

Course organization and program

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and Dr. Adrià Rovira García

Aim of this course

This Practical Course is partially based in the Master Courses we are teaching in different universities.

The original package available in <http://www.gage.upc.edu> has been updated and compiled in several sessions aimed to provide a basic background on **GNSS positioning techniques and algorithms**.

The Learning material is composed by a collection of slides for Theory & Laboratory exercises.

A book on GNSS Data Processing is given as complementary material.

The design and contents are focused on **the instrumental use of the concepts and techniques** involved in GNSS

Only a basic background on GNSS is needed. But **some mathematics and physics** are required.

Concepts introduced in the theory will be analysed in the laboratory sessions.

Theory

Fundamentals course

Lecture 1. Introduction to GNSS (1h)

Lecture 2. GNSS Architecture (1h)

Lecture 3. Overview of GNSS Positioning Techniques (3h)

Lecture 4. GNSS Time Reference Systems and Frames (3h)

Lecture 5. GNSS Measurements and Data Pre-processing (4h)

Lecture 6. Satellite Orbits and Clocks (3h)

Lecture 7. Code Pseudorange Modelling (4h)

Lecture 8. Solving Navigation Equations (4h)

Advanced course

Lecture 9. Precise Point Positioning (PPP) (3h)

Lecture 10. Augmentation Systems (3h)

Lecture 11. Differential Positioning with Code (4h)

Lecture 12. Differential Positioning with Carrier (4h)

Lecture 13. Ambiguity Resolution Techniques (4h)

Laboratory

Fundamentals course

- Tutorial 0. Introduction to gLAB tool suite (2h)
 - Tutorial 1. UNIX environment tools and skills (2h)
 - Tutorial 2. Measurements analysis and error budget (3h)
 - Tutorial 3. Model components analysis (3h)
 - Tutorial 4. Detailed code measurements modelling (3h)
 - Tutorial 5. Solving navigation equations (3h)
-

Advanced course

- Tutorial 6. Kinematic orbit estimation of a LEO satellite (4h)
- Tutorial 7. Differential positioning with code (4h)
- Tutorial 8. Carrier ambiguity fixing (4h)
- Tutorial 9. Differential positioning with carrier (4h)

Theory

Fundamentals course

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Advanced course

Lecture 9. Precise Point Positioning (PPP) (3h)

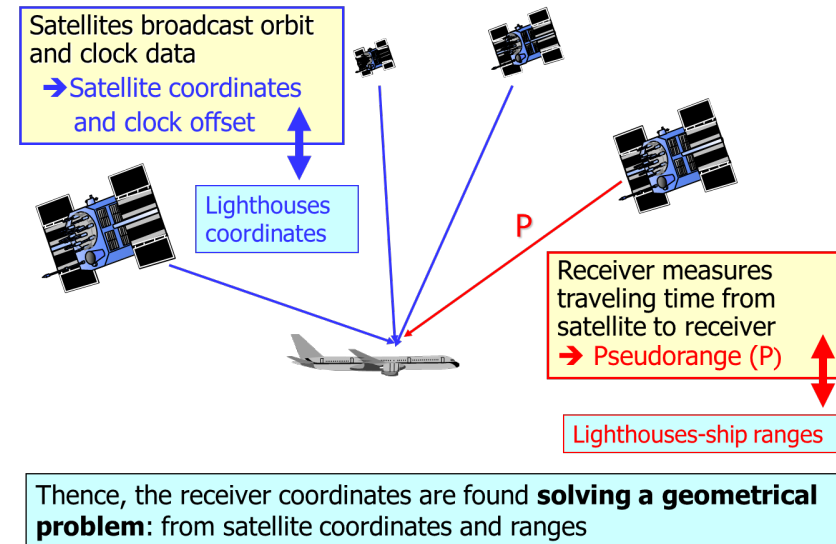
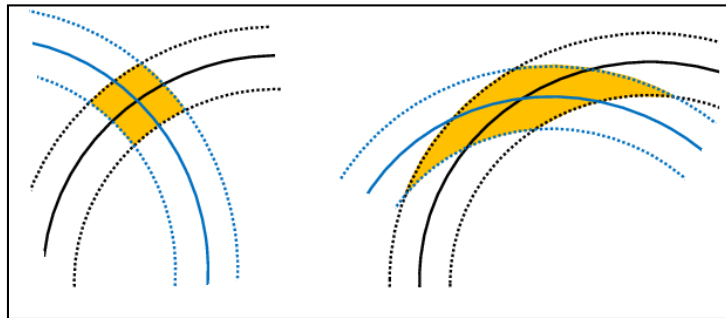
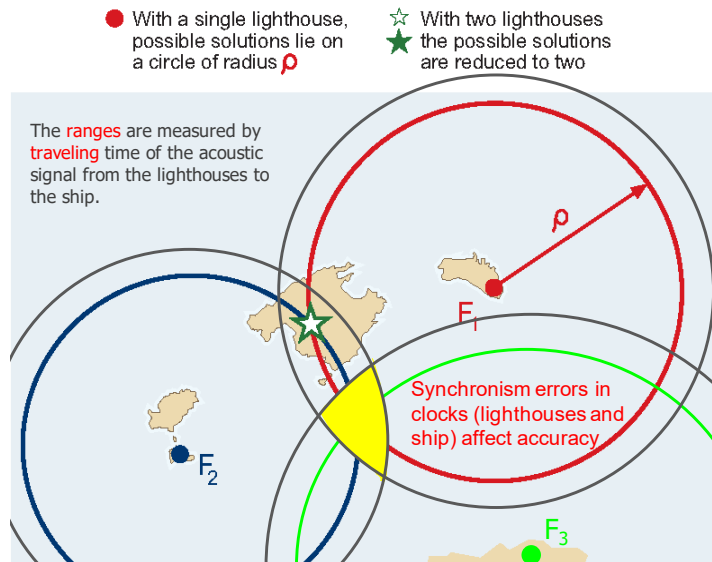
Lecture 10. Augmentation Systems (3h)

Lecture 11. Differential Positioning with Code (4h)

Lecture 12. Differential Positioning with Carrier (4h)

Lecture 13. Ambiguity Resolution Techniques (4h)

