]=$2}
  else {if (length(s[$3*1])!=0) {a=$3-s[$3*1]/3e8;
  printf "%s %s %14.8f %s %14.10f %14.10f %14.10f %14.10f %s \n",
  $1,$2,a,$4,$5,$6,$7,$8,$9}}}' > nada
mv nada 99mar23bell.a

cat ebre_clock 99mar23ebre.a | gawk '{if (NF==2){s[$1*1]=$2}
  else {if (length(s[$3*1])!=0) {a=$3-s[$3*1]/3e8;
  printf "%s %s %14.8f %s %14.10f %14.10f %14.10f %14.10f %s \n",
  $1,$2,a,$4,$5,$6,$7,$8,$9}}}' > nada
mv nada 99mar23ebre.a
```

   iv. Repeat the same steps in the former exercise (2) for positioning, in double differenced mode (with code and phase), the station `bell` relative to `ebre` with these new files `99mar23bell.a` and `99mar23bell.a`, whose

[117] These files are also available in the "`file`" directory.

observation epochs have already been adjusted to GPS time scale.

Note: For this item, use the following option in **GCAT**:
"`Satellite coordinates at emission: Geometric`"

Answer the following questions:

(a) Graphically represent the deviations `x,y,z`.
Execute:

```
gnuplot
plot "DDbell_ebre21.pos" u 2:3, "DDbell_ebre21.pos" u 2:4
     "DDbell_ebre21.pos" u 2:5
exit
```

(b) What is the order of the obtained errors?

5. Repeat the preceding exercise (2.) using precise orbits[118]. For that, the same steps must be followed, and take the same options for the program **GCAT** except from the file `MODEL` where you should select:

- File `[MODEL]`
  `[precise Orbits and clocks]`

  `[Satellite Clock interpolation]` in order to make it to continue calculating the observations even if satellite clocks are not available (in file sp3.tmp, they are given every five minutes, and the observations are every 30 seconds). Notice that these clock values will not be used, because they are canceled when working with double differences.

  `[Ionospheric refraction = NO]`, because we are working with ionosphere-free combination PC or LC.

  `[Satellite coordinates at emission: Using the PR]`, because pseudorange-based algorithm is used.

- Rename resulting file `DDbell_ebre21.mod`
  to be used later:
  `cp DDbell_ebre21.mod DDbell_ebre21_eci.mod`

(a) What is the order of the obtained discrepancies in `x,y,z`?

[118]The program **GCAT** uses the precise orbit files `99mar23bell.sp3` and 99mar23ebre.sp3, that must be available in the working directory.

(b) Compare these results with the ones obtained in the preceding exercise using broadcast orbits. Why are the results so similar?

6. [**"Fixing" versus "Floating" ambiguities**]

In this exercise, the method described in (Colombo et al., 1999) will be implemented in a simplified way to obtain the "exact value" of the phase ambiguities $\nabla\Delta LC$ for two stations separated about 100km. With repaired ambiguities (**fixed** to their exact values), the navigation solution will be calculated and will be compared with the one obtained estimating ambiguities as real numbers (i.e., **floating them**). As they are two very distant stations, the ionospheric refraction can not be assumed to be null between them, having the prediction of $\nabla\Delta STEC$ to be arranged. This can be obtained through a precise model of the ionosphere from a permanent station net (Hernández-Pajares et al., 2000).

- The file `DDbell_ebre21.obs` contains double differences of the observations of the station `bell` relative to the station `ebre` and to the satellite `PRN21`, according to the following format[119]

  [`bell` **ebre 21** `PRN iarc sec` $\nabla\Delta LC$ $\nabla\Delta LW$ $\nabla\Delta LI$]

- File `DDbell_ebre21_eci.mod` is identical to the one obtained previously in exercise 5 using precise orbits[120]. Its contents, as it has been already indicated before, is organized in the following fields[121]:

type sec PRN $\nabla\Delta$prefit σ_{obs} $\nabla\partial R/\partial x$ $\nabla\partial R/\partial y$ $\nabla\partial R/\partial z$ $\nabla\partial R/\partial t$ iarc

[119]This file has been obtained applying the script `DDobs.scr` to the `99mar23bell_ebre.s.gz` that contains the observations (without differentiating) of every station. It is enough to execute $\boxed{\text{DDobs.scr 99mar23bell\_ebre.s.gz}}$. See source code of this script for further details. Notation: $\nabla\Delta\diamondsuit = (\diamondsuit^j_{bell} - \diamondsuit^{21}_{bell}) - (\diamondsuit^j_{ebre} - \diamondsuit^{21}_{ebre})$.

[120]One has to emphasize that *wind-up* correction for the phase has not been introduced, neither *solid tides* (see GIPSY OASIS-II) for the receivers `bell` and `ebre`. Nevertheless, being stations at about 100Km, a great part of these errors will be canceled, not affecting the result too much. Also, the difference of antenna phase centers for the receivers of `bell` and `ebre` has been not taken into account, amounting to about 4 centimeters between them. In the same way, the considered tropospheric model is very elementary (see page 80).

[121]For format sake, "sec" time have been rounded to the nearest second. However, as can be verified in exercise 5, the values of prefit-residuals and partial derivatives have been calculated with **GCAT** option `Satellite coordinates at emission: Using PR`.

- File `DD_STEC_bell_ebre21.ion` contains the values of STEC (in meters of LI) of station `bell` relative to station `ebre` and to the satellite `PRN21`, according to the following format:

  [bell **ebre 21** PRN sec $\nabla\Delta$STEC].

(a) [FLOATING the bias $\nabla\Delta Bc$] (i.e., estimating as real numbers): Using the Kalman filter implemented in the program `kalman`, calculate the kinematic navigation solution[122] with LC and PC, and estimate the bias ($\nabla\Delta Bc$) of the arcs for the different satellites from the `DDbell_ebre21_eci.mod`.
Execute:

```
cp kalman.nml_DD_WN kalman.nml
cat DDbell_ebre21_eci.mod | kalman | grep B > DDbc
```

Note that the obtained file has the following format:
`DDbc= ["BIAS" sec bias_PRN01 bias_PRN02 ... bias_PRN32]`

i. Graphically represent the obtained values $\nabla\Delta Bc$ for the satellite PRN29.
Execute:

```
cat DDbc | gawk '{print $2,$31}' > DDbc_29
gnuplot
plot "DDbc_29"
exit
```

Notice that the values of $\nabla\Delta Bc$ for the satellite PRN29 are in field 31 (=29+2).
Is there any cycle-slip for this satellite? How much is the bias ($\nabla\Delta Bc$) at the end of every arc?

ii. Repeat the preceding analysis for every satellite and check that the estimations "at the end" of the different arcs correspond to the values in file `DDbell_ebre21.bc`.
Note: the file `DDbell_ebre21.bc` contains the next fields

[bell **ebre 21** sec0 sec1 PRN iarc sec0 sec1 $\nabla\Delta Bc$],

where `sec0` and `sec1` define the first and the last point of the arc.

(b) Starting from the files `DDbell_ebre21.obs`, `DDbell_ebre21.ion` and `DDbell_ebre21.bc`, generate the file `DDbell_ebre21.dat` with the help

[122] Modelate coordinates as "white-noise" (pure kinematic positioning) and "fix the receiver clock", according to the file parameters `kalman.nml_DD_WN`.

of the script `add.scr`[123], that will be the base to apply the resolution algorithm of the ambiguities defined in section 7.4 of the theory (page 147). The resultant file contents comes to be:
[`bell` **ebre 21** `PRN iarc sec` $\nabla\Delta LC$ $\nabla\Delta LW$ $\nabla\Delta LI$ $\nabla\Delta STEC$ $\nabla\Delta Bc$]

Execute:

```
add.scr DDbell_ebre21.obs DDbell_ebre21.ion DDbell_ebre21.bc
                                         > DDbell_ebre21.dat
```

(c) [`Calculating the "exact value" of the bias` $\nabla\Delta Bc$]
The program `ambisolv` implements the described equations in section 7.4 of the theory (page 147) to solve the ambiguities in L1 and L2 and calculate the exact value of the bias $\nabla\Delta Bc$. The output of this program provides, besides the exact values of $\nabla\Delta N1$, $\nabla\Delta N2$, $\nabla\Delta Bc$ of the solved ambiguities, a series of intermediate results of the calculation that allow us to continue and analyze the resolution process.
Edit the source code of the program `ambisolve`, identify the different fields of its OUTPUT, and generate the file `DDbc_fix` with the "exact" values of the ambiguities $\nabla\Delta Bc$ for the different arcs, in the next format:

$\boxed{\texttt{PRN sec } \nabla\Delta Bc}$

Execute:

```
textedit ambisolve.f

cat DDbell_ebre21.dat|ambisolv > DDbell_ebre21.amb
cat DDbell_ebre21.amb|gawk '{print $4,$6,$16}' > DDbc_fix
```

(d) [`FIXING the bias` $\nabla\Delta Bc$ `and "repairing"` $\nabla\Delta Lc$ `to their exact value`]
Select the observations corresponding to the phase LC from the file `DDbell_ebre21_eci.mod`, and correct the values $\nabla\Delta Lc$ with the "exact" values of the ambiguities $\nabla\Delta Bc$ previously obtained. Next, change the label "LC" to "PC" for the filter to deal with these repaired phases as codes[124]. Call the generated file `DDbell_ebre21_eci_fix.mod`.

Execute:

[123] See details in the header of the code.

[124] Notice that the phase measurements have been totally repaired. The cycle-slips have been eliminated and the ambiguities corrected with their exact values. In short, one has obtained unambiguous observables (as codes) and very accurately, because they are phase measurements.

```
grep LC DDbell_ebre21_eci.mod > LC.dat
cat DDbc_fix LC.dat|gawk '{if(NF==3) {Y[$1*1" "$2*1]=$3}
  else {if (length(Y[$3*1" "$2*1])!=0)
  {{$4=$4-Y[$3*1" "$2*1]; print $0}}}}' > nada
cat nada |sed 's/LC/PC/g' > DDbell_ebre21_eci_fix.mod
```

(e) Calculate the navigation solution "floating" ambiguities (use the file DDbell_ebre21_eci.mod). Repeat the calculations but using the file DDbell_ebre21_eci_fix.mod, with fixed ambiguities (to their exact value). Compare the obtained results.
Execute:

```
DDbell_ebre21_eci.mod    |kalman|grep X > DDbell_ebre21_eci.pos
DDbell_ebre21_eci_fix.mod|kalman|grep X > DDbell_ebre21_eci_fix.pos

gnuplot
plot "DDbell_ebre21_eci.pos" u 2:3 w linespoints 1,
     "DDbell_ebre21_eci_fix.pos" u 2:3 w linespoints 3

plot "DDbell_ebre21_eci.pos" u 2:4 w linespoints 1,
     "DDbell_ebre21_eci_fix.pos" u 2:4 w linespoints 3

plot "DDbell_ebre21_eci.pos" u 2:5 w linespoints 1,
     "DDbell_ebre21_eci_fix.pos" u 2:5 w linespoints 3
exit
```

7. Repeat the preceding exercise using broadcasted orbits and clocks instead of the precise ones. Use the file `DDbell_ebre21_eph.mod` (this file is identical to the one obtained in exercise 2 of this session). Execute:

Same steps as in the previous exercise, but with broadcast orbits and clocks.

Call the obtained files `DDbell_ebre21_eph.pos` and `DDbell_ebre21_eph_fix.pos`.

```
gnuplot
plot "DDbell_ebre21_eph.pos" u 2:3 w linespoints 1,
     "DDbell_ebre21_eph_fix.pos" u 2:3 w linespoints 3

plot "DDbell_ebre21_eph.pos" u 2:4 w linespoints 1,
     "DDbell_ebre21_eph_fix.pos" u 2:4 w linespoints 3

plot "DDbell_ebre21_eph.pos" u 2:5 w linespoints 1,
     "DDbell_ebre21_eph_fix.pos" u 2:5 w linespoints 3
exit
```

(a) Compare the obtained results with the broadcast and precise orbits.

(b) Why do we obtain more error in the coordinates x, z than in y? (all of them are expressed in the system WGS84)

8. With the aim of checking the ionosphere effect between `bell` and `ebre` when solving ambiguities, repeat exercise 5 assuming that $\nabla\Delta STEC = 0$.

   The following procedure is proposed:

   i. Generate the file `DDbell_ebre21.NO_ion` with null ionospheric corrections.
   Execute:

```
cat DDbell_ebre21.ion|awk '{$NF=0; print $0}'
                                > DDbell_ebre21.NO_ion
```

   ii. Repeat the same steps as in exercise 5, but using the file `DDbell_ebre21.NO_ion` for $\nabla\Delta STEC$.

   Answer the following questions:

   (a) Compare the obtained solution with the one in the preceding exercises.

   (b) Graphically represent the values of $\nabla\Delta STEC$.
   Execute:

```
gnuplot
set grid
set yrange[-.5:.5]
plot "DDbell_ebre21.ion" u 5:6
exit
```

   (c) What is the order of the double differences of the ionospheric refraction? Must we expect to solve the ambiguities if $\nabla\Delta STEC$ is neglected?

9. **Check that from the prefit-residuals[125] one can obtain an estimation of the receiver clock good enough to make the correction of clock in exercise 3.

   Make the process (for example) with the broadcast orbits. In this case, one could use the files `99mar23bell.a_PC.dmx`, `99mar23ebre.a_PC.dmx` generated in exercise 2[126].

[125] and without the need to solve navigation equations.

[126] These files are also available in the "`files`" directory.

The following procedure is proposed to be followed:

i. Generate again the original files:

```
zgrep bell 99mar23bell_ebre.s.gz > 99mar23bell.a
zgrep ebre 99mar23bell_ebre.s.gz > 99mar23ebre.a
```

ii. For every station (`bell`, `ebre`), generate a file with the prefit-residuals of the different observed satellites.These values provide an estimation (a bit noisy) of the clock offsets of these stations (select option `Satellite coordinates at emission: Geometric` in **GCAT**):

Execute:

```
cat 99mar23bell.a_PC.dmx| gawk '{print $1,$2,$3,$5}' > clock_bell
cat 99mar23ebre.a_PC.dmx| gawk '{print $1,$2,$3,$5}' > clock_ebre
```

Notice that the obtained values for the different satellites corresponding to the same station are very similar, although not identical . A refinement (which is not needed) could be taking its mean value for every epoch.

(a) Calculate the differences between the values of the obtained clocks from the prefit-residuals (files `clock_bell` and `clock_ebre`) and the ones obtained in the navigation solution contained in the files `99mar23bell.a_PC.pos` and `99mar23ebre.a_PC.pos`, generated in the preceding exercises.
Execute:

```
cat 99mar23bell.a_PC.pos clock_bell|gawk '{if (NF>4){T[$1*1]=$5}
 else {if (length(T[$2*1])!=0) print $2,$3,$4-T[$2*1]}}' > dT_bell

cat 99mar23ebre.a_PC.pos clock_ebre|gawk '{if (NF>4){T[$1*1]=$5}
 else {if (length(T[$2*1])!=0) print $2,$3,$4-T[$2*1]}}' > dT_ebre

gnuplot
plot "dT_bell" u 1:3,"dT_ebre" u 1:3
exit
```

Which is the order of the obtained differences? How much will this affect the correlation of time marks? How much will they affect the calculation of the double differences of the pseudorange?

(b) Why are the obtained differences so similar for the two stations? What can this values be attributed to?

iii. Adjust time marks of the registered epochs in the files:

```
cat clock_bell 99mar23bell.a | gawk '{if (NF==4){s[$2*1 $3*1]=$4}
  else {if (length(s[$3*1 $4*1])!=0) {a=$3-s[$3*1 $4*1]/3e8;
  printf "%s %s %14.8f %s %14.10f %14.10f %14.10f %14.10f %s \n",
  $1,$2,a,$4,$5,$6,$7,$8,$9}}}'>nada
cp nada 99mar23bell.a

cat clock_ebre 99mar23ebre.a | gawk '{if (NF==4){s[$2*1 $3*1]=$4}
  else {if (length(s[$3*1 $4*1])!=0) {a=$3-s[$3*1 $4*1]/3e8;
  printf "%s %s %14.8f %s %14.10f %14.10f %14.10f %14.10f %s \n",
  $1,$2,a,$4,$5,$6,$7,$8,$9}}}'>nada
cp nada 99mar23ebre.a
```

iv. Repeat the same steps as in the preceding exercise (3) for the double differences calculation and the coordinates estimation with these new files `99mar23bell.a` and `99mar23bell.a`:.

(c) Has the same result been obtained as in the navigation solution of exercise 2

Answers Session 7b Differential positioning
2.e
2.g
2.h
3.h
4.a
5.a
5.b
6.e
7.b
8.a
8.b

Appendix I: GPS constellation status

Navstar GPS Constellation Status
(08-01-12)

Blk II Seq	SVN	PRN Code	Internat. ID	NORAD Catalog Number	Orbit Plane Pos'n	Launch Date (UT)	Clock	Available	Decommissioned
Block I									
	01	04	1978-020A	10684		78-02-22		78-03-29	85-07-17
	02	07	1978-047A	10893		78-05-13		78-07-14	81-07-16
	03	06	1978-093A	11054		78-10-06		78-11-13	92-05-18
	04	08	1978-112A	11141		78-12-10		79-01-08	89-10-14
	05	05	1980-011A	11690		80-02-09		80-02-27	83-11-28
	06	09	1980-032A	11783		80-04-26		80-05-16	91-03-06
	07					81-12-18		Launch failure	
	08	11	1983-072A	14189		83-07-14		83-08-10	93-05-04
	09	13	1984-059A	15039		84-06-13		84-07-19	94-06-20
	10	12	1984-097A	15271		84-09-08		84-10-03	95-11-18
	11	03	1985-093A	16129		85-10-09		85-10-30	94-04-13
Block II									
II-1	14	14	1989-013A	19802		89-02-14		89-04-15	00-04-14
II-2	13	02	1989-044A	20061		89-06-10		89-08-10	04-05-12
II-3	16	16	1989-064A	20185		89-08-18		89-10-14	00-10-13
II-4	19	19	1989-085A	20302		89-10-21		89-11-23	01-09-11
II-5	17	17	1989-097A	20361		89-12-11		90-01-06	05-02-22
II-6	18	18	1990-008A	20452		90-01-24		90-02-14	00-08-18
II-7	20	20	1990-025A	20533		90-03-26		90-04-18	96-05-10
II-8	21	21	1990-068A	20724		90-08-02		90-08-22	03-01-27
II-9	15	15	1990-088A	20830		90-10-01		90-10-15	07-03-14
Block IIA									
II-10	23	32	1990-103A	20959	E-5	90-11-26	Rb2	90-12-10 23:45 UT	
II-11	24	24	1991-047A	21552	D-5	91-07-04	Cs4	91-08-30 04:44 UT	
II-12	25	25	1992-009A	21890	A-5	92-02-23	Rb1	92-03-24 11:00 UT	
II-13	28	28	1992-019A	21930		92-04-10		92-04-25	97-05

II-14	26	26	1992-039A	22014	F-5	92-07-07	Rb1	92-07-23 19:43 UT	
II-15	27	27	1992-058A	22108	A-4	92-09-09	Cs4	92-09-30 20:08 UT	
II-16	32	01	1992-079A	22231	F-6	92-11-22	Cs3	92-12-11 14:49 UT	
II-17	29	29	1992-089A	22275		92-12-18		93-01-05	07-10-23
II-18	22	22	1993-007A	22446		93-02-03		93-04-04	03-08-06
II-19	31	31	1993-017A	22581		93-03-30		93-04-13	05-10-24
II-20	37	07	1993-032A	22657		93-05-13		93-06-12	07-12-20
II-21	39	09	1993-042A	22700	A-1	93-06-26	Cs4	93-07-20 12:54 UT	
II-22	35	05	1993-054A	22779	B-5	93-08-30	Rb1	93-09-28 19:29 UT	
II-23	34	04	1993-068A	22877	D-4	93-10-26	Rb1	93-11-22 18:20 UT	
II-24	36	06	1994-016A	23027	C-5	94-03-10	Rb1	94-03-28 14:20 UT	
II-25	33	03	1996-019A	23833	C-2	96-03-28	Cs4	96-04-09 21:17 UT	
II-26	40	10	1996-041A	23953	E-3	96-07-16	Rb1	96-08-15 15:05 UT	
II-27	30	30	1996-056A	24320	B-2	96-09-12	Cs3	96-10-01 15:28 UT	
II-28	38	08	1997-067A	25030	A-3	97-11-06	Cs3	97-12-18 15:24 UT	

Block IIR

IIR-1	42	12				97-01-17		Launch failure
IIR-2	43	13	1997-035A	24876	F-3	97-07-23	Rb1	98-01-31 00:57 UT
IIR-3	46	11	1999-055A	25933	D-2	99-10-07	Rb1	00-01-03 15:02 UT
IIR-4	51	20	2000-025A	26360	E-1	00-05-11	Rb1	00-06-01 16:09 UT
IIR-5	44	28	2000-040A	26407	B-3	00-07-16	Rb2	00-08-17 13:51 UT
IIR-6	41	14	2000-071A	26605	F-1	00-11-10	Rb1	00-12-10 21:12 UT
IIR-7	54	18	2001-004A	26690	E-4	01-01-30	Rb1	01-02-15 15:51 UT
IIR-8	56	16	2003-005A	27663	B-1	03-01-29	Rb3	03-02-18 15:53 UT
IIR-9	45	21	2003-010A	27704	D-3	03-03-31	Rb3	03-04-12 05:27 UT
IIR-10	47	22	2003-058A	28129	E-2	03-12-21	Rb3	04-01-12 16:50 UT
IIR-11	59	19	2004-009A	28190	C-3	04-03-20	Rb3	04-04-05 17:06 UT
IIR-12	60	23	2004-023A	28361	F-4	04-06-23	Rb2	04-07-09 16:07 UT
IIR-13	61	02	2004-045A	28474	D-1	04-11-06	Rb3	04-11-22 16:23 UT

Block IIR-M

IIR-M-1	53	17	2005-038A	28874	C-4	05-09-26	Rb3	05-12-16 23:30 UT
IIR-M-2	52	31	2006-042A	29486	A-2	06-09-25	Rb3	06-10-12 22:53 UT
IIR-M-3	58	12	2006-052A	29601	B-4	06-11-17	Rb3	06-12-13 03:07 UT
IIR-M-4	55	15	2007-047A	32260	F-2	07-10-17	Rb3	07-10-31 22:46 UT
IIR-M-5	57	29	2007-062A	32384	C-1	07-12-20	Rb3	08-01-02 20:41 UT

Notes
-----

1. NORAD Catalog Number is also known as U.S. Space Command (USSPACECOM) object number and NASA catalog number.
2. No orbital plane position = satellite decommissioned from operational service.
3. Clock: Rb = Rubidium; Cs = Cesium.

```
4.  Selective Availability (S/A) had been enabled on Block II satellites during
    part of 1990; S/A off between about 10 August 1990 and 1 July 1991 due to
    Gulf crisis; standard level re-implemented on 15 November 1991;
    occasionally off for test and other purposes.  S/A was set to 0 on all
    satellites by presidential order on 2 May 2000 at approximately 04:00 UT.
5.  Anti-spoofing (A-S) was activated on 31 January 1994 at 00:00 UT on all
    Block II satellites (ref. NANU 050-94042); occasionally off for test and
    other purposes.
6.  Availability dates:
    These are the dates when a particular satellite is set healthy following
    launch. Typically these dates are days to weeks after the L-band
    transmitters first become active.
7.  Decommissioning dates:
    The decommissioning date for PRN06/SVN03 is the date of termination of
    operations of this satellite (ref. USNO) and is about 3 weeks later than
    other published dates for "deactivation".
    See earlier Navstar GPS Constellation Status reports for information on
    the decommissioning of inactive satellites. The PRN numbers of
    decommissioned satellites are re-assigned to new satellites.
8.  PRN number of SVN32 was changed from 32 to 01 on 28 January 1993.
9.  PRN05 and PRN06 are equipped with corner-cube reflectors for satellite
    laser ranging (SLR).  SLR tracking of the satellites will permit onboard
    clock errors and satellite ephemeris errors in GPS tracking to be
    differentiated.
10. PRN06/SVN36 moving to slot C-5 (was C-1), making way for PRN29/SVN57 (ref.
    NANU 2008003).
11. PRN07/SVN37 had been set unhealthy since 17 August 2007. Decommissioned
    from active service on 20 December 2007. L-band transmitters remain active
    for end-of-life testing (ref. NANUs 2007108, 2007169).
12. PRN29/SVN57 was launched on 20 December 2007 at 20:04 UT and set usable on
    2 January 2008 at 20:41 UT (ref. NANUs 2007170, 2007171, 2008001).
13. PRN32/SVN23, previously decommissioned, has been recommissioned for tests.
    After an initial test in December 2006, it has been continuously
    transmitting L-band signals since 2 April 2007. It was added to broadcast
    almanacs on 27 June 2007. It remains set unhealthy until further notice
    (ref. IGS and NANUs 2006155, 2007051, 2007081).
14. Constellation plot:
    <http://gge.unb.ca/Resources/GPSConstellationPlot.pdf>.
15. The next scheduled GPS satellite launches are (ref. NGA and Spaceflight
    Now):
    IIR-19/M-6                  NET 13 March 2008
    IIR-20/M-7                  NET 20 June 2008
    IIR-21/M-8                  NET 10 September 2008
    IIF-1                       NET January 2009
16. Compiled by Richard B. Langley, Dept. of Geodesy and Geomatics Engineering,
    University of New Brunswick.
```

Appendix II: RINEX-2.10 format description

This appendix contains the description of the "Receiver Independent Exchange Format" (RINEX) files. The symbol ** found in some margins indicates optional information (see **rinex2.10** included in satellites GLONASS and GEO in `http://www.ngs.noaa.gov/CORS/instructions2`).

```
1. THE PHILOSOPHY OF RINEX

The first proposal for the "Receiver Independent Exchange Format" RINEX has
been developed by the Astronomical Institute of the University of Berne for
the easy exchange of the GPS data to be collected during the large European
GPS campaign EUREF 89, which involved more than 60 GPS receivers of 4
different manufacturers. The governing aspect during the development was
the following fact:

Most geodetic processing software for GPS data use a well-defined set of
observables:

- the carrier-phase measurement at one or both carriers (actually being a
measurement on the beat frequency between the received carrier of the
satellite signal and a receiver-generated reference frequency).

- the pseudorange (code) measurement, equivalent to the difference of the
time of reception (expressed in the time frame of the receiver) and the
time of transmission (expressed in the time frame of the satellite) of a
distinct satellite signal.

- the observation time being the reading of the receiver clock at the
instant of validity of the carrier-phase and/or the code measurements.

Usually the software assumes that the observation time is valid for both
the phase AND the code measurements, AND for all satellites observed.

Consequently all these programs do not need most of the information that is
usually stored by the receivers:  They need phase, code, and time in the
above mentioned definitions, and some station-related information like
station name, antenna height, etc.
```

```
2. GENERAL FORMAT DESCRIPTION

Currently the format consists of six ASCII file types:
   1. Observation Data File
   2. Navigation Message File
   3. Meteorological Data File
   4. GLONASS Navigation Message File
   5. GEO Navigation Message File
   6. Satellite and Receiver Clock Date File

(The format definition of the clock files has been published in 1998
in a separate document by Jim Ray and Werner Gurtner, available at the IGS
Central Bureau Information System: ftp://igscb.jpl.nasa.gov/igscb/data/
format/rinex_clock.txt).

Each file type consists of a header section and a data section. The header
section contains global information for the entire file and is placed at
the beginning of the file. The header section contains header labels in
columns 61-80 for each line contained in the header section. These labels
are mandatory and must appear exactly as given in these descriptions and
examples.

The format has been optimized for mimimum space requirements independent
from the number of different observation types of a specific receiver by
indicating in the header the types of observations to be stored. In
computer systems allowing variable record lengths the observation records
may be kept as short as possible. Trailing blanks can be removed from the      |
records. The maximum record length is 80 bytes per record.                     |

Each Observation file and each Meteorological Data file basically contain
the data from one site and one session. RINEX Version 2 also allows to
include observation data from more than one site subsequently occupied by
a roving receiver in rapid static or kinematic applications. Although Version 2
allows to insert header records into the data field we do not recommend to
concatenate data of more than one receiver (or antenna) into the same file,
even if the data do not overlap in time.

If data from more than one receiver has to be exchanged it would not be
economical to include the identical satellite messages collected by the
different receivers several times. Therefore the Navigation Message File
from one receiver may be exchanged or a composite Navigation Message File
created containing non-redundant information from several receivers in
order to make the most complete file.

The format of the data records of the RINEX Version 1 Navigation Message
file is identical to the former NGS exchange format.
```

```
The actual format descriptions as well as examples are given in the Tables
at the end of the paper.


3. DEFINITION OF THE OBSERVABLES

GPS observables include three fundamental quantities that need to be defined:
Time, Phase, and Range.

TIME:

  The time of the measurement is the receiver time of the received signals.
  It is identical for the phase and range measurements and is identical for
  all satellites observed at that epoch. It is expressed in GPS time (not
  Universal Time).

PSEUDO-RANGE:

  The pseudo-range (PR) is the distance from the receiver antenna to the
  satellite antenna including receiver and satellite clock offsets (and
  other biases, such as atmospheric delays):

    PR  =  distance +
           c * (receiver clock offset - satellite clock offset +
                other biases)

  so that the pseudo-range reflects the actual behavior of the receiver
  and satellite clocks. The pseudo-range is stored in units of meters.

  See also clarifications for pseudoranges in mixed GPS/GLONASS files in
  chapter 8.1.

PHASE:

  The phase is the carrier-phase measured in whole cycles at both L1 and
  L2. The half-cycles measured by sqaring-type receivers must be converted
  to whole cycles and flagged by the wavelength factor in the header
  section.

  The phase changes in the same sense as the range (negative doppler). The
  phase observations between epochs must be connected by including the
  integer number of cycles. The phase observations will not contain any
  systematic drifts from intentional offsets of the reference oscillators.

The observables are not corrected for external effects like atmospheric
```

```
refraction, satellite clock offsets, etc.

If the receiver or the converter software adjusts the measurements using
the real-time-derived receiver clock offsets dT(r), the consistency of the
3 quantities phase / pseudo-range / epoch must be maintained, i.e. the
receiver clock correction should be applied to all 3 observables:

  Time(corr)  = Time(r)  -  dT(r)
  PR(corr)    =  PR(r)   -  dT(r)*c
  phase(corr) = phase(r) -  dT(r)*freq

DOPPLER:

The sign of the doppler shift as additional observable is defined as usual:
Positive for approaching satellites.



4. THE EXCHANGE OF RINEX FILES:

We recommend using the following naming convention for RINEX files:

        ssssdddf.yyt      ssss:    4-character station name designator
                           ddd:    day of the year of first record
                             f:    file sequence number within day
                                   0: file contains all the existing
                                      data of the current day
                            yy:    year
                             t:    file type:
                                   O: Observation file
                                   N: Navigation file
                                   M: Meteorological data file
                                   G: GLONASS Navigation file
                                   H: Geostationary GPS payload nav mess file

When data transmission times or storage volumes are critical we recommend
compressing the files prior to storage or transmission using the UNIX
"compress" und "uncompress" programs. Compatible routines are available on
VAX/VMS and PC/DOS systems, as well.
```

```
Proposed naming conventions for the compressed files:

File Types                      UNIX            VMS             DOS             |

Obs Files                       ssssdddf.yyO.Z  ssssdddf.yyO_Z  ssssdddf.yyY
Obs Files (Hatanaka compr)      ssssdddf.yyD.Z  ssssdddf.yyD_Z  ssssdddf.yyE
GPS Nav Files                   ssssdddf.yyN.Z  ssssdddf.yyN_Z  ssssdddf.yyX
GLONASS Nav File                ssssdddf.yyG.Z  ssssdddf.yyG_Z  ssssdddf.yyV
GEO Nav Files                   ssssdddf.yyH.Z  ssssdddf.yyH_Z  ssssdddf.yyU
Met Data Files                  ssssdddf.yyM.Z  ssssdddf.yyM_Z  ssssdddf.yyW
Clock Files (see sep.doc.)      ssssdddf.yyC.Z  ssssdddf.yyC_Z

References for the Hatanaka compression scheme: See e.g.

     ftp://igscb.jpl.nasa.gov/igscb/software/rnxcmp/docs/

     IGSMails 1525,1686,1726,1763,1785


5. RINEX VERSION 2 FEATURES


The following section contains features that have been introduced for RINEX
Version 2:


5.1 Satellite Numbers:

Version 2 has been prepared to contain GLONASS or other satellite systems'
observations. Therefore we have to be able to distinguish the satellites
of the different systems:  We precede the 2-digit satellite number with a
system identifier.

        snn                  s:    satellite system identifier
                                   G or blank : GPS
                                   R          : GLONASS
                                   S          : Geostationary signal payload
                                   T          : Transit

                            nn:    - PRN (GPS), slot number (GLONASS)
                                   - PRN-100 (GEO)
                                   - two-digit Transit satellite number

        Note: G is mandatory in mixed GPS/GLONASS files

                                   (blank default modified in April 1997)
```

5.2 Order of the Header Records:

As the record descriptors in columns 61-80 are mandatory, the programs reading a RINEX Version 2 header are able to decode the header records with formats according to the record descriptor, provided the records have been first read into an internal buffer.

We therefore propose to allow free ordering of the header records, with the following exceptions:

- The "RINEX VERSION / TYPE" record must be the first record in a file

- The default "WAVELENGTH FACT L1/2" record must precede all records defining wavelength factors for individual satellites

- The "# OF SATELLITES" record (if present) should be immediately followed by the corresponding number of "PRN / # OF OBS" records. (These records may be handy for documentary purposes. However, since they may only be created after having read the whole raw data file we define them to be optional.

5.3 Missing Items, Duration of the Validity of Values

Items that are not known at the file creation time can be set to zero or blank or the respective record may be completely omitted. Consequently items of missing header records will be set to zero or blank by the program reading RINEX files. Trailing blanks may be truncated from the record. Each value remains valid until changed by an additional header record.

5.4 Event Flag Records

The "number of satellites" also corresponds to the number of records of the same epoch followed. Therefore it may be used to skip the appropriate number of records if certain event flags are not to be evaluated in detail.

5.5 Receiver Clock Offset

A large number of users asked to optionally include a receiver-derived clock offset into the RINEX format. In order to remove uncertainties if the data (epoch, pseudorange, phase) have been previously corrected or not by the reported clock offset, RINEX Version 2.10 requests a clarifying (new) header record.

It would then be possible to reconstruct the original observations if necessary.

As the output format for the receiver-derived clock offset is limited to nanoseconds the offset should be rounded to the nearest nanosecond before it is used to correct the observables in order to guarantee correct reconstruction.

6. ADDITIONAL HINTS AND TIPS

6.1 Version 1 / Version 2

Programs developed to read RINEX Version 1 files have to verify the version number. Version 2 files may look different (version number, END OF HEADER record, receiver and antenna serial number alphanumeric) even if they do not use any of the new features

6.2 Leading Blanks in CHARACTER fields

We propose that routines to read RINEX Version 2 files automatically delete leading blanks in any CHARACTER input field. Routines creating RINEX Version 2 files should also left-justify all variables in the CHARACTER fields.

6.3 Variable-length Records

DOS, and other, files may have variable record lengths, so we recommend to first read each observation record into a 80-character blank string and decode the data afterwards. In variable length records, empty data fields at the end of a record may be missing, especially in the case of the optional receiver clock offset.

6.4 Blank Fields

In view of future modifications we recommend to carefully skip any fields currently defined to be blank (Format fields nX), because they may be assigned to new contents in future versions.

```
6.5 2-Digit Years

RINEX version 2 stores the years of data records with two digits only. The
header of observation files contains a TIME OF FIRST OBS record with the full
four-digit year, the GPS nav messages contain the GPS week numbers. From these
two data items the unambiguous year can easily be reconstructed.

A hundred-year ambiguity occurs in the met data and GLONASS and GEO nav
messages: Instead of introducing a new TIME OF FIRST OBS header line it is
safeto stipulate that any two-digit years in RINEX Version 1 and Version 2.xx
files are understood to represent

                80-99:  1980-1999
                00-79:  2000-2079

Full 4-digit year fields could then be defined by a future RINEX version 3.


6.6 Fit Interval

Bit 17 in word 10 of subframe 2 is a "fit interval" flag which indicates the
curve-fit interval used by the GPS Control Segment in determining the
ephemeris parameters, as follows (see ICD-GPS-200, 20.3.3.4.3.1):

                0 = 4 hours
                1 = greater than 4 hours.

Together with the IODC values and Table 20-XII the actual fit interval can be
determined. The second value in the last record of each message shall contain
the fit interval in hours determined using IODC, fit flag, and Table 20-XII,
according to the Interface Document ICD-GPS-200.


6.7 Satellite Health

The health of the signal components (bits 18 to 22 of word three in subframe
one) are now (Version 2.10) included into the health value reported in the
second field of the sixth nav mess records.

A program reading RINEX files could easily decide if bit 17 only or all bits
(17-22) have been written:                                                    |

RINEX Value:   0    Health OK
RINEX Value:   1    Health not OK (bits 18-22 not stored)
RINEX Value: >32    Health not OK (bits 18-22 stored)
```

```
6.8 Transmission Time of Message (Navigation message file)

The transmission time of message can be shortly before midnight
Saturday/Sunday, the TOE and TOC of the message already in the next week.
As the reported week in the RINEX nav message (BROADCAST ORBIT - 5 record)
goes with ToE (this is different from the GPS week in the original satellite
message!), the transmission time of message should be reduced by 604800
(i.e., will become negative) to also refer to the same week.


7. RINEX UNDER ANTISPOOFING (AS)

Some receivers generate code delay differences between the first and second
frequency using cross-correlation techniques when AS is on and may recover
the phase observations on L2 in full cycles. Using the C/A code delay on
L1 and the observed difference it is possible to generate a code delay
observation for the second frequency.

Other receivers recover P code observations by breaking down the Y code
into P and W code.

Most of these observations may suffer from an increased noise level. In
order to enable the postprocessing programs to take special actions, such
AS-infected observations are flagged using bit number 2 of the Loss of Lock
Indicators (i.e. their current values are increased by 4).


8. GLONASS Extensions

8.1 RINEX Observation File

8.1.1 Time System Identifier

The original RINEX Version 2 needed one major supplement, the explicit
definition of the time system:

GLONASS is basically running on UTC (or, more precisely, GLONASS system time
linked to UTC(SU)), i.e. the time tags are given in UTC and not GPS time.
In order to remove possible misunderstandings and ambiguities, the header
records "TIME OF FIRST OBS" and (if present) "TIME OF LAST OBS" in GLONASS and
GPS observation files _can_, in mixed GLONASS/GPS observation files _must_
contain a time system identifier defining the system that all time tags in the
file are referring to: "GPS" to identify GPS time, "GLO" to identify the
GLONASS UTC time system. Pure GPS files default to GPS and pure GLONASS files
default to GLO.
```

```
Format definitions see Table A1.

Hence, the two possible time tags differ by the current number of leap seconds.

In order to have the current number of leap seconds available we recommend
to include a LEAP SECOND line into the RINEX header.

If there are known non-integer biases between the "GPS receiver clock"
and "GLONASS receiver clock" in the same receiver, they should be applied.
In this case the respective code and phase observations have to be corrected,
too (c * bias if expressed in meters).

Unknown such biases will have to be solved for during the post processing

The small differences (modulo 1 second) between GLONASS system time, UTC(SU),
UTC(USNO) and GPS system time have to be dealt with during the post-processing
and not before the RINEX conversion. It may also be necessary to solve for
remaining differences during the post-processing.

8.1.2 Pseudorange Definition

The pseudorange (code) measurement is defined to be equivalent to the
difference of the time of reception (expressed in the time frame of the
receiver) and the time of transmission (expressed in the time frame of the
satellite) of a distinct satellite signal.

If a mixed-mode GPS/GLONASS receiver refers all pseudorange observations to
one receiver clock only,

- the raw GLONASS pseudoranges will show the current number of leap seconds
  between GPS time and GLONASS time if the receiver clock is running in the
  GPS time frame

- the raw GPS pseudoranges will show the negative number of leap seconds
  between GPS time and GLONASS time if the receiver clock is running in the
  GLONASS time frame

In order to avoid misunderstandings and to keep the code observations within
the format fields, the pseudoranges must be corrected in this case as follows:

  PR(GPS) := PR(GPS) + c * leap_seconds   if generated with a receiver clock
                                          running in the GLONASS time frame

  PR(GLO) := PR(GLO) - c * leap_seconds   if generated with a receiver clock
                                          running in the GPS time frame
```

to remove the contributions of the leap seconds from the pseudoranges.

"leap_seconds" is the actual number of leap seconds between GPS and GLONASS (UTC) time, as broadcast in the GPS almanac and distributed in Circular T of BIPM.

8.1.3 More Than 12 Satellites per Epoch

The format of the epoch / satellite line in the observation record part of the RINEX Observation files has only been defined for up to 12 satellites per epoch. We explicitly define now the format of the continuation lines, see Table A2.

8.2 RINEX Navigation Files for GLONASS

As the GLONASS navigation message differs in contents from the GPS message too much, a special GLONASS navigation message file format has been defined.

The header section and the first data record (epoch, satellite clock information) is similar to the GPS navigation file. The following records contain the satellite position, velocity and acceleration, the clock and frequency biases as well as auxiliary information as health, satellite frequency (channel), age of the information.

The corrections of the satellite time to UTC are as follows:

```
  GPS    : Tutc = Tsv - af0  -  af1  *(Tsv-Toc) - ... -  A0  - ... - leap_sec
  GLONASS: Tutc = Tsv + TauN - GammaN*(Tsv-Tb)        + TauC
```

```
***  In order to use the same sign conventions for the GLONASS corrections
     as in the GPS navigation files,  the broadcast GLONASS values are
     stored as:
                  -TauN, +GammaN, -TauC.
```

The time tags in the GLONASS navigation files are given in UTC (i.e. _not_ Moscow time or GPS time).

Filenaming convention: See above.

9. RINEX Extensions for Geostationary Satellites (GPS Signal Payloads)

With the implementation of GNSS programs, GPS-like ranging measurements can be performed on geostationary navigation payloads.

```
RINEX Version 2.10 defines the necessary extensions to handle such data in
RINEX files for data exchange and postprocessing purposes.


9.1 RINEX Observation Files for GEO Satellites

A new satellite system identifier has been defined for the geostationary
GPS signal payloads: "S", to be used in the RINEX VERSION / TYPE header line
and in the satellite identifier ’snn’, nn being the GEO PRN number minus 100.

                   e.g.: PRN = 120 --> ’snn’ = "S20"

In mixed dual frequency GPS satellite / single frequency GEO payload
observation files the fields for the second frequency observations of GEO
satellites remain blank, are set to zero values or (if last in the record)     |
can be truncated.                                                              |

The time system identifier of GEO satellites generating GPS signals defaults   |
to GPS time.                                                                   |


9.2 RINEX Navigation Message Files for GEO Satellites

As the GEO broadcast orbit format differs from the GPS message a special GEO
navigation message file format has been defined which is nearly identical with
the GLONASS nav mess file format.

The header section contains informations about the generating program,
comments, and the difference between the GEO system time and UTC.

The first data record contains the epoch and satellite clock information,
the following records contain the satellite position, velocity and
acceleration and auxiliary information such as health, age of the data, etc.

The time tags in the GEO navigation files are given in the GPS time frame,
i.e. not UTC.

The corrections of the satellite time to UTC are as follows:

  GEO    : Tutc = Tsv - aGf0  -  aGf1 *(Tsv-Toe) -  W0  - leap_sec

W0 being the correction to transform the GEO system time to UTC. Toe, aGf0,
aGf1 see below in the format definition tables.
*  References for the definition of the accuracy and health codes still have *
*  to be defined.                                                            *
*  Help is needed here by colleagues working with such GEO data!             *
```

10. REFERENCES

Evans, A. (1989): "Summary of the Workshop on GPS Exchange Formats." Proceedings of the Fifth International Geodetic Symposium on Satellite Systems, pp. 917ff, Las Cruces.

Gurtner, W., G. Mader, D. Arthur (1989): "A Common Exchange Format for GPS Data." CSTG GPS Bulletin Vol.2 No.3, May/June 1989, National Geodetic Survey, Rockville.

Gurtner, W., G. Mader (1990): "The RINEX Format: Current Status, Future Developments." Proceedings of the Second International Symposium of Precise Positioning with the Global Positioning system, pp. 977ff, Ottawa.

Gurtner, W., G. Mader (1990): "Receiver Independent Exchange Format Version 2." CSTG GPS Bulletin Vol.3 No.3, Sept/Oct 1990, National Geodetic Survey, Rockville.

Gurtner, W. (1994): "RINEX: The Receiver-Independent Exchange Format." GPS World, Volume 5, Number 7, July 1994.

11. RINEX VERSION 2.10 FORMAT DEFINITIONS AND EXAMPLES

```
 +----------------------------------------------------------------------------+
 |                                 TABLE A1                                   |
 |           GPS OBSERVATION DATA FILE - HEADER SECTION DESCRIPTION           |
 +--------------------+------------------------------------------+------------+
 |   HEADER LABEL     |               DESCRIPTION                |   FORMAT   |
 |  (Columns 61-80)   |                                          |            |
 +--------------------+------------------------------------------+------------+
 |RINEX VERSION / TYPE| - Format version (2.10)                  | F9.2,11X,  |
 |                    | - File type ('O' for Observation Data)   |   A1,19X,  |
 |                    | - Satellite System: blank or 'G': GPS    |   A1,19X   |
 |                    |                     'R': GLONASS         |            |
 |                    |                     'S': Geostationary   |            |
 |                    |                          signal payload  |            |
 |                    |                     'T': NNSS Transit    |            |
 |                    |                     'M': Mixed           |            |
 +--------------------+------------------------------------------+------------+
 |PGM / RUN BY / DATE | - Name of program creating current file  |     A20,   |
 |                    | - Name of agency  creating current file  |     A20,   |
 |                    | - Date of file creation                  |     A20    |
 +--------------------+------------------------------------------+------------+
*|COMMENT             | Comment line(s)                          |     A60    |*
```

```
 +--------------------+------------------------------------------+------------+
 |MARKER NAME         | Name of antenna marker                   |     A60    |
 +--------------------+------------------------------------------+------------+
*|MARKER NUMBER       | Number of antenna marker                 |     A20    |*
 +--------------------+------------------------------------------+------------+
 |OBSERVER / AGENCY   | Name of observer / agency                |   A20,A40  |
 +--------------------+------------------------------------------+------------+
 |REC # / TYPE / VERS | Receiver number, type, and version       |    3A20    |
 |                    | (Version: e.g. Internal Software Version)|            |
 +--------------------+------------------------------------------+------------+
 |ANT # / TYPE        | Antenna number and type                  |    2A20    |
 +--------------------+------------------------------------------+------------+
 |APPROX POSITION XYZ | Approximate marker position (WGS84)      |   3F14.4   |
 +--------------------+------------------------------------------+------------+
 |ANTENNA: DELTA H/E/N| - Antenna height: Height of bottom       |   3F14.4   |
 |                    |   surface of antenna above marker        |            |
 |                    | - Eccentricities of antenna center       |            |
 |                    |   relative to marker to the east         |            |
 |                    |   and north (all units in meters)        |            |
 +--------------------+------------------------------------------+------------+
 |WAVELENGTH FACT L1/2| - Default wavelength factors for         |            |
 |                    |   L1 and L2                              |    2I6,    |
 |                    |   1:  Full cycle ambiguities             |            |
 |                    |   2:  Half cycle ambiguities (squaring)  |            |
 |                    |   0 (in L2): Single frequency instrument |            |
 |                    |                                          |            |
 |                    | - zero or blank                          |     I6     |
 |                    |                                          |            |
 |                    | The default wavelength factor line is    |            |
 |                    | required and must preceed satellite-     |            |
 |                    | specific lines.                          |            |
 +--------------------+------------------------------------------+------------+
*|WAVELENGTH FACT L1/2| - Wavelength factors for L1 and L2       |    2I6,    |*
 |                    |   1:  Full cycle ambiguities             |            |
 |                    |   2:  Half cycle ambiguities (squaring)  |            |
 |                    |   0 (in L2): Single frequency instrument |            |
 |                    | - Number of satellites to follow in list |    I6,     |
 |                    |   for which these factors are valid.     |            |
 |                    | - List of PRNs (satellite numbers with   | 7(3X,A1,I2)|
 |                    |   system identifier)                     |            |
 |                    |                                          |            |
 |                    | These opional satellite specific lines   |            |
 |                    | may follow, if they identify a state     |            |
 |                    | different from the default values.       |            |
 |                    |                                          |            |
 |                    | Repeat record if necessary.              |            |
```

```
 +--------------------+------------------------------------------+------------+
 |# / TYPES OF OBSERV | - Number of different observation types  |     I6,    |
 |                    |   stored in the file                     |            |
 |                    | - Observation types                      |  9(4X,A2)  |
 |                    |                                          |            |
 |                    |   If more than 9 observation types:      |            |
 |                    |     Use continuation line(s)             |6X,9(4X,A2) |
 |                    |                                          |            |
 |                    | The following observation types are      |            |
 |                    | defined in RINEX Version 2.10:           |            |
 |                    |                                          |            |
 |                    | L1, L2: Phase measurements on L1 and L2  |            |
 |                    | C1    : Pseudorange using C/A-Code on L1 |            |
 |                    | P1, P2: Pseudorange using P-Code on L1,L2|            |
 |                    | D1, D2: Doppler frequency on L1 and L2   |            |
 |                    | T1, T2: Transit Integrated Doppler on    |            |
 |                    |         150 (T1) and 400 MHz (T2)        |            |
 |                    | S1, S2: Raw signal strengths or SNR      |            |
 |                    |         values as given by the receiver  |            |
 |                    |         for the L1,L2 phase observations |            |
 |                    |                                          |            |
 |                    | Observations collected under Antispoofing|            |
 |                    | are converted to "L2" or "P2" and flagged|            |
 |                    | with bit 2 of loss of lock indicator     |            |
 |                    | (see Table A2).                          |            |
 |                    |                                          |            |
 |                    | Units : Phase       : full cycles        |            |
 |                    |         Pseudorange : meters             |            |
 |                    |         Doppler     : Hz                 |            |
 |                    |         Transit     : cycles             |            |
 |                    |         SNR etc     : receiver-dependent |            |
 |                    |                                          |            |
 |                    | The sequence of the types in this record |            |
 |                    | has to correspond to the sequence of the |            |
 |                    | observations in the observation records  |            |
 +--------------------+------------------------------------------+------------+
*|INTERVAL            | Observation interval in seconds          |   F10.3    |*
 +--------------------+------------------------------------------+------------+
 |TIME OF FIRST OBS   | - Time of first observation record       | 5I6,F13.7, |
 |                    |   (4-digit-year, month,day,hour,min,sec) |            |
 |                    | - Time system: GPS (=GPS time system)    |   5X,A3    |
 |                    |                GLO (=UTC time system)    |            |
 |                    |   Compulsory in mixed GPS/GLONASS files  |            |
 |                    |   Defaults: GPS for pure GPS files       |            |
 |                    |             GLO for pure GLONASS files   |            |
 +--------------------+------------------------------------------+------------+
```

```
*|TIME OF LAST OBS    | - Time of last  observation record       | 5I6,F13.7, |*
 |                    |   (4-digit-year, month,day,hour,min,sec) |            |
 |                    | - Time system: Same value as in          |   5X,A3    ||
 |                    |                 TIME OF FIRST OBS record |            ||
 +--------------------+------------------------------------------+------------+
*|RCV CLOCK OFFS APPL | Epoch, code, and phase are corrected by  |     I6     |*
 |                    | applying the realtime-derived receiver   |            |
 |                    | clock offset: 1=yes, 0=no; default: 0=no |            |
 |                    | Record required if clock offsets are     |            |
 |                    | reported in the EPOCH/SAT records        |            |
 +--------------------+------------------------------------------+------------+
*|LEAP SECONDS        | Number of leap seconds since 6-Jan-1980  |     I6     |*
 |                    | Recommended for mixed GPS/GLONASS files  |            |
 +--------------------+------------------------------------------+------------+
*|# OF SATELLITES     | Number of satellites, for which          |     I6     |*
 |                    | observations are stored in the file      |            |
 +--------------------+------------------------------------------+------------+
*|PRN / # OF OBS      | PRN (sat.number), number of observations |3X,A1,I2,9I6|*
 |                    | for each observation type indicated      |            |
 |                    | in the "# / TYPES OF OBSERV" - record.   |            |
 |                    |                                          |            |
 |                    |   If more than 9 observation types:      |            |
 |                    |   Use continuation line(s)               |   6X,9I6   |
 |                    |                                          |            |
 |                    | This record is (these records are)       |            |
 |                    | repeated for each satellite present in   |            |
 |                    | the data file                            |            |
 +--------------------+------------------------------------------+------------+
 |END OF HEADER       | Last record in the header section.       |    60X     |
 +--------------------+------------------------------------------+------------+

                        Records marked with * are optional


 +----------------------------------------------------------------------------+
 |                                  TABLE A2                                  |
 |            GPS OBSERVATION DATA FILE - DATA RECORD DESCRIPTION             |
 +-------------+--------------------------------------------------+------------+
 | OBS. RECORD | DESCRIPTION                                      |   FORMAT   |
 +-------------+--------------------------------------------------+------------+
 | EPOCH/SAT   | - Epoch :                                        |            |
 |     or      |   - year (2 digits, padded with 0 if necessary) |  1X,I2.2,  |
 | EVENT FLAG  |   - month,day,hour,min,                          |  4(1X,I2), |
 |             |   - sec                                          |   F11.7,   |
 |             |                                                  |            |
 |             | - Epoch flag 0: OK                               |   2X,I1,   |
```

```
|             |               1: power failure between        |            |
|             |                  previous and current epoch   |            |
|             |              >1: Event flag                   |            |
|             | - Number of satellites in current epoch       |    I3,     |
|             | - List of PRNs (sat.numbers with system       | 12(A1,I2), |
|             |   identifier, see 5.1) in current epoch       |            |
|             | - receiver clock offset (seconds, optional)   |   F12.9    |
|             |                                               |            |
|             |   If more than 12 satellites: Use continuation|    32X,    |
|             |   line(s)                                     | 12(A1,I2)  |
|             |                                               |            |
|             | If epoch flag  2-5:                           |            |
|             |                                               |            |
|             |   - Event flag:                               |  [2X,I1,]  |
|             |     2: start moving antenna                   |            |
|             |     3: new site occupation (end of kinem. data)|           |
|             |        (at least MARKER NAME record follows)  |            |
|             |     4: header information follows             |            |
|             |     5: external event (epoch is significant,  |            |
|             |        same time frame as observation time tags)|          |
|             |                                               |            |
|             |   - "Number of satellites" contains number of |    [I3]    |
|             |     special records to follow.                |            |
|             |     Maximum number of records: 999            |            |
|             |                                               |            |
|             |   - For events without significant epoch the  |            ||
|             |     epoch fields can be left blank            |            ||
|             |                                               |            |
|             | If epoch flag = 6:                            |            |
|             |     6: cycle slip records follow to optionally|            |
|             |        report detected and repaired cycle slips|           |
|             |        (same format as OBSERVATIONS records;  |            |
|             |         slip instead of observation; LLI and  |            |
|             |         signal strength blank or zero)        |            |
+-------------+-----------------------------------------------+------------+
|OBSERVATIONS | - Observation      | rep. within record for   |  m(F14.3,  |
|             | - LLI              | each obs.type (same seq  |    I1,     |
|             | - Signal strength  | as given in header)      |    I1)     |
|             |                                               |            |
|             | If more than 5 observation types (=80 char):  |            |
|             | continue observations in next record.         |            |
|             |                                               |            |
|             | This record is (these records are) repeated for|           |
|             | each satellite given in EPOCH/SAT - record.   |            |
|             |                                               |            |
|             | Observations:                                 |            |
```

```
 |             |   Phase  : Units in whole cycles of carrier     |            |
 |             |   Code   : Units in meters                      |            |
 |             | Missing observations are written as 0.0         |            |
 |             | or blanks.                                      |            |
 |             |                                                 |            |
 |             | Phase values overflowing the fixed format F14.3 |            |
 |             | have to be clipped into the valid interval (e.g.|            |
 |             | add or subtract 10**9), set LLI indicator.      |            |
 |             |                                                 |            |
 |             | Loss of lock indicator (LLI). Range: 0-7        |            |
 |             |  0 or blank: OK or not known                    |            |
 |             |  Bit 0 set : Lost lock between previous and     |            |
 |             |              current observation: cycle slip    |            |
 |             |              possible                           |            |
 |             |  Bit 1 set : Opposite wavelength factor to the  |            |
 |             |              one defined for the satellite by a |            |
 |             |              previous WAVELENGTH FACT L1/2 line.|            |
 |             |              Valid for the current epoch only.  |            |
 |             |  Bit 2 set : Observation under Antispoofing     |            |
 |             |              (may suffer from increased noise)  |            |
 |             |                                                 |            |
 |             |  Bits 0 and 1 for phase only.                   |            |
 |             |                                                 |            |
 |             | Signal strength projected into interval 1-9:    |            |
 |             |  1: minimum possible signal strength            |            |
 |             |  5: threshold for good S/N ratio                |            |
 |             |  9: maximum possible signal strength            |            |
 |             |  0 or blank: not known, don't care              |            |
 +-------------+-------------------------------------------------+------------+


 +----------------------------------------------------------------------------+
 |                                TABLE A3                                    |
 |         GPS NAVIGATION MESSAGE FILE - HEADER SECTION DESCRIPTION           |
 +--------------------+------------------------------------------+------------+
 |    HEADER LABEL    |               DESCRIPTION                |   FORMAT   |
 |  (Columns 61-80)   |                                          |            |
 +--------------------+------------------------------------------+------------+
 |RINEX VERSION / TYPE| - Format version (2.10)                  | F9.2,11X,  |
 |                    | - File type ('N' for Navigation data)    |   A1,19X   |
 +--------------------+------------------------------------------+------------+
 |PGM / RUN BY / DATE | - Name of program creating current file  |     A20,   |
 |                    | - Name of agency  creating current file  |     A20,   |
 |                    | - Date of file creation                  |     A20    |
 +--------------------+------------------------------------------+------------+
*|COMMENT             | Comment line(s)                          |     A60    |*
```

```
 +--------------------+-----------------------------------------+------------+
*|ION ALPHA           | Ionosphere parameters A0-A3 of almanac  |  2X,4D12.4 |*
 |                    | (page 18 of subframe 4)                 |            |
 +--------------------+-----------------------------------------+------------+
*|ION BETA            | Ionosphere parameters B0-B3 of almanac  |  2X,4D12.4 |*
 +--------------------+-----------------------------------------+------------+
*|DELTA-UTC: A0,A1,T,W| Almanac parameters to compute time in UTC| 3X,2D19.12,|*
 |                    | (page 18 of subframe 4)                 |     2I9    |
 |                    | A0,A1: terms of polynomial              |            |
 |                    | T    : reference time for UTC data      |            |
 |                    | W    : UTC reference week number.       |            |
 |                    |        Continuous number, not mod(1024)! |            |
 +--------------------+-----------------------------------------+------------+
*|LEAP SECONDS        | Delta time due to leap seconds          |     I6     |*
 +--------------------+-----------------------------------------+------------+
 |END OF HEADER       | Last record in the header section.      |    60X     |
 +--------------------+-----------------------------------------+------------+

                        Records marked with * are optional


 +----------------------------------------------------------------------------+
 |                                 TABLE A4                                   |
 |           GPS NAVIGATION MESSAGE FILE - DATA RECORD DESCRIPTION            |
 +--------------------+-----------------------------------------+------------+
 |    OBS. RECORD     | DESCRIPTION                             |   FORMAT   |
 +--------------------+-----------------------------------------+------------+
 |PRN / EPOCH / SV CLK| - Satellite PRN number                  |     I2,    |
 |                    | - Epoch: Toc - Time of Clock            |            |
 |                    |          year (2 digits, padded with 0  |            |
 |                    |                if necessary)            |  1X,I2.2,  |
 |                    |          month                          |   1X,I2,   |
 |                    |          day                            |   1X,I2,   |
 |                    |          hour                           |   1X,I2,   |
 |                    |          minute                         |   1X,I2,   |
 |                    |          second                         |    F5.1,   |
 |                    | - SV clock bias       (seconds)         |  3D19.12   |
 |                    | - SV clock drift      (sec/sec)         |            |
 |                    | - SV clock drift rate (sec/sec2)        |            |
 +--------------------+-----------------------------------------+------------+
 | BROADCAST ORBIT - 1| - IODE Issue of Data, Ephemeris         | 3X,4D19.12 |
 |                    | - Crs                 (meters)          |            |
 |                    | - Delta n             (radians/sec)     |            |
 |                    | - M0                  (radians)         |            |
 +--------------------+-----------------------------------------+------------+
 | BROADCAST ORBIT - 2| - Cuc                 (radians)         | 3X,4D19.12 |
```

```
|                    | - e Eccentricity                         |            |
|                    | - Cus                 (radians)          |            |
|                    | - sqrt(A)             (sqrt(m))          |            |
+--------------------+------------------------------------------+------------+
| BROADCAST ORBIT - 3| - Toe Time of Ephemeris                  | 3X,4D19.12 |
|                    |                       (sec of GPS week)  |            |
|                    | - Cic                 (radians)          |            |
|                    | - OMEGA               (radians)          |            |
|                    | - CIS                 (radians)          |            |
+--------------------+------------------------------------------+------------+
| BROADCAST ORBIT - 4| - i0                  (radians)          | 3X,4D19.12 |
|                    | - Crc                 (meters)           |            |
|                    | - omega               (radians)          |            |
|                    | - OMEGA DOT           (radians/sec)      |            |
+--------------------+------------------------------------------+------------+
| BROADCAST ORBIT - 5| - IDOT                (radians/sec)      | 3X,4D19.12 |
|                    | - Codes on L2 channel                    |            |
|                    | - GPS Week # (to go with TOE)            |            |
|                    |   Continuous number, not mod(1024)!      |            |
|                    | - L2 P data flag                         |            |
+--------------------+------------------------------------------+------------+
| BROADCAST ORBIT - 6| - SV accuracy         (meters)           | 3X,4D19.12 |
|                    | - SV health        (bits 17-22 w 3 sf 1) |            |
|                    | - TGD                 (seconds)          |            |
|                    | - IODC Issue of Data, Clock              |            |
+--------------------+------------------------------------------+------------+
| BROADCAST ORBIT - 7| - Transmission time of message        *) | 3X,4D19.12 |
|                    |         (sec of GPS week, derived e.g.   |            |
|                    |    from Z-count in Hand Over Word (HOW)  |            |
|                    | - Fit interval        (hours)            |            |
|                    |         (see ICD-GPS-200, 20.3.4.4)      |            |
|                    |   Zero if not known                      |            |
|                    | - spare                                  |            |
|                    | - spare                                  |            |
+--------------------+------------------------------------------+------------+

*) Adjust the Transmission time of message by -604800 to refer to the reported
   week, if necessary


+----------------------------------------------------------------------------+
|                                 TABLE A5                                   |
|           METEOROLOCICAL DATA FILE - HEADER SECTION DESCRIPTION            |
+--------------------+------------------------------------------+------------+
|   HEADER LABEL     |               DESCRIPTION                |   FORMAT   |
|  (Columns 61-80)   |                                          |            |
```

```
 +--------------------+------------------------------------------+------------+
 |RINEX VERSION / TYPE| - Format version (2.10)                  | F9.2,11X,  |
 |                    | - File type ('M' for Meteorological Data)|   A1,39X   |
 +--------------------+------------------------------------------+------------+
 |PGM / RUN BY / DATE | - Name of program creating current file  |     A20,   |
 |                    | - Name of agency  creating current file  |     A20,   |
 |                    | - Date of file creation                  |     A20    |
 +--------------------+------------------------------------------+------------+
*|COMMENT             | Comment line(s)                          |     A60    |*
 +--------------------+------------------------------------------+------------+
 |MARKER NAME         | Station Name                             |     A60    |
 |                    | (preferably identical to MARKER NAME in  |            |
 |                    |  the associated Observation File)        |            |
 +--------------------+------------------------------------------+------------+
*|MARKER NUMBER       | Station Number                           |     A20    |*
 |                    | (preferably identical to MARKER NUMBER in|            |
 |                    |  the associated Observation File)        |            |
 +--------------------+------------------------------------------+------------+
 |# / TYPES OF OBSERV | - Number of different observation types  |     I6,    |
 |                    |   stored in the file                     |            |
 |                    | - Observation types                      |   9(4X,A2) |
 |                    |                                          |            |
 |                    | The following meteorological observation |            |
 |                    | types are defined in RINEX Version 2:    |            |
 |                    |                                          |            |
 |                    | PR : Pressure (mbar)                     |            |
 |                    | TD : Dry temperature (deg Celsius)       |            |
 |                    | HR : Relative Humidity (percent)         |            |
 |                    | ZW : Wet zenith path delay (millimeters) |            |
 |                    |      (for WVR data)                      |            |
 |                    | ZD : Dry component of zenith path delay  |            |
 |                    |      (millimeters)                       |            ||
 |                    | ZT : Total zenith path delay             |            |
 |                    |      (millimeters)                       |            ||
 |                    |                                          |            |
 |                    | The sequence of the types in this record |            |
 |                    | must correspond to the sequence of the   |            |
 |                    | measurements in the data records         |            |
 |                    |                                          |            |
 |                    | If more than 9 observation types are     |            |
 |                    | being used, use continuation lines with  |            |
 |                    | format (6X,9(4X,A2))                     |            |
 |                    |                                          |            |
 +--------------------+------------------------------------------+------------+
 |SENSOR MOD/TYPE/ACC | Description of the met sensor            |            |
 |                    | - Model (manufacturer)                   |   A20,     |
```

```
|                    | - Type                                 |   A20,6X,  |
|                    | - Accuracy (same units as obs values)  |   F7.1,4X, |
|                    | - Observation type                     |   A2,1X    |
|                    | Record is repeated for each observation|            |
|                    | type found in # / TYPES OF OBSERV record|           |
+--------------------+----------------------------------------+------------+
|SENSOR POS XYZ/H    | Approximate position of the met sensor |            |
|                    | - Geocentric coordinates X,Y,Z   (ITRF |  3F14.4,   |
|                    | - Ellipsoidal height H        or WGS-84)|  1F14.4,  |
|                    | - Observation type                     |  1X,A2,1X  |
|                    | Set X,Y,Z to zero if not known.        |            |
|                    | Make sure H refers to ITRF or WGS-84!  |            |
|                    | Record required for barometer,         |            |
|                    | recommended for other sensors.         |            |
+--------------------+----------------------------------------+------------+
|END OF HEADER       | Last record in the header section.     |    60X     |
+--------------------+----------------------------------------+------------+

                      Records marked with * are optional


+----------------------------------------------------------------------------+
|                                 TABLE A6                                   |
|             METEOROLOGICAL DATA FILE - DATA RECORD DESCRIPTION             |
+-------------+--------------------------------------------------+------------+
| OBS. RECORD | DESCRIPTION                                      |   FORMAT   |
+-------------+--------------------------------------------------+------------+
| EPOCH / MET | - Epoch in GPS time (not local time!)            |            |
|             |     year (2 digits, padded with 0 if necessary) |  1X,I2.2,  |
|             |     month,day,hour,min,sec                       | 5( 1X,I2), |
|             |                                                  |            |
|             |   The 2-digit years in RINEX Version 1 and 2.xx |            |
|             |   files are understood to represent              |            |
|             |   80-99:  1980-1999   and   00-79:  2000-2079    |            |
|             |                                                  |            |
|             | - Met data in the same sequence as given in the |   mF7.1    |
|             |   header                                         |            |
|             |                                                  |            |
|             | More than 8 met data types: Use continuation    |4X,10F7.1,3X|
|             | lines                                            |            |
+-------------+--------------------------------------------------+------------+


+----------------------------------------------------------------------------+
|                                 TABLE A7                                   |
|                     GPS OBSERVATION DATA FILE - EXAMPLE                    |
```

```
+----------------------------------------------------------------------------+

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
BLANK OR G = GPS,  R = GLONASS,  T = TRANSIT,  M = MIXED    COMMENT
XXRINEXO V9.9       AIUB                24-MAR-01 14:43     PGM / RUN BY / DATE
EXAMPLE OF A MIXED RINEX FILE                               COMMENT
A 9080                                                      MARKER NAME
9080.1.34                                                   MARKER NUMBER
BILL SMITH          ABC INSTITUTE                           OBSERVER / AGENCY
X1234A123           XX                  ZZZ                 REC # / TYPE / VERS
234                 YY                                      ANT # / TYPE
  4375274.       587466.      4589095.                      APPROX POSITION XYZ
         .9030         .0000         .0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     1     2     6   G14   G15   G16   G17   G18   G19      WAVELENGTH FACT L1/2
     0                                                      RCV CLOCK OFFS APPL
     4    P1    L1    L2    P2                              # / TYPES OF OBSERV
    18.000                                                  INTERVAL
  2001     3    24    13    10   36.0000000                 TIME OF FIRST OBS
                                                            END OF HEADER
 01  3 24 13 10 36.0000000  0  3G12G 9G 6                            -.123456789
  23629347.915            .300 8         -.353    23629364.158
  20891534.648           -.120 9         -.358    20891541.292
  20607600.189           -.430 9          .394    20607605.848
 01  3 24 13 10 50.0000000  4  4
     1     2     2   G 9   G12                              WAVELENGTH FACT L1/2
  *** WAVELENGTH FACTOR CHANGED FOR 2 SATELLITES ***        COMMENT
      NOW 8 SATELLITES HAVE WL FACT 1 AND 2!                COMMENT
                                                            COMMENT
 01  3 24 13 10 54.0000000  0  5G12G 9G 6R21R22                      -.123456789
  23619095.450      -53875.632 8    -41981.375    23619112.008
  20886075.667      -28688.027 9    -22354.535    20886082.101
  20611072.689       18247.789 9     14219.770    20611078.410
  21345678.576       12345.567 5
  22123456.789       23456.789 5
 01  3 24 13 11  0.0000000  2  1
            *** FROM NOW ON KINEMATIC DATA! ***             COMMENT
 01  3 24 13 11 48.0000000  0  4G16G12G 9G 6                         -.123456789
  21110991.756       16119.980 7     12560.510    21110998.441
  23588424.398     -215050.557 6   -167571.734    23588439.570
  20869878.790     -113803.187 8    -88677.926    20869884.938
  20621643.727       73797.462 7     57505.177    20621649.276
                            3  4
A 9080                                                      MARKER NAME
```

```
9080.1.34                                                   MARKER NUMBER
         .9030         .0000         .0000                  ANTENNA: DELTA H/E/N
          --> THIS IS THE START OF A NEW SITE <--           COMMENT
 01  3 24 13 12  6.0000000  0  4G16G12G 6G 9                         -.123456987
  21112589.384       24515.877 6     19102.763 3  21112596.187
  23578228.338     -268624.234 7   -209317.284 4  23578244.398
  20625218.088       92581.207 7     72141.846 4  20625223.795
  20864539.693     -141858.836 8   -110539.435 5  20864545.943
 01  3 24 13 13  1.2345678  5  0
                            4  1
        (AN EVENT FLAG WITH SIGNIFICANT EPOCH)              COMMENT
 01  3 24 13 14 12.0000000  0  4G16G12G 9G 6                         -.123456012
  21124965.133       89551.30216     69779.62654  21124972.2754
  23507272.372     -212616.150 7   -165674.789 5  23507288.421
  20828010.354     -333820.093 6   -260119.395 5  20828017.129
  20650944.902      227775.130 7    177487.651 4  20650950.363
                            4  1
           *** ANTISPOOFING ON G 16 AND LOST LOCK           COMMENT
 01  3 24 13 14 12.0000000  6  2G16G 9
                 123456789.0      -9876543.5
                         0.0           -0.5
                            4  2
           ---> CYCLE SLIPS THAT HAVE BEEN APPLIED TO       COMMENT
                THE OBSERVATIONS                            COMMENT
 01  3 24 13 14 48.0000000  0  4G16G12G 9G 6                         -.123456234
  21128884.159      110143.144 7     85825.18545  21128890.7764
  23487131.045     -318463.297 7   -248152.72824  23487146.149
  20817844.743     -387242.571 6   -301747.22925  20817851.322
  20658519.895      267583.67817    208507.26234  20658525.869
                            4  4
         ***   SATELLITE G 9   THIS EPOCH ON WLFACT 1 (L2)  COMMENT
         *** G 6 LOST LOCK AND THIS EPOCH ON WLFACT 2 (L2)  COMMENT
                (OPPOSITE TO PREVIOUS SETTINGS)             COMMENT

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|


+------------------------------------------------------------------------------+
|                                   TABLE A8                                   |
|                     GPS NAVIGATION MESSAGE FILE - EXAMPLE                    |
+------------------------------------------------------------------------------+

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           N: GPS NAV DATA                         RINEX VERSION / TYPE
XXRINEXN V2.10      AIUB                3-SEP-99 15:22      PGM / RUN BY / DATE
```

```
EXAMPLE OF VERSION 2.10 FORMAT                              COMMENT
     .1676D-07   .2235D-07  -.1192D-06  -.1192D-06          ION ALPHA
     .1208D+06   .1310D+06  -.1310D+06  -.1966D+06          ION BETA
     .133179128170D-06  .107469588780D-12   552960     1025 DELTA-UTC: A0,A1,T,W
    13                                                      LEAP SECONDS
                                                            END OF HEADER
 6 99  9  2 17 51 44.0 -.839701388031D-03 -.165982783074D-10  .000000000000D+00
     .910000000000D+02  .934062500000D+02  .116040547840D-08  .162092304801D+00
     .484101474285D-05  .626740418375D-02  .652112066746D-05  .515365489006D+04
     .409904000000D+06 -.242143869400D-07  .329237003460D+00 -.596046447754D-07
     .111541663136D+01  .326593750000D+03  .206958726335D+01 -.638312302555D-08
     .307155651409D-09  .000000000000D+00  .102500000000D+04  .000000000000D+00
     .000000000000D+00  .000000000000D+00  .000000000000D+00  .910000000000D+02
     .406800000000D+06  .000000000000D+00
13 99  9  2 19  0  0.0  .490025617182D-03  .204636307899D-11  .000000000000D+00
     .133000000000D+03 -.963125000000D+02  .146970407622D-08  .292961152146D+01
    -.498816370964D-05  .200239347760D-02  .928156077862D-05  .515328476143D+04
     .414000000000D+06 -.279396772385D-07  .243031939942D+01 -.558793544769D-07
     .110192796930D+01  .271187500000D+03 -.232757915425D+01 -.619632953057D-08
    -.785747015231D-11  .000000000000D+00  .102500000000D+04  .000000000000D+00
     .000000000000D+00  .000000000000D+00  .000000000000D+00  .389000000000D+03
     .410400000000D+06  .000000000000D+00

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

+------------------------------------------------------------------------------+
|                                   TABLE A9                                   |
|                      METEOROLOGICAL DATA FILE - EXAMPLE                      |
+------------------------------------------------------------------------------+
----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           METEOROLOGICAL DATA                     RINEX VERSION / TYPE
XXRINEXM V9.9       AIUB                3-APR-96 00:10      PGM / RUN BY / DATE
EXAMPLE OF A MET DATA FILE                                  COMMENT
A 9080                                                      MARKER NAME
     3    PR    TD    HR                                    # / TYPES OF OBSERV
PAROSCIENTIFIC      740-16B                       0.2    PR SENSOR MOD/TYPE/ACC
HAENNI                                            0.1    TD SENSOR MOD/TYPE/ACC
ROTRONIC            I-240W                        5.0    HR SENSOR MOD/TYPE/ACC
        0.0           0.0           0.0        1234.5678 PR SENSOR POS XYZ/H
                                                            END OF HEADER
 96  4  1  0  0 15  987.1   10.6   89.5
 96  4  1  0  0 30  987.2   10.9   90.0
 96  4  1  0  0 45  987.1   11.6   89.0

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|
```

```
 +----------------------------------------------------------------------------+
 |                                TABLE A10                                   |
 |        GLONASS NAVIGATION MESSAGE FILE - HEADER SECTION DESCRIPTION        |
 +--------------------+------------------------------------------+------------+
 |    HEADER LABEL    |               DESCRIPTION                |   FORMAT   |
 |  (Columns 61-80)   |                                          |            |
 +--------------------+------------------------------------------+------------+
 |RINEX VERSION / TYPE| - Format version (2.10)                  | F9.2,11X,  |#
 |                    | - File type ('G' = GLONASS nav mess data)|   A1,39X   |
 +--------------------+------------------------------------------+------------+
 |PGM / RUN BY / DATE | - Name of program creating current file  |     A20,   |
 |                    | - Name of agency  creating current file  |     A20,   |
 |                    | - Date of file creation (dd-mmm-yy hh:mm)|     A20    |
 +--------------------+------------------------------------------+------------+
*|COMMENT             | Comment line(s)                          |     A60    |*
 +--------------------+------------------------------------------+------------+
*|CORR TO SYSTEM TIME | - Time of reference for system time corr |            |*
 |                    |   (year, month, day)                     |     3I6,   |
 |                    | - Correction to system time scale (sec)  |  3X,D19.12 |
 |                    |   to correct GLONASS system time to      |            |
 |                    |   UTC(SU)                         (-TauC)|            |
 +--------------------+------------------------------------------+------------+
*|LEAP SECONDS        | Number of leap seconds since 6-Jan-1980  |     I6     |*
 +--------------------+------------------------------------------+------------+
 |END OF HEADER       | Last record in the header section.       |    60X     |
 +--------------------+------------------------------------------+------------+

                         Records marked with * are optional
```

```
 +----------------------------------------------------------------------------+
 |                                TABLE A11                                   |
 |         GLONASS NAVIGATION MESSAGE FILE - DATA RECORD DESCRIPTION          |
 +--------------------+------------------------------------------+------------+
 |    OBS. RECORD     | DESCRIPTION                              |   FORMAT   |
 +--------------------+------------------------------------------+------------+
 |PRN / EPOCH / SV CLK| - Satellite number:                      |     I2,    |
 |                    |       Slot number in sat. constellation  |            |
 |                    | - Epoch of ephemerides             (UTC) |            |
 |                    |     - year (2 digits, padded with 0,     |   1X,I2.2, |
 |                    |                if necessary)             |            |
 |                    |     - month,day,hour,minute,             |  4(1X,I2), |
 |                    |     - second                             |    F5.1,   |
```

```
 |                    | - SV clock bias (sec)            (-TauN)|   D19.12,  |
 |                    | - SV relative frequency bias    (+GammaN)|   D19.12,  |
 |                    | - message frame time                (tk)|   D19.12   |
 |                    |   (0 .le. tk .lt. 86400 sec of day UTC)  |            |
 |                    |                                         |            |
 |                    |   The 2-digit years in RINEX 1 and 2.xx  |            |
 |                    |   files are understood to represent      |            |
 |                    |   80-99: 1980-1999  and  00-79: 2000-2079|            |
 +--------------------+-----------------------------------------+------------+
 | BROADCAST ORBIT - 1| - Satellite position X      (km)         | 3X,4D19.12 |
 |                    | -          velocity X dot  (km/sec)     |            |
 |                    | -          X acceleration  (km/sec2)    |            |
 |                    | -          health (0=OK)            (Bn)|            |
 +--------------------+-----------------------------------------+------------+
 | BROADCAST ORBIT - 2| - Satellite position Y      (km)         | 3X,4D19.12 |
 |                    | -          velocity Y dot  (km/sec)     |            |
 |                    | -          Y acceleration  (km/sec2)    |            |
 |                    | -          frequency number (1-24)      |            |
 +--------------------+-----------------------------------------+------------+
 | BROADCAST ORBIT - 3| - Satellite position Z      (km)         | 3X,4D19.12 |
 |                    | -          velocity Z dot  (km/sec)     |            |
 |                    | -          Z acceleration  (km/sec2)    |            |
 |                    | - Age of oper. information  (days)   (E) |            |
 +--------------------+-----------------------------------------+------------+
```