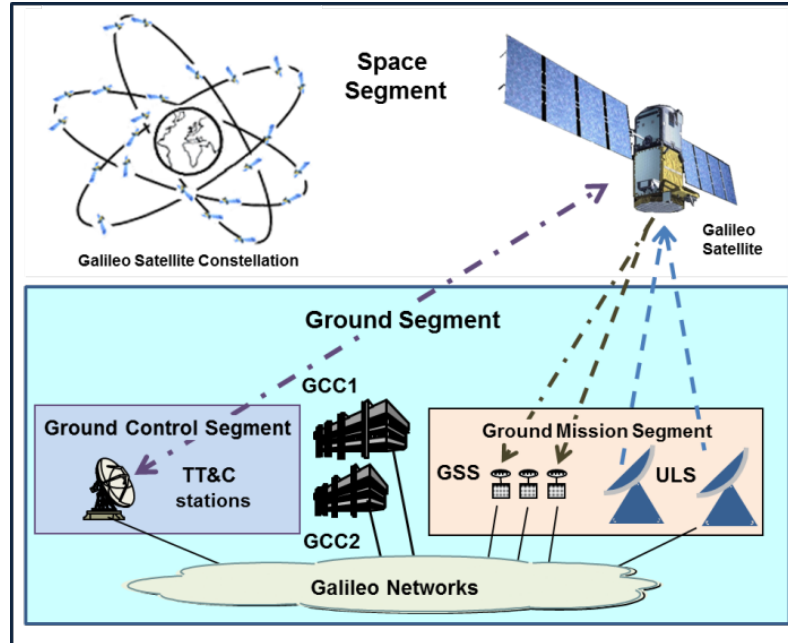
Lecture 1. Introduction to GNSS



Introduction to the DGNSS (1h)

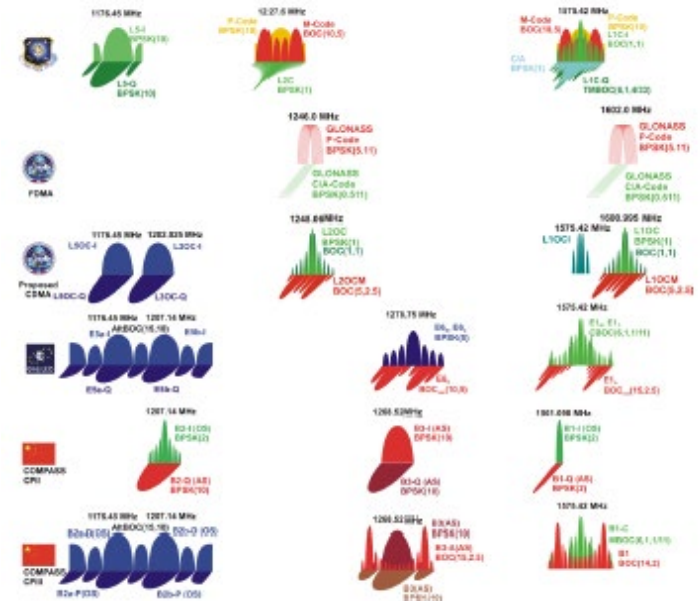
- An Intuitive Approach to GNSS Positioning.
- A Deeper Analysis of 2D Pseudorange-Based Positioning.
- Translation to 3D GNSS Positioning.

Lecture 2. GNSS Architecture



GNSS Architecture (1h)

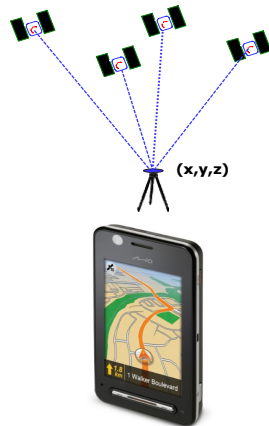
- GNSS concept: Historical review.
- GNSS segments.
- GNSS signals
- Similarities and differences between GNSS.
- The more satellites the better?



Lecture 3. Overview of GNSS positioning techniques

Standalone (code) positioning

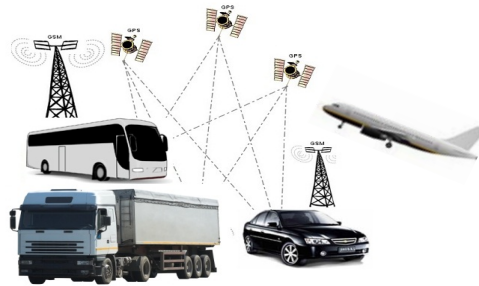
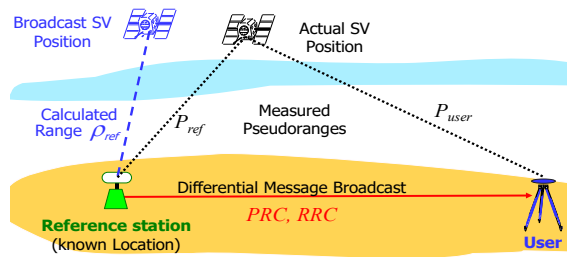
Standard Point Positioning (SPP)



Few meters.
World wide.
Single epoch.

Code based Differential positioning (DGNSS)

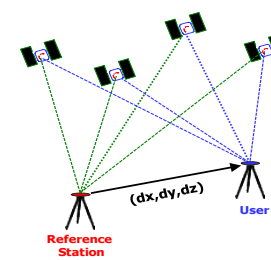
← Improved Accuracy →



Carrier based Differential positioning

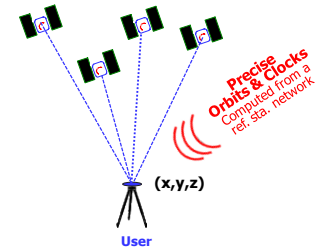
← High Accuracy →

Relative Positioning (RTK)

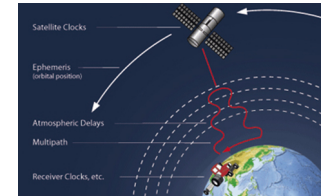


Few centimetres.
Local Area (few km).
Few seconds.

Precise Point Positioning (PPP)