```
+------------------------------------------------------------------------------+
|                                  TABLE A12                                   |
|                 GLONASS NAVIGATION MESSAGE FILE - EXAMPLE                    |
+------------------------------------------------------------------------------+

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           GLONASS NAV DATA                        RINEX VERSION / TYPE
ASRINEXG V1.1.0 VM  AIUB                19-FEB-98 10:42     PGM / RUN BY / DATE
STATION ZIMMERWALD                                          COMMENT
  1998     2    16    0.379979610443D-06                    CORR TO SYSTEM TIME
                                                            END OF HEADER
 3 98  2 15  0 15  0.0 0.163525342941D-03 0.363797880709D-11 0.108000000000D+05
    0.106275903320D+05-0.348924636841D+00 0.931322574615D-09 0.000000000000D+00
   -0.944422070313D+04 0.288163375854D+01 0.931322574615D-09 0.210000000000D+02
    0.212257280273D+05 0.144599342346D+01-0.186264514923D-08 0.300000000000D+01
 4 98  2 15  0 15  0.0 0.179599039257D-03 0.636646291241D-11 0.122400000000D+05
    0.562136621094D+04-0.289074897766D+00-0.931322574615D-09 0.000000000000D+00
   -0.236819248047D+05 0.102263259888D+01 0.931322574615D-09 0.120000000000D+02
    0.762532910156D+04 0.339257907867D+01 0.000000000000D+00 0.300000000000D+01
```

```
11 98  2 15  0 15  0.0-0.559808686376D-04-0.272848410532D-11 0.108600000000D+05
   -0.350348437500D+04-0.255325126648D+01 0.931322574615D-09 0.000000000000D+00
    0.106803754883D+05-0.182923507690D+01 0.000000000000D+00 0.400000000000D+01
    0.228762856445D+05 0.447064399719D+00-0.186264514923D-08 0.300000000000D+01
12 98  2 15  0 15  0.0 0.199414789677D-04-0.181898940355D-11 0.108900000000D+05
    0.131731816406D+05-0.143945598602D+01 0.372529029846D-08 0.000000000000D+00
    0.171148715820D+05-0.118937969208D+01 0.931322574615D-09 0.220000000000D+02
    0.135737919922D+05 0.288976097107D+01-0.931322574615D-09 0.300000000000D+01

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|


+------------------------------------------------------------------------------+
|                                  TABLE A13                                   |
|                     GLONASS OBSERVATION FILE - EXAMPLE                       |
+------------------------------------------------------------------------------+

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           OBSERVATION DATA    R (GLONASS)         RINEX VERSION / TYPE
XXRINEXO V1.1       AIUB                27-AUG-93 07:23     PGM / RUN BY / DATE
TST1                                                        MARKER NAME
VIEWEG              BRAUNSCHWEIG                            OBSERVER / AGENCY
100                 XX-RECEIVER         1.0                 REC # / TYPE / VERS
101                 XX-ANTENNA                              ANT # / TYPE
  3844808.114    715426.767   5021804.854                   APPROX POSITION XYZ
        1.2340         .0000         .0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     2    C1    L1                                          # / TYPES OF OBSERV
    10.000                                                  INTERVAL
  1993     8    23    14    24   40.0490000     GLO         TIME OF FIRST OBS
                                                            END OF HEADER
 93  8 23 14 24 40.0490000  0  3  2R01R21
  23986839.824       20520.565 5
  23707804.625       19937.231 5
  23834065.096       -9334.581 5
 93  8 23 14 24 50.0490000  0  3  2R01R21
  23992341.033       49856.525 5
  23713141.002       48479.290 5
  23831189.435      -24821.796 5
 93  8 23 14 25   .0490000  0  3  2R01R21
  23997824.854       79217.202 5
  23718494.110       77092.992 5
  23828329.946      -40219.918 5
 93  8 23 14 25 10.0490000  0  5  2R05R17R01R21
  24003328.910      108602.422 5
```

```
  24933965.449      -19202.780 5
  22203326.578       -2987.327 5
  23723851.686      105777.849 5
  23825485.526      -55529.205 5
 93  8 23 14 25 20.0490010  0  5  2R05R17R01R21
  24008828.023      138012.178 5
  24927995.616      -51188.500 5
  22202547.907       -7213.298 5
  23729236.758      134533.636 5
  23822662.277      -70749.590 5
 93  8 23 14 25 30.0490000  0  5  2R05R17R01R21
  24014330.779      167446.477 5
  24922041.288      -83151.666 5
  22201767.457      -11388.909 5
  23734633.024      163360.131 5
  23819848.894      -85881.102 5

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|
```

```
+------------------------------------------------------------------------------+
|                                  TABLE A14                                   |
|                MIXED GPS/GLONASS OBSERVATION FILE - EXAMPLE                  |
+------------------------------------------------------------------------------+
```

```
----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
YYRINEXO V2.8.1 VM  AIUB                 6-FEB-00 13:59     PGM / RUN BY / DATE
TST2                                                        MARKER NAME
001-02-A                                                    MARKER NUMBER
JIM                 Y-COMPANY                               OBSERVER / AGENCY
1                   YY-RECEIVER         2.0.1               REC # / TYPE / VERS
1                   GEODETIC L1                             ANT # / TYPE
  3851178.1849   -80151.4072  5066671.1013                  APPROX POSITION XYZ
        1.2340        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     0                                                WAVELENGTH FACT L1/2
     2    C1    L1                                          # / TYPES OF OBSERV
    10.000                                                  INTERVAL
    11                                                      LEAP SECONDS
  2000     2     6    11    53    0.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER
 00  2  6 11 53  0.0000000  0 14G23G07G02G05G26G09G21R20R19R12R02R11
                                R10R03
  22576523.586   -11256947.60212
  22360162.704   -16225110.75413
```

```
 24484865.974    14662682.882 2
 21950524.331   -13784707.24912
 22507304.252     9846064.848 2
 20148742.213   -20988953.712 4
 22800149.591   -16650822.70012
 19811403.273   -25116169.741 3
 23046997.513    -3264701.688 2
 22778170.622  -821857836.745 1
 22221283.991  -988088156.884 2
 19300913.475   -83282658.19013
 20309075.579  -672668843.84713
 23397403.484  -285457101.34211
00  2  6 11 53 10.0000000  0 14G23G07G02G05G26G09G21R20R19R12R02R11
                                R10R03
 22578985.016   -11244012.910 2
 22359738.890   -16227337.841 2
 24490324.818    14691368.710 2
 21944376.706   -13817012.849 2
 22512598.731     9873887.580 2
 20147322.111   -20996416.338 4
 22798942.949   -16657163.594 2
 19812513.509   -25110234.795 3
 23053885.702    -3227854.397 2
 22770607.029  -821898566.774 1
 22222967.297  -988079145.989 2
 19297913.736   -83298710.38413
 20313087.618  -672647337.04113
 23392352.454  -285484291.40311

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|
```

```
 +----------------------------------------------------------------------------+
 |                                 TABLE A15                                  |
 |     GEOSTATIONARY NAVIGATION MESSAGE FILE - HEADER SECTION DESCRIPTION     |
 +--------------------+------------------------------------------+------------+
 |    HEADER LABEL    |               DESCRIPTION                |   FORMAT   |
 |  (Columns 61-80)   |                                          |            |
 +--------------------+------------------------------------------+------------+
 |RINEX VERSION / TYPE| - Format version (2.10)                  | F9.2,11X,  |
 |                    | - File type ('H' = GEO nav mess data)    |   A1,39X   |
 +--------------------+------------------------------------------+------------+
 |PGM / RUN BY / DATE | - Name of program creating current file  |     A20,   |
 |                    | - Name of agency  creating current file  |     A20,   |
 |                    | - Date of file creation (dd-mmm-yy hh:mm)|     A20    |
 +--------------------+------------------------------------------+------------+
```

```
*|COMMENT            | Comment line(s)                          |     A60    |*
 +--------------------+------------------------------------------+------------+
*|CORR TO SYSTEM TIME | - Time of reference for system time corr |            |*
 |                    |   (year, month, day)                     |    3I6,    |
 |                    | - Correction to transform the GEO system | 3X,D19.12  |
 |                    |   time to UTC                        (W0)|            |
 +--------------------+------------------------------------------+------------+
*|LEAP SECONDS        | Number of leap seconds since 6-Jan-1980  |     I6     |*
 +--------------------+------------------------------------------+------------+
 |END OF HEADER       | Last record in the header section.       |    60X     |
 +--------------------+------------------------------------------+------------+

                        Records marked with * are optional


 +----------------------------------------------------------------------------+
 |                                 TABLE A16                                  |
 |      GEOSTATIONARY NAVIGATION MESSAGE FILE - DATA RECORD DESCRIPTION       |
 +--------------------+------------------------------------------+------------+
 |    OBS. RECORD     | DESCRIPTION                              |   FORMAT   |
 +--------------------+------------------------------------------+------------+
 |PRN / EPOCH / SV CLK| - Satellite number (PRN - 100)           |     I2,    |
 |                    | - Epoch of ephemerides (GPS)       (Toe) |            |
 |                    |     - year (2 digits, padded with 0      |            |
 |                    |                if necessary)             |   1X,I2.2, |
 |                    |     - month,day,hour,minute,             |  4(1X,I2), |
 |                    |     - second                             |    F5.1,   |
 |                    | - SV clock bias (sec)              (aGf0)|   D19.12,  |
 |                    | - SV relative frequency bias       (aGf1)|   D19.12,  |
 |                    | - message frame time (sec of day GPS)    |   D19.12   |
 +--------------------+------------------------------------------+------------+
 | BROADCAST ORBIT - 1| - Satellite position X      (km)         | 3X,4D19.12 |
 |                    | -           velocity X dot  (km/sec)     |            |
 |                    | -           X acceleration  (km/sec2)    |            |
 |                    | -           health (0=OK)                |            |
 +--------------------+------------------------------------------+------------+
 | BROADCAST ORBIT - 2| - Satellite position Y      (km)         | 3X,4D19.12 |
 |                    | -           velocity Y dot  (km/sec)     |            |
 |                    | -           Y acceleration  (km/sec2)    |            |
 |                    | - Accuracy code             (URA, meters)|            ||
 +--------------------+------------------------------------------+------------+
 | BROADCAST ORBIT - 3| - Satellite position Z      (km)         | 3X,4D19.12 |
 |                    | -           velocity Z dot  (km/sec)     |            |
 |                    | -           Z acceleration  (km/sec2)    |            |
 |                    | - spare                                  |            |
 +--------------------+------------------------------------------+------------+
```

```
+------------------------------------------------------------------------------+
|                                  TABLE A17                                   |
|                  MIXED GPS/GEO OBSERVATION FILE - EXAMPLE                    |
+------------------------------------------------------------------------------+

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           OBSERVATION DATA    M (MIXED)           RINEX VERSION / TYPE
RinExp V.2.0.2      TESTUSER            00-02-04 09:30      PGM / RUN BY / DATE
                                                            COMMENT
The file contains L1 pseudorange and phase data of the      COMMENT
geostationary AOR-E satellite (PRN 120 = S20)               COMMENT
                                                            COMMENT
TLSE D                                                      MARKER NAME
ESTB                TESTAGENCY                              OBSERVER / AGENCY
SGL98030069         Novatel Millennium  HW3-1 SW 4.45/2.3   REC # / TYPE / VERS
                    ASH701073.1                             ANT # / TYPE
  4629365.0750   112100.1790  4371619.4160                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     4    C1    L1    L2    P2                              # / TYPES OF OBSERV
     1                                                      INTERVAL
  2000     1    13    14    45    0.000000      GPS         TIME OF FIRST OBS
  2000     1    13    15     0    0.000000      GPS         TIME OF LAST OBS
     0                                                      RCV CLOCK OFFS APPL
                                                            END OF HEADER
 00 01 13 14 45  0.0000000  0  8G25G17G06G05G24G29G30S20             0.000535140
  21839900.207     -236148.877 9   -184047.71049  21839901.4384
  25151926.413     -161002.900 9   -125509.72447  25151935.8274
  20531103.515      763336.059 9    594797.53149  20531105.0114
  23001624.801     -432989.642 9   -337436.50348  23001628.1684
  23610349.510     -384890.728 9   -299952.38848  23610354.3504
  23954474.398     -151982.173 9   -118480.96847  23954481.1994
  20622367.016     -332628.466 9   -259214.55249  20622367.8754
  38137559.506      335849.135 9
 00 01 13 14 45  1.0000000  0  8G25G17G06G05G24G29G30S20             0.000535144
  21839500.278     -238250.743 9   -185685.52549  21839501.4814
  25151246.148     -164576.503 9   -128294.33947  25151256.2614
  20531084.382      763235.849 9    594719.44849  20531085.8784
  23002123.430     -430369.237 9   -335394.62748  23002126.7114
  23610670.127     -383205.864 9   -298639.51048  23610674.9834
  23955051.773     -148948.417 9   -116117.00748  23955058.5034
  20622558.579     -331621.765 9   -258430.11049  20622559.4574
  38137558.783      335846.284 9
```

```
 00 01 13 14 45  2.0000000  0  8G25G17G06G05G24G29G30S20            0.000535144
  21839100.418     -240352.173 9   -187323.00449  21839101.6534
  25150565.890     -168150.148 9   -131078.97647  25150576.2144
  20531065.378      763136.116 9    594641.73549  20531066.8984
  23002622.082     -427748.683 9   -333352.63648  23002625.3444
  23610990.819     -381520.461 9   -297326.20848  23610995.8424
  23955629.062     -145914.531 9   -113752.94748  23955636.5544
  20622750.161     -330614.723 9   -257645.40149  20622751.0554
  38137558.365      335843.457 9

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

+------------------------------------------------------------------------------+
|                                  TABLE A18                                   |
|                   GEO NAVIGATION MESSAGE FILE - EXAMPLE                      |
+------------------------------------------------------------------------------+

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|

     2.10           H: GEO NAV MSG DATA                     RINEX VERSION / TYPE
SuP v. 1.4          TESTUSER            04-02-00 10:04      PGM / RUN BY / DATE
                                                            COMMENT
The file contains navigation message data of the            COMMENT
geostationary AOR-E satellite (PRN 120 = S20)               COMMENT
                                                            COMMENT
                                                            END OF HEADER
20 00 01 13 14 46 24.0  .209547579288D-07 -.545696821064D-11  .532351280000D+05
     .406131052800D+08  .150625000000D+01  .875000000000D-04  .000000000000D+00
    -.112454290400D+08  .308125000000D+01 -.112500000000D-03  .400000000000D+01
     .781616000000D+05  .959600000000D+01 -.437500000000D-03  .000000000000D+00
20 00 01 13 14 48 00.0  .204890966415D-07 -.545696821064D-11  .533161280000D+05
     .406132503200D+08  .151500000000D+01  .875000000000D-04  .000000000000D+00
    -.112451338400D+08  .307000000000D+01 -.125000000000D-03  .400000000000D+01
     .790812000000D+05  .955600000000D+01 -.437500000000D-03  .000000000000D+00
20 00 01 13 14 49 36.0  .195577740669D-07 -.545696821064D-11  .533981280000D+05
     .406133961600D+08  .152375000000D+01  .875000000000D-04  .000000000000D+00
    -.112448396800D+08  .305875000000D+01 -.125000000000D-03  .400000000000D+01
     .799968000000D+05  .951600000000D+01 -.437500000000D-03  .000000000000D+00
20 00 01 13 14 51 12.0  .190921127796D-07 -.545696821064D-11  .534791280000D+05
     .406135428800D+08  .153250000000D+01  .875000000000D-04  .000000000000D+00
    -.112445465600D+08  .304687500000D+01 -.125000000000D-03  .400000000000D+01
     .809084000000D+05  .947600000000D+01 -.437500000000D-03  .000000000000D+00

----|---1|0---|---2|0---|---3|0---|---4|0---|---5|0---|---6|0---|---7|0---|---8|
```

Appendix III: some data files

File 95oct18casa___r0.rnx

```
     2              OBSERVATION DATA    GPS                 RINEX VERSION / TYPE
srx/v1.8.1.4        BAI                 95/10/19 03:18:22   PGM / RUN BY   / DATE
CASA                                                        MARKER NAME
4087-S                                                      MARKER NUMBER
gn2                 jpl                                     OBSERVER / AGENCY
138                 ROGUE SNR-8000      95.03.08            REC # / TYPE /  VERS
                    DORNE MARGOLIN T                        ANT # / TYPE
 -2444431.2031 -4428688.6270  3875750.1442                  APPROX POSITION  XYZ
      0.163000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
    30                                                      INTERVAL
     1     1     0                                          WAVELENGTH FACT L1/2
     4    L1    L2    P1    P2                              # / TYPES OF  OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:   >500  >100   >50   >10    >5    >0   bad   n/a      COMMENT
 sig:      9     8     7     6     5     4     1     0      COMMENT
  1995    10    18    00    00   00.000000                  TIME OF FIRST OBS
  1995    10    18    23    59   30.000000                  TIME OF LAST OBS
    24                                                      # OF SATELLITES
    01   783   783   783   783                              PRN / # OF OBS
    02   878   878   878   878                              PRN / # OF OBS
    04   925   925   925   925                              PRN / # OF OBS
    05   746   746   746   746                              PRN / # OF OBS
    06   762   762   762   762                              PRN / # OF OBS
    07   793   793   793   793                              PRN / # OF OBS
    09   907   907   907   907                              PRN / # OF OBS
    14   739   739   739   739                              PRN / # OF OBS
    15   973   973   973   973                              PRN / # OF OBS
    16   936   936   936   936                              PRN / # OF OBS
    17   848   848   848   848                              PRN / # OF OBS
    18   740   740   740   740                              PRN / # OF OBS
    19   883   883   883   883                              PRN / # OF OBS
    20   876   876   876   876                              PRN / # OF OBS
    21   871   871   871   871                              PRN / # OF OBS
    22   891   891   891   891                              PRN / # OF OBS
    23   835   835   835   835                              PRN / # OF OBS
    24   737   737   737   737                              PRN / # OF OBS
    25   874   874   874   874                              PRN / # OF OBS
```

```
   26  1046  1046  1046  1046                              PRN / # OF OBS
   27   843   843   843   843                              PRN / # OF OBS
   28   956   956   956   956                              PRN / # OF OBS
   29   836   836   836   836                              PRN / # OF OBS
   31  1041  1041  1041  1041                              PRN / # OF OBS
                                                           END OF HEADER
95 10 18 00 00 00.0000000  0  6 14 15 18 22 25 29
-20141789.28908 -15694892.26208  20764791.10308  20764791.88908
-10156688.05308  -7914296.97108  23025606.13308  23025608.42008
 -1005974.21907   -783874.88007  24656587.15107  24656589.16307
-12846588.72508 -10010318.02408  22508513.35408  22508514.93708
-15501368.59408 -12078973.35808  22258999.20508  22258999.63208
 -8778399.37908  -6840304.85208  22409115.47708  22409115.63508
95 10 18 00 00 30.0000000  0  6 14 15 18 22 25 29
-20180843.59808 -15725324.18808  20757359.26608  20757360.16208
-10059627.94808  -7838665.76408  23044076.11208  23044078.37308
 -1089522.05507   -848977.05707  24640688.39407  24640690.63507
-12918063.69808 -10066012.75108  22494912.01508  22494913.64808
-15427158.70008 -12021147.48308  22273121.01508  22273121.20808
 -8834322.07108  -6883880.95608  22398473.85408  22398474.05408
95 10 18 00 01 00.0000000  0  6 14 15 18 22 25 29
-20219463.08808 -15755417.29708  20750010.06208  20750010.98808
 -9962343.71908  -7762859.92208  23062588.65308  23062591.02308
 -1173023.79007   -914043.30307  24624798.01507  24624800.80007
-12989135.97508 -10121393.69808  22481387.45608  22481388.93408
-15352642.34508 -11963082.79508  22287301.00108  22287301.34708
 -8890128.47608  -6927366.45408  22387854.08208  22387854.42908
95 10 18 00 01 30.0000000  0  6 14 15 18 22 25 29
-20257638.92508 -15785164.69808  20742745.40308  20742746.30108
 -9864838.43508  -7686881.82608  23081143.25108  23081145.55608
 -1256465.81607   -979063.01707  24608919.43807  24608922.07207
-13059811.85808 -10176465.76608  22467938.31708  22467939.90208
-15277821.67508 -11904780.98508  22301538.84908  22301539.27308
 -8945816.58908  -6970759.77208  22377257.01008  22377257.22508
95 10 18 00 02 00.0000000  0  6 14 15 18 22 25 29
-20295366.06008 -15814562.46208  20735566.15808  20735566.93908
 -9767115.42008  -7610734.07008  23099739.47908  23099741.64008
 -1339857.31807  -1044043.36207  24593050.72507  24593053.16807
-13130100.61608 -10231236.17508  22454562.75208  22454564.08708
-15202710.66308 -11846252.93808  22315832.09408  22315832.42008
 -9001386.17408  -7014060.73808  22366682.49008  22366682.61908
```

File 95oct18casa___r0.eph

```
     2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
srx/v1.8.1.4        BAI                 95/10/19 03:18:35   PGM / RUN BY / DATE
CASA                                                        COMMENT
 -2444431.2031 -4428688.6270  3875750.1442                  COMMENT
                                                            END OF HEADER
14 95 10 18 00 51 44.0 1.129414886236D-05 1.136868377216D-13 0.000000000000D+00
    1.730000000000D+02-5.175000000000D+01 4.375182243902D-09-5.836427291652D-01
   -2.712011337280D-06 2.427505562082D-03 8.568167686462D-06 5.153718931198D+03
    2.623040000000D+05 4.470348358154D-08 1.698435481558D+00 1.676380634308D-08
    9.636381916043D-01 2.153437500000D+02 3.056960010495D+00-8.030691653399D-09
   -5.178787145843D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.730000000000D+02
    2.592180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
18 95 10 18 00 51 44.0-3.725290298462D-06-2.273736754432D-13 0.000000000000D+00
    2.120000000000D+02 2.618750000000D+01 4.973421448680D-09-5.133230702863D-01
    1.206994056702D-06 6.170925335027D-03 9.013339877129D-06 5.153748090744D+03
    2.623040000000D+05-1.303851604462D-07 2.681992356113D+00 4.842877388000D-08
    9.431056887089D-01 1.989062500000D+02 1.455299735875D+00-8.303560162325D-09
    2.064371703653D-10 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00-1.862645149231D-09 2.120000000000D+02
    2.642280000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
22 95 10 18 00 51 44.0 2.556233666837D-04 2.842170943040D-12 0.000000000000D+00
    3.900000000000D+01-4.750000000000D+00 4.843058875699D-09 6.055024966723D-01
   -1.490116119385D-08 8.348581031896D-03 7.729977369308D-06 5.153607444763D+03
    2.623040000000D+05 2.421438694000D-08-1.496151018112D+00 1.378357410431D-07
    9.486056799085D-01 2.211875000000D+02-6.486990867494D-02-8.231771457751D-09
   -3.639437311458D-10 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 3.900000000000D+01
    2.654280000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
29 95 10 18 00 51 44.0 1.882202923298D-06 4.547473508865D-13 0.000000000000D+00
    2.180000000000D+02 2.471875000000D+01 4.634478758928D-09-3.024837471822D+00
    1.160427927971D-06 4.891381249763D-03 9.119510650635D-06 5.153622446060D+03
    2.623040000000D+05 4.656612873077D-08 2.687151563471D+00 1.676380634308D-08
    9.536605855970D-01 1.977500000000D+02-1.863201500290D+00-7.982475359282D-09
    2.396528396456D-10 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00-9.313225746155D-10 2.180000000000D+02
    2.658180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
04 95 10 18 02 00  0.0 4.873285070062D-05 1.818989403546D-12 0.000000000000D+00
    1.480000000000D+02-2.778125000000D+01 4.546617956316D-09-2.539773236047D+00
   -1.389533281326D-06 3.546471474692D-03 4.887580871582D-06 5.153623073578D+03
    2.664000000000D+05 1.490116119385D-08 6.257514816552D-01 3.911554813385D-08
    9.688215766103D-01 2.906875000000D+02-1.130434761111D+00-8.163197172786D-09
    1.571494030463D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.480000000000D+02
    2.596680000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

File 95oct18casa.a

```
casa 291    0 14  -3832855.061  -3832852.989  20764791.103  20764791.889  1
casa 291    0 15  -1932753.473  -1932752.152  23025606.133  23025608.420  1
casa 291    0 18   -191430.529   -191430.252  24656587.151  24656589.163  1
casa 291    0 22  -2444624.551  -2444621.901  22508513.354  22508514.937  1
casa 291    0 25  -2949812.363  -2949808.662  22258999.205  22258999.632  1
casa 291    0 29  -1670473.859  -1670472.308  22409115.477  22409115.635  1
casa 291   30 14  -3840286.848  -3840284.776  20757359.266  20757360.162  1
casa 291   30 15  -1914283.549  -1914282.239  23044076.112  23044078.373  1
casa 291   30 18   -207329.153   -207328.868  24640688.394  24640690.635  1
casa 291   30 22  -2458225.787  -2458223.122  22494912.015  22494913.648  1
casa 291   30 25  -2935690.690  -2935686.992  22273121.015  22273121.208  1
casa 291   30 29  -1681115.594  -1681114.037  22398473.854  22398474.054  1
casa 291   60 14  -3847635.893  -3847633.821  20750010.062  20750010.988  1
casa 291   60 15  -1895770.976  -1895769.678  23062588.653  23062591.023  1
casa 291   60 18   -223219.005   -223218.710  24624798.015  24624800.800  1
casa 291   60 22  -2471750.391  -2471747.715  22481387.456  22481388.934  1
casa 291   60 25  -2921510.699  -2921507.003  22287301.001  22287301.347  1
casa 291   60 29  -1691735.199  -1691733.640  22387854.082  22387854.429  1
casa 291   90 14  -3854900.513  -3854898.440  20742745.403  20742746.301  1
casa 291   90 15  -1877216.337  -1877215.051  23081143.251  23081145.556  1
casa 291   90 18   -239097.495   -239097.188  24608919.438  24608922.072  1
casa 291   90 22  -2485199.565  -2485196.877  22467938.317  22467939.902  1
casa 291   90 25  -2907272.799  -2907269.105  22301538.849  22301539.273  1
casa 291   90 29  -1702332.295  -1702330.732  22377257.010  22377257.225  1
casa 291  120 14  -3862079.748  -3862077.674  20735566.158  20735566.939  1
casa 291  120 15  -1858620.266  -1858618.992  23099739.479  23099741.640  1
casa 291  120 18   -254966.370   -254966.052  24593050.725  24593053.168  1
casa 291  120 22  -2498575.070  -2498572.370  22454562.752  22454564.087  1
casa 291  120 25  -2892979.649  -2892975.958  22315832.094  22315832.420  1
casa 291  120 29  -1712906.835  -1712905.270  22366682.490  22366682.619  1
casa 291  150 14  -3869175.843  -3869173.768  20728470.119  20728470.864  1
casa 291  150 15  -1839983.374  -1839982.111  23118376.275  23118378.699  1
casa 291  150 18   -270826.629   -270826.296  24577190.126  24577193.081  1
casa 291  150 22  -2511880.571  -2511877.859  22441257.344  22441258.630  1
casa 291  150 25  -2878634.795  -2878631.106  22330177.069  22330177.149  1
casa 291  150 29  -1723459.581  -1723458.013  22356129.631  22356129.929  1
casa 291  180 14  -3876191.090  -3876189.014  20721455.105  20721455.758  1
casa 291  180 15  -1821306.327  -1821305.074  23137053.370  23137055.648  1
casa 291  180 18   -286680.271   -286679.921  24561336.623  24561339.450  1
casa 291  180 22  -2525117.232  -2525114.507  22428020.316  22428021.802  1
casa 291  180 25  -2864239.845  -2864236.158  22344571.874  22344572.165  1
casa 291  180 29  -1733987.305  -1733985.732  22345601.764  22345602.227  1
casa 291  210 14  -3883124.276  -3883122.201  20714521.919  20714522.686  1
casa 291  210 15  -1802589.714  -1802588.473  23155769.979  23155772.345  1
```

File 95oct18.b

```
01 291 64800 26560710.417338371  3.318603849D-03  9.545863611D-01 -20.896243479119998 -1.353453453D+00  1.066747756D+00
01 291 72000 26560720.404535621  3.318444942D-03  9.545886827D-01 -21.421333508132399 -1.353299314D+00  2.116753977D+00
01 291 75104 26560733.497989449  3.317848895D-03  9.545894551D-01 -21.647705449028965 -1.353219885D+00  2.569409848D+00
01 291 79200 26560733.301396008  3.317846450D-03  9.545902597D-01 -21.946422979773799 -1.353215379D+00 -3.116355992D+00
01 291 82304 26560761.552602175  3.316573216D-03  9.545908698D-01 -22.172795006982365 -1.353426746D+00 -2.663409650D+00
02 291 14400 26560437.008677519  1.485944062D-02  9.484849160D-01 -21.462377253220196 -2.501756070D+00  2.337358114D+00
02 291 17504 26560413.692240424  1.486066693D-02  9.484844566D-01 -21.688749397461766 -2.501768192D+00  2.790112915D+00
02 291 21600 26560413.672573983  1.486066658D-02  9.484836827D-01 -21.987467191532598 -2.501768024D+00 -2.895637510D+00
02 291 24704 26560411.706610158  1.486079418D-02  9.484829937D-01 -22.213839834629165 -2.501767430D+00 -2.442893929D+00
02 291 36000 26560381.509273152  1.486041850D-02  9.484799933D-01 -23.037647675268399 -2.501667688D+00 -7.953793623D-01
02 291 43200 26560391.653688494  1.486092095D-02  9.484774361D-01 -23.562740586229800 -2.501675582D+00  2.548099460D-01
04 291 7200 26559830.7845155522  3.546471474D-03  9.688215766D-01 -18.800443269153600 -1.130434761D+00 -2.539773236D+00
04 291 14400 26559832.829102169  3.546654945D-03  9.688212986D-01 -19.325534642588497 -1.130492091D+00 -1.489503489D+00
04 291 28800 26559835.109611436  3.546280786D-03  9.688217506D-01 -20.375718605143099 -1.130687663D+00  6.111150725D-01
04 291 31904 26559838.923550639  3.546478110D-03  9.688221208D-01 -20.602091663708666 -1.130648478D+00  1.063833774D+00
04 291 36000 26559838.903894715  3.546462161D-03  9.688222159D-01 -20.900810437934296 -1.130650294D+00  1.661289368D+00
04 291 39104 26559831.393962409  3.546592546D-03  9.688223124D-01 -21.127183266821667 -1.130760478D+00  2.114157644D+00
04 291 50400 26559829.015163395  3.546697436D-03  9.688230058D-01 -21.950993777285798 -1.130708762D+00 -2.521412462D+00
05 291 36000 26561271.374434988  1.767611713D-03  9.511125875D-01 -23.022652421169397 -1.905754870D+00  3.061358450D+00
05 291 39104 26561295.123773131  1.766463974D-03  9.511117770D-01 -23.249024939918967 -1.906010860D+00 -2.768849022D+00
05 291 43200 26561295.104116678  1.766466768D-03  9.511106389D-01 -23.547743401079799 -1.906009690D+00 -2.171445158D+00
05 291 46304 26561268.877612184  1.766639645D-03  9.511095944D-01 -23.774115960790368 -1.905250549D+00 -1.719483398D+00
```

File 95oct18.clocks

```
01 291 64800  6.078188307583D-04 -2.160049916711D-12  0.000000000000D+00
01 291 72000  6.078034639359D-04 -2.160049916711D-12  0.000000000000D+00
01 291 75104  6.078099831939D-04 -2.046363078989D-12  0.000000000000D+00
01 291 79200  6.077880971134D-04 -2.160049916711D-12  0.000000000000D+00
01 291 82304  6.077950820327D-04 -2.046363078989D-12  0.000000000000D+00
02 291 14400 -2.010073512793D-04 -2.728484105319D-12  0.000000000000D+00
02 291 17504 -2.010175958276D-04 -2.728484105319D-12  0.000000000000D+00
02 291 21600 -2.010273747146D-04 -2.728484105319D-12  0.000000000000D+00
02 291 24704 -2.010376192629D-04 -2.728484105319D-12  0.000000000000D+00
02 291 36000 -2.010688185692D-04 -2.728484105319D-12  0.000000000000D+00
02 291 43200 -2.010888420045D-04 -2.728484105319D-12  0.000000000000D+00
04 291 7200   4.873285070062D-05  1.818989403546D-12  0.000000000000D+00
04 291 14400  4.874588921666D-05  1.818989403546D-12  0.000000000000D+00
04 291 28800  4.877196624875D-05  1.818989403546D-12  0.000000000000D+00
04 291 31904  4.877336323261D-05  1.818989403546D-12  0.000000000000D+00
04 291 36000  4.878453910351D-05  1.818989403546D-12  0.000000000000D+00
04 291 39104  4.878640174866D-05  1.818989403546D-12  0.000000000000D+00
04 291 50400  4.880642518401D-05  1.818989403546D-12  0.000000000000D+00
05 291 36000  4.221685230732D-06  2.160049916711D-12  0.000000000000D+00
05 291 39104  4.241243004799D-06  2.273736754432D-12  0.000000000000D+00
05 291 43200  4.237517714500D-06  2.160049916711D-12  0.000000000000D+00
```

File 97jan09coco___r0.rnx

```
     2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX  AUSLIG              10-JAN-97 10:19     PGM / RUN BY /  DATE
Australian Regional GPS Network (ARGN)  - COCOS ISLAND      COMMENT
BIT 2 OF LLI(+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
     -0.000000000103          HARDWARE CALIBRATION (S)      COMMENT
     -0.000000054663          CLOCK OFFSET (S)              COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100    93.05.25 / 2.8.33.2  REC # / TYPE  / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
    30                                                      INTERVAL
  1997     1     9     0     7   30.000000                  TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094
  24610920.702     -924108.174 6   -720085.67045  24610920.0404
  20718775.074   -18605935.474 9 -14498133.97346  20718775.6074
  20842713.610   -19083282.892 9 -14870090.55546  20842713.4814
 97  1  9  0  8  0.0000000  0  7  1 25  9  5 23 17  6
  22133910.078   -14236004.699 8 -11092992.05945  22133910.3834
  22655271.274   -11599563.608 7  -9038621.81245  22655271.4994
  22612558.689   -12856969.398 7 -10018409.02845  22612559.6234
  22734737.216   -11602224.451 7  -9040689.79045  22734738.4484
  24604536.459     -957654.791 6   -746225.94245  24604534.7294
  20705341.319   -18676530.135 9 -14553142.81046  20705341.9424
  20853116.611   -19028615.135 9 -14827492.32746  20853116.7804
 97  1  9  0  8 30.0000000  0  7  1 25  9  5 23 17  6
  22140190.352   -14203001.317 8 -11067275.16945  22140191.1924
  22638420.202   -11688115.262 7  -9107623.10345  22638419.4574
  22630271.840   -12763886.669 7  -9945877.07545  22630271.8724
  22738365.024   -11583159.458 7  -9025833.98645  22738365.9064
  24598097.384     -991492.176 6   -772592.83444  24598100.2324
  20691984.567   -18746719.862 9 -14607836.11046  20691985.4654
  20863557.500   -18973748.285 9 -14784738.95746  20863557.4954
```

File 1995-10-18.eci

```
10 1995 10 17 11 59 50.00 -4570.23221 22343.26956-14207.54335-1.72076536-2.11129485-2.69102630  0
10 1995 10 17 12 14 50.00 -6076.58950 20262.45361-16504.48943-1.62197124-2.50659329-2.40599804  0
10 1995 10 17 12 29 50.00 -7481.50029 17843.44999-18525.86601-1.49559158-2.86174702-2.07941433  0
10 1995 10 17 12 44 50.00 -8760.91454 15125.21520-20236.44381-1.34342728-3.17056668-1.71621512  0
10 1995 10 17 12 59 50.00 -9892.60246 12152.01364-21605.74702-1.16772081-3.42746450-1.32201601  0
10 1995 10 17 13 14 50.00-10856.54635  8972.82673-22608.63840 -.97114245-3.62756433 -.90305463  0
10 1995 10 17 13 29 50.00-11635.31641  5640.66287-23225.84923 -.75676842-3.76681175 -.46612187  0
10 1995 10 17 13 44 50.00-12214.42283  2211.76924-23444.44031 -.52804991-3.84208133 -.01847656  0
10 1995 10 17 13 59 50.00-12582.63548 -1255.25026-23258.17795 -.28877162-3.85127741  .43225753  0
10 1995 10 17 14 14 50.00-12732.26139 -4700.40323-22667.80937 -.04299943-3.79342382  .87821269  0
10 1995 10 17 14 29 50.00-12659.36983 -8063.35089-21681.22129  .20498341-3.66873770 1.31141546  0
10 1995 10 17 14 44 50.00-12363.95517-11284.50749-20313.46705  .45075120-3.47868207 1.72394031  0
10 1995 10 17 14 59 50.00-11850.02752-14306.17561-18586.64947  .68982067-3.22599211 2.10807561  0
10 1995 10 17 15 14 50.00-11125.62301-17073.69143-16529.65060  .91774452-2.91467036 2.45649755  0
10 1995 10 17 15 29 50.00-10202.72709-19536.55031-14177.70322 1.13021063-2.54994727 2.76244628  0
10 1995 10 17 15 44 50.00 -9097.10708-21649.48026-11571.80544 1.32314404-2.13820471 3.01989776  0
10 1995 10 17 15 59 50.00 -7828.05306-23373.42988 -8757.98524 1.49280782-1.68686222 3.22372395  0
10 1995 10 17 16 14 50.00 -6418.03012-24676.43742 -5786.42870 1.63589859-1.20422731 3.36983413  0
10 1995 10 17 16 29 50.00 -4892.24860-25534.35055 -2710.49242 1.74963251 -.69931413 3.45529027  0
10 1995 10 17 16 44 50.00 -3278.16264-25931.37064   414.37415 1.83181793 -.18163605 3.47839058  0
10 1995 10 17 16 59 50.00 -1604.91090-25860.40172  3531.76545 1.88091096  .33901976 3.43871688  0
10 1995 10 17 17 14 50.00    97.28443-25323.19163  6585.48190 1.89605185  .85282690 3.33714327  0
10 1995 10 17 17 29 50.00  1797.74176-24330.26159  9520.68224 1.87708037 1.35015382 3.17580594  0
10 1995 10 17 17 44 50.00  3465.94881-22900.62882 12284.99177 1.82453030 1.82178812 2.95803585  0
10 1995 10 17 17 59 50.00  5072.17706-21061.33515 14829.53493 1.73960365 2.25914209 2.68825859  0
10 1995 10 17 18 14 50.00  6588.06462-18846.80158 17109.86454 1.62412670 2.65443288 2.37186634 -1
10 1995 10 17 18 29 50.00  7987.15251-16298.03436 19086.76631 1.48049086 3.00083151 2.01506919 -1
10 1995 10 17 18 44 50.00  9245.36193-13461.71203 20726.92236 1.31158136 3.29257802 1.62473248 -1
10 1995 10 17 18 59 50.00 10341.40344-10389.18488 22003.42517 1.12069781 3.52506128 1.20820762 -1
10 1995 10 17 19 14 50.00 11257.11227 -7135.41784 22896.13851  .91147002 3.69486457  .77316262 -1
10 1995 10 17 19 29 50.00 11977.70696 -3757.90718 23391.90817  .68777251 3.79977931  .32741851 -1
10 1995 10 17 19 44 50.00 12491.97165  -315.59775 23484.62963  .45364057 3.83879043 -.12120431 -1
10 1995 10 17 19 59 50.00 12792.36449  3132.17550 23175.18337  .21319046 3.81203791 -.56502617 -1
10 1995 10 17 20 14 50.00 12875.05649  6526.70234 22471.25091 -.02945500 3.72075910 -.99663264 -1
10 1995 10 17 20 29 50.00 12739.90672  9810.85330 21387.02618 -.27023373 3.56721646-1.40898282 -1
10 1995 10 17 20 44 50.00 12390.37990 12929.96577 19942.83688 -.50520455 3.35461499-1.79550097 -1
10 1995 10 17 20 59 50.00 11833.41338 15832.65263 18164.69040 -.73059985 3.08701340-2.15015272 -1
10 1995 10 17 21 14 50.00 11079.24001 18471.52918 16083.75775 -.94287097 2.76923174-2.46750705 -1
10 1995 10 17 21 29 50.00 10141.17301 20803.85646 13735.80739-1.13872723 2.40675827-2.74278554 -1
10 1995 10 17 21 44 50.00  9035.35855 22792.10089 11160.59986-1.31516896 2.00565703-2.97190111 -1
10 1995 10 17 21 59 50.00  7780.50085 24404.41158  8401.25153-1.46951558 1.57247723-3.15148768 -1
10 1995 10 17 22 14 50.00  6397.56390 25615.01701  5503.57511-1.59942963 1.11416481-3.27892274 -1
10 1995 10 17 22 29 50.00  4909.45323 26404.54312  2515.40245-1.70293732  .63797619-3.35234411 -1
10 1995 10 17 22 44 50.00  3340.68028 26760.25448  -514.10547-1.77844646  .15139338-3.37066210 -1
10 1995 10 17 22 59 50.00  1717.01172 26676.21948 -3535.15778-1.82476230 -.33795994-3.33356778 -1
10 1995 10 17 23 14 50.00    65.10540 26153.40000 -6498.03893-1.84110144 -.82240282-3.24153789 -1
```

Appendix IV: list of Programs

FORTRAN programs

Below is the list of some FORTRAN programs used in the practical sessions of the book. These programs use subroutines to invert and multiply matrix, finding examples of them in the book *Numerical Recipes*.

- **Program cart2esf:**

It makes the change from Cartesian coordinates to Spherical.

```
c2345678901234567
      implicit double precision (a-h,o-z)
c ------------------------------------------------------------
c              (x,y,z) ---->|cart2esf| ---> (r, alfa, delta)
c                                              in degrees
c     Example:
c             Execute: echo "1 0 0"|cart2esf
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------

        pi=3.1415926535898d0

10      continue
        read (*,*,end=100) x,y,z
        r=dsqrt(x**2+y**2+z**2)
        alfa=datan2(y,x)
        if (alfa.lt.0.d0) alfa=alfa+2.d0*pi
        delta=datan2(z,dsqrt(x**2+y**2))
        print *, r, alfa*180.d0/pi, delta*180.d0/pi
        goto 10

100     continue
        end
```

- **Program esf2cart:**

  It makes the change from Spherical coordinates to Cartesian.

```
c234567
      implicit double precision (a-h,o-z)

c -------------------------------------------------------------
c
c        (r, alfa, delta)  ---->|esf2cart| ---> (x,y,z)
c           in degrees
c      Example:
c           Execute: echo "1 90 90"|esf2cart
c
c       @gAGE (Research group of Astronomy and GEomatics).
c -------------------------------------------------------------

        pi=3.1415926535898d0

10      continue

        read (*,*,end=100) r,a,d

        a=a*pi/180.d0
        d=d*pi/180.d0

        x=r*dcos(a)*dcos(d)
        y=r*dsin(a)*dcos(d)
        z=r*dsin(d)
        write(*,'(f16.9,1x,f16.9,1x,f16.9)') x,y,z
        goto 10

100     continue
end
```

- **Program ymdUT2sid:**

Calculation of the Julian day and sidereal time starting from year, month, day, and hour.

```
c234567890
        implicit double precision (a-h,o-z)
        double precision jd
c       ------------------------------------------------------
c       It calculates the JULIAN DAY  and the SIDEREAL MEAN
c       time for a determined date and hour (i.e., the
c       hour angle at the Aries point):
c       year month day hour(UT)-->|ymdUT2sid|--> jd sid(hh.hh)
c       Execute:
c         echo "1978 11 13 0"| ymdUT2sid
c         result:
c                    2443825.5    3.45038611041   3  27  1.39000
c
c       @gAGE (Research group of Astronomy and GEomatics).
c       ------------------------------------------------------
        read(*,*) xy,xm,xd,xt
        if (xm.le.2.d0) then
           xy=xy-1.d0
           xm=xm+12.d0
        endif
        jd=int(365.25d0*xy)+int(30.6001d0*(xm+1.d0))+xd
     *     +xt/24.d0+1720981.5d0
        tt=(jd-2451545.d0)/36525.d0
        sid1=24110.54841d0+8640184.812866d0*tt
     *       +0.093104d0*tt**2-(6.2d-6)*(tt**3)
        sid1=(sid1/3600.d0+xt)
        sid1=dmod(sid1,24.d0)
        if (sid1.lt.0.d0) sid1=sid1+24.d0
        ih=int(sid1)
        xmm=(sid1-dble(ih))*60.d0
        mm=int(xmm)
        xss=(xmm-dble(mm))*60.d0
        write(*,'(f10.1,1x,f16.11,1x,i3,1x,i3,1x,f8.5)')
     *        jd, sid1, ih,mm,xss
        end
```

- **Program wgs2eq.f:**
  It makes the change from terrestrial coordinates (WGS84) to equatorial.

```
c234567
      implicit double precision (a-h,o-z)
      dimension r(3), rp(3)

c --------------------------------------------------------------
c
c          (t_sid,x,y,z)  ---->|wgs2eq| ---> (x,y,z)
c              WGS84                      equatorial coord.
c         x => Greenwich                   x => Aries
c         z => North Pole                  z => North Pole
c
c   Example:
c            echo "3.460 16336.506 7596.636 -19390.923"|wgs2eq
c
c       @gAGE (Research group of Astronomy and GEomatics).
c --------------------------------------------------------------

        pi=3.1415926535898d0

        read (*,*) ts,r
        ts=ts*pi/12.d0
        call rot3(-ts,r,rp)

        write(*,'(f12.5,1x,f12.5,1x,f12.5)') rp

        end
c ---------------------------------------------------------------

        subroutine rot3(ang,r,rp)
        implicit double precision (a-h,o-z)
        dimension r(3),rp(3)

        rp(1)=cos(ang)*r(1)+sin(ang)*r(2)
        rp(2)=-sin(ang)*r(1)+cos(ang)*r(2)
        rp(3)=r(3)
        end
c ---------------------------------------------------------------
```

- **Program eq2wgs:**

  It makes the change from equatorial coordinates to terrestrial.

```
c234567
      implicit double precision (a-h,o-z)
      dimension r(3), rp(3)

c -----------------------------------------------------------
c
c         (t_sid,x,y,z)  -----> |eq2wgs| --------> (x,y,z)
c      equatorial coord.                              WGS84
c         x => Aries                          x => Greenwich
c         z => North Pole                     z => North Pole
c   Example:
c    echo "3.460 4099.155 17543.866 -19390.923"|eq2wgs
c
c    @gAGE (Research group of Astronomy and GEomatics).
c -----------------------------------------------------------

        pi=3.1415926535898d0

        read (*,*) ts,r
        ts=ts*pi/12.d0
        call rot3(ts,r,rp)

        write(*,'(f12.5,1x,f12.5,1x,f12.5)') rp

        end
c -----------------------------------------------------------

        subroutine rot3(ang,r,rp)
        implicit double precision (a-h,o-z)
        dimension r(3),rp(3)

        rp(1)=cos(ang)*r(1)+sin(ang)*r(2)
        rp(2)=-sin(ang)*r(1)+cos(ang)*r(2)
        rp(3)=r(3)
        end
c -----------------------------------------------------------
```

- **Programe wgs2eq_ts:**

The same as in wgs2eq, but the sidereal time is calculated by the program itself.

```
c234567
      implicit double precision (a-h,o-z)
      dimension r(3), rp(3)

c -------------------------------------------------------------
c
c       (yy mm dd UT,x,y,z)  ---->|wgs2eq_ts| ---> (x,y,z)
c           WGS84                            equatorial coord.
c        x => Greenwich                       x => Aries
c        z => North Pole                      z => North Pole
c
c  Example:
c  echo "1998 11 13 0 16336.5 7596.6 -19390.9 "|wgs2eq_ts
c
c    @gAGE (Research group of Astronomy and GEomatics).
c -------------------------------------------------------------

        pi=3.1415926535898d0

        read (*,*)  ay,am,ad,ah,r

c       Sidereal time calculation
        call sid0TU(ay,am,ad,ts)
        ts=ts+1.00273790934d0*ah
        ts=dmod(ts,24.d0)
c       print *, ay,am,ad,ah,ts
        ts=ts*pi/12.d0

c       Coordinate transformation
        call rot3(-ts,r,rp)

        write(*,'(f12.5,1x,f12.5,1x,f12.5)') rp

        end

c _____________________________________________________________
```

```
      subroutine rot3(ang,r,rp)
      implicit double precision (a-h,o-z)
      dimension r(3),rp(3)

      rp(1)=dcos(ang)*r(1)+dsin(ang)*r(2)
      rp(2)=-dsin(ang)*r(1)+dcos(ang)*r(2)
      rp(3)=r(3)
      end
c ------------------------------------------------------------
      subroutine sid0TU(xy,xm,xd,sid)
      implicit double precision (a-h,o-z)
      double precision jd

c     ----------------------------------------------------
c     year month day -->|sid0TU|--> sid(hh.hh)
c     ----------------------------------------------------

      if (xm.le.2.) then
         xy=xy-1.d0
         xm=xm+12.d0
      endif

c     Julian day (jd)
      jd=int(365.25d0*xy)+int(30.6001d0*(xm+1.d0))+xd+
     * +1720981.5d0

c     Sidereal time calculation at 0h TU(sid)
      tt=(jd-2451545.d0)/36525.d0

      sid=24110.54841d0+8640184.812866d0*tt
     *     +0.093104d0*tt**2-(6.2d-6)*(tt**3)
      sid=sid/3600.d0
      sid=dmod(sid,24.d0)
      if (sid.lt.0.d0) sid=sid+24.d0

      end
c ------------------------------------------------------------
```

- **Program eq2wgs_ts:**

The same as in eq2wgs, but the sidereal time is calculated by the program itself.

```
c234567
      implicit double precision (a-h,o-z)
      dimension r(3), rp(3)

c ------------------------------------------------------------
c
c   (yy,mm,dd,hh,x,y,z)  -----> |eq2wgs_ts| ------> (x,y,z)
c     equatorial coord.                               WGS84
c      x => Aries                                 x => Greenwich
c      z => North Pole                          z => North Pole
c
c Example:
c     echo "1998 11 13 0 4099.15 17543.86 -19390.92"|eq2wgs_ts
c
c    @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------

        pi=3.1415926535898d0
10      continue
        read(*,*,end=100) ay,am,ad,ah,r

c       Sidereal time calculation
        call sid0TU(ay,am,ad,ts)
c        ts=ts+(365.2422d0/364.2422d0)*ah
        ts=ts+1.00273790934d0*ah
        ts=dmod(ts,24.d0)

c        print *, ay,am,ad,ah,ts
        ts=ts*pi/12.d0
c       Coordinate transformation
        call rot3(ts,r,rp)
c       print *, ts
        write(*,'(f12.5,1x,f12.5,1x,f12.5)') rp
        goto 10

100     continue
        end
```

```
c -----------------------------------------------------------
      subroutine rot3(ang,r,rp)
      implicit double precision (a-h,o-z)
      dimension r(3),rp(3)

      rp(1)=dcos(ang)*r(1)+dsin(ang)*r(2)
      rp(2)=-dsin(ang)*r(1)+dcos(ang)*r(2)
      rp(3)=r(3)

      end
c -----------------------------------------------------------
      subroutine sid0TU(xy,xm,xd,sid)
      implicit double precision (a-h,o-z)
      double precision jd

c     Sidereal time calculation at 0h TU

c     ----------------------------------------------------
c     year month day  -->|sid0TU|-->  sid(hh.hh)
c     ----------------------------------------------------

      if (xm.le.2.d0) then
         xy=xy-1.d0
         xm=xm+12.d0
      endif

c     Julian day (jd)
      jd=int(365.25d0*xy)+int(30.6001d0*(xm+1.d0))+xd
     * +1720981.5d0

c     print *,xy,xm,xd,jd
c     Sidereal time calculation at 0h TU (sid)
      tt=(jd-2451545.d0)/36525.d0

      sid=24110.54841d0+8640184.812866d0*tt
     *     +0.093104d0*tt**2-(6.2d-6)*(tt**3)
      sid=sid/3600.d0
      sid=dmod(sid,24.d0)
      if (sid.lt.0.d0) sid=sid+24.d0
      end
c -----------------------------------------------------------
```

- **Program orb2xyz:**

It calculates coordinates (x, y, z) from the orbital elements.

```
cx234567
      implicit double precision (a-h,o-z)
       dimension p(6),r0(3),r(3),rot1(9),rot2(9),
     * rot3(9)

c   ..........................................................
c
c
c  sat,day,seg,[a,e,i,OMEGA,omega,Mo]-->|orb2xyz|-->isat,iday,
c                                                  time,
c                                                  r_sat_topo
c
c   NOTE:
c   z => North Pole
c   If W (asc. node long.) referring to Aries  => x to Aries
c   If W (asc. node long. ) referring to Green. => x to Green.
c
c       Note: in the files *eph and *b
c             W is referring to Greenwich
c
c      @gAGE (Research group of Astronomy and GEomatics).
c   ----------------------------------------------------------

        do i=1,3
          r0(i)=0.d0
        enddo

10      continue

        read (*,*,end=100) isat,id,t,p
        call kepler(p(6),p(2),ex)
        r0(1)=p(1)*(1.d0-p(2)*dcos(ex))
        xv=datan2(dsqrt(1.d0-p(2)**2)*dsin(ex),dcos(ex)-p(2))
        call rotate(3,-p(5)-xv,rot3)
        call rotate(1,-p(3),rot1)
        call prod(rot1,3,3,rot3,3,3,rot2)
        call rotate(3,-p(4),rot3)
```

```
        call prod(rot3,3,3,rot2,3,3,rot1)
        call prod(rot1,3,3,r0,3,1,r)

        write(*,*) isat,id,t,r
        goto 10

100     end


c  -----------------------------------------------------------
       subroutine kepler(xm,e,ex)
       implicit double precision (a-h,o-z)

        eps=1.d-12
        ex=xm

10     dex=xm-(ex-e*dsin(ex))
       ex=ex+dex
       if (dex .gt. eps) goto 10
       return
       end
c  -----------------------------------------------------------
```

- **Program ele_orb2rv:**

It calculates the position and velocity from the orbital elements.

```
cx234567
      implicit double precision (a-h,o-z)
       dimension p(6),r0(3),r(3),v(3),rot1(9),rot2(9),
     * rot3(9), rot(9)

c   ..........................................................
c
c
c     [a,e,i,OMEGA,omega,M] ---> |ele_orb2rv| ----> [r,v]
c                                               (equatorial)
c                                               x => Aries
c                                               z => North Pole
c     (units: km and rad)             (units: km and km/s)
c
c     NOTE:  TERRESTRIAL coordinates!!: ---------------------
c   If W (asc. node long.) referred to Aries  => x to Aries
c   If W (asc. node long.) referred to Green. => x to Green.
c     WARNING: in this last case the velocity would correspond to
c              an inertial system rotating an angle=t_sider.
c              about z axis.
c
c       Note: in the files *eph and *b
c              W is referring to Greenwich
c
c   ----------------------------------------------------------
c     Note: M=n(t-T)  with n=dsqrt(gm/a**3)
c   ..........................................................
c
c   Example1:
c   echo "26549. 0.014 0.946 -1.246 -2.476 0.781" |ele_orb2rv
c
c   Result:
c        -15366.34 -2287.86 -21205.69 .99 -3.77 -.36
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------
```

```
         gm=398600.5d0
         c=299792458.d0

10       continue
         read (*,*,end=100) p
         call kepler(p(6),p(2),ex)
         rr=p(1)*(1.d0-p(2)*dcos(ex))
         xv=datan2(dsqrt(1.d0-p(2)**2)*dsin(ex),dcos(ex)-p(2))
         call rotate(3,-p(5),rot3)
         call rotate(1,-p(3),rot1)
         call prod(rot1,3,3,rot3,3,3,rot2)
         call rotate(3,-p(4),rot3)
         call prod(rot3,3,3,rot2,3,3,rot)

c        Computing position r:
         r0(1)=rr*dcos(xv)
         r0(2)=rr*dsin(xv)
         r0(3)=0.d0
         call prod(rot,3,3,r0,3,1,r)

c        Computing velocity v:
         xna=dsqrt(gm/p(1))
         b=p(1)*dsqrt(1.d0-p(2)**2)
         v(1)=xna/rr*(rot(4)*b*dcos(ex)-rot(1)*p(1)*dsin(ex))
         v(2)=xna/rr*(rot(5)*b*dcos(ex)-rot(2)*p(1)*dsin(ex))
         v(3)=xna/rr*(rot(6)*b*dcos(ex)-rot(3)*p(1)*dsin(ex))
         write(*,*) r,v
         goto 10

100      end
c -------------------------------------------------------------
        subroutine kepler(xm,e,ex)
        implicit double precision (a-h,o-z)
        eps=1.d-12
        ex=xm
10      dex=xm-(ex-e*dsin(ex))
        ex=ex+dex
        if (dex .gt. eps) goto 10
        return
        end
```

- **Program rv2ele_orb:**

  It calculates the orbital elements taking the position and velocity of the satellite as inputs.

```
cx234567
      implicit double precision (a-h,o-z)
      dimension r(3),v(3),c(3),ve(3)

c   .........................................................
c
c
c    [r,v]  ------> |rv2ele_orb| -->  [a,e,i,OMEGA,omega,M]
c    (equatorial)
c     x => Aries
c     z => North Pole
c    (units: km and km/s)            (units: km and rad)
c
c   ---------------------------------------------------------
c     Note: M=n(t-T) with n=dsqrt(gm/a**3)
c   .........................................................
c
c
c  Example1:
c  echo "-15334. -2312. -21208. .994 -3.770 -.359" |rv2ele_orb
c
c  Result:
c         26549.521 .015 .947 -1.247 -2.477 .782
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c -----------------------------------------------------------


        gm=398600.5d0
        pi=3.1415926535898d0

10      continue
        read (*,*,end=100) r,v

        rr=dsqrt(r(1)**2+r(2)**2+r(3)**2)
```

```
      vv=dsqrt(v(1)**2+v(2)**2+v(3)**2)

      c(1)=r(2)*v(3)-r(3)*v(2)
      c(2)=r(3)*v(1)-r(1)*v(3)
      c(3)=r(1)*v(2)-r(2)*v(1)
      cc=dsqrt(c(1)**2+c(2)**2+c(3)**2)

      ve(1)=(-c(2)*v(3)+c(3)*v(2))/gm-r(1)/rr

      ve(2)=(-c(3)*v(1)+c(1)*v(3))/gm-r(2)/rr
      ve(3)=(-c(1)*v(2)+c(2)*v(1))/gm-r(3)/rr

      ex=dsqrt(ve(1)**2+ve(2)**2+ve(3)**2)
      a=1.d0/(2.d0/rr-vv**2/gm)
      Wg=datan2(c(1),-c(2))
      xi=dacos(c(3)/cc)
      wp=datan2(ve(3)/dsin(xi),(ve(1)+dsin(Wg)*ve(3)*
     *   dcos(xi)/dsin(xi))/dcos(Wg))
      E=dacos((1.d0-rr/a)/ex)
      control=r(1)*v(1)+r(2)*v(2)+r(3)*v(3)
      if (control.lt.0.d0) E=-E
      xn=dsqrt(gm/a**3)
      xM=E-ex*dsin(E)

c     TT=t-xM/xn
c     if (TT.lt.0.d0) TT=TT+2.d0*pi/xn

      write(*,'(f16.9,1x,f16.9,1x,f16.9,1x,f16.9,1x,
     * f16.9,1x,f16.9)') a,ex,xi,Wg,wp,xM
      goto 10

100   continue

      end
```

- **Program lms:**

It solves an overdimensioned equation system using the least mean square method with weights.

```
c234567890

        program lms

        implicit double precision (a-h,o-z)
        parameter (nmc=100)
        dimension a(nmc),ay(nmc),aa((nmc**2+nmc)/2),x(nmc)

c -----------------------------------------------------------
c       Given the system  Y=AX with covariances (data noise)
c                        P=diag(sig(Y1)**2,...,sig(Yk)**2)
c       it calculates the minimum variance solution:
c                        X^=inv[A'*inv(P)*A]*[A'*inv(P)*Y]
c ...........................................................
c
c
c       The data file must have the following format
c
c
c       n     <----------- number of components of vector X
c       a(1,1).....a(1,n) y(1) sig_y(1)   <---- standard error
c         :          :     :      :
c         :          :     :      :
c       a(k,1).....a(k,n) y(k) sig_y(k)
c         :          :     :      :
c                                    <----- Admits any number of rows
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ...........................................................
c
c   Execute:   cat file.dat | lms
c
c ------------------------------------------------------------


        read(*,*,end=100) n
```

```
10      continue
        read(*,*,end=100) (a(i),i=1,n),y,sig
        do 20 i=1,n
        ay(i)=ay(i)+a(i)*y/(sig**2)
        aa(i*(i+1)/2)=aa(i*(i+1)/2)+a(i)*a(i)/(sig**2)
        do 30 j=i+1,n
        aa(j*(j-1)/2+i)=aa(j*(j-1)/2+i)+a(i)*a(j)/(sig**2)
30      continue
20      continue
        goto 10


100     continue

        call invsp(aa,n,ier)
        if (ier.eq.1) print *, "Warning: sigular matrix"
        call prod(aa,n,n,ay,n,1,x)

        write(*,*) (x(i),i=1,n)

        end
```

- **Program** `kalman0.f`

```
      program kalman0
      implicit double precision (a-h,o-z)
      double precision fi_x,fi_y,fi_z,fi_t
      parameter (nmax=10)
      dimension P((nmax**2+nmax)/2),xfi(nmax),Q(nmax),
     * a(nmax),y(nmax),x(nmax)
      character*4 itype
      namelist /parameters/fi_x,fi_y,fi_z,fi_t,
     *  Pxx,Pyy,Pzz,Ptt,Qxx,Qyy,Qzz,Qtt
c ==============================================================
c
c    cat file.dat --> |kalman0| --->  time dx dy dz dt
c                         ^
c           kalman.nml___|
c
c   ------------------------------------------------------------
c   It applies to the linear system:  Y= A x, i.e.:
c
c
c    [  ]   [                                  ] [dx ]
c    [yi] = [(x0-xs)/ro (y0-ys)/ro (z0-zs)/ro 1] [dy ]
c    [  ]   [                                  ] [dz ]
c    [  ]   [                                  ] [cdt]
c
c    being:
c                 [1/(sigma_y1)^2                      ]
c                 [                                    ]
c              W= [        1/(sigma_yi)^2              ]
c                 [                                    ]
c                 [                      1/(sigma_yn)^2 ]
c
c
c    [Pxx         ]       [Qxx         ]       [fi_x            ]
c P0=[   Pyy      ]     Q=[   Qyy      ]    fi=[    fi_y        ]
c    [      Pzz   ]       [      Qzz   ]       [        fi_z    ]
c    [         Ptt]       [         Qtt]       [            fi_t]
c
c    (See filter equations in chapter 6)
c  .............................................................
```

```
c   The values of [y(n),sigma_y,(x0-xs)/ro,(y0-ys)/ro,(z0-zs)/ro]
c   are obtained from the file "file.dat":
c
c
c   Example of file: file.dat
c  -----------------------------------------------------------------
c  [itype time PRN y(n) sigma_y (x0-xs)/ro (y0-ys)/ro (z0-zs)/ro 1 ]
c                    (meters)
c                    .....................
c    PC   900  03 5934.730 10.0 -0.557881  0.398805 -0.727820  1.0
c    PC   900  17 5939.028 10.0  0.058012  0.613973 -0.787191  1.0
c    PC  1800  22 5933.606 10.0 -0.657670  0.369129 -0.656667  1.0
c                    .....................
c
c  The values of Pxx, Pyy, Pzz, Ptt, Qxx, Qyy, Qzz, Qtt,
c   fi_x, fi_y, fi_z, fi_t, are established through the namelist
c   kalman.nml:
c
c
c  ... kalman.nml ...
c  $parameters
c    Pxx=1.d+8
c    Pyy=1.d+8    (m2)
c    Pzz=1.d+8
c    Ptt=9.d+16
c    fi_x=1.d0     ctt ==> fi=1; Q=0
c    fi_y=1.d0     wn  ==> fi=0; Q=sigma**2
c    fi_z=1.d0     rw  ==> fi=1; Q=sigma**2 * dt
c    fi_t=0.d0
c    Qxx=0.d0
c    Qyy=0.d0     (m2)
c    Qzz=0.d0
c    Qtt=9.d+16
c  $end
c  .................
c
c     @gAGE (Research group of Astronomy and GEomatics).
c =================================================================
```

```
c   Initialization values..................
          nobs=0
          do i=1,nmax
           a(i)=0.d0
           x(i)=0.d0
           y(i)=0.d0
           xfi(i)=0.d0
           Q(i)=0.d0
          enddo
          do i=1,(nmax**2+nmax)/2
           P(i)=0.d0
          enddo
c   .......................................

        open (10,file="kalman.nml")
        read (10,nml=parameters)
        close(10)

c   Kalman FILTER declaration matrix ......
c      State transition matrix.....
          xfi(1)=fi_x
          xfi(2)=fi_y
          xfi(3)=fi_z
          xfi(4)=fi_t
c      A priori covariance values (in meters)...
c      (sig_dx_i=1.d3m; sig_dt=1.d4m)
          P(1)=Pxx
          P(3)=Pyy
          P(6)=Pzz
          P(10)=Ptt
c      Process noise matrix (in meters).......
          Q(1)=Qxx
          Q(2)=Qyy
          Q(3)=Qzz
          Q(4)=Qtt
c   ...........................................

        nvar=4
```

```
c    ================================================
c    BEGIN fordward propagation (of a priori data):
c       Computing the prediction from the a priori values
c       x^_(1)=xfi*x^(0); P_(1)=xfi*P(0)*xfi'+Q
c
c       NOTE:
c       1) x^(0)=0 ==> x^_(1)=0.
c
c          Let's: x:= x^_(1)=0  (x was initialized as "0")
c
c
c       2) The matrix "xfi" and "Q" are assumed to be
c           diagonal, and stored as n-dim vectors..

        do i=1,nvar
          P(i*(i+1)/2)= P(i*(i+1)/2)*xfi(i)+Q(i)
        enddo

c    END of fordward propagation (of a priori data).
c    ================================================


c         P:=inv[P_(n)] ...........
           call invsp(P,nvar,ier)
c        .........................


c    BEGIN MAIN LOOP +++++++++++++++++++++++++++++++++++++++++++

c    :::Begin data loop ::::::::::::::::::::::::::::::::::::::::::::
        nf=0
10      read (*,*,end=900) itype,tt,isat,yy,sigma_y,(a(j),j=1,nvar)
        nf=nf+1
        if (nf.eq.1) tt0=tt
        if (tt .gt. tt0) goto 200
25      continue
c       print *,"AA",tt,isat,yy,sigma_y,itype,(a(j),j=1,nvar)

c       -------------------------------------------
```

```
c       PREPARING  matrix and vector for ESTIMATION
c       Building the vector and matrix: .......
c             y:=A'(n)*W*Y(n)
c             P:=inv(P_(n))+A'(n)*W*A(n)
c             (where W <--> 1/sigma_y**2)
c       ......................................
         do j=1,nvar
          y(j)=y(j)+a(j)*yy/sigma_y**2
          do i=1,j
            P(j*(j-1)/2+i)=P(j*(j-1)/2+i)+
     *      a(i)*a(j)/sigma_y**2
          enddo
         enddo
c       -------------------------------------------

        nobs=nobs+1
        tt0=tt
        goto 10

c   ::::::::::::::::::::::::::::::::::::::::::::::End Data loop :::
200     continue

c    ESTIMATION ==============================================

c       P:=  P(n)=inv[inv(P_(n))+A'(n)*W*A(n)]..........
        call invsp(P,nvar,ier0)
c       ...............................................

c       x:= x^(n)=P(n)*[inv(P_(n))*x^_(n)+A'(n)*W*Y(n)]
        do i=1,nvar
          x(i)=0.d0
          do k=1,nvar
             if (k.lt.i) ik=k+i*(i-1)/2
             if (k.ge.i) ik=i+k*(k-1)/2
             x(i)=x(i)+P(ik)*y(k)
          enddo
        enddo
c       ..................................
c    End estimation
c    =========================================================
```

```
c    ...PRINT WKALMAN ESTIMATION..........................
        write(*,'(a4,1x,f8.2,4(1x,f10.5))') "XYZT",tt0,(x(i),i=1,4)

c    =========================================================
c    BEGIN fordward propagation:

c       x:= x^_(n+1)=fi*x^(n) ..........
         do i=1,nvar
          x(i)=x(i)*xfi(i)
         enddo
c       ................................

c       P:= P_(n+1)=fi*P(n)*(fi)'+Q ...............
         do i=1,nvar
          P(i*(i+1)/2)= P(i*(i+1)/2)*xfi(i)+Q(i)
         enddo
c       ...........................................
c    END of fordward propagation.
c    =========================================================

c       PREPARING  matrix and vector for ESTIMATION ---------
c       P:=inv[P_(n+1)] ...........
         call invsp(P,nvar,ier)
c       .........................

c       y:=inv(P_(n+1))*x^_(n+1) ..........
         do i=1,nvar
          y(i)=0.d0
          do k=1,nvar
             if (k.lt.i) ik=k+i*(i-1)/2
             if (k.ge.i) ik=i+k*(k-1)/2
             y(i)=y(i)+P(ik)*x(k)
          enddo
         enddo
c       -------------------------------------------------------

        tt0=tt
        goto 25
c  +++++++++++++++++++++++++++++++++++++++++++++++ END of Main LOOP
900     continue
         end
```

- **Program** `kalman.f`

```
      program kalman

      implicit double precision (a-h,o-z)
      double precision fi_x,fi_y,fi_z,fi_t
      parameter (nsat=32,nmax=nsat+4)
      dimension P((nmax**2+nmax)/2),AWA((nmax**2+nmax)/2),
     * xfi(nmax),Q(nmax),a(nmax),y(nmax),AWy(nmax),PIx(nmax),
     * x(nmax),iarc0(nmax)
      character*1 itype
      namelist /parameters/fi_x,fi_y,fi_z,fi_t,
     *  Pxx,Pyy,Pzz,Ptt,Qxx,Qyy,Qzz,Qtt

c =======================================================================
c
c    cat file.dat --> |kalman| --->  XYZT: time dx dy dz dt
c                        ^           BIAS: time b01 ... b32
c                        |
c           kalman.nml___|
c
c
c   --------------------------------------------------------------------
c
c   It applies to the linear system:  Y= A x, i.e.:
c
c  [  ]   [                                                    ] [dx ]
c  [  ]   [                                                    ] [dy ]
c  [  ]   [                                                    ] [dz ]
c  [  ] = [                                                    ] [dt ]
c  [yi]   [(x0-xs)/ro (y0-ys)/ro (z0-zs)/ro 1  0  ...  1  ...  0] [b1 ]
c  [  ]   [                                        ^           ] [...]
c  [  ]   [                                        |           ] [bk ]
c  [  ]   [                                        |           ] [...]
c  [  ]   [                                        |           ] [b32]
c                 coeff. for "bk" bias                 |
c                 if observation [yi] corresponds_____|
c                 to the satellite with PRN=k
c
c
```

```
c
c    where:
c           [b1 ... bk ... b32] are the bias of the phase arcs
c                               for the satellites PRN01,...,PRN32
c
c
c    being:
c                    [1/(sigma_y1)^2                      ]
c                    [     ...                            ]
c                 W= [        1/(sigma_yi)^2              ]
c                    [                    ...             ]
c                    [                      1/(sigma_yn)^2 ]
c
c
c
c        [Pxx                    ]        [Qxx                    ]
c        [   Pyy                 ]        [   Qyy                 ]
c        [      Pzz              ]        [      Qzz              ]
c    P0= [         Ptt           ]     Q= [         Qtt           ]
c        [            Pb1        ]        [            Qb1        ]
c        [               ...     ]        [               ...     ]
c        [                  Pb32 ]        [                  Qb32 ]
c
c
c
c
c        [fi_x                          ]    - If a cycle-slip is produced
c        [    fi_y                      ]      in the sat. PRN=k:
c        [        fi_z                  ]       fi_bk=0, Qbk= 9e16 m2
c    fi= [            fi_t              ]
c        [                fi_b1         ]    - If no cycle-slip is produced
c        [                     ...      ]       fi_bk=1, Qbk= 0
c        [                        fi_b32]
c                                            * "iarc" allows us to identify
c                                               cycle-slips.
c                                               If "iarc" changes=>cycle-slip
c
c
c
c    (See equations of Kalman filter in chapter 6)
c
```

```
c  ...........................................................
c   The values of [y(n),sigma_y,(x0-xs)/ro,(y0-ys)/ro,(z0-zs)/ro]
c   are obtained from the file "file.dat":
c
c
c   Example of file "file.dat"
c  -----------------------------------------------------------------
c  [itype time PRN y(n) sigma_y (x0-xs)/ro (y0-ys)/ro (z0-zs)/ro 1 iarc]
c                     (meters)
c                    .....................
c    PC   900  03 5934.730 10.0 -0.557881  0.398805 -0.727820  1.0  1
c    LC   900  03 5935.241  0.1 -0.557881  0.398805 -0.727820  1.0  1
c    PC   900  17 5939.028 10.0  0.058012  0.613973 -0.787191  1.0  2
c    LC   900  17 5938.107  0.1  0.058012  0.613973 -0.787191  1.0  2
c    PC  1800  22 5933.606 10.0 -0.657670  0.369129 -0.656667  1.0  1
c    LC  1800  22 5932.513  0.1 -0.657670  0.369129 -0.656667  1.0  1
c                    .....................
c
c  The values of Pxx, Pyy, Pzz, Ptt, Qxx, Qyy, Qzz, Qtt,
c  fi_x, fi_y, fi_z, fi_t, are established through the
c  namelist kalman.nml:
c
c  ... kalman.nml ...
c  $parameters
c    Pxx=1.d+8
c    Pyy=1.d+8    (m2)
c    Pzz=1.d+8
c    Ptt=9.d+16
c    fi_x=1.d0     ctt ==> fi=1; Q=0
c    fi_y=1.d0     wn  ==> fi=0; Q=sigma**2
c    fi_z=1.d0     rw  ==> fi=1; Q=sigma**2 * dt
c    fi_t=0.d0
c    Qxx=0.d0
c    Qyy=0.d0     (m2)
c    Qzz=0.d0
c    Qtt=9.d+16
c  $end
c  .................
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ===================================================================
```

```
c   Initialization values..................
          do i=1,nmax
           a(i)=0.d0
           x(i)=0.d0
           y(i)=0.d0
           AWy(i)=0.d0
           xfi(i)=0.d0
           Q(i)=0.d0
           iarc0(i)=0
          enddo
          do i=1,(nmax**2+nmax)/2
           P(i)=0.d0
           AWA(i)=0.d0
          enddo
c   .......................................

        open (10,file="kalman.nml")
        read (10,nml=parameters)
        close(10)

c   Kalman FILTER declaration matrix ......
c      State transition matrix.....
          xfi(1)=fi_x
          xfi(2)=fi_y
          xfi(3)=fi_z
          xfi(4)=fi_t
c      A priori covariance values (in meters)...
          P(1)=Pxx
          P(3)=Pyy
          P(6)=Pzz
          P(10)=Ptt
c      Process noise matrix (in meters).......
          Q(1)=Qxx
          Q(2)=Qyy
          Q(3)=Qzz
          Q(4)=Qtt
c      Arc bias:
          Pbias=9.d-16
          Qbias=9.d+16
```

```
c   ...........................................
        nvar=nmax

        do i=5,nvar
          P(i*(i+1)/2)=Pbias
          xfi(i)=1.d0
        enddo


c    BEGIN MAIN LOOP +++++++++++++++++++++++++++++++++++++++++++

c    :::Begin data loop :::::::::::::::::::::::::::::::::::::::::
        nf=0
10      read (*,*,end=900) itype,tt,isat,yy,sigma_y,
     *                     (a(j),j=1,4),iarc
        nf=nf+1
        if (nf.eq.1) tt0=tt


c       Completing the Design Matrix ...............
        do i=5,nvar
          a(i)=0.d0
        enddo
        if (itype.eq."L") then
          i=isat+4
          a(i)=1.d0
          if (iarc.ne.iarc0(isat)) then
            xfi(i)=0.d0
            Q(i)=Qbias
          endif
          iarc0(isat)=iarc
c          print *, isat,Q(i),xfi(i),iarc,iarc0(isat)
        endif

        i=isat+4
c       ..........................................

c       .........................................
        if (tt .gt. tt0) goto 200
25      continue
```

```
c       ------------------------------------------
c       PREPARING  matrix and vector for ESTIMATION
c       Building the vector and matrix: .......
c             AWy:=A'(n)*W*Y(n)
c             AWA:=inv(P_(n))+A'(n)*W*A(n)
c             (where W <--> 1/sigma_y**2)
c       .......................................

         do j=1,nvar
          AWy(j)=AWy(j)+a(j)*yy/sigma_y**2
          do i=1,j
            AWA(j*(j-1)/2+i)=AWA(j*(j-1)/2+i)+
     *      a(i)*a(j)/sigma_y**2
          enddo
         enddo
c       ------------------------------------------

        tt0=tt
        goto 10

c   ::::::::::::::::::::::::::::::::::::::::::::::End Data loop :::

200     continue


c    =====================================================
c    BEGIN fordward propagation:

c       x:= x^_(n)=fi(n)*x^(n-1) ..........
         do i=1,nvar
          x(i)=x(i)*xfi(i)
         enddo
c       ................................
c
c       P:= P_(n)=fi(n)*P(n-1)*fi(n)'+Q(n) ...............
         do i=1,nvar
          P(i*(i+1)/2)= P(i*(i+1)/2)*xfi(i)+Q(i)
         enddo
c       ............................................
c    END of fordward propagation.
```

```
c    ========================================================
c    ESTIMATION ==============================================

c       P:=inv[P_(n)] ...........
         call invsp(P,nvar,ier)
c       .........................
c       PIx:=inv(P_(n))*x^_(n) ..........
         do i=1,nvar
          PIx(i)=0.d0
          do k=1,nvar
             if (k.lt.i) ik=k+i*(i-1)/2
             if (k.ge.i) ik=i+k*(k-1)/2
             PIx(i)=PIx(i)+P(ik)*x(k)
          enddo
         enddo
c      -------------------------------------------------------
c      P(n)=inv[inv(P_(n))+A'(n)*W(n)*A(n)]==>  P:=inv[P + AWA]
        do i=1,nvar*(nvar+1)/2
         P(i)=P(i)+AWA(i)
        enddo
        call invsp(P,nvar,ier0)
c       ..............................................

c      x^(n)=P(n)*[inv(P_(n))*x^_(n)+A'(n)*W(n)*Y(n)]
c      ==> x:=P(n)*[PIx + AWy]

        do i=1,nvar
         y(i)=PIx(i)+AWy(i)
        enddo

        do i=1,nvar
          x(i)=0.d0
          do k=1,nvar
             if (k.lt.i) ik=k+i*(i-1)/2
             if (k.ge.i) ik=i+k*(k-1)/2
             x(i)=x(i)+P(ik)*y(k)
          enddo
        enddo
c       ..................................
c    End estimation
c    ========================================================
```

```
c    ...PRINT KALMAN ESTIMATION..........................
        write(*,'(a4,1x,f8.2,4(1x,f10.5))') "XYZT",tt0,(x(i),i=1,4)
        write(*,'(a4,1x,f8.2,32(1x,f8.3))') "BIAS",tt0,(x(i),i=5,nvar)
c    .......................................................


c       Reinitializing variables for next iteration ...........
        tt0=tt
        do i=5,nvar
          xfi(i)=1.d0
          Q(i)=0.d0
        enddo
        do i=1,nvar
         AWy(i)=0.d0
        enddo
        do i=1,nvar*(nvar+1)/2
         AWA(i)=0.d0
        enddo
        goto 25
c       .....................................................
c  ++++++++++++++++++++++++++++++++++++++++++++++ END of Main LOOP

900     continue
         end
```