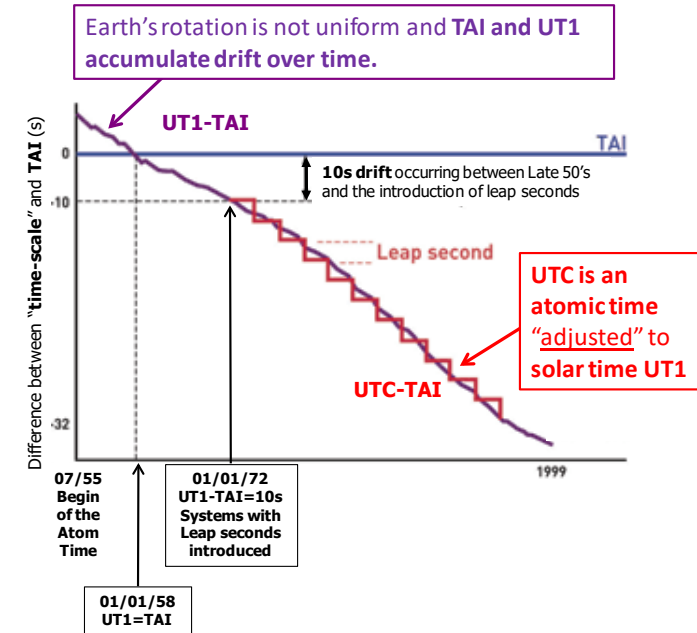
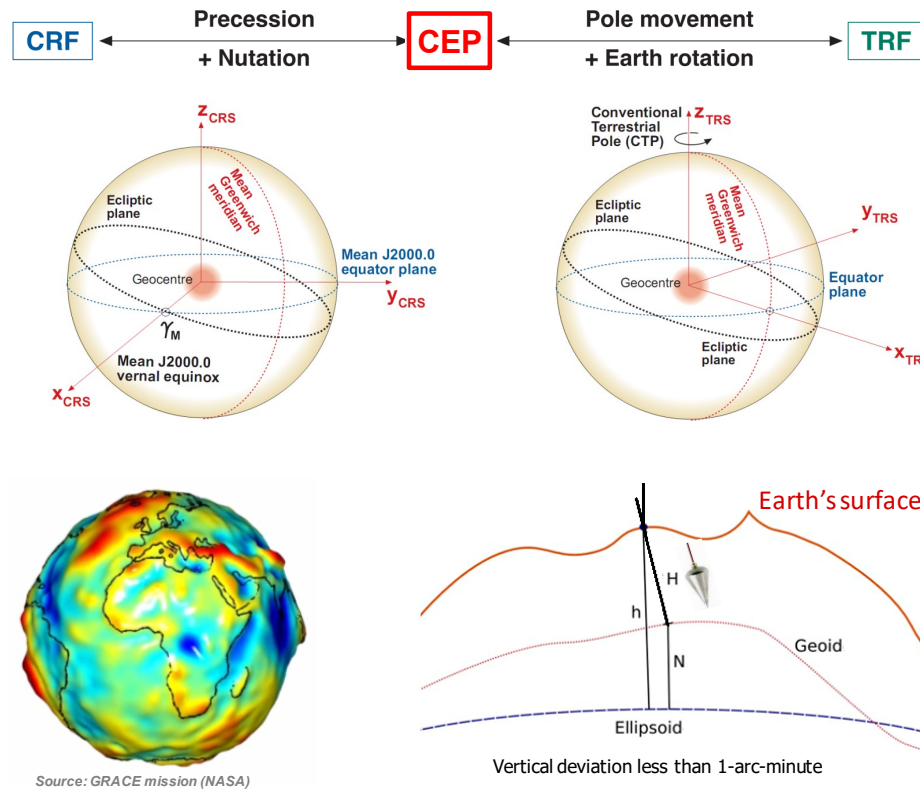
cm – dm level.
World wide.
Best part of one hour.



Overview of GNSS positioning techniques (3h)

- Standalone positioning.
- Code based Differential positioning (DGNSS, GBAS, SBAS).
- Carrier based Differential positioning (RTK, NRTK, PPP).
- Commercial Services.

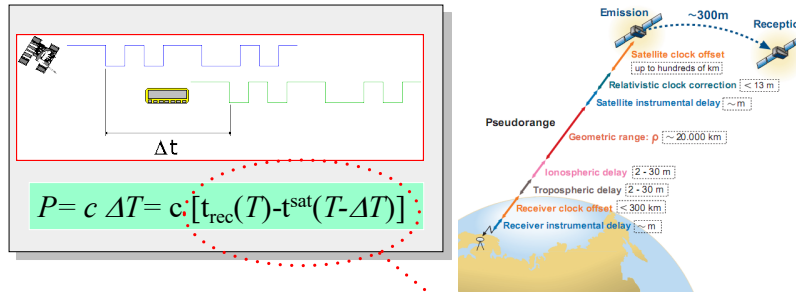
Lecture 4. GNSS Time and Reference Systems & Frames



Review of Coordinate and time references (3h)

- Fundamentals of Coordinate systems and Frames. Transformations:
 - Geodetic coordinates. Geoid, Datum Coordinate conversions
 - GNSS Reference Frames
- Fundamentals of Time References:
 - Earth's rotation times. Sidereal time. Atomic Times. GNSS times.

Lecture 5. GNSS measurements and Data Pre-processing

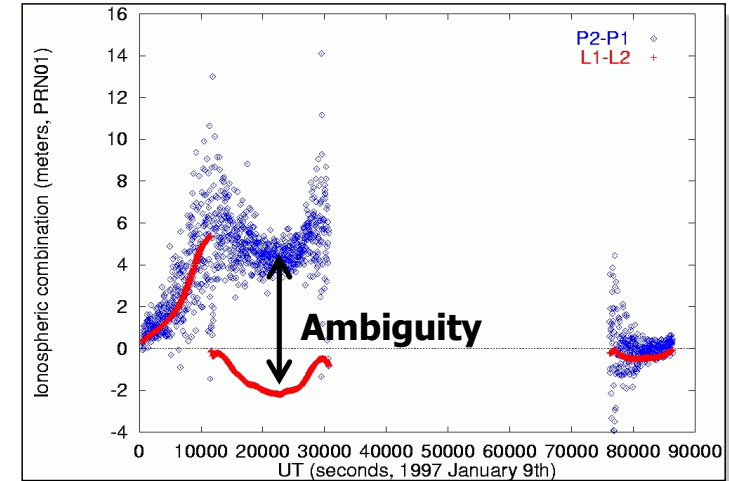


$$P_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta$$

Geometric range Clock offsets

$$\sum \delta = Trop_{rec}^{sat} + Ion_{rec}^{sat} + K_{rec} + K^{sat} + \epsilon$$