- **Program** `ambisolv.f`

```
      program ambisolv
      implicit double precision (a-h,o-z)
      character*4 sta,staR

c   ---------------------------------------------------------------
c
c
c   [sta,staR,isatR,isat,sec,DDLc,DDLw,DDLi,DDSTEC,DDBc] --->
c
c                                    |ambisolv| ---> [N1,N2,Nw,Lc]
c
c
c   ==> [sta,staR,isatR,isat,iarc,sec,DDxNw,DDNw,DDxN1,DDN1,DDN2,
c         1   2    3     4     5   6    7    8     9 a  10   11
c
c                        DDLi,DDLi_r,DDSTEC,DDBc,DDBc_r,DDLc,DDLc_r]
c                         12   13       14    15    16    17   18
c
c  .................................................................
c
c     where:
c     -----
c
c      "DD" indicates double differences.
c
c      DDNw:   exact value (integer) of wide-lane ambiguity
c      DDxNw:  approximate value (before rounding up) of
c                                                wide-lane ambiguity
c
c      DDN1:   exact value (integer) of L1 ambiguity
c      DDxN1:  approximate value (before rounding up) of
c                                                    L1 ambiguity
c
c      DDN2 :  exact value (integer) of L2 ambiguity
c
c      DDLi  : Ionospheric combination before repairing (fixing
c                                                      ambiguities)
c
```

```
c      DDLi_r: Repaired ionospheric combination (i.e., with
c                      ambiguities fixed to their exact values)
c      DDSTEC: Prediction of STEC used to repair the ambiguities
c
c      DDBc  : Bias in ionosphere-free combination before repairing
c
c      DDBc_r: Bias in repaired ionosphere-free combination
c
c      DDLc  : Ionosphere-free combination before repairing
c      DDLc_r: Repaired ionosphere-free combination
c
c
c     Note:
c        DDLi,DDSTEC are expressed in m_LI
c
c        Bc is the bias of the Lc phase, estimated by the filter
c        Lc=Lc_inambigua+Bc
c
c
c    Execute:
c    --------
c            cat DDbell_ebre_21.dat |ambisolv > DDbell_ebre_21.amb
c
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c   -----------------------------------------------------------------


c      ----Constants ------------------
        f1=1.54d0*1.023d9
        f2=1.20d0*1.023d9
        c=299792458.d0
        xlambda1=c/f1
        xlambda2=c/f2
        xlambdaC=c/(f1+f2)
        xlambdaW=c/(f1-f2)
        gamma=(77.d0/60.d0)**2
        fact=(77.d0/60.d0)/((77.d0/60.d0)**2-1.d0)
c       --------------------------------
```

```
10      continue
        read(*,*,end=100) sta,staR,isatR,isat,iarc,sec,
     *                    DDLc,DDLw,DDLi,DDSTEC,DDBc

        xNw=(DDLw-DDLc-DDSTEC*fact+DDBc)/xlambdaW
        Nw=nint(xNw)

        xN1=(DDLi-DDSTEC-Nw*xlambda2)/(xlambda1-xlambda2)
        N1=nint(xN1)
c        print *,"AA",isat,iarc,sec,xNw,Nw,xN1,N1,DDBc


c       Reconstruction of Li_unambiguous (DDLi_r): .........
        DDLi_r=DDLi-dble(N1)*(xlambda1-xlambda2)-dble(Nw)*xlambda2
        err_DDLi=DDLi_r-DDSTEC
c       ....................................................

        N2=N1-Nw

        DDBc_r=(gamma*xlambda1*dble(N1)-xlambda2*dble(N2))/
     *       (gamma-1.d0)


c       Reconstruction of Lc_unambiguous (DDLc_r): ........
        DDLc_r=DDLc-DDBc_r
c       ...................................................


        write(*,'(2(a4,1x),3(i2,1x),1x,f8.2,1x,f9.4,1x,i5,1x,
     *   f9.4,2(1x,i5),3(1x,f6.4),2x,2(1x,f9.4),2(1x,f14.4))')
     *   sta,staR,isatR,isat,iarc,sec,xNw,Nw,xN1,N1,N2,DDLi,
     *   DDLi_r,DDSTEC,DDBc,DDBc_r,DDLc,DDLc_r


        goto 10
100     continue

        end
```

- **Program** `coord_ems_P.f`

```
c234567
       program coord_ems_P
       implicit double precision (a-h,o-z)
       dimension a(38)

c  ---------------------------------------------------------------------
c      Applying the PSEUDORANGE algorithm, this program computes the
c      satellite coordinates (WGS-84) at the EMISSION time, in the ECEF
c      system at the RECEPTION time. It provides also the difference
c      between the reception time (in the receiver clock) and the
c      emission time (in GPS time).
c
c
c      INPUT:
c      -----
c       - Signal reception time (in the receiver clock)
c           iyear: year
c           idoy:  Day-Of-year
c           sec:   Seconds of day
c           P:     Pseudorange measurement
c           (x_sta,y_sta,z_sta): receiver coordinates
c                                WGS'84 (in meters)
c
c       - Navigation message (broadcast data)
c         (in RINEX format)
c
c           [sat, year,mon,day,h,m,sec, a0,a1,a2     ]
c           [                    IODE,Crs,dn,xMo,    ]
c           [                    Cuc, e, Cus,a12,    ]
c           [                    toe,Cic,Omgg,Cis,   ]
c           [                    xIo,Crc,omgp,Omgd   ]
c           [                    xIDOT,xx,GPS_Week,xx]
c           [                    SVac,SVh,TGD,IODC   ]
c           [                    xx, xx , xx ,xx     ]
c
c     OUTPUT:
c     ------
c         Satellite coordinates (WGS84) at EMISSION time
c         in ECEF system, tied to the Earth at the RECEPTION time.
```

```
c
c     dt: t_recepction[receiver_clock] - t_emission[t_GPS] (seconds)
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c -------------------------------------------------------------
c
c Example of execution:
c
c a) Create file "datos_ems_P" with the following data (all data in a
c    single line. Fields must be separated by blank spaces!!!).
c
c 1998 286 38230 23585247.703 4789031  176612 4195008
c 14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
c +1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
c -3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
c +2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
c +9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
c -1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
c +3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
c +2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
c
c
c b) Execute:
c
c       cat datos_ems_P | coord_ems_P
c
c
c
c -----------------------------------------------------------------------------



c       Parameters Declaration : ..........
c       - Light speed (m/s)
           c=299792458.d0
c       - WGS-84 Earth rotation rate (rad/s)
          om_e=7.2921151467d-5
c       ...................................
```

```
      read(*,*,end=10) iyear,idoy,sec,P,x_sta,y_sta,z_sta,a
10    continue



c   ==> GPS satellite clock Offset with GPS time:

        toc=a(5)*3600.d0+a(6)*60.d0+a(7)
        dt_sat=a(8)+a(9)*(sec-toc)+a(10)*(sec-toc)**2


c   ==> EMISION time  (in GPS time)<==
        sec1=sec-P/c-dt_sat


c   ==> Satellite coordinates at EMISION time (in GPS time)
c       [in the ECEF reference system tied to the Earth at
c        EMISION time]
        call orbit(iyear,idoy,sec1,a,x0,y0,z0,Ek)


c   ==> Satellite coordinates at the EMISION time (in GPS time)
c       [in the ECEF reference system tied to the Earth at
c        RECEPTION time]

c       ... Geometric Range (in seconds)
        dt=dsqrt((x0-x_sta)**2+(y0-y_sta)**2+(z0-z_sta)**2)/c

c       Coordinate transformation to the ECEF system tied to
c       the Earth at RECEPTION time.
c       .... Earth Rotation during the time "dt"
        x=x0+y0*om_e*dt
        y=y0-x0*om_e*dt
        z=z0
c       ...................................................

        write(*,'(f4.1,1x,f14.4,1x,f14.4,1x,f14.4,f14.4,1x,f9.3)'),
     *  a(1),sec1,x,y,z,sec1-sec

        end
```

- **Subroutine** `prod.f`

```
      subroutine prod(A,m,l,B,ll,n,C)
      implicit DOUBLE PRECISION (A-H,O-Z)
      dimension A(*),B(*),C(*)

c ------------------------------------------------------------
c      It computes the product of a mxl matrix A and a llxn
c      matrix B. The result is the mxn matrix C. The matrix
c      must be a general matrix, Fortran vectored (by columns).
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------

      do i=1,m*n
         C(i)=0.d0
      enddo

      if (l.ne.ll) then
         print *, "ERROR: dimension matrix"
       goto 100
      endif

       do 10 i=1,m
         do 20 j=1,n
           C(i+m*(j-1))=0.d0
           do 30 k=1,l
             C(i+m*(j-1))=C(i+m*(j-1))+A(i+m*(k-1))*B(k+l*(j-1))
30         enddo
20       enddo
10     enddo

100   continue
      return
      end
```

- **Subroutine** `rotate.f`

```
      subroutine rotate(iaxis,angle,rot)
      implicit DOUBLE PRECISION (A-H,O-Z)
      dimension rot(9)

c -----------------------------------------------------------
c     It provides the (Fortran vectored) rotation matrix "rot"
c     for a given angle "angle" around the coordinate axis
c     [1<=>x], [2<=>y],[3<=>z].
c
c     @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------

      do i=1,9
          rot(i)=0.d0
      enddo

      if (iaxis .eq. 1) then
          rot(1)=1.d0
          rot(5)=dcos(angle)
          rot(6)=-dsin(angle)
          rot(8)=dsin(angle)
          rot(9)=dcos(angle)
        elseif (iaxis .eq. 2) then
          rot(5)=1.d0
          rot(1)=dcos(angle)
          rot(7)=-dsin(angle)
          rot(3)=dsin(angle)
          rot(9)=dcos(angle)
        elseif (iaxis .eq. 3) then
          rot(9)=1.d0
          rot(1)=dcos(angle)
          rot(2)=-dsin(angle)
          rot(4)=dsin(angle)
          rot(5)=dcos(angle)
      endif

      return
      end
```

- **Subroutine** `invsp.f`

```
      subroutine invsp(A,n,ier)

c ------------------------------------------------------------
c       It calculates the INVERSE of a MATRIX nxn SIMETRIC
c       AND POSITIVE DEFINED (the matrix must be vectorized
c       -as simetric- by columns) If the matrix is not positive
c       defined, the calculation  stops and gives an error
c       output: ier=1.
c       NOTE: the matrix A is replaced by its inverse.
c
c       NOTICE that
c        the inverse of a generic matrix can be calculated by:
c        inv(A)=inv(A'*A)*A' (where A'*A is sim. and pos. def.)
c
c      @gAGE (Research group of Astronomy and GEomatics).
c -------------------------------------------------------------

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION a(*)

c       CHOLESKY decomposition  (A=T'*T)

      call chol(A,n,ier)
c       print *, "CHOLESKY", (a(i),i=1,20)

c       Inverse of the triangular Cholesky matrix {inv(T)}
      do 100 l=1,n
      i=n-l+1
      a(i+i*(i-1)/2)=1.d0/a(i+i*(i-1)/2)
      do 110 ll=i+1,n
      j=n-ll+i+1
      s=0.d0
      do 120 k=i+1,j
      s=s+a(i+k*(k-1)/2)*a(k+j*(j-1)/2)
120   continue
      a(i+j*(j-1)/2)=-s*a(i+i*(i-1)/2)
110   continue
100   continue
```

```
c       Inverse of matrix A   {inv(A)=inv(T)*inv(T)'}
        do 200 i=1,n
        do 210 j=i,n
        s=0.d0
        do 220 k=j,n
        s=s+a(i+k*(k-1)/2)*a(j+k*(k-1)/2)
220     continue
        a(i+j*(j-1)/2)=s
210     continue
200     continue

        return
        end
```

- **Subroutine** `chol.f`

```
        subroutine chol(A,n,ier)

        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION a(*)

c -----------------------------------------------------------
c       It calculates the CHOLESKY decomposition of matrix A
c       -> nxn simetric and positively defined: (A=T'*T)
c       --In the output, A is replaced by T-- (the matrix must
c       be vectorized --as simetric-- by columns). If the
c       matrix is not positively defined, the calculation stops
c       and an error output is produced: ier=1 (or  ier=2 if
c       its determinant is null).
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------
        ier=0
        do 30 i=1,n
        do 20 j=i,n
        sum=a(i+j*(j-1)/2)
        do 10 k=i-1,1,-1
        sum=sum-a(k+i*(i-1)/2)*a(k+j*(j-1)/2)
10      continue
        if (i.eq.j) then
          if(sum.le.0.d0) then
                ier=1
                if (sum.eq.0) ier=2
                goto 100
          endif
          a(i+i*(i-1)/2)=dsqrt(sum)
          else
          a(i+j*(j-1)/2)=sum/a(i+i*(i-1)/2)
        endif
20      continue
30      continue

100     return
        end
```

- **Subroutine** `orbit.f`

```
      subroutine orbit(iyear,idoy,sec,a,x,y,z,Ek)
      implicit double precision (a-h,o-z)
      dimension a(38)

c -------------------------------------------------------------
c     This subroutine calculates the coordinates (WGS84) of a
c     GPS satellite for a given epoch (iyear,idoy,sec
c     --GPS-time--), taking as input the navigation message
c     parameters.
c
c     INPUT:
c     -----
c      - epoch for which the coordinates must be calculated.
c          iyear: year
c          idoy:  day of year
c          sec:   seconds in the day
c      - Navigation message (broadcast data)
c        (according to RINEX format)
c
c          [sat, year,mon,day,h,m,sec, a0,a1,a2     ]
c          [                    IODE,Crs,dn,xMo,    ]
c          [                    Cuc, e, Cus,a12,    ]
c          [                    toe,Cic,Omgg,Cis,   ]
c          [                    xIo,Crc,omgp,Omgd   ]
c          [                    xIDOT,xx,GPS_Week,xx]
c          [                    SVac,SVh,TGD,IODC   ]
c          [                    xx, xx , xx ,xx     ]
c
c    OUTPUT:
c    ------
c           Satellite coordinates in the WGS-84 reference
c           system tied to the Earth
c           (x,y,z): in meters
c           Ek: eccentric anomaly
c
c    @gAGE (Research group of Astronomy and GEomatics).
c -------------------------------------------------------------
```

```
c       Parameters declaration .....
c       Light speed (m/s)
        c=299792458.d0
c       WGS-84 Earth Univ. Grav. parameter (m3/s2)
        xmu=3.986005d+14
c       WGS-84 Earth rotation rate (rad/s)
        om_e=7.2921151467d-5
        pi=3.1415926535898d0
c       ............................
c       GPS Navigation message parameters:
        xIODE=a(11)
        Crs=a(12)
        dn=a(13)
        xMo=a(14)
        Cuc=a(15)
        e=a(16)
        Cus=a(17)
        a12=a(18)
        toe=a(19)
        Cic=a(20)
        Omgg=a(21)
        Cis=a(22)
        xIo=a(23)
        Crc=a(24)
        omgp=a(25)
        Omgd=a(26)
        xId=a(27)
        iGPSweek=int(a(29))
c       ........................................
c       Computing the GPS_week (nw) and second (sW) of week
        xy=dble(iyear)
c       In RINEX" format year is given with only two digits
        if (xy.lt.100.d0) then
          if (xy.lt.80.d0) then
            xy=xy+2000.d0
           else
            xy=xy+1900.d0
          endif
        endif
```

```
c       GPS day: (1980jan6.0 => JD=2444244.5 => id_GPS=1.0)
        id_GPS=int(365.25d0*(xy-1.d0))+idoy-722835
c       Day of week:
        idw=mod(id_GPS,7)
c       Number of GPS week:
        nw=(id_GPS-idw)/7
c       seconds of week:
        sw=dble(idw)*86400.d0+sec

c       Control of GPS WEEK
        if (nw.ne.iGPSweek) print *,"ERROR: week",iGPSweek,nw

c       Time from current ephemeris epoch
        tk=sw-toe
        if(tk.gt.302400.d0)  tk=tk-604800.d0
        if(tk.lt.-302400.d0) tk=tk+604800.d0
c       Control of age of orbit data
        if (tk.gt.7200.d0) print *,"WARNING tk=", tk," > 7200sec"

c       True anomaly fk:
        xMk=xMo+(dsqrt(xmu)/(a12**3)+dn)*tk
        call nsteffensen(xMk,e,Ek)
        fk=datan2(dsqrt(1.d0-e**2)*dsin(Ek),dcos(Ek)-e)

c       Arg. of Latitude uk,radius rk, inclination ik:
        uk=omgp+fk+Cuc*dcos(2.d0*(omgp+fk))+
     *       Cus*dsin(2.d0*(omgp+fk))
        rk=(a12**2)*(1.d0-e*dcos(Ek))+Crc*dcos(2.d0*(omgp+fk))+
     *     Crs*dsin(2.d0*(omgp+fk))
        xIk=xIo+xId*tk+Cic*dcos(2.d0*(omgp+fk))+
     *      Cis*dsin(2.d0*(omgp+fk))

c       Position in orbital plane
        xp=rk*dcos(uk)
        yp=rk*dsin(uk)

c       Longitude of ascending node xlmk:
        xlmk=Omgg+(Omgd-om_e)*tk-om_e*toe
```

```
c        CT-System coordinates
         x=xp*dcos(xlmk)-yp*dcos(xIk)*dsin(xlmk)
         y=xp*dsin(xlmk)+yp*dcos(xIk)*dcos(xlmk)
         z=yp*dsin(xIk)

         return
         end


c        -----------------------------------------------------------

        subroutine nsteffensen(xm,e,ex)
        implicit double precision (a-h,o-z)

c       Algorithm to accelerate convergence of
c       Newton-Rapson Method.
c       Equations of type p=g(p)   (==> E=M+e*sin(E))
c       The method requires that g'(p)<>1  (==> p single root)

         tol=1.d-15

         xm=datan2(dsin(xm),dcos(xm))
         p=xm

10       continue
         p0=p
         p1=xm+e*dsin(p0)
         p2=xm+e*dsin(p1)
         dd=dabs(p2-2.d0*p1+p0)
         if(dd.lt.tol) goto 100
         p=p0-(p1-p0)**2/(p2-2.d0*p1+p0)
         if(dabs(p-p0).gt.tol) goto 10

100      continue
         ex=p

        return
        end
```

- **Subroutine** `rec2ems.f`

```
      subroutine rec2ems(iyear,idoy,sec,x_sta,y_sta,z_sta,a,x,y,z,dt)
      implicit double precision (a-h,o-z)
      dimension a(38)

c  -----------------------------------------------------------------
c     Applying GEOMETRIC ALGORITHM, this subroutine calculates
c     the coordinates (WGS-84) of a satellite at the "emission"
c     time of the signal, and in the system tied to the Earth
c     at the "reception" time, taking as input the reception time
c     (iyear,idoy,sec -GPS-time-) and navigation message parameters.
c     It also provides the difference between emission and reception
c     times.
c
c     NOTE: Any offset in the receiver clock affecting the
c           reception time will be translated to the emission time
c           by the geometric algorithm.
c
c
c     INPUT:
c     -----
c      -  Instant when signal is received
c          iyear: year
c          idoy:  day of year
c          sec:   seconds in the day
c          (x_sta,y_sta,z_sta): approximate coordinates of
c                         receiver position (WGS'84, in meters)
c
c      - Navigation message (broadcast data)
c        (according to RINEX format)
c
c           [sat, year,mon,day,h,m,sec, a0,a1,a2     ]
c           [                    IODE,Crs,dn,xMo,    ]
c           [                    Cuc, e, Cus,a12,    ]
c           [                    toe,Cic,Omgg,Cis,   ]
c           [                    xIo,Crc,omgp,Omgd   ]
c           [                    xIDOT,xx,GPS_Week,xx]
c           [                    SVac,SVh,TGD,IODC   ]
c           [                    xx, xx , xx ,xx     ]
c
```

```
c
c    OUTPUT:
c    ------
c           Satellite coordinates at emission epoch in the
c           reference system tied to the Earth at "reception"
c           time (x,y,z), WGS84, in meters
c
c           dt: t_reception - t_emission (seconds)
c
c     NOTE:
c     A simple example program [coord_ems.f] about how to use this
c     subroutine follows
c
c ====================== coord_ems.f ============================
c234567
c      program coord_ems
c      implicit double precision (a-h,o-z)
c      dimension a(38)
c
c
c ---------------------------------------------------------------
c a) Create file "data_ems" with the following data
c    (see "rec2ems.f"), arranged in one single line (fields
c     must be separated with a blank space!!):
c
c 1998 286 38230 4789031  176612 4195008
c 14 98 10 13 12 0 0 +5.6545250E-06 +9.09494701E-13 +0.00000000E+00
c +1.280000000E+02 -6.10000000E+01 +4.38125402E-09 +8.198042513E-01
c -3.313645720E-06 +1.09227513E-03 +5.67547976E-06 +5.153795101E+03
c +2.160000000E+05 -6.33299350E-08 +1.00409621E+00 -3.725290298E-09
c +9.736580013E-01 +2.74031250E+02 +2.66122811E+00 -8.081050495E-09
c -1.457203524E-10 +1.00000000E+00 +9.79000000E+02 +0.000000000E+00
c +3.200000000E+01 +0.00000000E+00 -2.32830643E-09 +1.280000000E+02
c +2.088180000E+05 +0.00000000E+00 +0.00000000E+00 +0.000000000E+00
c
c
c b) Execute:
c
c       cat data_ems | coord_ems
c
c ---------------------------------------------------------------
```

```
c
c
c
c      read(*,*,end=10) iyear,idoy,sec,x_sta,y_sta,z_sta,a
c10    continue
c
c      call rec2ems(iyear,idoy,sec,x_sta,y_sta,z_sta,a,x,y,z,dt)
c
c      write(*,'(f4.1,1x,f14.4,1x,f14.4,1x,f14.4,f14.4,1x,f5.3)'),
c     * a(1),sec,x,y,z,dt
c
c      end
c
c
c
c     ----------------- compilation -------------------
c      f77 -c orbit.f
c      f77 -c rec2ems.f
c      f77 -o coord_ems  coord_ems.f  rec2ems.o orbit.o
c     -------------------------------------------------
c ================================================================
c     ......................................................
c      @gAGE (Research group of Astronomy and GEomatics).
c  ---------------------------------------------------------------


c       CALCULATION OF SATELLITE COORDINATES
c       Parameter statement: ...........
c       - Maximum number of iterations
           nit_max=10
c       - Tolerance
           tol=1.d-3
c       - Velocity of light (m/s)
           c=299792458.d0
c       - Rotation velocity of the Earth (rad/s)
          om_e=7.2921151467d-5
c       .....................................
```

```
c   ==> at RECEPTION instant <==
        call orbit(iyear,idoy,sec,a,x,y,z,Ek)


c   ==> at EMISSION instant <==
        nit=0
        sec1=sec

60      continue
c       ... Geometric satellite-station distance (in time units)
        dt=dsqrt((x-x_sta)**2+(y-y_sta)**2+(z-z_sta)**2)/c
        sec=sec1-dt

c       .... coordinate calculation for "t:=sec1-dt"
        call orbit(iyear,idoy,sec,a,x0,y0,z0,Ek)

        ctl=dsqrt((x0-xa)**2+(y0-ya)**2+(z0-za)**2)

        xa=x0
        ya=y0
        za=z0

c       Coordinate transformation to the system tied to the Earth
c       at "recepcion" instant
c       .... Earth rotation during the time "dt"
        x=x0+y0*om_e*dt
        y=y0-x0*om_e*dt
        z=z0
c       ...................................................
        nit=nit+1
        if (ctl.gt.tol) then
          if (nit.lt.nit_max) then
             goto 60
            else
             print *, "ERROR: the algorithm does not converge ",nit
           endif
        endif

        return
        end
```

- **Subroutine** `klob.f`

```
      subroutine klob(t,x_sta,y_sta,z_sta,
     *                 x_sat,y_sat,z_sat,
     *                 alpha0,alpha1,alpha2,alpha3,
     *                 beta0,beta1,beta2,beta3,Tiono)

      implicit double precision (a-h,o-z)

c ------------------------------------------------------------
c  Klobuchar model implementation for the calculation of the
c  ionospheric delay.
c
c  INPUT:
c        t   : observation epoch (seconds of day)
c        (x_sta,y_sta,z_sta): receiver coordinates (WGS84), meters
c        (x_sat,y_sat,z_sat): satellite coordinates (WGS84), meters
c
c        Klobuchar coefficients (file klobuchar.dat):
c           alpha0,alpha1,alpha
c           beta0,beta1,beta2,beta3
c  OUTPUT:
c        Tiono: ionospheric delay (meters L1)
c
c
c      @gAGE (Research group of Astronomy and GEomatics).
c ------------------------------------------------------------

c   ... parameter declarations ......
      c=299792458.d0
      pi=3.1415926535898d0
c   .................................


c  ============================================================
c  Calculate the geodetic user longitude and latitude
      call car2geo(x_sta,y_sta,z_sta,ulon,ulat,h)

c  Calculate the elevation and azimuth of receiver-satelite ray
      x1=x_sat-x_sta
```

```
        y1=y_sat-y_sta
        z1=z_sat-z_sta
        rho=dsqrt(x1**2+y1**2+z1**2)
        slat=dsin(ulat)
        slon=dsin(ulon)
        clat=dcos(ulat)
        clon=dcos(ulon)
        G1=(-slat*clon*x1-slat*slon*y1+clat*z1)/rho
        G2=(-slon*x1+clon*y1)/rho
        G3=( clat*clon*x1+clat*slon*y1+slat*z1)/rho
        elev=dasin(G3)
        azim=datan2(G2,G1)
c  ============================================================

c   Calculate the Earth-centered angle fm (in semicircles): ........
     fm=0.0137d0/(elev/pi+0.11d0)-0.022d0

c   Compute the subionospheric latitude, xilat (in semicircles): ...
     xilat=ulat/pi+fm*dcos(azim)
     if (xilat.gt. 0.416d0) xilat= 0.416d0
     if (xilat.lt.-0.416d0) xilat=-0.416d0

c   Compute the subionospheric longitude, xilon (in semicircles): ..
     xilon=ulon/pi+fm*dsin(azim)/dcos(xilat*pi)

c   Find the Geomagnetic latitude gmlat (in semicircles): ...
     gmlat=xilat+0.064d0*dcos((xilon-1.617d0)*pi)

c   Find the local time at subionspheric point: tsub (in sec.)
     tsub=4.32d4*xilon+t
     if (tsub .gt. 86400.d0) tsub=tsub-86400.d0
     if (tsub .lt. 0.d0) tsub=tsub+86400.d0

c   Convert to slant time delay (compute the slant factor F):
     F=1.d0+16.d0 *(0.53d0-elev/pi)**3

c  Compute the ionospheric time delay sTEC (in meters of delay):
    alpha=alpha0+alpha1*gmlat+alpha2*gmlat**2+alpha3*gmlat**3
    beta= beta0+beta1*gmlat+beta2*gmlat**2+beta3*gmlat**3
        if (alpha.lt.0.d0) alpha=0.d0
        if (beta.lt.72000.d0) beta=72000.d0
```

```
x=2.d0*pi*(tsub-50400.d0)/beta

if (dabs(x).le.pi/2.d0) then
     y=alpha*(1.d0-x**2/2.d0+x**4/24.d0)
  else
       y=0.d0
endif

Tiono=F*(5.d-9+y)*c

    return
    end
```

- **Subroutine** `car2geo.f`

```
c234567890

       subroutine car2geo(x,y,z,xlon,xlat,h)
       implicit double precision (a-h,o-z)

c -----------------------------------------------------------
c      Conversion from Cartesian coordinates (x,y,z)
c      to ellipsoidal (fi,lambda,h) for WGS84
c
c      sta x y z ---> |car2geo| ---> xlon,xlat, h
c
c
c      xlon: geodetic longitude
c      xlat: geodetic latitude
c      h:  height over the ellipsoid
c
c
c     @gAGE (Research group of Astronomy and GEomatics).
c -----------------------------------------------------------

c ....value declaration.......................
       tol=1.d-9
       pi=3.1415926535898d0
c        WGS84 parameters (in meters)
         a=6378137.d0
         f=1.d0/298.257223563
         b=a*(1.d0-f)
         e2=(a**2-b**2)/a**2
c      ..............................................


       xl=datan2(y,x)
       p=dsqrt(x**2+y**2)
       fi=datan(z/p/dsqrt(1.d0-e2))
       fia=fi
20     continue
       xn=a**2/dsqrt((a*dcos(fi))**2+(b*dsin(fi))**2)
       h=p/dcos(fi)-xn
```

```
          fi=datan(z/p/(1.d0-e2*xn/(xn+h)))
          if(dabs(fi-fia).gt.tol) then
             fia=fi
             goto 20
          endif

          xlon=xl
          xlat=fi

          return
          end


c          xn=a**2/dsqrt((a*dcos(fi))**2+(b*dsin(fi))**2)
c          x=(xn+h)*dcos(fi)*dcos(xl)
c          y=(xn+h)*dcos(fi)*dsin(xl)
c          z=(b**2/a**2*xn+h)*dsin(fi)
c          write(*,'(a4,3(1x,f15.4))') "ORG", x,y,z
```

scripts

- **script** DDbell_ebre21.scr

```
#!/bin/tcsh -f

# This script calculates double differences for the model components
# of station bell regarding to ebre and satellite PRN21.
# It is executed over files "*.dmx" generated by GCAT program
# I.e.:
#
# rover= bell
# sta_ref= ebre
# sat_ref= 21
#
# Dbell_21=L[bell,sat_i]-L[bell,21]
# Debre_21=L[ebre,sat_j]-L[ebre,21]
#
# DDbell_ebre21=Dbell_21-Debre_21
# ...........................
#
# Execute:
#  DDbell_ebre21_GCAT.scr PC 99mar23bell.a_PC.dmx 99mar23ebre.a_PC.dmx
#
#     @gAGE (Research group of Astronomy and GEomatics).
# ------------------------------------------------------------------
set type=$1

cat $2|awk '{print $1,"'$type'",int($2+0.5),$3,$5,$6,$7,$8,$9,$NF}'
                                                         > bell_GCAT
cat $3|awk '{print $1,"'$type'",int($2+0.5),$3,$5,$6,$7,$8,$9,$NF}'
                                                         > ebre_GCAT

# 1. SATELLITE SELECTION:
# Select the satellite PRN21:
cat bell_GCAT | gawk '{if ($4==21) print $0}' > bell_21
cat ebre_GCAT | gawk '{if ($4==21) print $0}' > ebre_21

# Select satellites different from PRN21:
cat bell_GCAT | gawk '{if ($4!=21) print $0}' > bell_n21
cat ebre_GCAT | gawk '{if ($4!=21) print $0}' > ebre_n21
```

```
# 2. SINGLE DIFFERENCES:
# Single differences for every station, referring to satellite PRN21
cat bell_21 bell_n21 | gawk '{if ($4==21) {r[$2 $3]=$5;x[$2 $3]=$6;
 y[$2 $3]=$7;z[$2 $3]=$8;t[$2 $3]=$9;i[$2 $3]=$10};
 {if ($4!=21 && length(r[$2 $3])!=0) printf "%s %s %6i %02i %16.12f
   %16.12f %16.12f %16.12f %16.12f %3i \n",$1,$2,$3,$4,$5-r[$2 $3],
   $6-x[$2 $3],$7-y[$2 $3],$8-z[$2 $3],$9-t[$2 $3],$10+i[$2 $3]}}'
                                                            > Dbell_21

cat ebre_21 ebre_n21 | gawk '{if ($4==21) {r[$2 $3]=$5;i[$2 $3]=$10};
 {if ($4!=21 && length(r[$2 $3])!=0) printf "%s %s %6i %02i %16.12f
  %16.12f %16.12f %16.12f %16.12f %3i \n",$1,$2,$3,$4,$5-r[$2 $3],
                                 0,0,0,0,$10+i[$2 $3]}}' > Debre_21

# 3. DOUBLE DIFFERENCES:
cat Debre_21 Dbell_21 | gawk '{if ($1=="ebre") {r[$2 $3 $4]=$5;
  x[$2 $3 $4]=$6;y[$2 $3 $4]=$7;z[$2 $3 $4]=$8;t[$2 $3 $4]=$9;
  i[$2 $3 $4]=$10} else {if (length(r[$2 $3 $4])!=0)
  printf "%s %6i %02i %16.12f %16.12f %16.12f %16.12f %16.12f %3i \n",
  $2,$3,$4,$5-r[$2 $3 $4],$6-x[$2 $3 $4],$7-y[$2 $3 $4],
  $8-z[$2 $3 $4],$9-t[$2 $3 $4],$10+i[$2 $3 $4]}}'>DDbell_ebre21_GCAT

# 4. END: write results in the corresponding format for "kalman.f"
#           program input :
#     [itype sec PRN DDprefit sigma Ddx Ddy Ddz Ddt iarc]
#
#    NOTE:1) sigma corresponds to the measurement noise (in this
#            case, double diferenciated code observations).
#            Taking: sigma=10m (PC)
#         2) iarc is the number of phase arc (only took into
#            account for phase measurements, and it is used for
#            identifying instants where cycle-slips have happened)

cat DDbell_ebre21_GCAT |gawk '{s=10;if ($1=="LC") {s=0.01};
  printf "%s %6i %02i %16.12f %6.3f %16.12f %16.12f %16.12f %16.12f
                     %3i \n", $1,$2,$3,$4,s,$5,$6,$7,$8,$9}' > nada
mv nada  "DDbell_ebre21_"$type".mod"

rm bell_GCAT ebre_GCAT bell_21 ebre_21 bell_n21 ebre_n21 Dbell_21
                                      Debre_21  DDbell_ebre21_GCAT
```

- **script** `add.scr`

```
#!/bin/csh -f

# -------------------------------------------------------------------
# This script generates a unique file from files:
#     DDbell_ebre21.obs DDbell_ebre21.ion  DDbell_ebre21.bc
#
#
# Execute:
#
#  add.scr DDbell_ebre21.obs DDbell_ebre21.ion  DDbell_ebre21.bc
#                                              > DDbell_ebre21.dat
#
#
#  Input files:
#  --------------
#
#   DDbell_ebre21.obs= [bell ebre 21 isat iarc sec DDLc DDLw DDLi]
#
#   DDbell_ebre21.ion= [bell ebre 21 isat sec DDSTEC]
#
#   DDbell_ebre21.bc = [bell ebre 21 isat iarc sec0 sec1 DDbc]
#
#       where
#              DDbc: bias in DDLc observable
#              sec0: first point of DDLc arc
#              sec1: last point of DDLc arc
#
#
#  OUTPUT:   DDbell_ebre21.dat
#  ------
#
#         [sta,staR,isatR,isat,iarc,sec,DDLc,DDLw,DDLi,DDSTEC,DDBc]
#
#
#
#
#      @gAGE (Research group of Astronomy and GEomatics).
# --------------------------------------------------------------------
```

```
set DDobs=$1
set DDion=$2
set DDbc=$3

cat $DDion $DDobs|gawk '{if (NF==6) {I[$4" "$5]=$6}
              else{if (length(I[$4" "$6])!=0)
                {printf "%s %8.4f \n", $0,I[$4" "$6]}}}' > nada.dat

cat $DDbc nada.dat|gawk '{if (NF==8) {t0[$4" "$5]=$6;
     t1[$4" "$5]=$7;Bc[$4" "$5]=$8}
             else{s0=t0[$4" "$5];s1=t1[$4" "$5];
                  if ($6>=s0 && $6<=s1)
                       {printf "%s %8.4f \n", $0,Bc[$4" "$5]}}}'


rm nada.dat
```

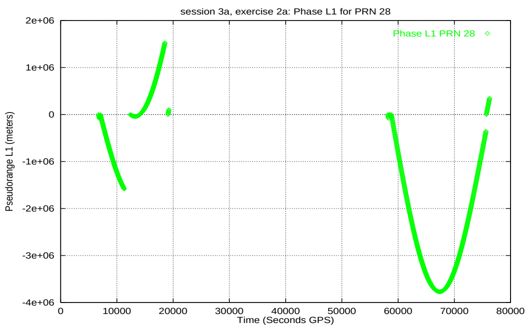
- **script** `DDobs.scr`

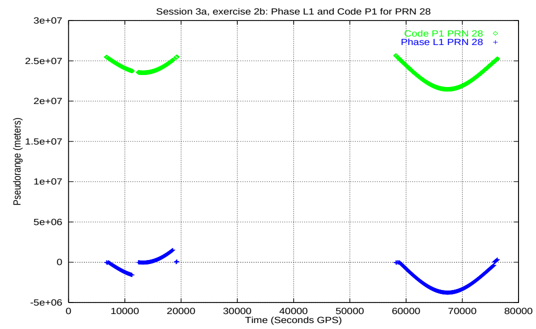
```
#!/bin/csh -f

# -----------------------------------------------------------------
#
# Execute:
#
#  DDobs.scr  99mar23bell_ebre.s_1_30_ninja.gz
#
#  NOTE: Use a zipped file "*.gz"
#
#  Input file:
#  --------------
#   99mar23bell_ebre.s_1_30_ninja.gz:
#                  [sta,idoy,sec,isat,xLC,xLI,PC,PI,iarc]
#
#
#  OUTPUT:   DDbell_ebre21.obs
#  ------
#
#           [sta,staR,isatR,isat,iarc,sec,DDLc,DDLw,DDLi]
#
#
#
#   @gAGE (Research group of Astronomy and GEomatics).
# ----------------------------------------------------------------

set file=$1

set sta="bell"
set sta_ref="ebre"
set sat_ref=21

zgrep -v $sta_ref $file > nsb.tmp
zgrep $sta_ref $file > nse.tmp
gawk '{if ($4=="'$sat_ref'") print $0}' nsb.tmp >  nada.tmp
gawk '{if ($4!="'$sat_ref'") print $0}' nsb.tmp >> nada.tmp
gawk '{if ($4=="'$sat_ref'") print $0}' nse.tmp >> nada.tmp
gawk '{if ($4!="'$sat_ref'") print $0}' nse.tmp >> nada.tmp
```