Tropospheric delay Ionospheric delay Instrumental delays noise

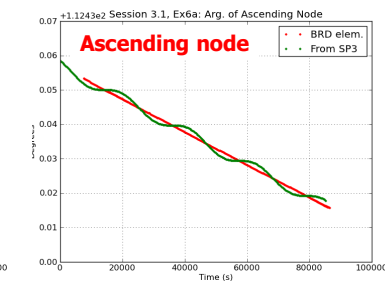
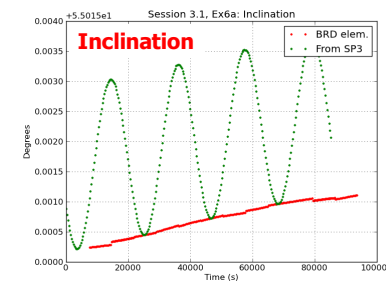
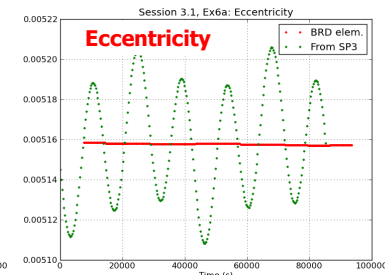
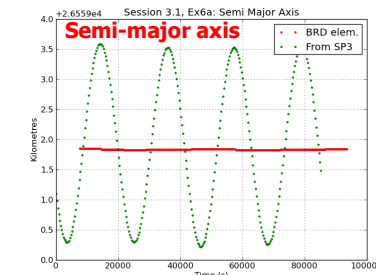
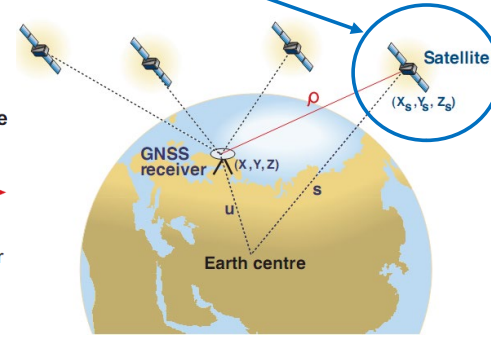
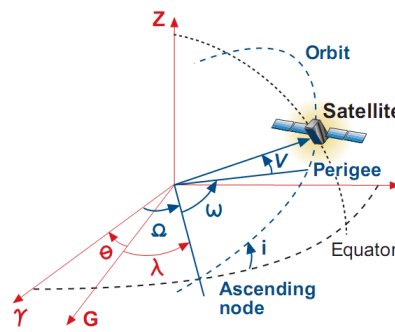
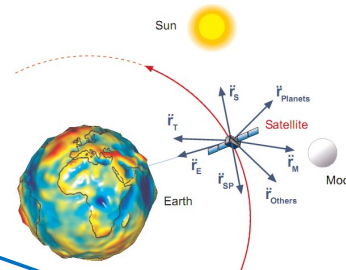
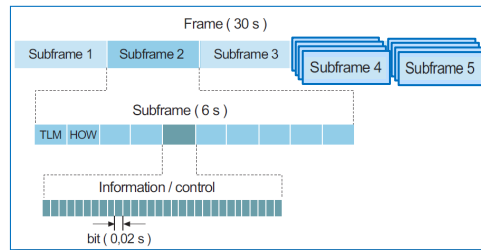


GNSS measurements and Data Pre-processing (4h)

- Review of GNSS measurements.
- Linear combination of measurements.
- Carrier cycle-slips detection.
- Carrier smoothing of code pseudorange.
- Code multipath.

Lecture 6. Satellite orbits and clocks

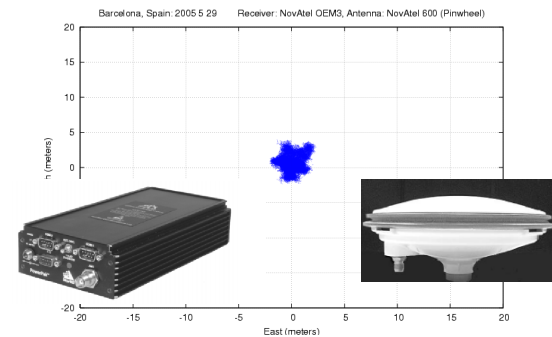
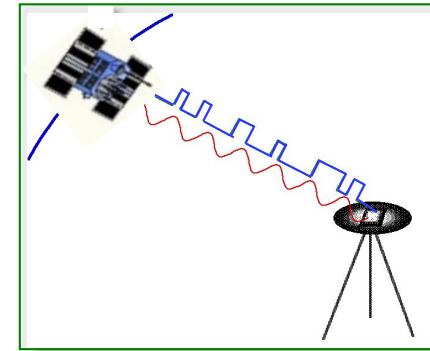
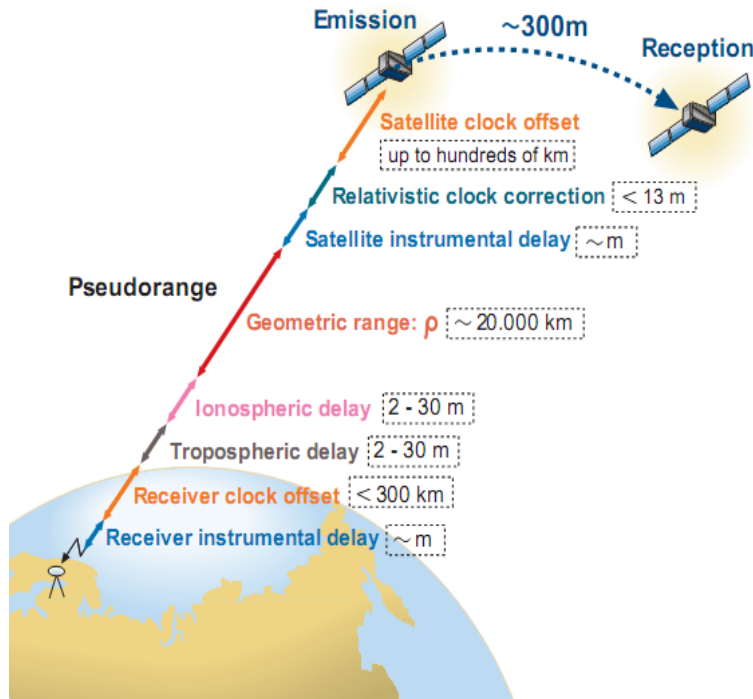
gAGE



Satellite Orbits and clocks (3h)

- Elliptic orbit: Keplerian elements.
- Perturbed Keplerian elements. Osculating orbit.
- GNSS satellite coordinates and clock computation and accuracy.
 - From broadcast navigation message.
 - From precise products.

Lecture 7. Code pseudorange modelling



$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c \left(d\bar{t}^{sat} + \Delta rel^{sat} \right) + Trop_{rec}^{sat} + Ion_{rec}^{sat} + TGD^{sat}$$

Code pseudorange modelling (3h)

- Linear model and Pfit-Residuals.
- Code measurements modelling.
- Example of computation of modelled pseudorange.

Lecture 8. Solving navigation equations

For each satellite in view

Iono+Tropo+TGD...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

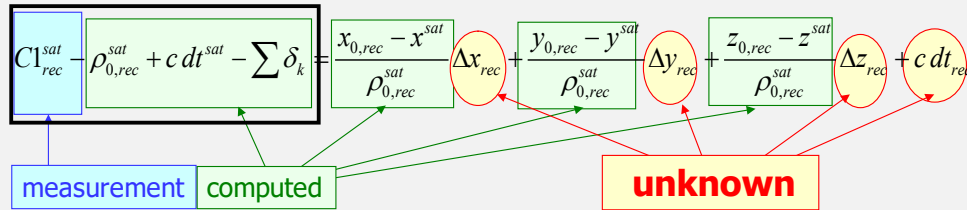
Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c(dt_{rec} - dt^{sat}) + \sum \delta_k$$

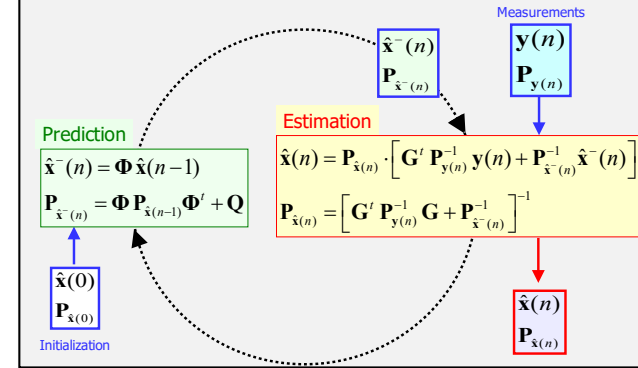
where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)



Kalman filter (see kalman.f)



Solving navigation equations (4h)

- Linear Model: navigation equations.
- Least Squares solution.
- Weighted Least Squares and Minimum Variance Estimator.
- Kalman filter.
 - Examples for static and kinematic positioning.

Theory

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Advanced course

Lecture 9. Precise Point Positioning (PPP) (3h)

Lecture 10. Augmentation Systems (3h)

Lecture 11. Differential Positioning with Code (4h)

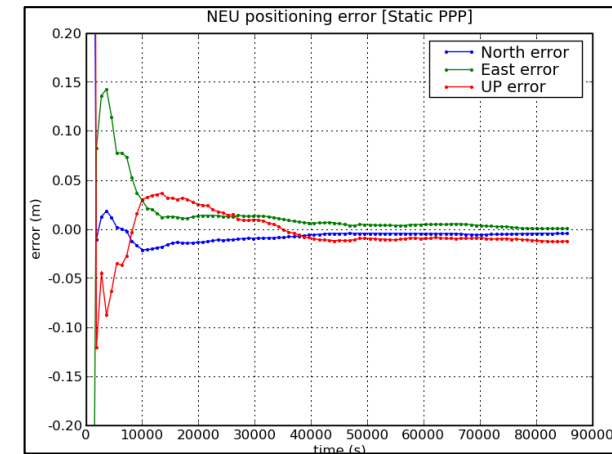
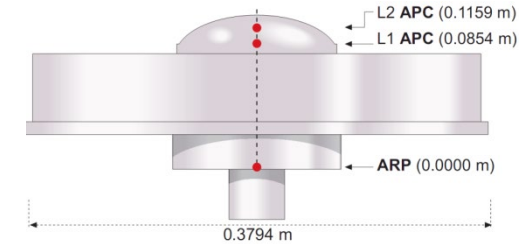
Lecture 12. Differential Positioning with Carrier (4h)

Lecture 13. Ambiguity Resolution Techniques (4h)

Lecture 9. Precise Point Positioning

$$\begin{bmatrix} \text{Prefit}(P_C)^1 \\ \text{Prefit}(L_C)^1 \\ \dots \\ \text{Prefit}(P_C)^n \\ \text{Prefit}(L_C)^n \end{bmatrix} = \begin{bmatrix} \frac{x_{o,rec} - x^1}{\rho_{0,rec}^1} & \frac{y_{o,rec} - y^1}{\rho_{0,rec}^1} & \frac{z_{o,rec} - z^1}{\rho_{0,rec}^1} & 1 & M_{wet}^1 & 0 & \dots & \dots & 0 \\ \frac{x_{o,rec} - x^1}{\rho_{0,rec}^1} & \frac{y_{o,rec} - y^1}{\rho_{0,rec}^1} & \frac{z_{o,rec} - z^1}{\rho_{0,rec}^1} & 1 & M_{wet}^1 & 1 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{x_{o,rec} - x^n}{\rho_{0,rec}^n} & \frac{y_{o,rec} - y^n}{\rho_{0,rec}^n} & \frac{z_{o,rec} - z^n}{\rho_{0,rec}^n} & 1 & M_{wet}^n & 0 & \dots & \dots & 0 \\ \frac{x_{o,rec} - x^n}{\rho_{0,rec}^n} & \frac{y_{o,rec} - y^n}{\rho_{0,rec}^n} & \frac{z_{o,rec} - z^n}{\rho_{0,rec}^n} & 1 & M_{wet}^n & 0 & \dots & \dots & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ cdt_{rec} \\ \Delta Tr_{Z,wet} \\ B_C^1 \\ \dots \\ B_C^n \end{bmatrix}$$

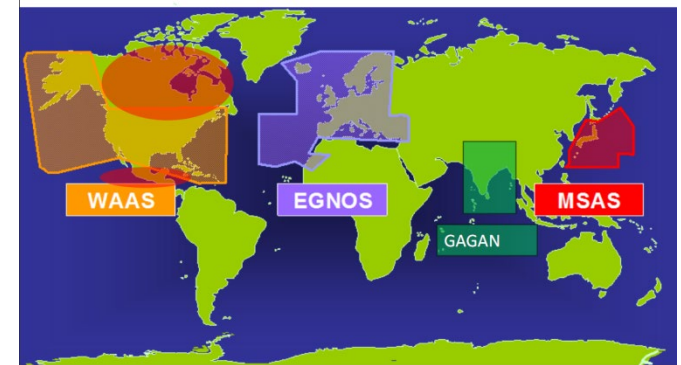
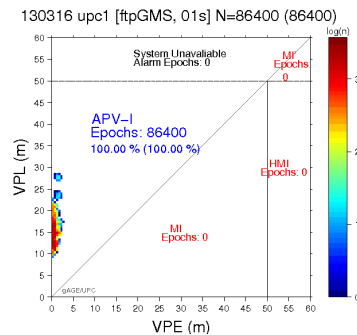
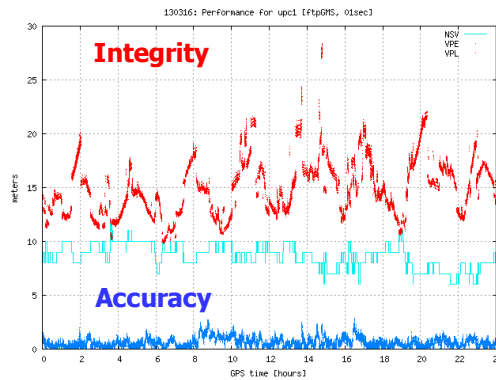
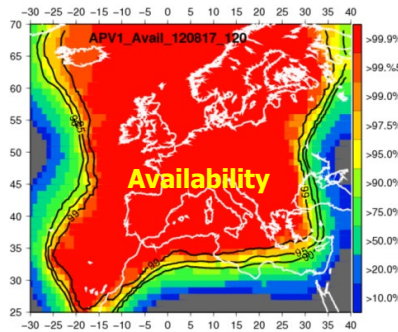
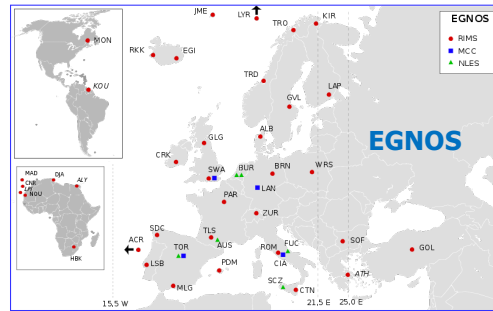
$$\begin{aligned} \text{Prefit}(P_C)^k &= P_C^k - \rho_0^k + cdt^k - Trop_0^k \\ \text{Prefit}(L_C)^k &= L_C^k - \rho_0^k + cdt^k - Trop_0^k - \lambda_N \omega^k \end{aligned}$$