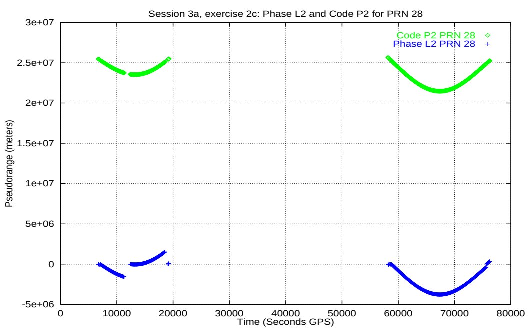
```
# Computing single differences:
# -----------------------------
#
#  Dbell_21=L[bell,sat_i]-L[bell,21]
#  Debre_21=L[ebre,sat_j]-L[ebre,21]
#
#   Format: [sta isatR isat,iarc,sec,DLc,DLw,DLi]


cat nada.tmp | gawk 'BEGIN{a="'$sat_ref'";g=(77/60)**2;f=sqrt(g)/(g-1)}
 {w=$5+f*$6;if ($4==a) {LC[$1" "$3]=$5;LI[$1" "$3]=$6;LW[$1" "$3]=w;
                                                        A[$1" "$3]=$9}
  else
  {if (length(LC[$1" "$3])!=0) printf "%s %2i   %02i %2i %6i %16.4f
     %16.4f %10.4f \n", $1,a,$4,$9+A[$1" "$3],$3,$5-LC[$1" "$3],
     w-LW[$1" "$3], $6-LI[$1" "$3]}}' > "D_sta_"$sat_ref


# Computing double differences:
# -----------------------------
#
# DDbell_ebre21=Dbell_21-Debre_21
#
#  Format: [sta staR isatR isat,iarc,sec,DDLc,DDLw,DDLi]


cat "D_sta_"$sat_ref | gawk '{if (length(LC[$5" "$3])!=0)
 {printf "%s %s %s   %s %2i %8i %16.4f %16.4f %10.4f \n",
   s[$5" "$3],$1,$2,$3,$4+A[$5" "$3],$5,LC[$5" "$3]-$6,LW[$5" "$3]-$7,
   LI[$5" "$3]-$8} else {LC[$5" "$3]=$6;LW[$5" "$3]=$7;LI[$5" "$3]=$8;
   A[$5" "$3]=$4;s[$5" "$3]=$1}}' > "DD"$sta"_"$sta_ref""$sat_ref".obs"

rm nsb.tmp nse.tmp nada.tmp
#rm  "D_sta_"$sat_ref
```

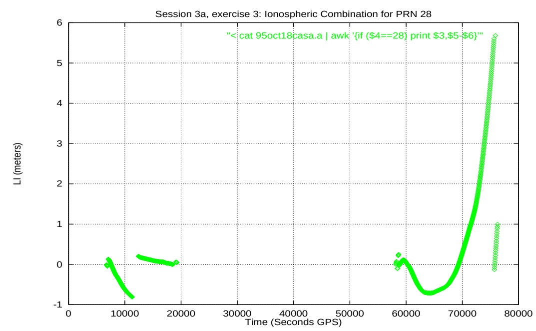
Appendix V: exercise graphs

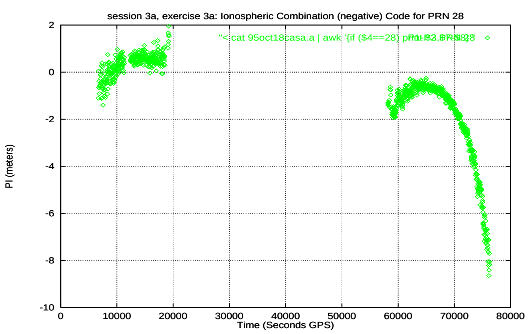
Graphs Session 3a

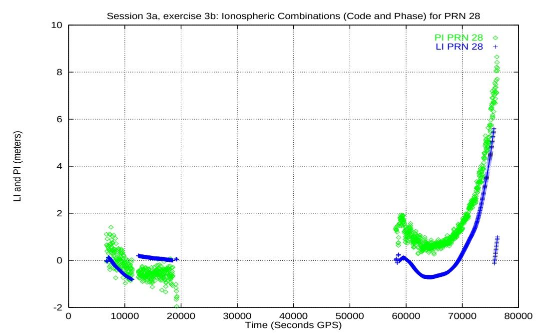


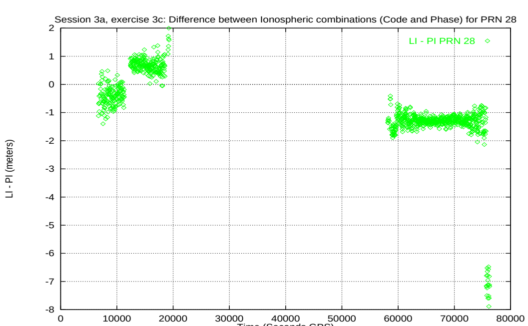


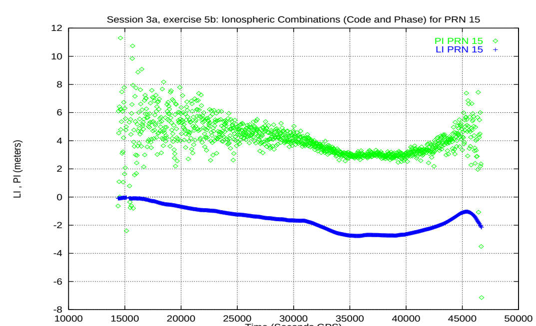


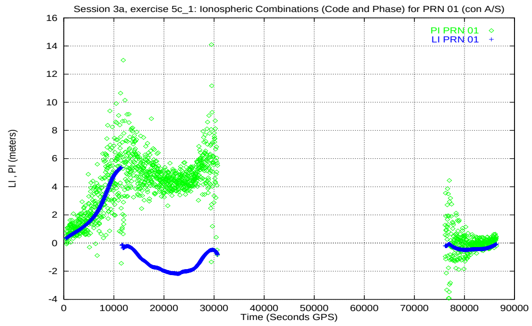


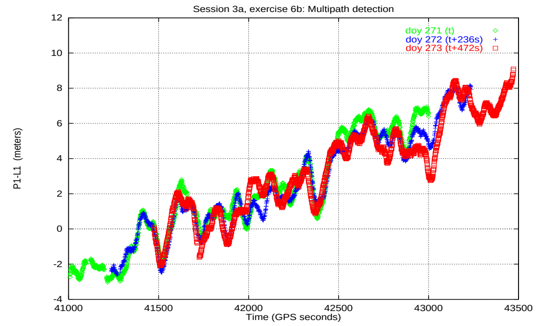




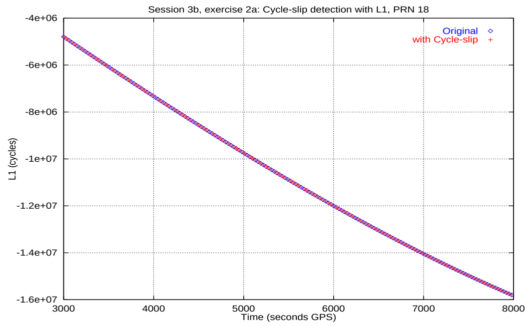


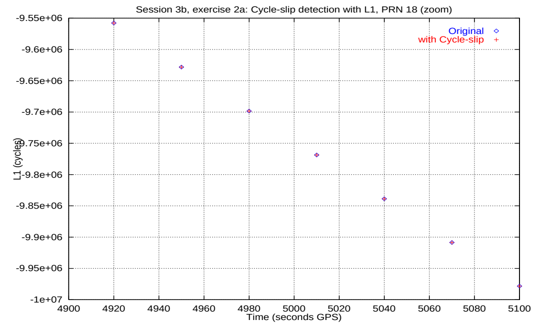


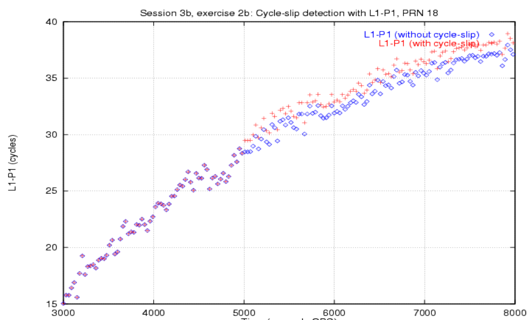


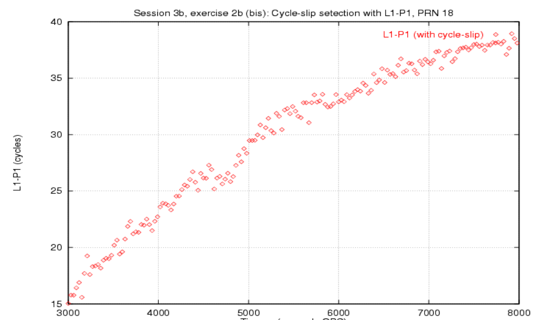


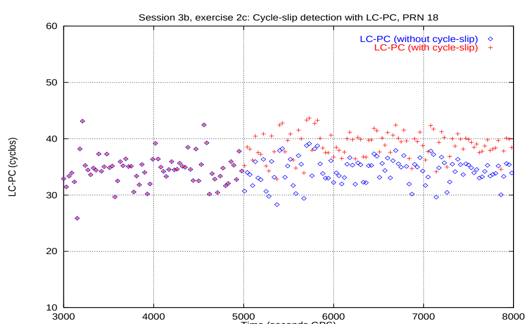
Graphs Session 3b

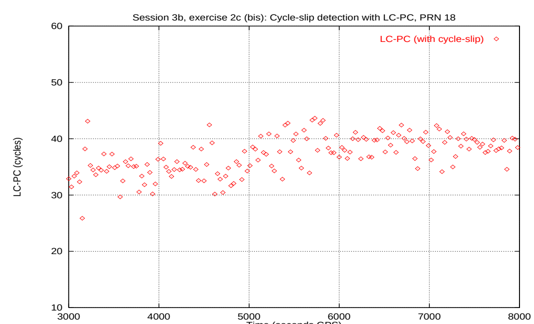


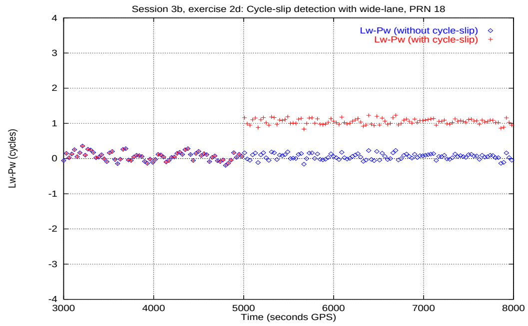


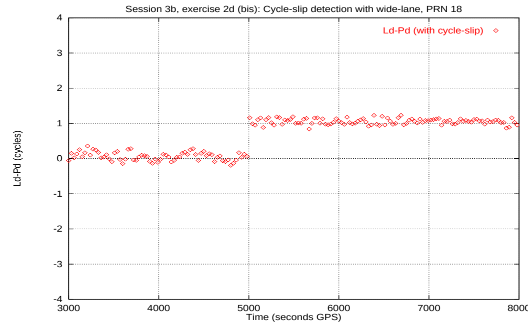


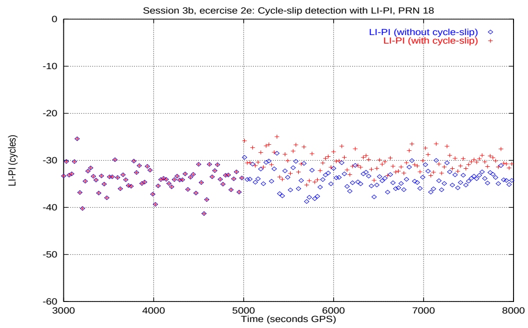


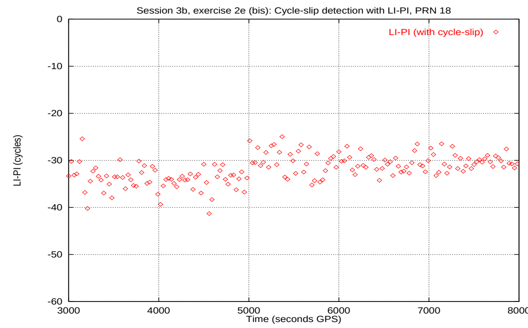


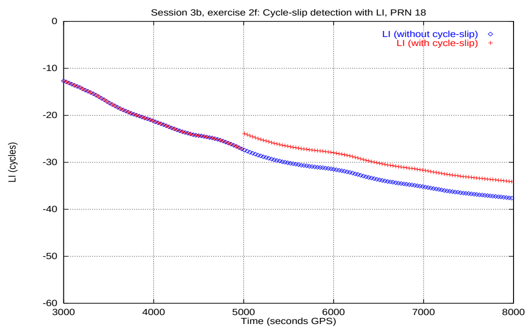


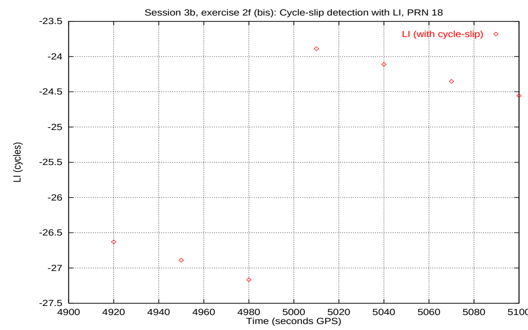




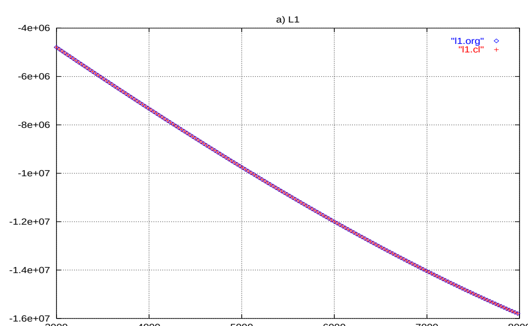


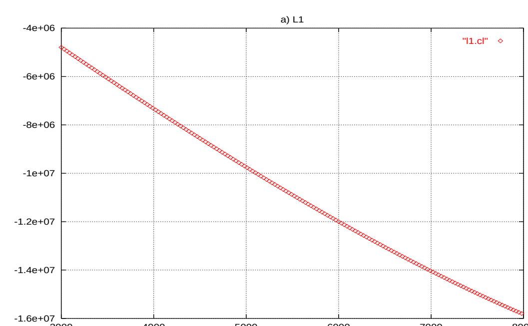


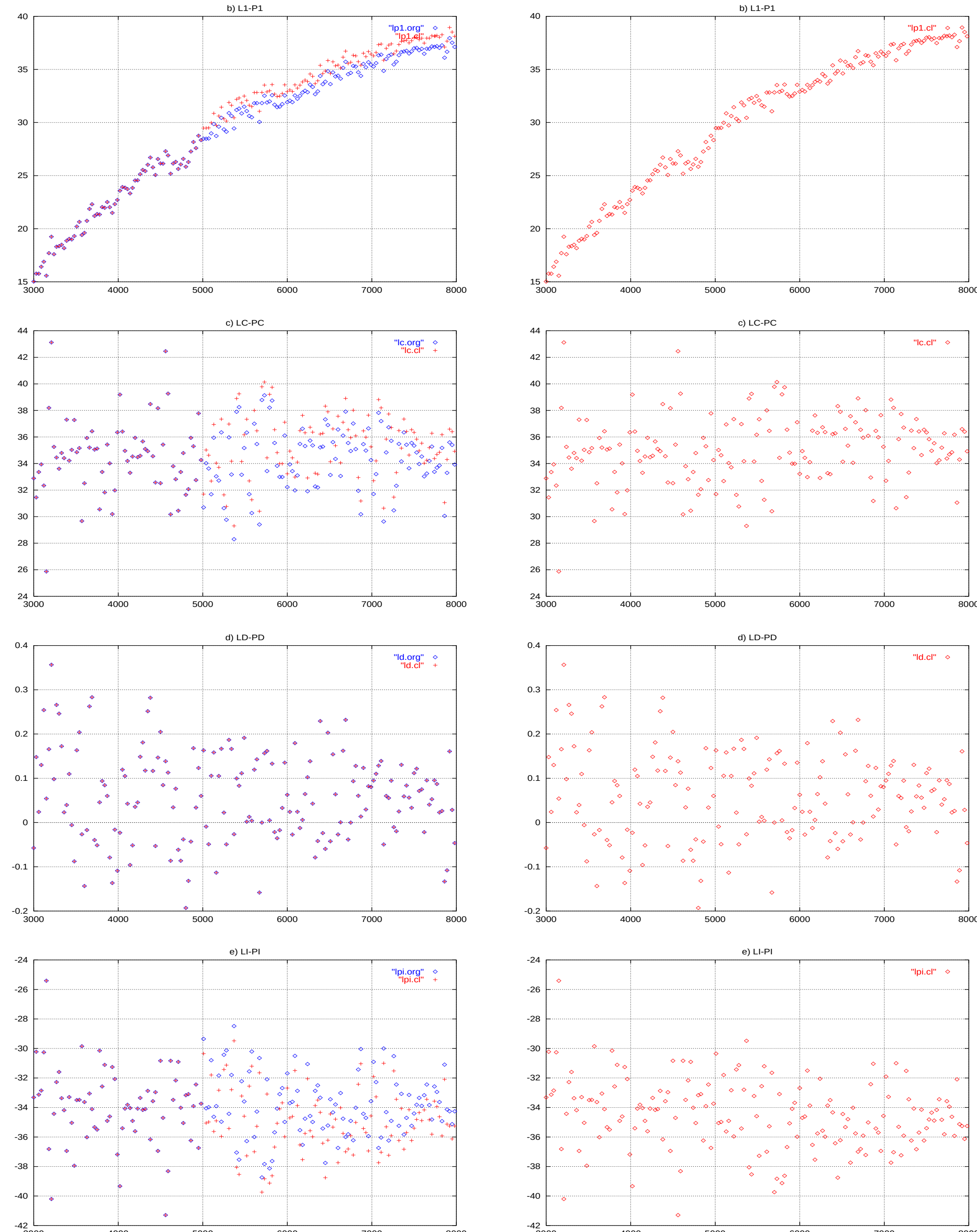


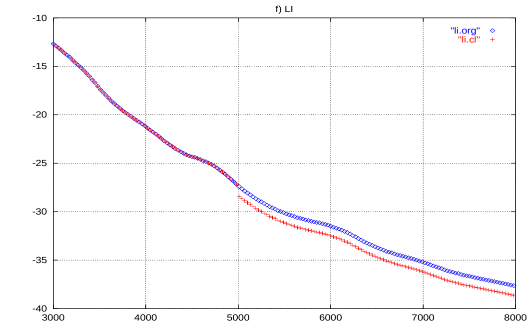


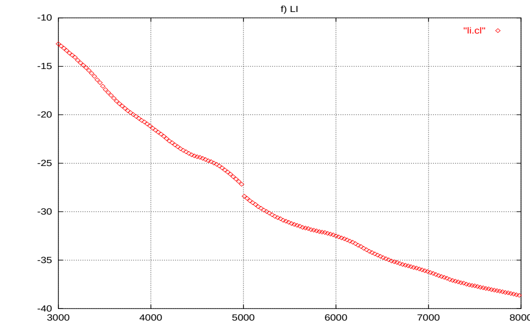
Graphs Session 3b (script exercise 3)



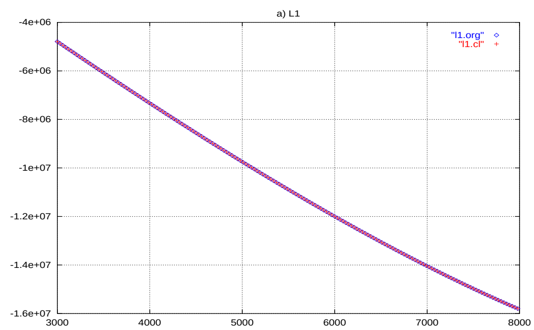


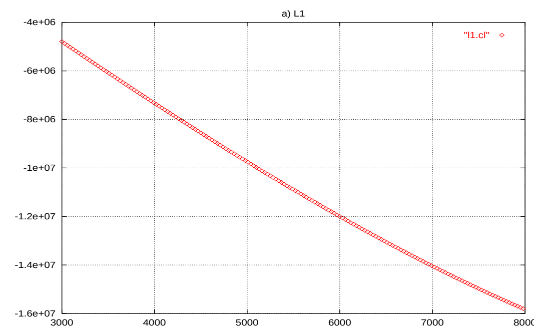

b) L1-P1
"lp1.org"
"lp1.cl"
b) L1-P1
"lp1.cl"
c) LC-PC
"lc.org"
"lc.cl"
c) LC-PC
"lc.cl"
d) LD-PD
"ld.org"
"ld.cl"
d) LD-PD
"ld.cl"
e) LI-PI
"lpi.org"
"lpi.cl"
e) LI-PI
"lpi.cl"

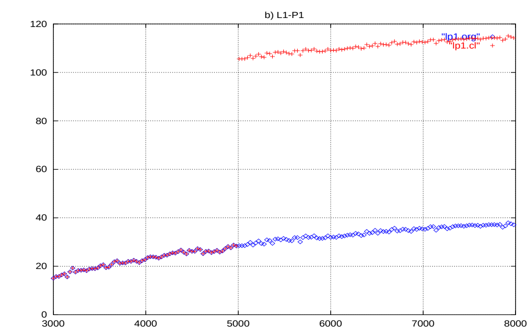


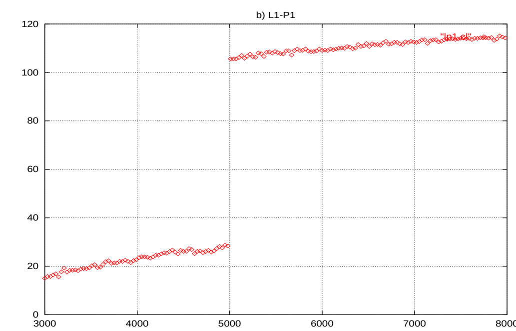


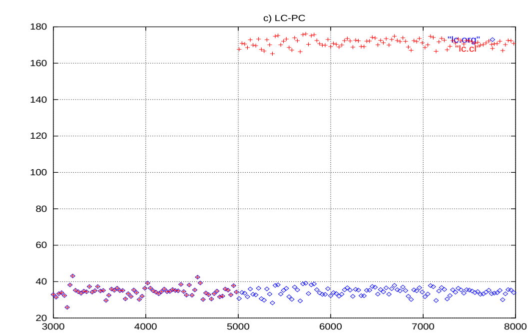
Graphs Session 3b (script exercise 5)

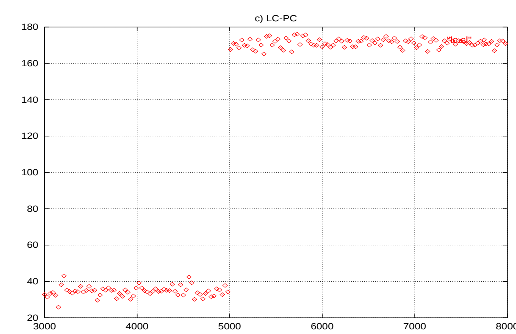


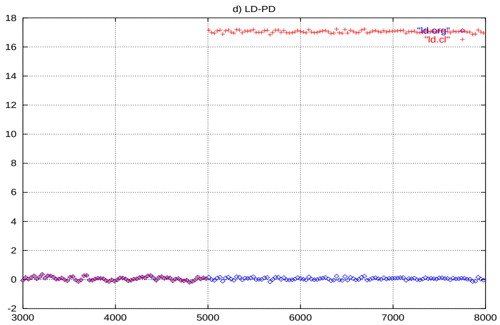


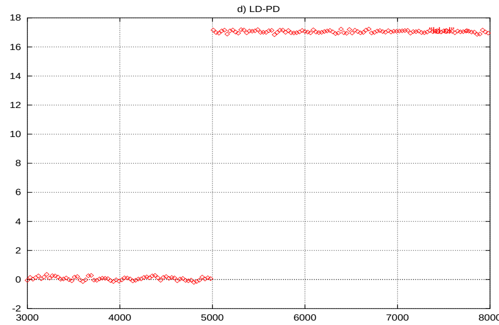


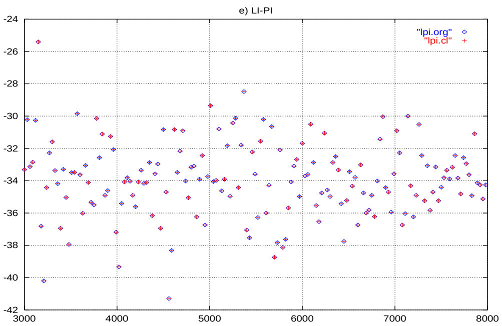


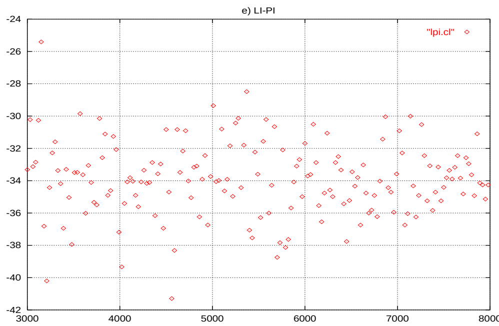


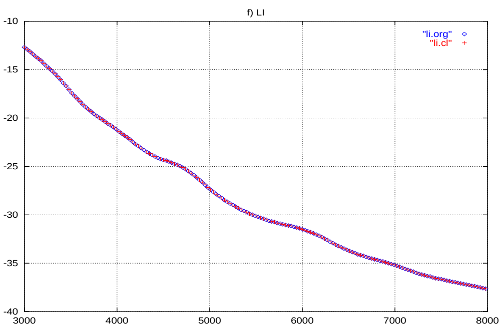


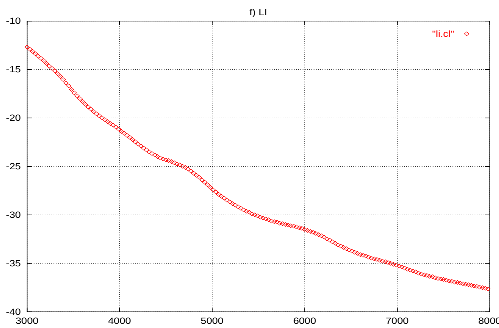




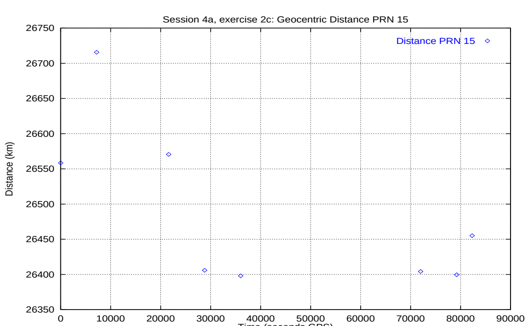


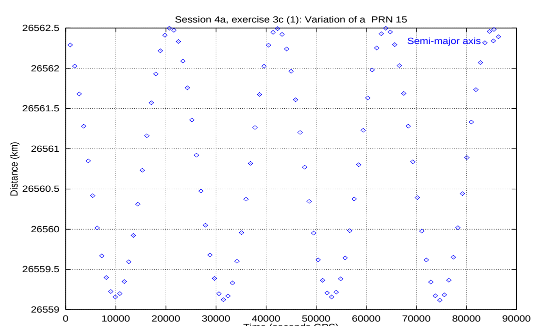


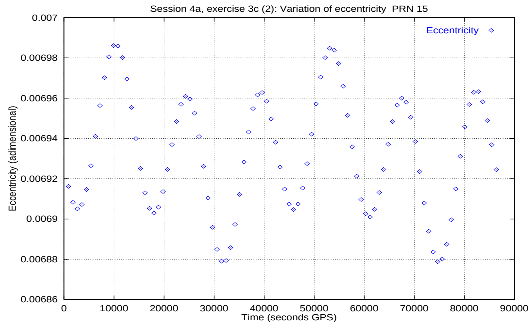




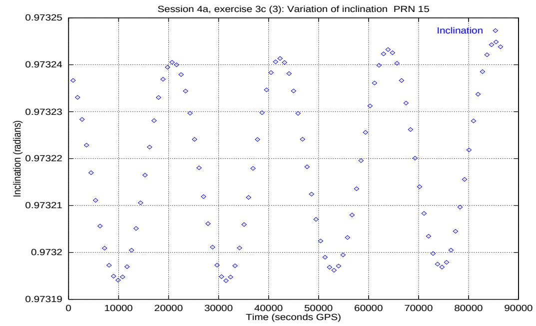
Graphs Session 4a



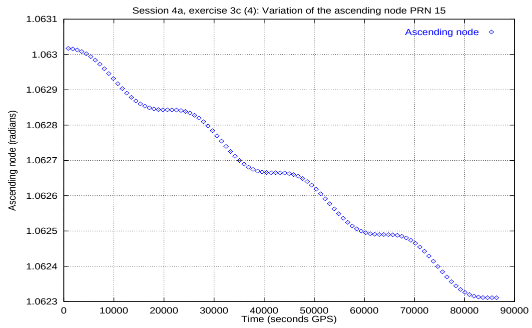




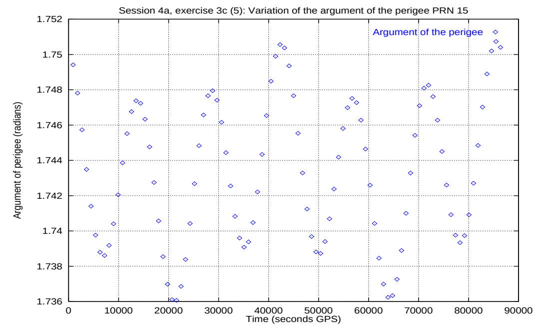
Session 4a, exercise 3c (2): Variation of eccentricity PRN 15
Eccentricity
Eccentricity (adimensional)
Time (seconds GPS)
0.007
0.00698
0.00696
0.00694
0.00692
0.0069
0.00688
0.00686
0
10000
20000
30000
40000
50000
60000
70000
80000
90000



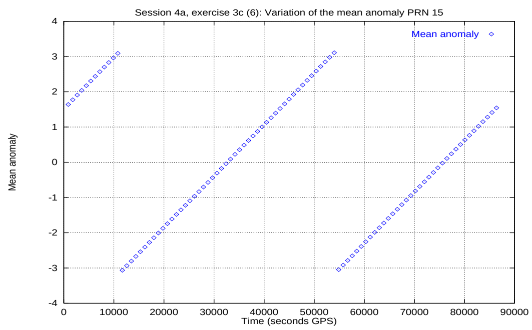
Session 4a, exercise 3c (3): Variation of inclination PRN 15
Inclination
Inclination (radians)
Time (seconds GPS)
0.97325
0.97324
0.97323
0.97322
0.97321
0.9732
0.97319
0
10000
20000
30000
40000
50000
60000
70000
80000
90000



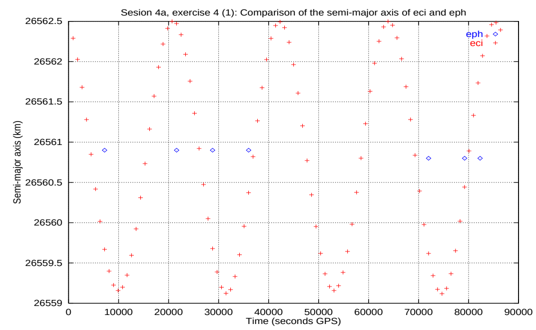
Session 4a, exercise 3c (4): Variation of the ascending node PRN 15
Ascending node
Ascending node (radians)
Time (seconds GPS)
1.0631
1.063
1.0629
1.0628
1.0627
1.0626
1.0625
1.0624
1.0623
0
10000
20000
30000
40000
50000
60000
70000
80000
90000



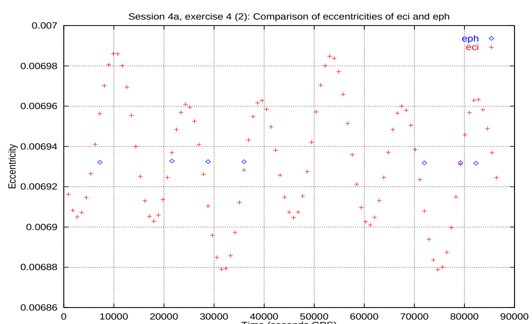
Session 4a, exercise 3c (5): Variation of the argument of the perigee PRN 15
Argument of the perigee
Argument of perigee (radians)
Time (seconds GPS)
1.752
1.75
1.748
1.746
1.744
1.742
1.74
1.738
1.736
0
10000
20000
30000
40000
50000
60000
70000
80000
90000



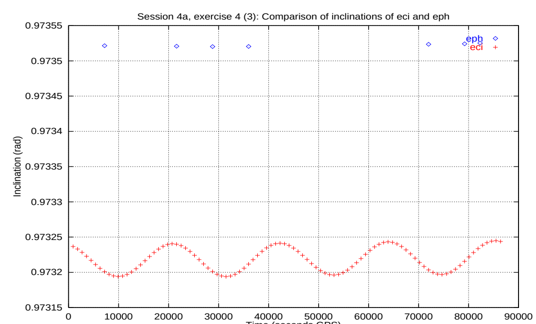
Session 4a, exercise 3c (6): Variation of the mean anomaly PRN 15
Mean anomaly
Mean anomaly
Time (seconds GPS)
4
3
2
1
0
-1
-2
-3
-4
0
10000
20000
30000
40000
50000
60000
70000
80000
90000



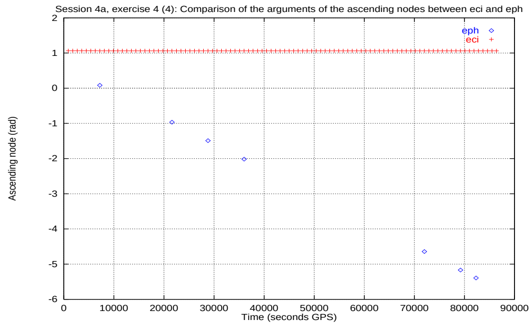
Session 4a, exercise 4 (1): Comparison of the semi-major axis of eci and eph
eph
eci
Semi-major axis (km)
Time (seconds GPS)
26562.5
26562
26561.5
26561
26560.5
26560
26559.5
26559
0
10000
20000
30000
40000
50000
60000
70000
80000
90000

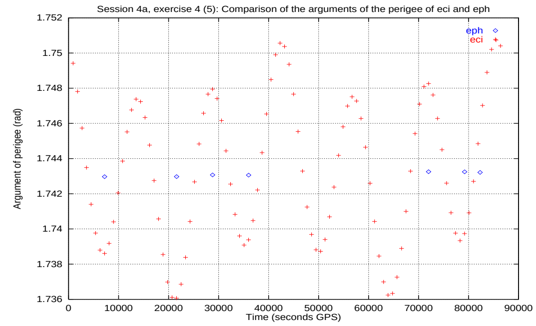


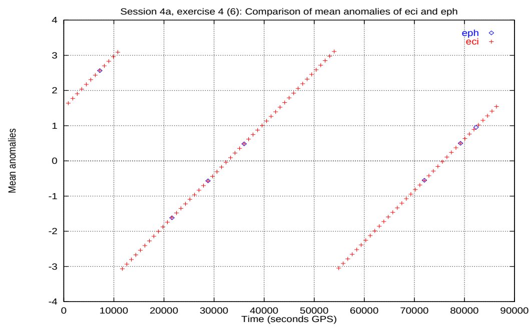
Session 4a, exercise 4 (2): Comparison of eccentricities of eci and eph
eph
eci
Eccentricity
Time (seconds GPS)
0.007
0.00698
0.00696
0.00694
0.00692
0.0069
0.00688
0.00686
0
10000
20000
30000
40000
50000
60000
70000
80000
90000

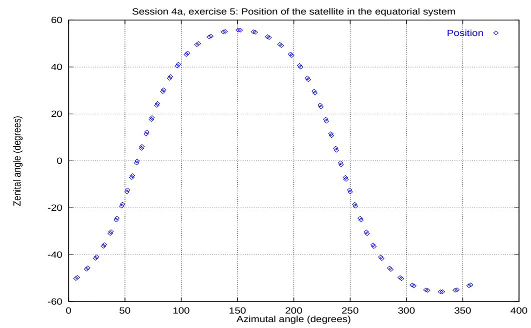


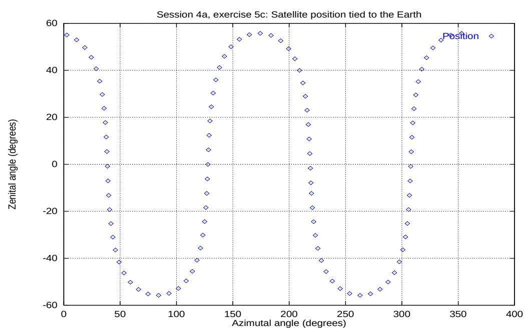
Session 4a, exercise 4 (3): Comparison of inclinations of eci and eph
eph
eci
Inclination (rad)
Time (seconds GPS)
0.97355
0.9735
0.97345
0.9734
0.97335
0.9733
0.97325
0.9732
0.97315
0
10000
20000
30000
40000
50000
60000
70000
80000
90000



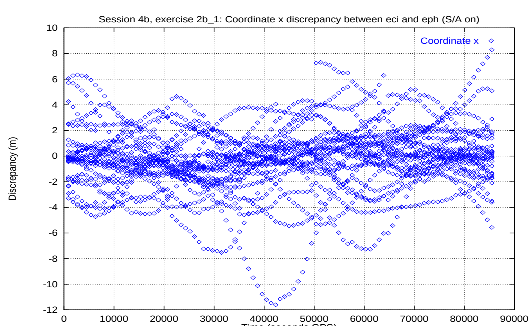


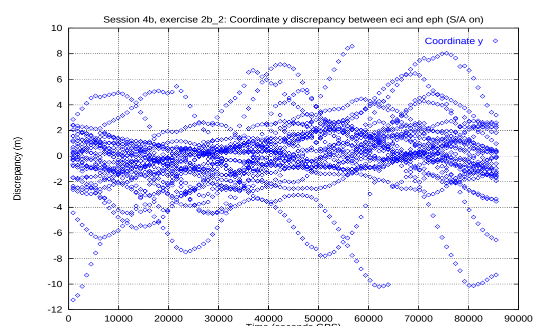


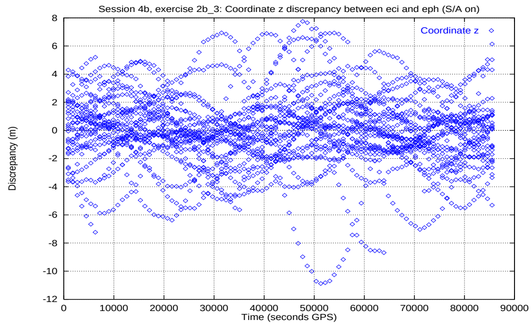


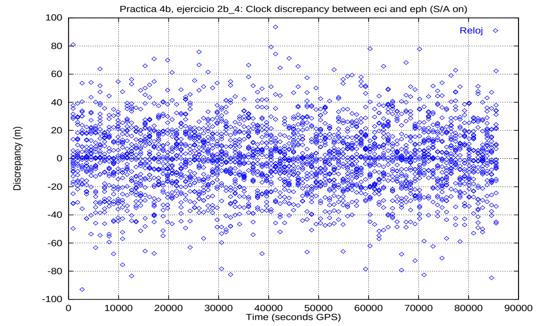


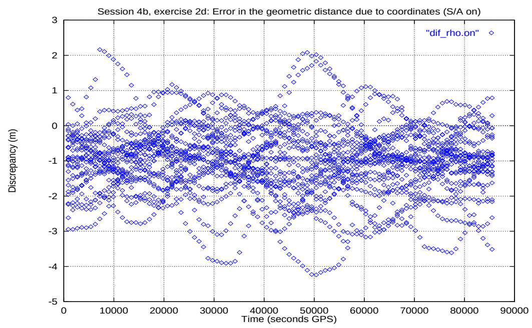
Graphs Session 4b

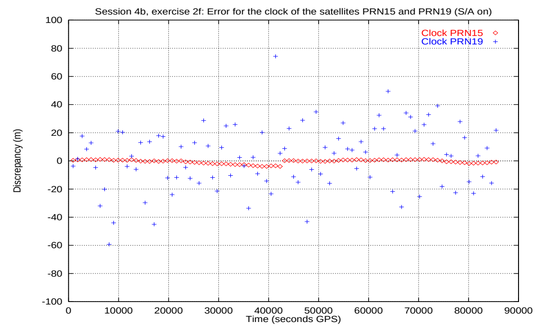


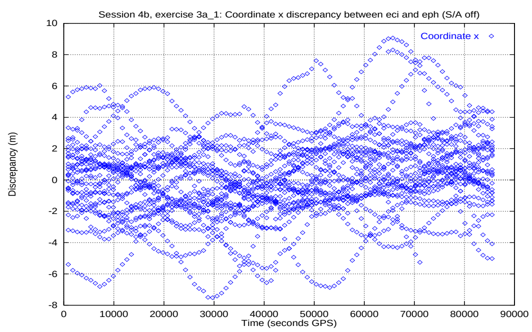


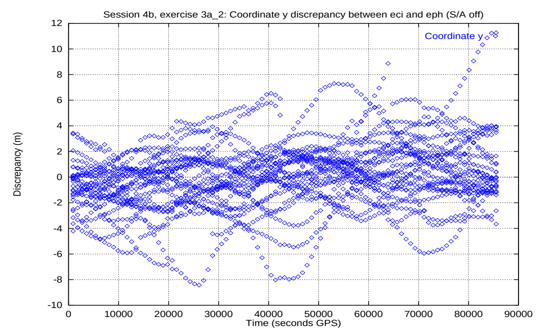

Session 4b, exercise 2b_3: Coordinate z discrepancy between eci and eph (S/A on)
Coordinate z
Discrepancy (m)
Time (seconds GPS)