Precise Point Positioning (3h)

- Additional model terms for PPP
- Linear observation model for PPP.
- Parameter estimation: Floating ambiguities
- Accelerating filter convergence with precise ionospheric corrections (Fast-PPP)

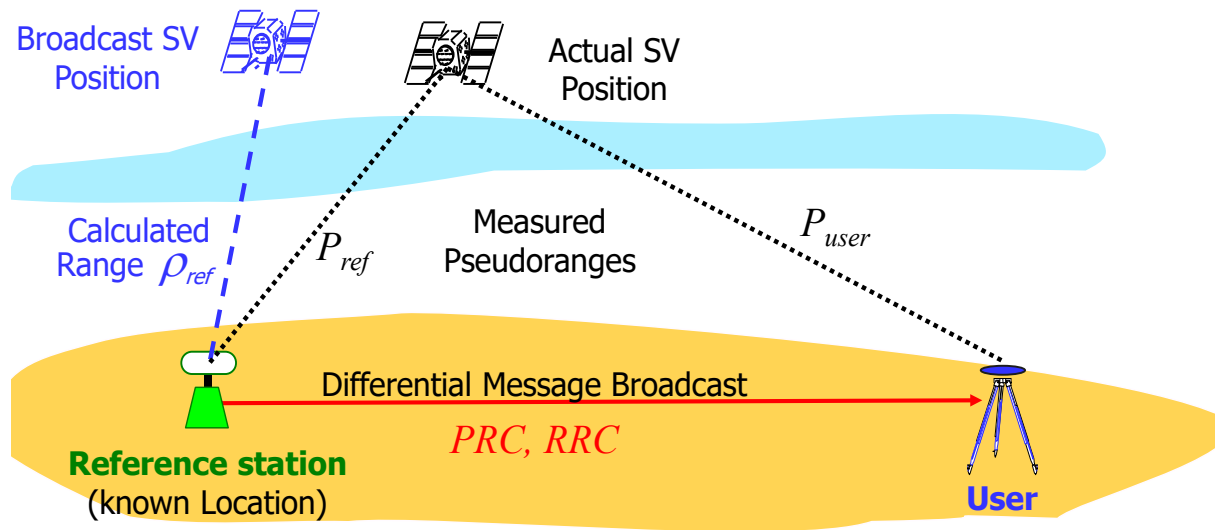
Lecture 10. Augmentation systems



Augmentation systems (3h)

- Introduction to augmentation systems:
 - What augmentation is? Why augmentation systems?
- Ground-Based Augmentation Systems (GBAS).
- Satellite-Based Augmentation Systems (SBAS).
- Aircraft-Based Augmentation Systems (ABAS): RAIM and ARAIM.

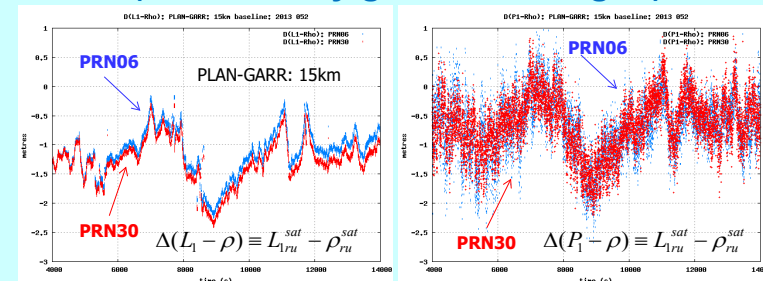
Lecture 11. Differential positioning with code



Differential positioning with code (4h)

- Single differences of measurements.
- Linear Model for DGNSS.
- Geographic decorrelation of the errors
- Error mitigation and 'short' baseline concept.
- Differential Code based positioning.

Single-Difference of measurements (corrected by geometric range!!)



Dif. Wind-up: Very small

$$\Delta(L_1 - \rho) \equiv L_{ru}^j - \rho_{ru}^j = c \delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + v_{l,ru}^j$$

$$\Delta(P_1 - \rho) \equiv P_{ru}^j - \rho_{ru}^j = c \delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + v_{p,ru}^j$$

Dif. Receiver clock:
Main variations Common
for all satellites

Dif. Tropo. and Iono. :
Small variations

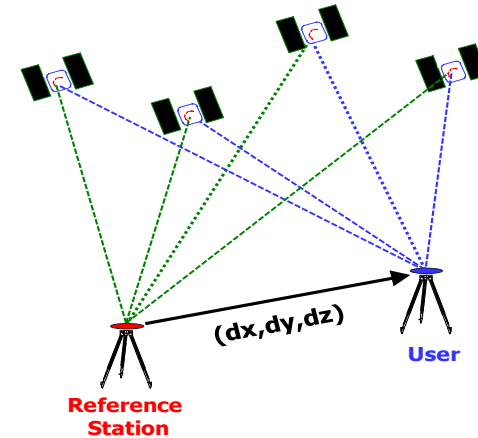
Dif. Instrumental delays and carrier ambiguities:
constant

Lecture 12. Differential positioning with carrier



Differential Positioning with DD of Carriers (4h)

- Double differences (DD) of measurements.
- Linear Model for carrier based positioning in DD.
- Relative Positioning
- The role of geometric diversity.



Relative Positioning

Thence, the double differences of ranges are:

$$\rho_{ru}^{jk} = \rho_{ru}^k - \rho_{ru}^j = -(\hat{\mathbf{p}}_u^k - \hat{\mathbf{p}}_u^j) \cdot \mathbf{r}_{ru} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru}$$

As commented before, for short baselines (e.g. less than 10km), we can assume that ephemeris and propagation errors cancel, thence:

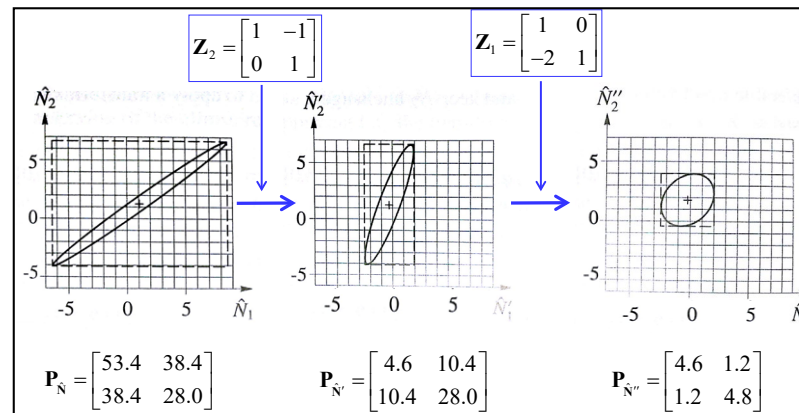
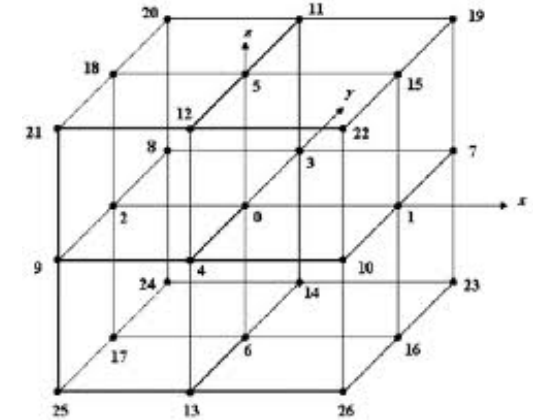
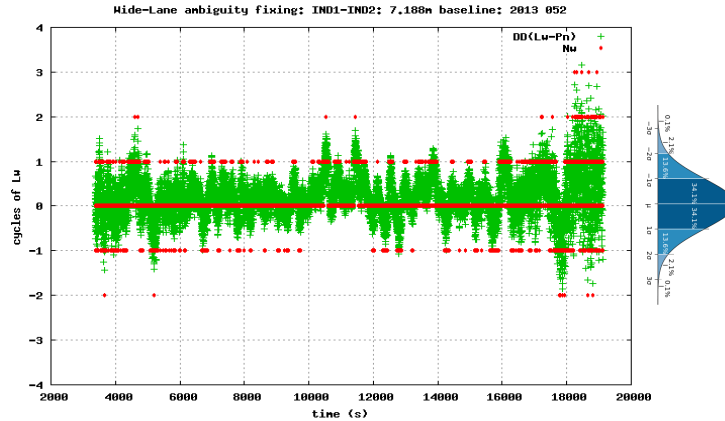
$$\begin{aligned} P_{ru}^{jk} &= \rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + v_{p,ru}^{jk} & P_{ru}^{jk} &= \rho_{ru}^{jk} + v_{p,ru}^{jk} \\ L_{ru}^{jk} &= \rho_{ru}^{jk} + T_{ru}^j - I_{ru}^{jk} + \lambda \omega_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{l,ru}^{jk} & L_{ru}^{jk} &= \rho_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{l,ru}^{jk} \end{aligned}$$

$$\begin{aligned} P_{ru}^{jk} &= -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru} + v_{p,ru}^{jk} \\ L_{ru}^{jk} &= -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru} + \lambda N_{ru}^{jk} + v_{l,ru}^{jk} \end{aligned}$$

Note that these equations allows a direct estimation of the baseline, **without needing an accurate knowledge of the reference station coordinates.**

Lecture 13. Ambiguity Resolution Techniques

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Ambiguity Resolution Techniques (4h)

- Resolving ambiguities one at a time
 - Single- Dual- and There-frequency measurements.
- Resolving ambiguities as a set
 - Search techniques, LAMBDA method.

LAMBDA software package

Matlab implementation, Version 3.0

Sandra Verhagen and Bofeng Li

TU Delft

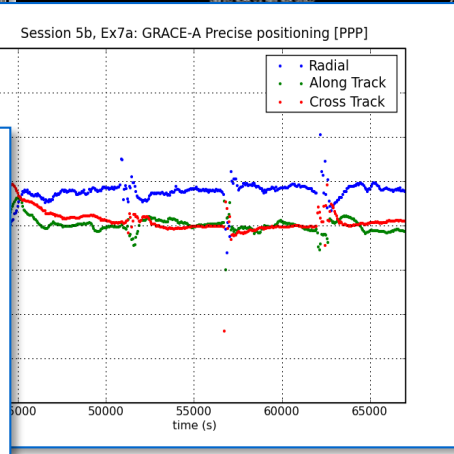
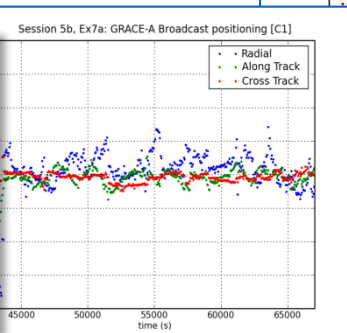
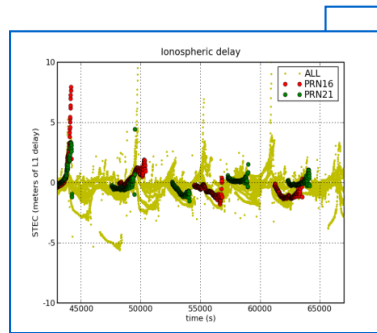
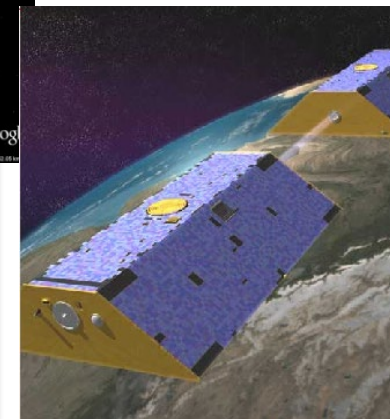
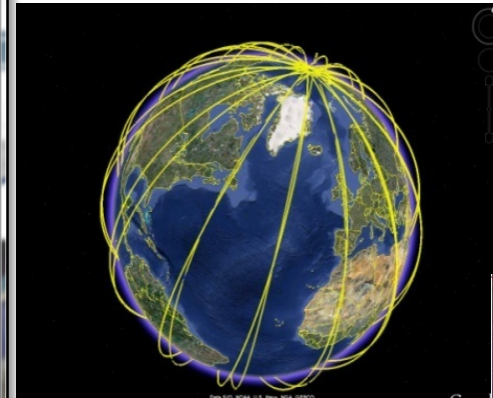
Mathematical Geodesy and Positioning, Delft University of Technology

Curtin University



Laboratory Sessions

Please remember that you will need **your laptop** for the laboratory session **with permissions to install software**