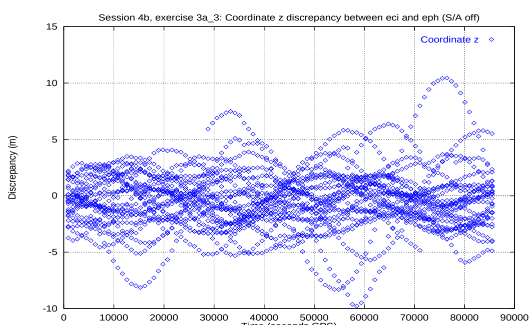

Practica 4b, ejercicio 2b_4: Clock discrepancy between eci and eph (S/A on)
Reloj
Discrepancy (m)
Time (seconds GPS)

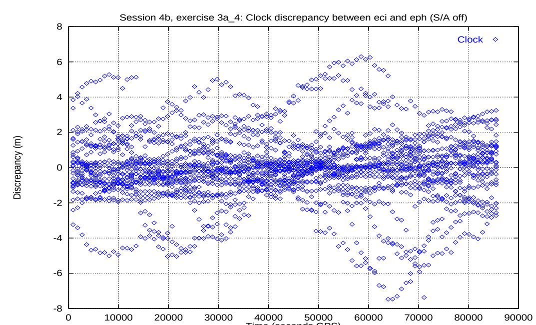

Session 4b, exercise 2d: Error in the geometric distance due to coordinates (S/A on)
"dif_rho.on"
Discrepancy (m)
Time (seconds GPS)

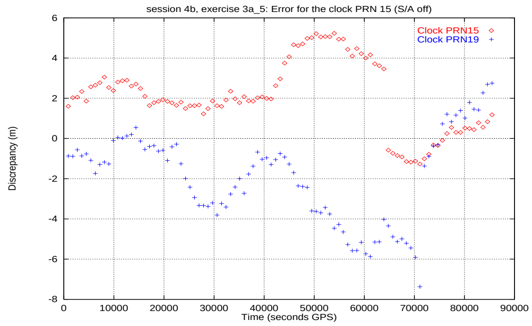

Session 4b, exercise 2f: Error for the clock of the satellites PRN15 and PRN19 (S/A on)
Clock PRN15
Clock PRN19
Discrepancy (m)
Time (seconds GPS)

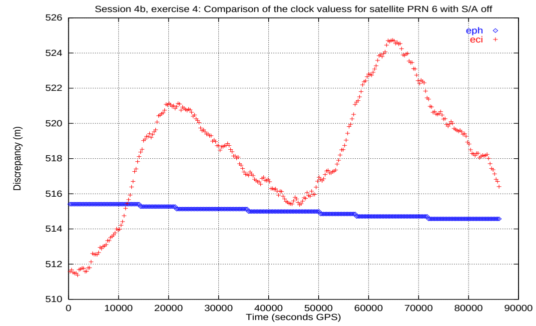

Session 4b, exercise 3a_1: Coordinate x discrepancy between eci and eph (S/A off)
Coordinate x
Discrepancy (m)
Time (seconds GPS)

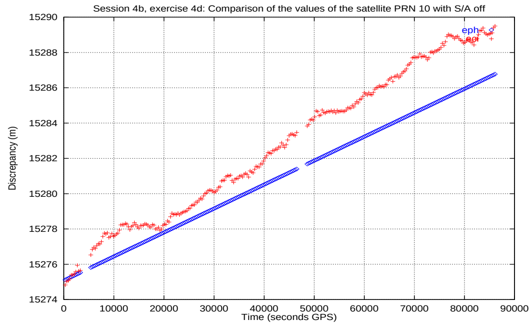

Session 4b, exercise 3a_2: Coordinate y discrepancy between eci and eph (S/A off)
Coordinate y
Discrepancy (m)
Time (seconds GPS)

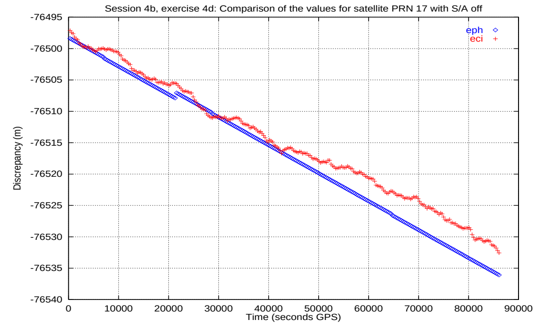

Session 4b, exercise 3a_3: Coordinate z discrepancy between eci and eph (S/A off)
Coordinate z
Discrepancy (m)
Time (seconds GPS)

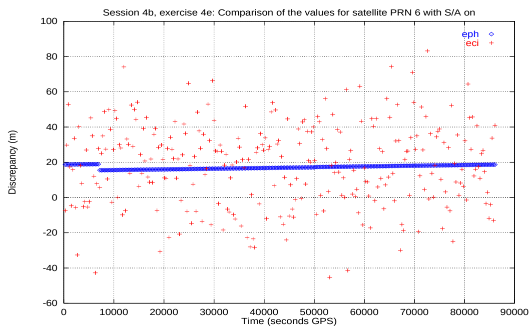

Session 4b, exercise 3a_4: Clock discrepancy between eci and eph (S/A off)
Clock
Discrepancy (m)
Time (seconds GPS)

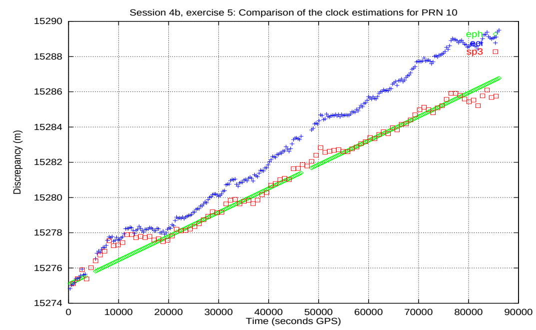

session 4b, exercise 3a_5: Error for the clock PRN 15 (S/A off)
Clock PRN15
Clock PRN19
Discrepancy (m)
Time (seconds GPS)

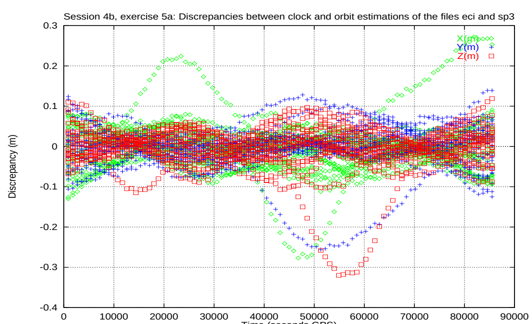

Session 4b, exercise 4: Comparison of the clock valuess for satellite PRN 6 with S/A off
eph
eci
Discrepancy (m)
Time (seconds GPS)

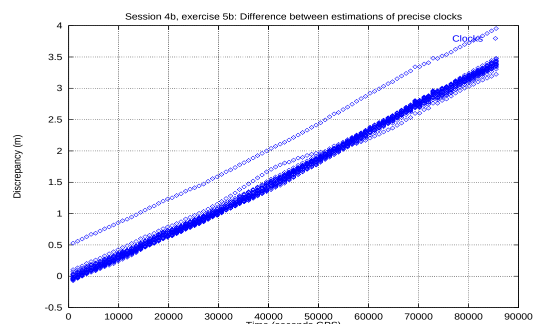

Session 4b, exercise 4d: Comparison of the values of the satellite PRN 10 with S/A off
eph
eci
Discrepancy (m)
Time (seconds GPS)


Session 4b, exercise 4d: Comparison of the values for satellite PRN 17 with S/A off
eph
eci
Discrepancy (m)
Time (seconds GPS)

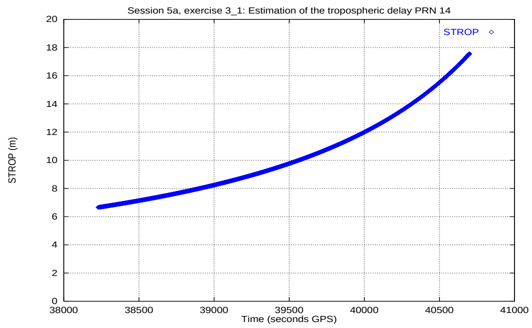

Session 4b, exercise 4e: Comparison of the values for satellite PRN 6 with S/A on
eph
eci
Discrepancy (m)
Time (seconds GPS)

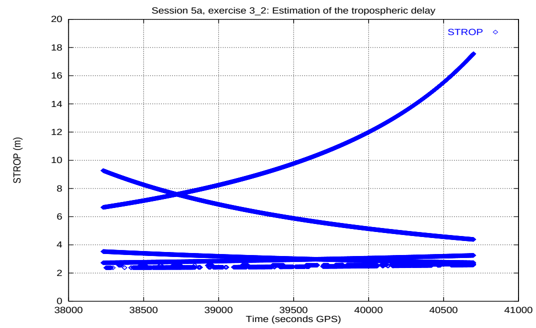

Session 4b, exercise 5: Comparison of the clock estimations for PRN 10
eph
eci
sp3
Discrepancy (m)
Time (seconds GPS)

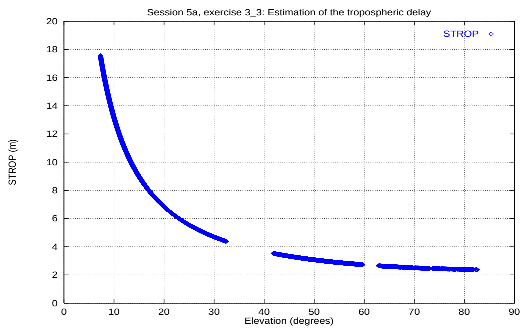

Session 4b, exercise 5a: Discrepancies between clock and orbit estimations of the files eci and sp3
X(m)
Y(m)
Z(m)
Discrepancy (m)
Time (seconds GPS)

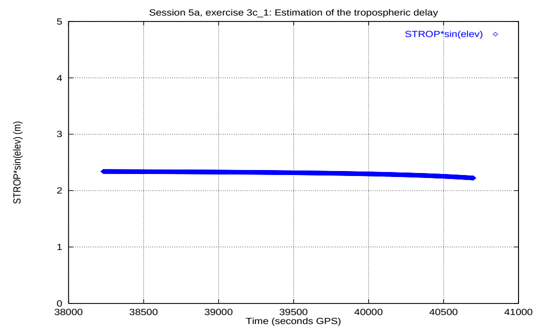

Session 4b, exercise 5b: Difference between estimations of precise clocks
Clocks
Discrepancy (m)
Time (seconds GPS)

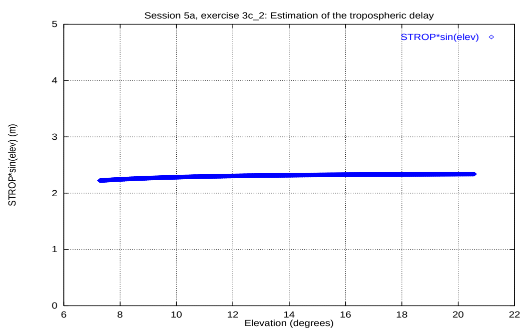
Graphs Session 5a

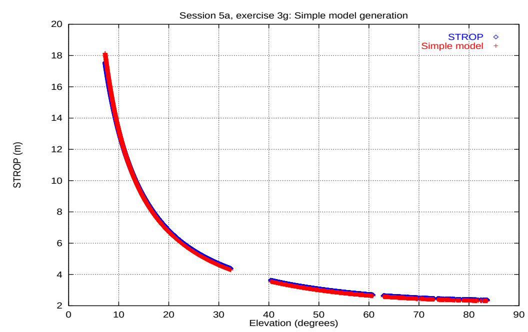


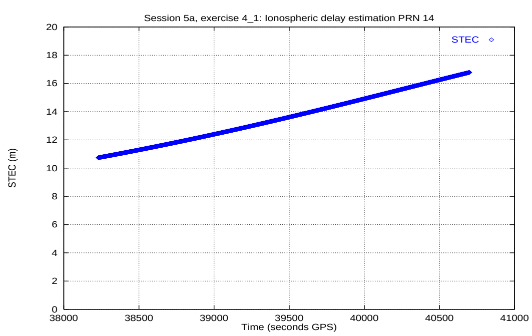


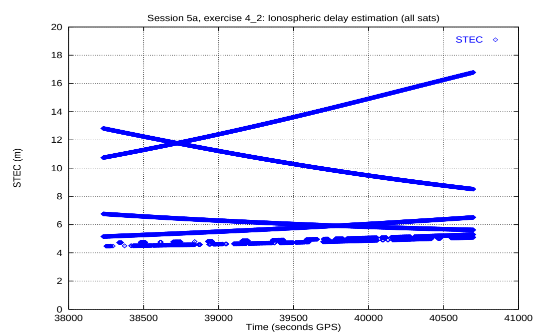


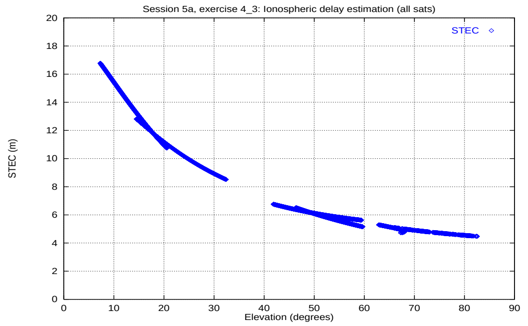


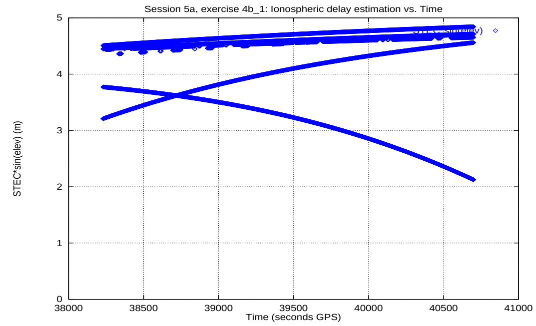


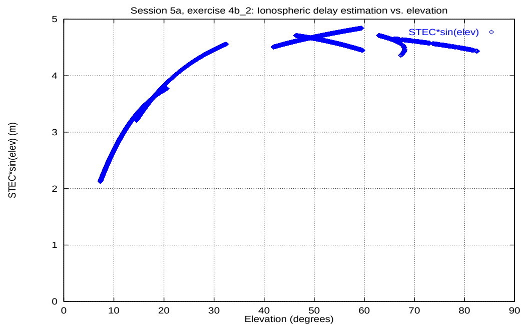


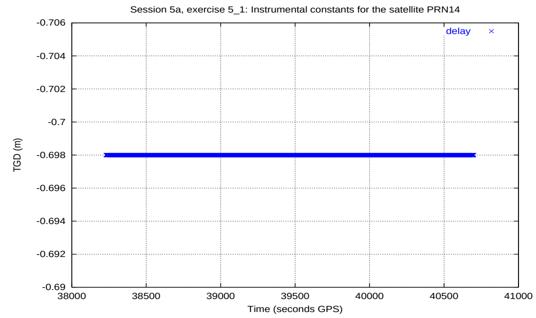


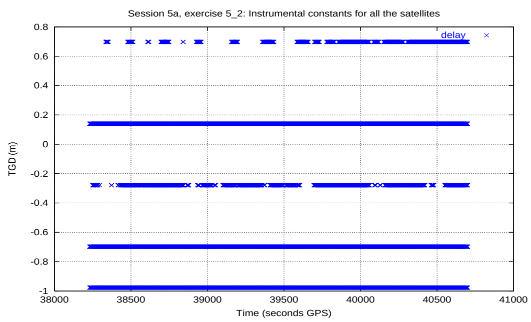


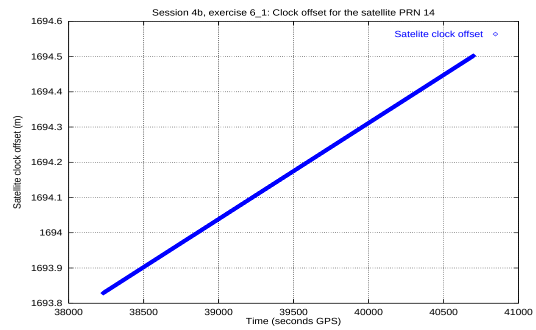

Session 5a, exercise 4_3: Ionospheric delay estimation (all sats)
STEC
STEC (m)
Elevation (degrees)
0
2
4
6
8
10
12
14
16
18
20
0 10 20 30 40 50 60 70 80 90

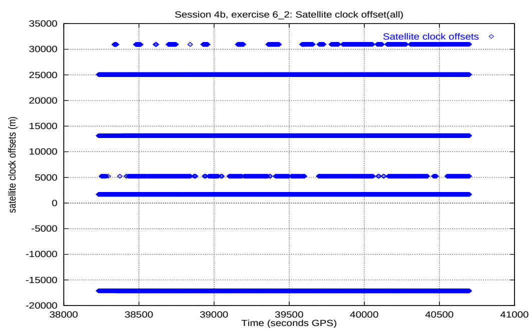

Session 5a, exercise 4b_1: Ionospheric delay estimation vs. Time
STEC*sin(elev) (m)
Time (seconds GPS)
0
1
2
3
4
5
38000 38500 39000 39500 40000 40500 41000


Session 5a, exercise 4b_2: Ionospheric delay estimation vs. elevation
STEC*sin(elev)
STEC*sin(elev) (m)
Elevation (degrees)
0
1
2
3
4
5
0 10 20 30 40 50 60 70 80 90

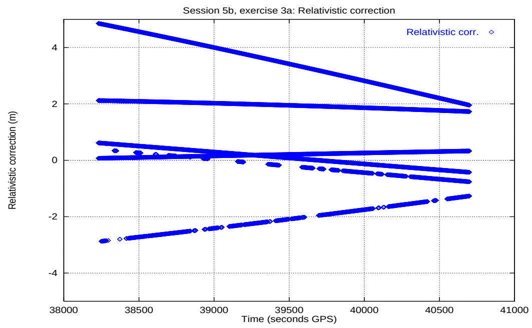

Session 5a, exercise 5_1: Instrumental constants for the satellite PRN14
delay
TGD (m)
Time (seconds GPS)
-0.706
-0.704
-0.702
-0.7
-0.698
-0.696
-0.694
-0.692
-0.69
38000 38500 39000 39500 40000 40500 41000

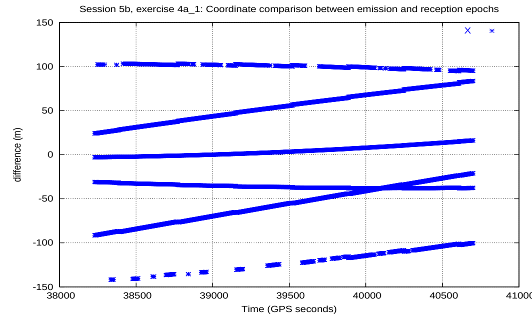

Session 5a, exercise 5_2: Instrumental constants for all the satellites
delay
TGD (m)
Time (seconds GPS)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
38000 38500 39000 39500 40000 40500 41000

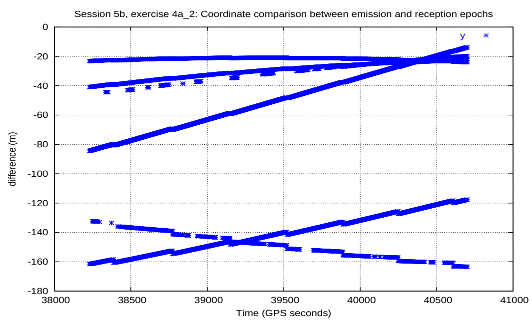

Session 4b, exercise 6_1: Clock offset for the satellite PRN 14
Satelite clock offset
Satellite clock offset (m)
Time (seconds GPS)
1694.6
1694.5
1694.4
1694.3
1694.2
1694.1
1694
1693.9
1693.8
38000 38500 39000 39500 40000 40500 41000

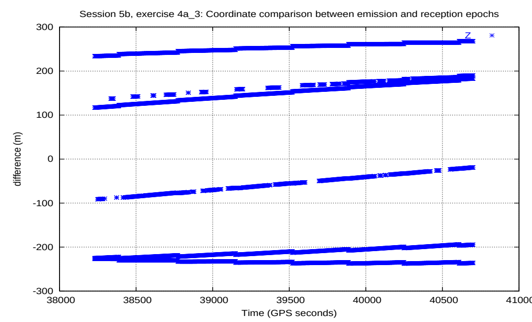

Session 4b, exercise 6_2: Satellite clock offset(all)
Satellite clock offsets
satellite clock offsets (m)
Time (seconds GPS)
35000
30000
25000
20000
15000
10000
5000
0
-5000
-10000
-15000
-20000
38000 38500 39000 39500 40000 40500 41000

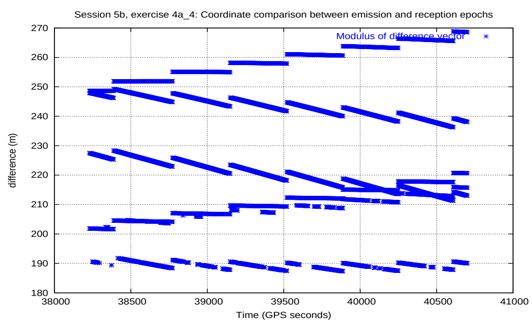
Graphs Session 5b

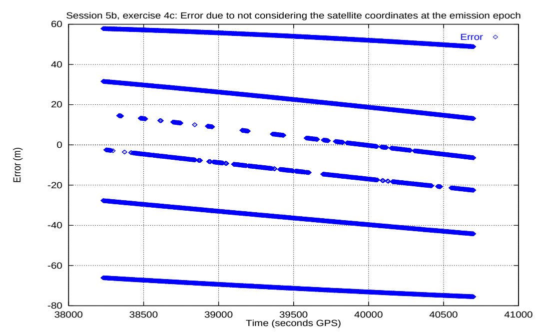


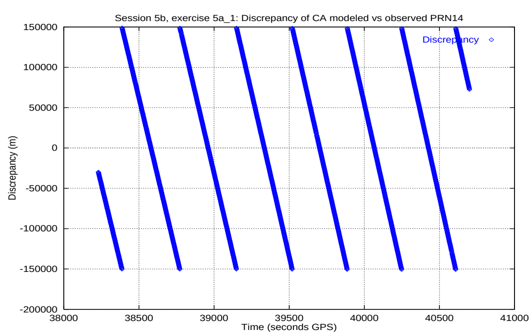


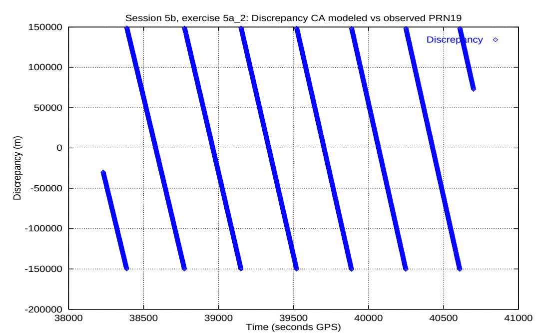


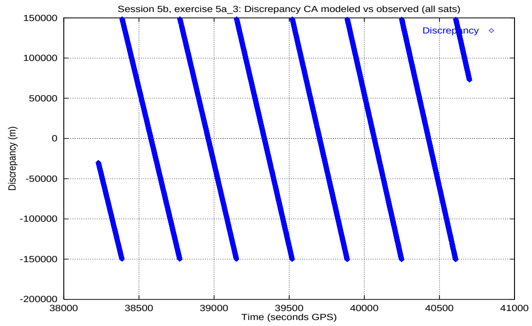


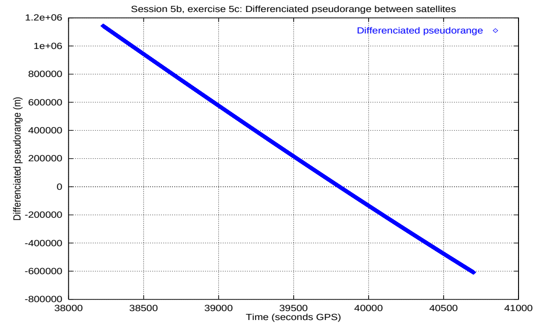


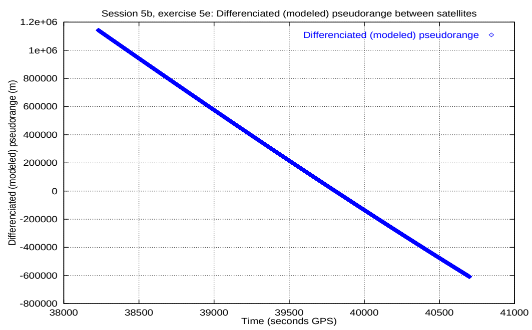


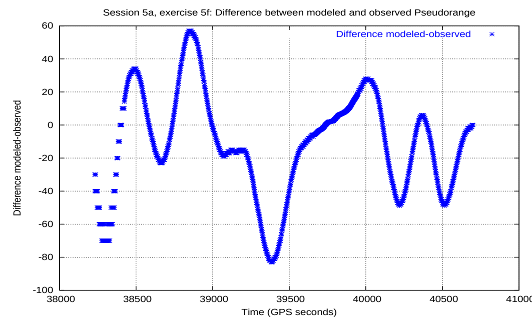




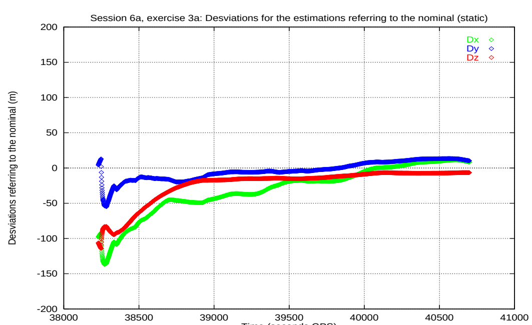


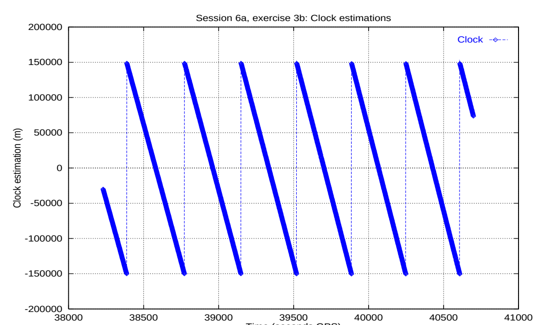


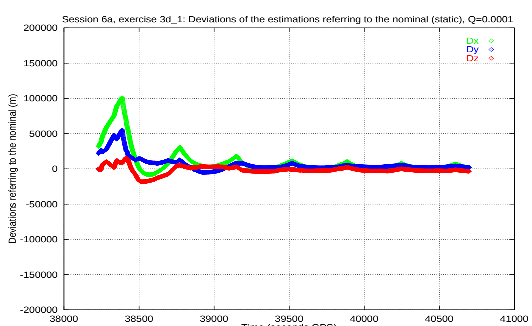


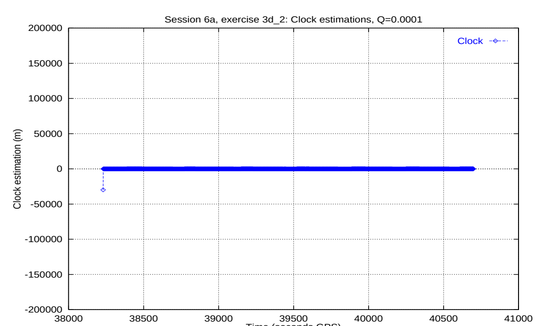


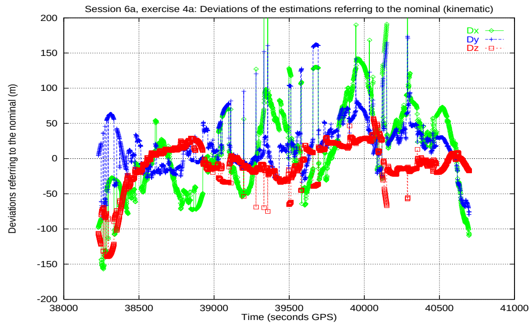
Graphs Session 6a

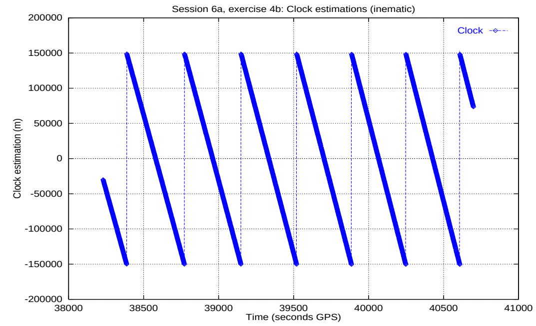


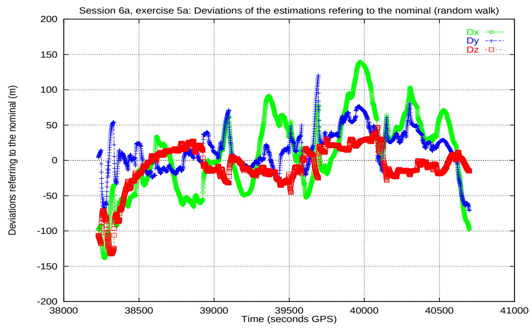


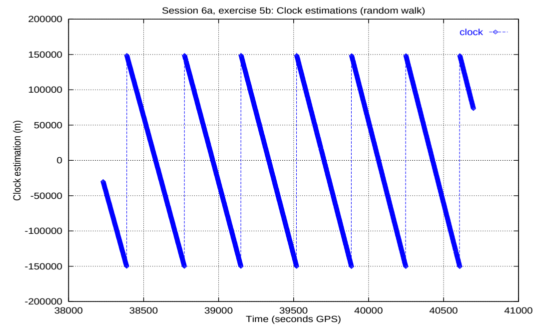


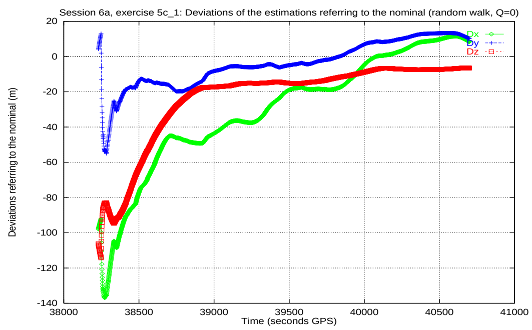


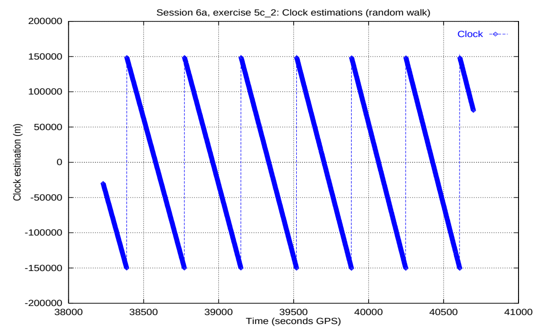

Session 6a, exercise 4a: Deviations of the estimations referring to the nominal (kinematic)
Deviations referring to the nominal (m)
Time (seconds GPS)
Dx
Dy
Dz

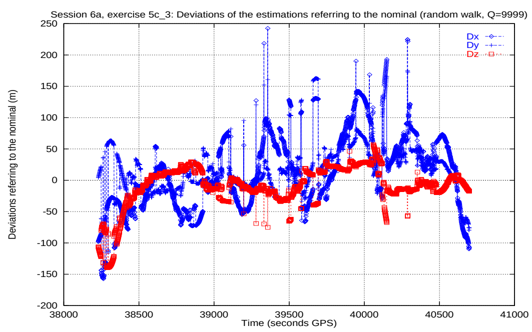

Session 6a, exercise 4b: Clock estimations (inematic)
Clock estimation (m)
Time (seconds GPS)
Clock

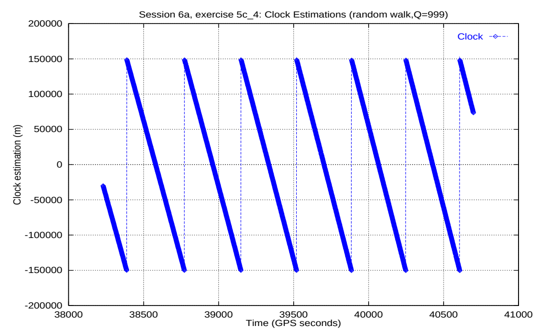

Session 6a, exercise 5a: Deviations of the estimations refering to the nominal (random walk)
Deviations referring to the nominal (m)
Time (seconds GPS)
Dx
Dy
Dz

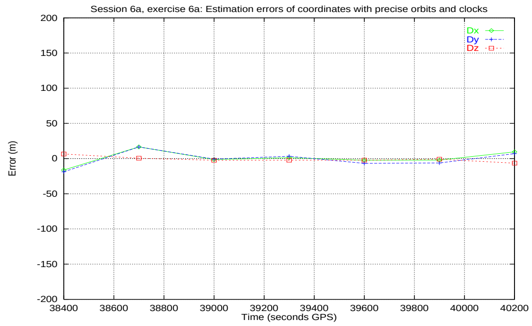

Session 6a, exercise 5b: Clock estimations (random walk)
Clock estimation (m)
Time (seconds GPS)
clock

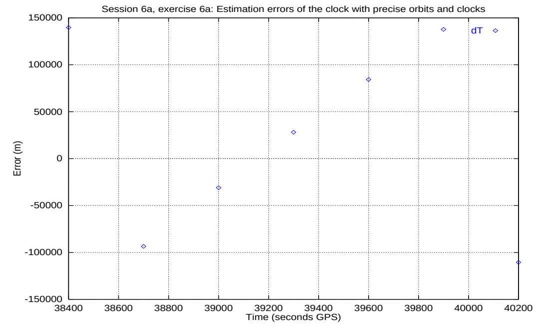

Session 6a, exercise 5c_1: Deviations of the estimations referring to the nominal (random walk, Q=0)
Deviations referring to the nominal (m)
Time (seconds GPS)
Dx
Dy
Dz

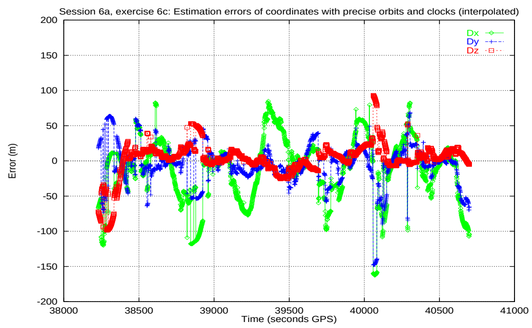

Session 6a, exercise 5c_2: Clock estimations (random walk)
Clock estimation (m)
Time (seconds GPS)
Clock

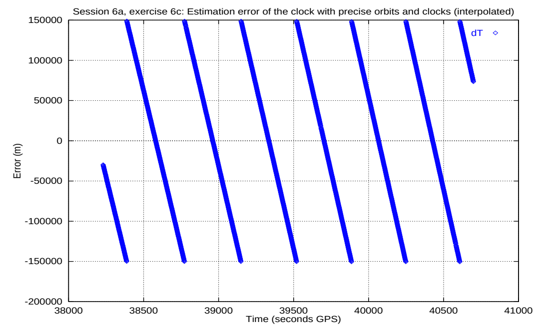

Session 6a, exercise 5c_3: Deviations of the estimations referring to the nominal (random walk, Q=9999)
Deviations referring to the nominal (m)
Time (seconds GPS)
Dx
Dy
Dz

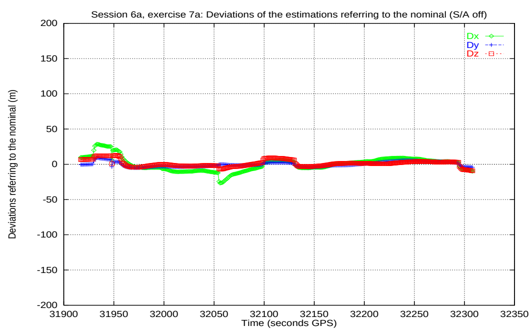

Session 6a, exercise 5c_4: Clock Estimations (random walk,Q=999)
Clock estimation (m)
Time (GPS seconds)
Clock

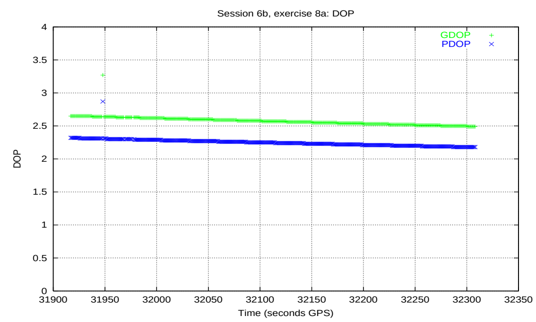

Session 6a, exercise 6a: Estimation errors of coordinates with precise orbits and clocks
Error (m)
Time (seconds GPS)
Dx
Dy
Dz

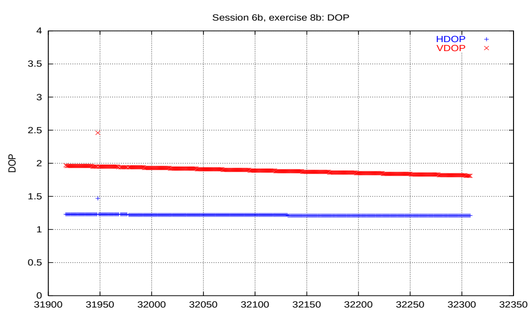

Session 6a, exercise 6a: Estimation errors of the clock with precise orbits and clocks
Error (m)
Time (seconds GPS)
dT


Session 6a, exercise 6c: Estimation errors of coordinates with precise orbits and clocks (interpolated)
Error (m)
Time (seconds GPS)
Dx
Dy
Dz

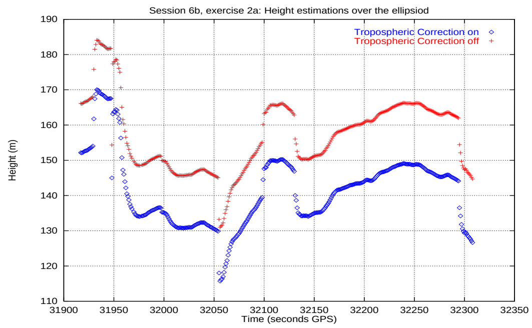

Session 6a, exercise 6c: Estimation error of the clock with precise orbits and clocks (interpolated)
Error (m)
Time (seconds GPS)
dT

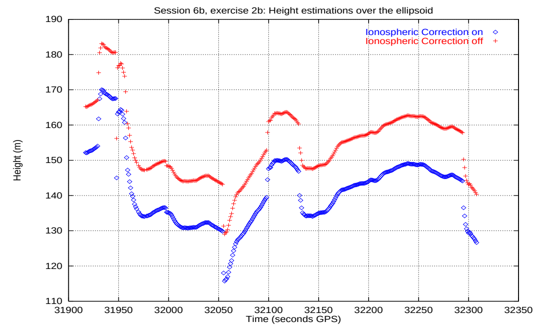

Session 6a, exercise 7a: Deviations of the estimations referring to the nominal (S/A off)
Deviations referring to the nominal (m)
Time (seconds GPS)
Dx
Dy
Dz

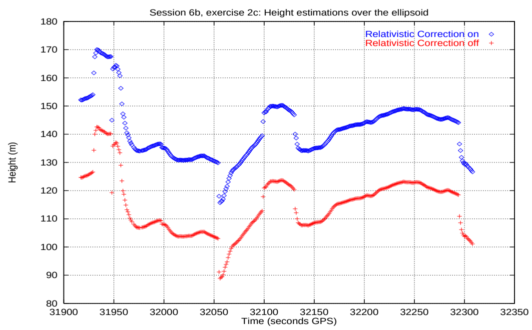

Session 6b, exercise 8a: DOP
DOP
Time (seconds GPS)
GDOP
PDOP

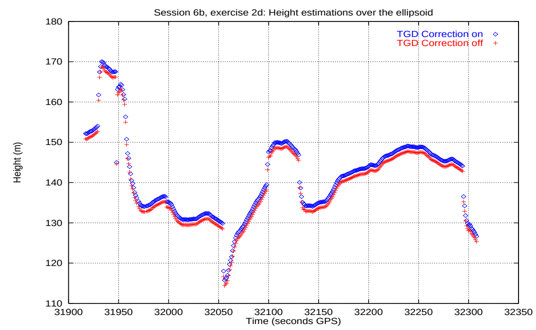

Session 6b, exercise 8b: DOP
DOP
Time (seconds GPS)
HDOP
VDOP

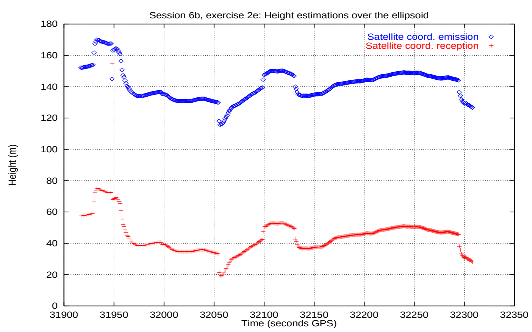
Graphs Session 6b

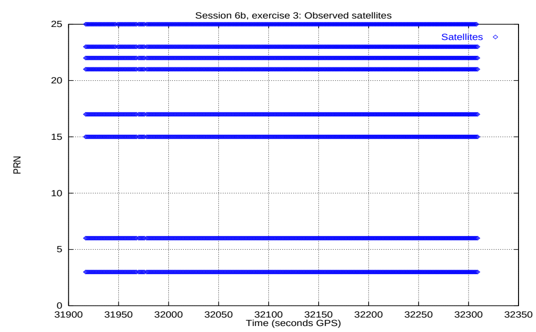


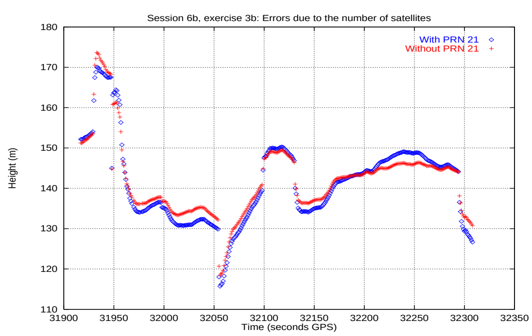


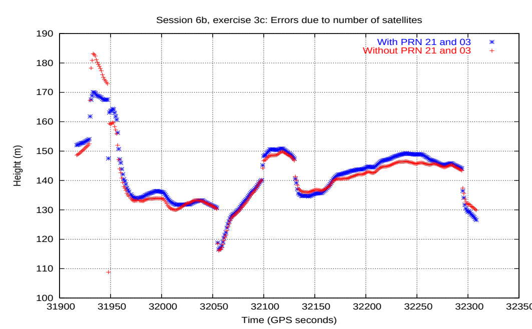


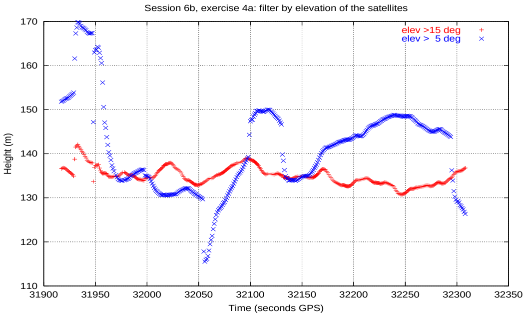


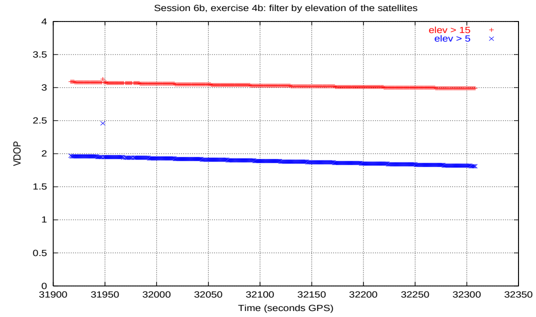


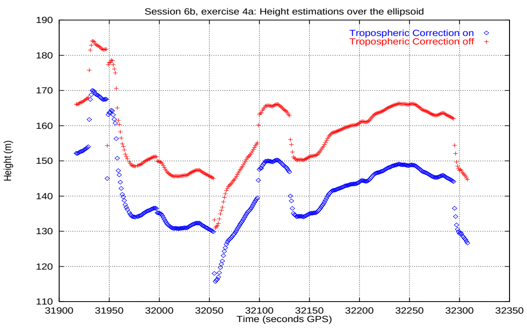


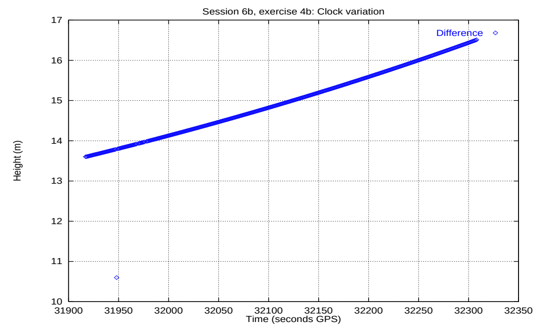


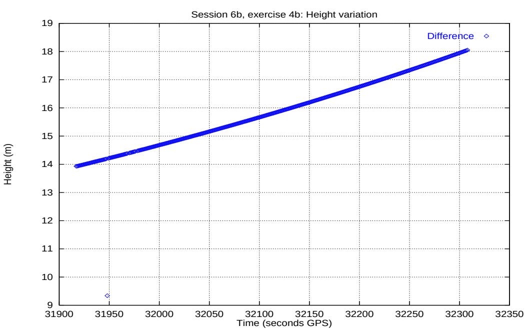


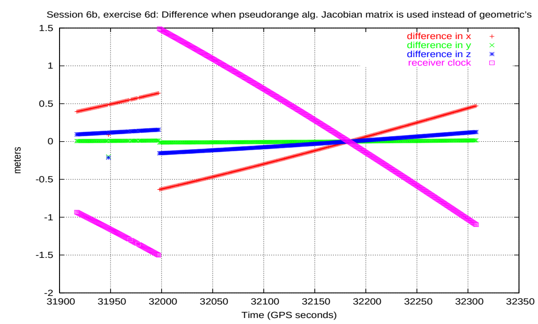

Session 6b, exercise 4a: filter by elevation of the satellites
elev >15 deg
elev > 5 deg
Height (m)
Time (seconds GPS)


Session 6b, exercise 4b: filter by elevation of the satellites
elev > 15
elev > 5
VDOP
Time (seconds GPS)

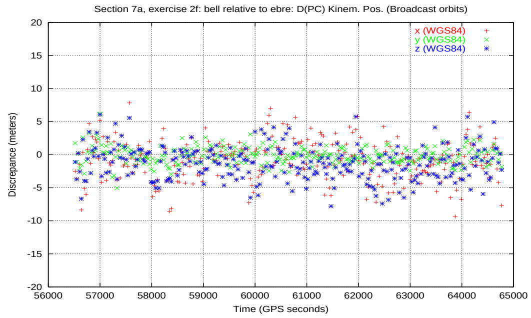

Session 6b, exercise 4a: Height estimations over the ellipsoid
Tropospheric Correction on
Tropospheric Correction off
Height (m)
Time (seconds GPS)

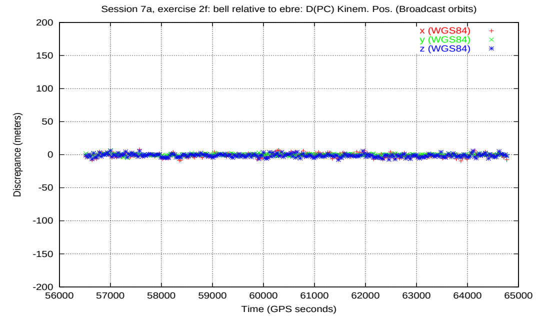

Session 6b, exercise 4b: Clock variation
Difference
Height (m)
Time (seconds GPS)

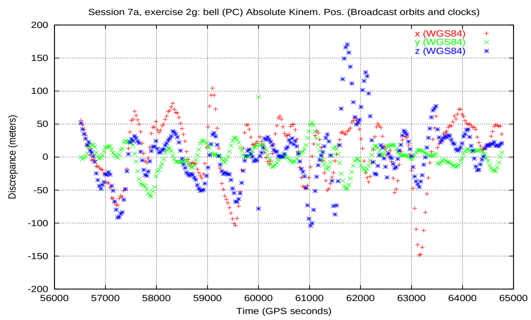

Session 6b, exercise 4b: Height variation
Difference
Height (m)
Time (seconds GPS)

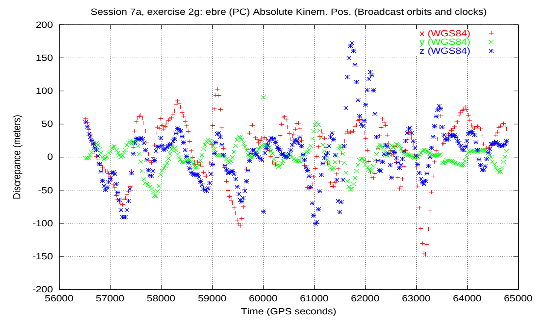

Session 6b, exercise 6d: Difference when pseudorange alg. Jacobian matrix is used instead of geometric's
difference in x
difference in y
difference in z
receiver clock
meters
Time (GPS seconds)

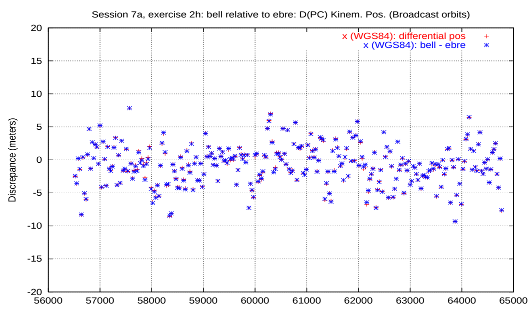
Graphs Session 7a

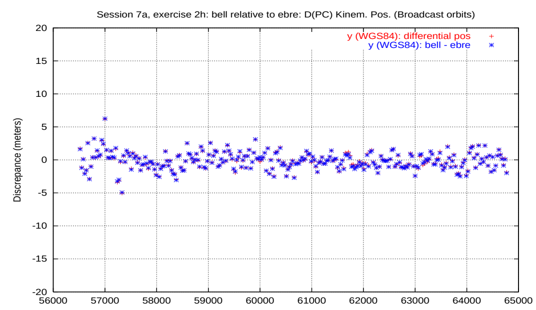


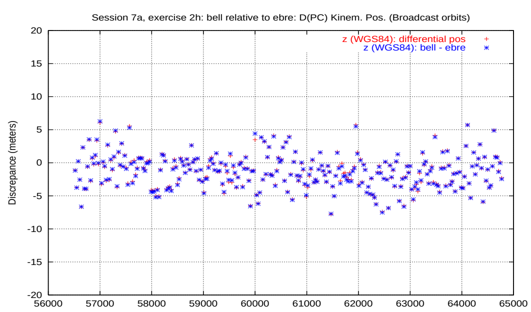


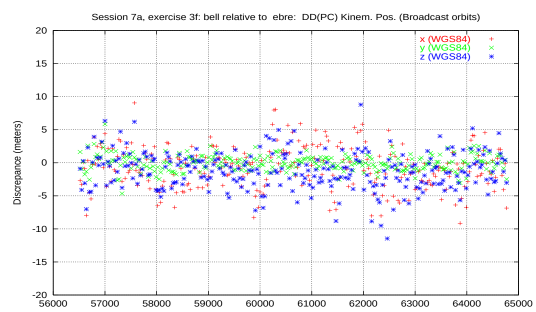


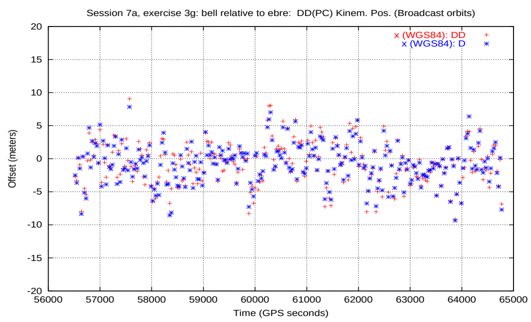


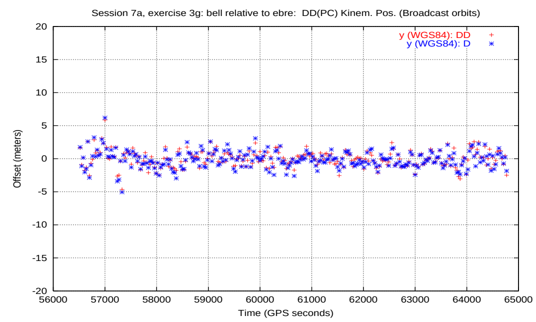


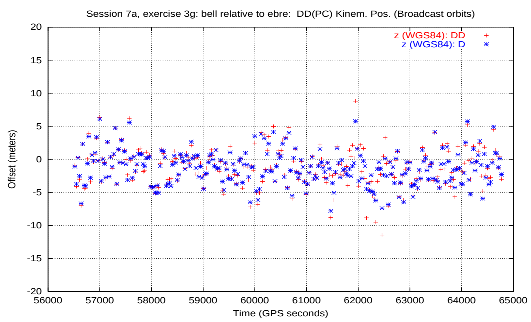


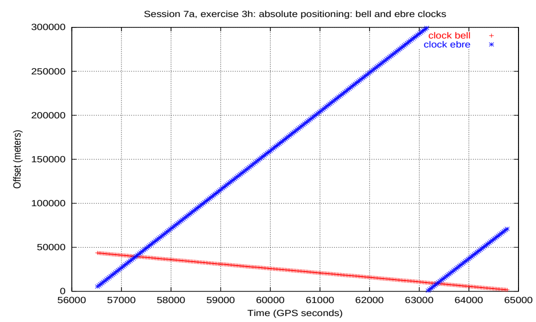


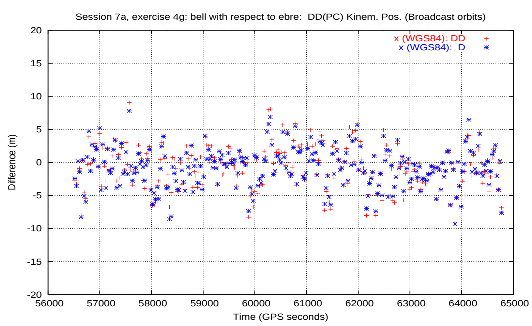


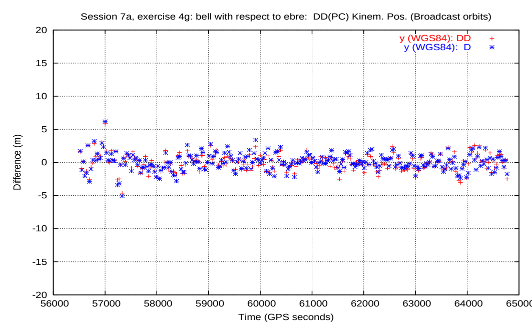

Session 7a, exercise 3g: bell relative to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
x (WGS84): DD
x (WGS84): D
Offset (meters)
Time (GPS seconds)

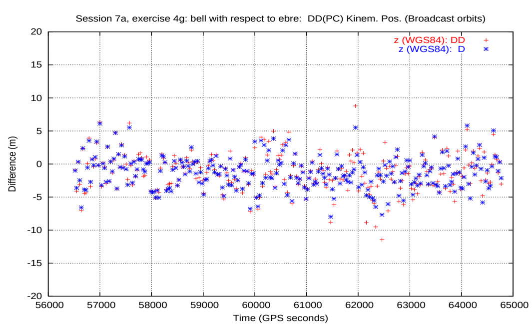

Session 7a, exercise 3g: bell relative to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
y (WGS84): DD
y (WGS84): D
Offset (meters)
Time (GPS seconds)

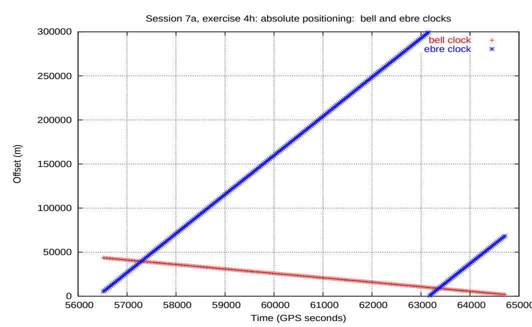

Session 7a, exercise 3g: bell relative to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
z (WGS84): DD
z (WGS84): D
Offset (meters)
Time (GPS seconds)

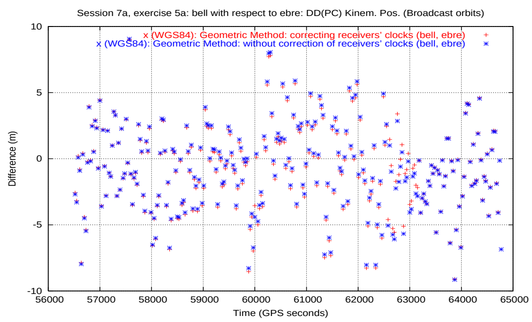

Session 7a, exercise 3h: absolute positioning: bell and ebre clocks
clock bell
clock ebre
Offset (meters)
Time (GPS seconds)

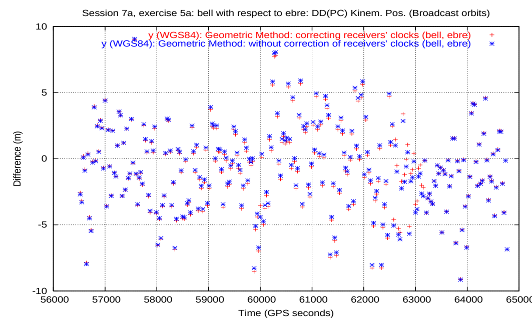

Session 7a, exercise 4g: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
x (WGS84): DD
x (WGS84): D
Difference (m)
Time (GPS seconds)

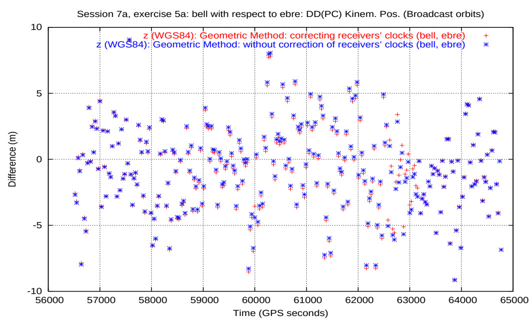

Session 7a, exercise 4g: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
y (WGS84): DD
y (WGS84): D
Difference (m)
Time (GPS seconds)

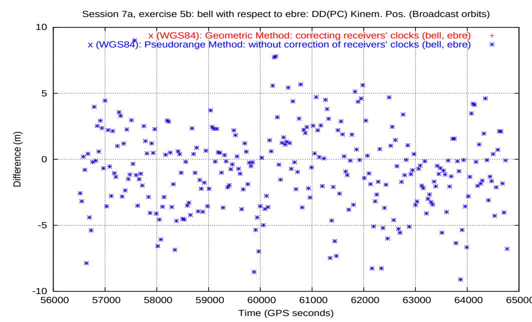

Session 7a, exercise 4g: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
z (WGS84): DD
z (WGS84): D
Difference (m)
Time (GPS seconds)

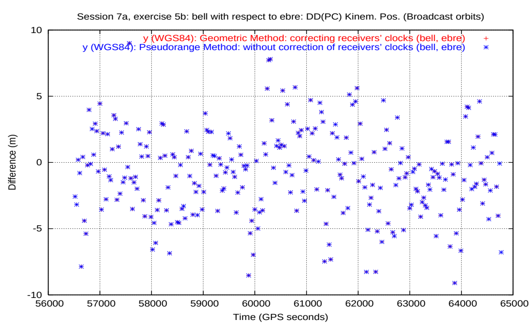

Session 7a, exercise 4h: absolute positioning: bell and ebre clocks
bell clock
ebre clock
Offset (m)
Time (GPS seconds)

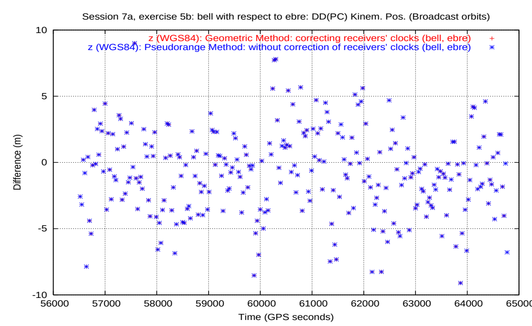

Session 7a, exercise 5a: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
x (WGS84): Geometric Method: correcting receivers' clocks (bell, ebre)
x (WGS84): Geometric Method: without correction of receivers' clocks (bell, ebre)
Difference (m)
Time (GPS seconds)


Session 7a, exercise 5a: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
y (WGS84): Geometric Method: correcting receivers' clocks (bell, ebre)
y (WGS84): Geometric Method: without correction of receivers' clocks (bell, ebre)
Difference (m)
Time (GPS seconds)

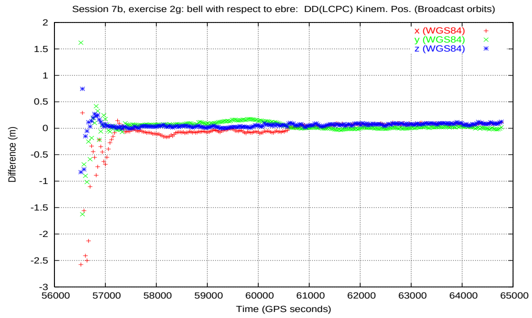

Session 7a, exercise 5a: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
z (WGS84): Geometric Method: correcting receivers' clocks (bell, ebre)
z (WGS84): Geometric Method: without correction of receivers' clocks (bell, ebre)
Difference (m)
Time (GPS seconds)

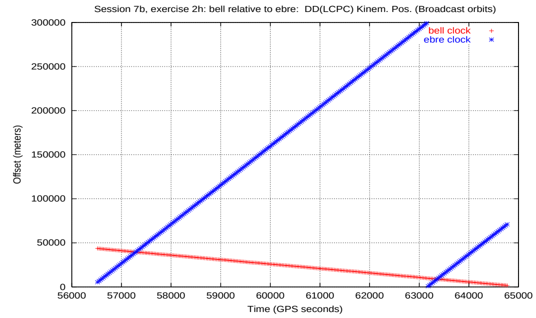

Session 7a, exercise 5b: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
x (WGS84): Geometric Method: correcting receivers' clocks (bell, ebre)
x (WGS84): Pseudorange Method: without correction of receivers' clocks (bell, ebre)
Difference (m)
Time (GPS seconds)

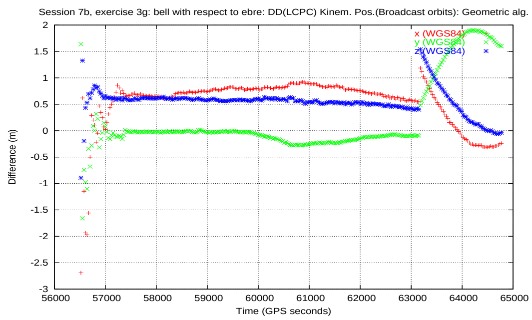

Session 7a, exercise 5b: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
y (WGS84): Geometric Method: correcting receivers' clocks (bell, ebre)
y (WGS84): Pseudorange Method: without correction of receivers' clocks (bell, ebre)
Difference (m)
Time (GPS seconds)

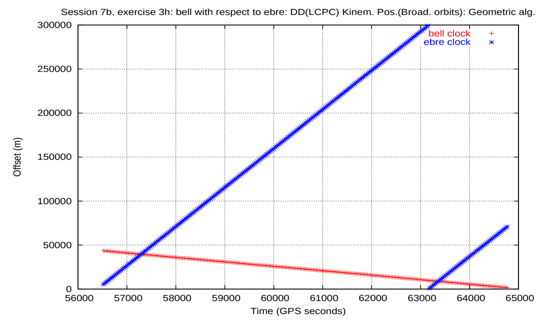

Session 7a, exercise 5b: bell with respect to ebre: DD(PC) Kinem. Pos. (Broadcast orbits)
z (WGS84): Geometric Method: correcting receivers' clocks (bell, ebre)
z (WGS84): Pseudorange Method: without correction of receivers' clocks (bell, ebre)
Difference (m)
Time (GPS seconds)

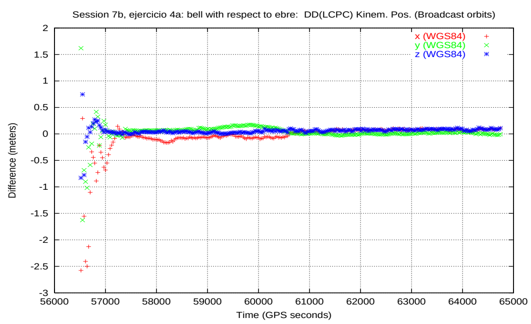
Graphs Session 7b

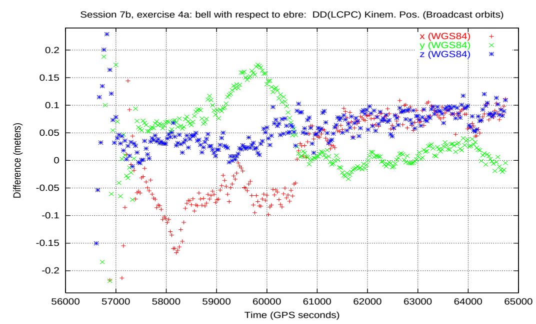


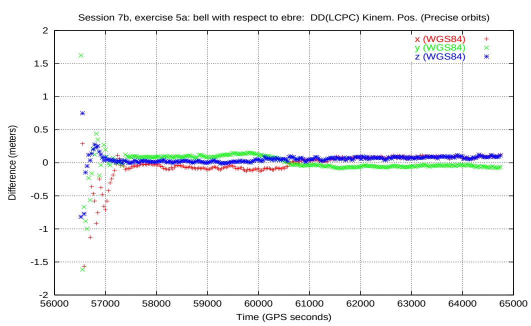


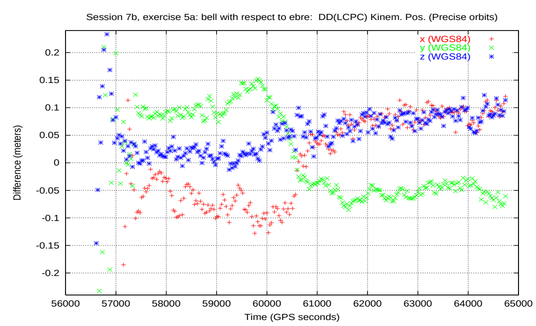


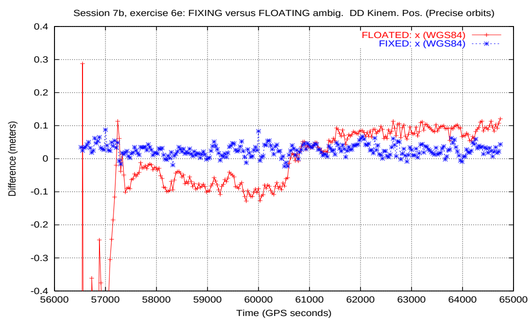


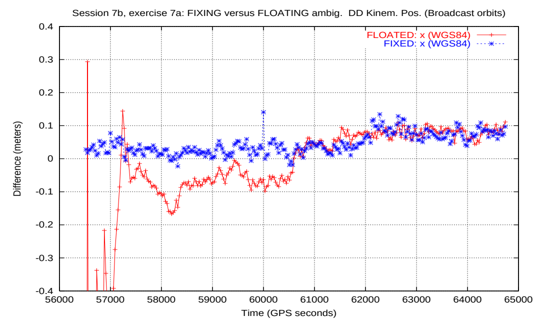


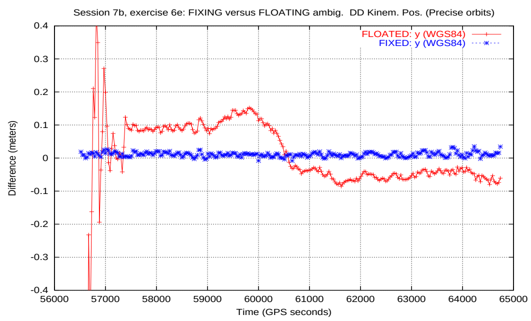


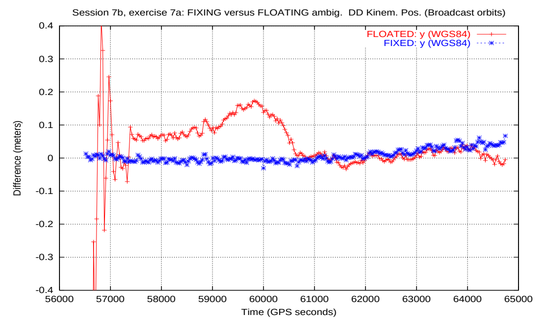


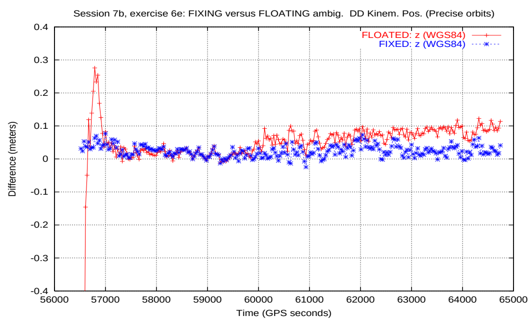


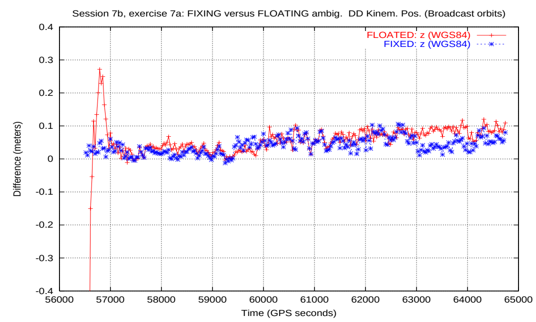

Session 7b, exercise 6e: FIXING versus FLOATING ambig. DD Kinem. Pos. (Precise orbits)
FLOATED: x (WGS84)
FIXED: x (WGS84)
Difference (meters)
Time (GPS seconds)

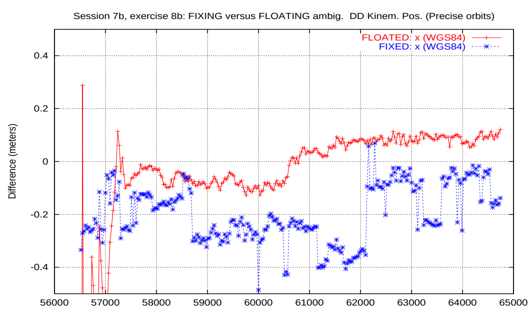

Session 7b, exercise 7a: FIXING versus FLOATING ambig. DD Kinem. Pos. (Broadcast orbits)
FLOATED: x (WGS84)
FIXED: x (WGS84)
Difference (meters)
Time (GPS seconds)

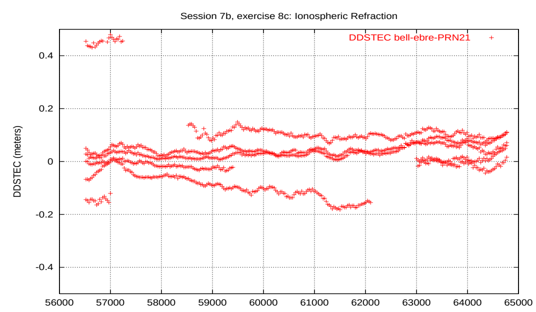

Session 7b, exercise 6e: FIXING versus FLOATING ambig. DD Kinem. Pos. (Precise orbits)
FLOATED: y (WGS84)
FIXED: y (WGS84)
Difference (meters)
Time (GPS seconds)

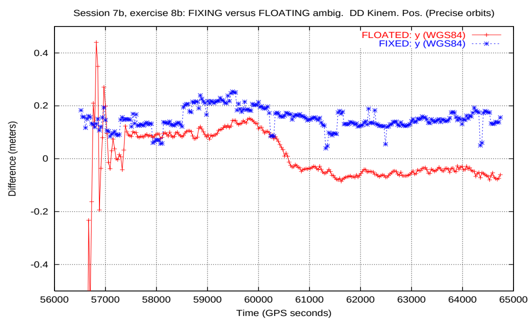

Session 7b, exercise 7a: FIXING versus FLOATING ambig. DD Kinem. Pos. (Broadcast orbits)
FLOATED: y (WGS84)
FIXED: y (WGS84)
Difference (meters)
Time (GPS seconds)

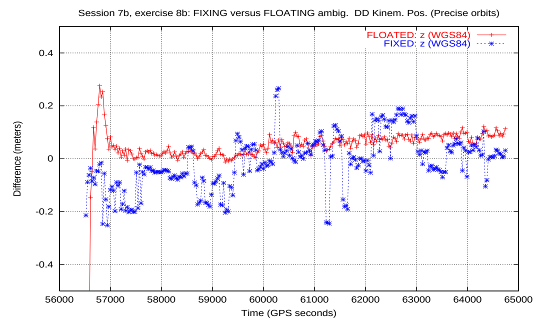

Session 7b, exercise 6e: FIXING versus FLOATING ambig. DD Kinem. Pos. (Precise orbits)
FLOATED: z (WGS84)
FIXED: z (WGS84)
Difference (meters)
Time (GPS seconds)

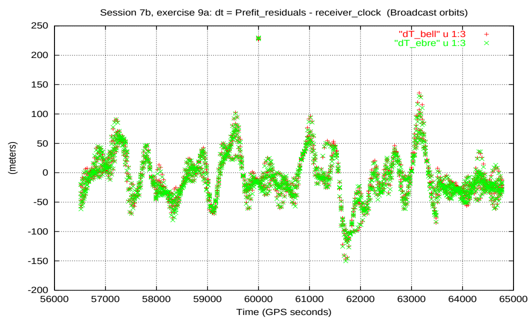

Session 7b, exercise 7a: FIXING versus FLOATING ambig. DD Kinem. Pos. (Broadcast orbits)
FLOATED: z (WGS84)
FIXED: z (WGS84)
Difference (meters)
Time (GPS seconds)

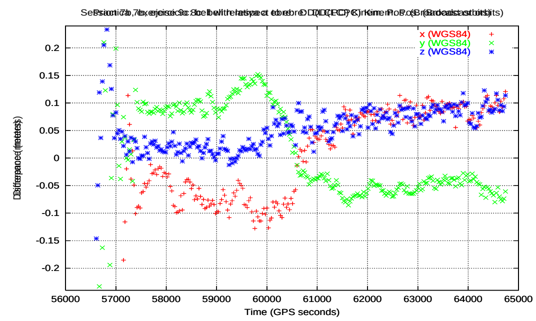

Session 7b, exercise 8b: FIXING versus FLOATING ambig. DD Kinem. Pos. (Precise orbits)
FLOATED: x (WGS84)
FIXED: x (WGS84)
Difference (meters)
Time (GPS seconds)


Session 7b, exercise 8c: Ionospheric Refraction
DDSTEC bell-ebre-PRN21
DDSTEC (meters)
Time (GPS seconds)


Session 7b, exercise 8b: FIXING versus FLOATING ambig. DD Kinem. Pos. (Precise orbits)
FLOATED: y (WGS84)
FIXED: y (WGS84)
Difference (meters)
Time (GPS seconds)


Session 7b, exercise 8b: FIXING versus FLOATING ambig. DD Kinem. Pos. (Precise orbits)
FLOATED: z (WGS84)
FIXED: z (WGS84)
Difference (meters)
Time (GPS seconds)


Session 7b, exercise 9a: dt = Prefit_residuals - receiver_clock (Broadcast orbits)
"dT_bell" u 1:3
"dT_ebre" u 1:3
(meters)
Time (GPS seconds)


x (WGS84)
y (WGS84)
z (WGS84)
Time (GPS seconds)

Solutions to exercises

The solutions to exercises are provided in a series of ASCII blocks that can be obtained through electronic mail. They can be asked for by e-mail to:

`jaume@ma4.upc.edu`

This solutions sheets (`solutions_sheets`) are organized in such a way that making "`cut`" and "`paste`", you may comfortably execute the commands of each exercise.

The `solutions_sheets` directory also may be found in the CDROM that comes with the book, inside directory `PDGPS`.

Inside that CDROM is also available:

- An already installed version of software (`directory PDGPS` in CDROM), with programs and directories already organized to do the book exercises.
- A not-installed version with the source code of all programs (directory `SOFTWARE_SOURCES` in CDROM), except for GCAT, only available as executable.

Software installation

Directory `SOFTWARE_SOURCES` available in CDROM contains book software organized in the following subdirectories:

- `README_install`: file with installation instructions for book software.
- `soft_install`: this directory contains source code of programs, and should be installed following README_install instructions.
- `programs`: in this directory are executable programs for exercises.
- `working`: directory (initially empty) where exercises will be done.

NOTE: in **PDGPS** directory a "running version" of software is supplied, organized according to directories scheme used in the book (i.e., `files`, `programs`, `working`[127]).

Minimum requisites for software install

- Available Personal Computer (PC) (486 or higher, with 8Mb RAM and at least 5Mb free in hard disk), running LINUX operative system and able to work in graphic mode –Xwindows–. Of course, given that source code is available, installation may be done on any UNIX platform.
- Also it should be installed:
  - FORTRAN compiler `f77` (if `g77` compiler is used, files `config` in `subroutines` directory and `Makefile` in `programs` directory should be modified –it would be enough to substitute word `f77` for `g77`–)[128]
  - languages: `awk` or `gawk` and `perl`

[127] In `working` directory is where exercises will be done, and is initially empty.

[128] On the other hand, some FORTRAN compilers do not recognize instructions `dsin, dcos, datan2, etc.`, specific for double-precision work. If that is the case, it would be enough to substitute those functions for their equivalents `sin, cos, atan2, etc.` in programs' source code.

– `gnuplot` graphical representation environment

FORTRAN compiler, programming languages and `gnuplot` program are part of any standard LINUX distribution, and usually they are installed by default.

For any consult or remark, please write to:
`jaume@ma4.upc.edu`

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