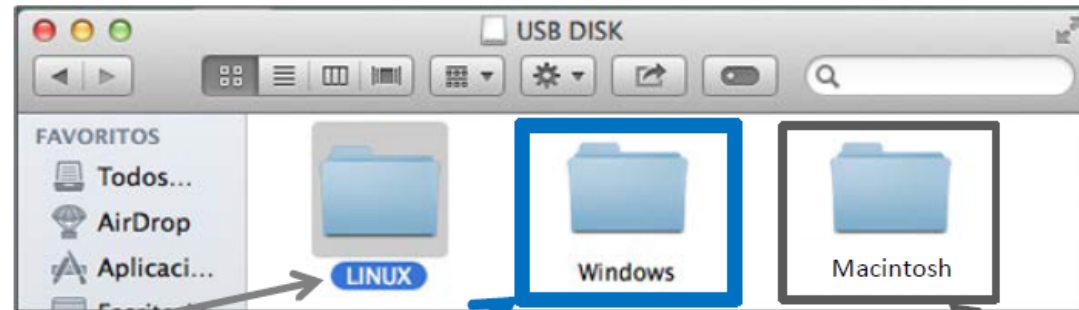



Software tools





This tutorial has been designed to be executed under **UNIX (Linux) Operative System** (OS), which is a very powerful and robust environment.

Nevertheless, the necessary tools are provided for **Windows or Macintosh** users to install this software and to emulate a UNIX command line shell over Windows.



 **Linux** users can install the **native version** of the software

 **Windows** users can install the windows version of **gLAB** and the **Cygwin** emulator of a Linux command shell.

 **Macintosh** users can install the software with the ".dmg" file.

Laboratory

Fundamentals course

Tutorial 0. Introduction to gLAB tool suite (2h)

Tutorial 1. UNIX environment tools and skills (2h)

Tutorial 2. Measurements analysis and error budget (3h)

Tutorial 3. Model components analysis (3h)

Tutorial 4. Detailed code measurements modelling (3h)

Tutorial 5. Solving navigation equations (3h)

Advanced course

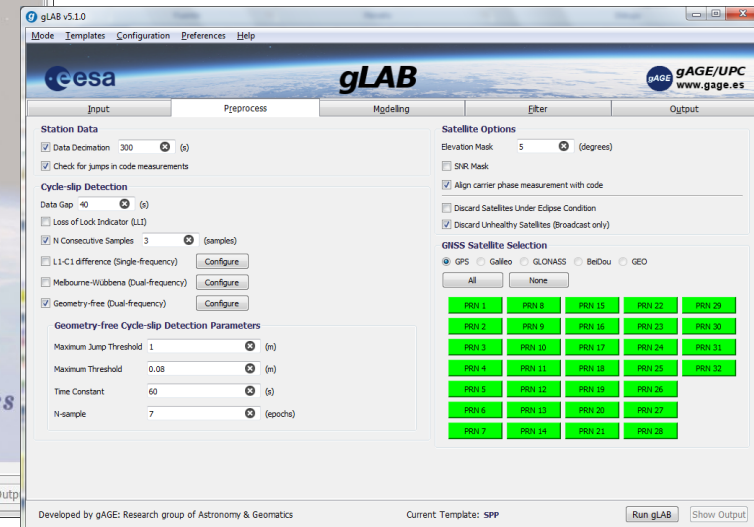
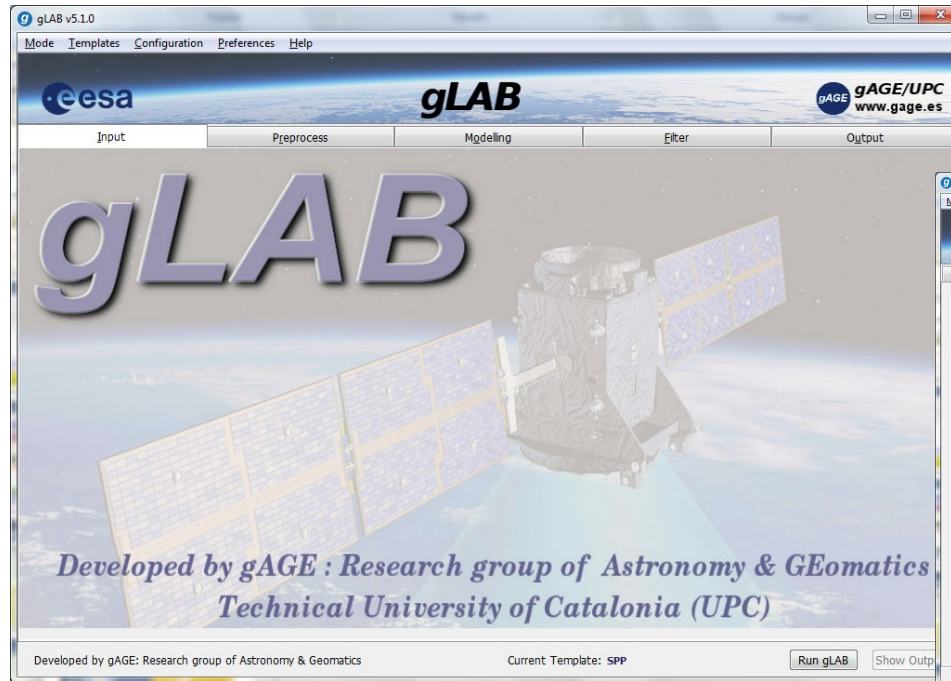
Tutorial 6. Kinematic orbit estimation of a LEO satellite (4h)

Tutorial 7. Differential positioning with code (4h)

Tutorial 8. Carrier ambiguity fixing (4h)

Tutorial 9. Differential positioning with carrier (4h)

Laboratory 0. Introduction to gLAB tool suite

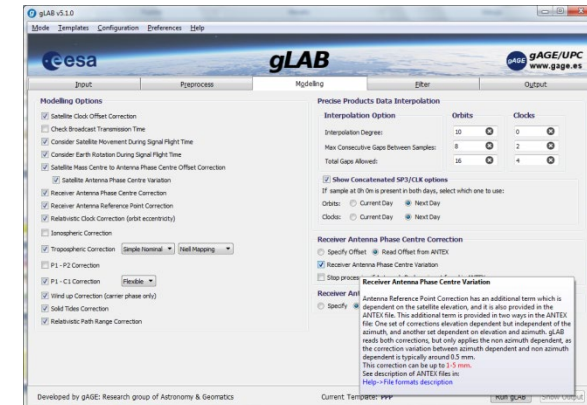


Introduction to gLAB tool suite (2h)

The ESA/UPC GNSS LABORatory tool suit (gLAB) will be introduced.

Some examples of gLAB capabilities and usage will be shown to get an additional training for the next laboratory sessions.

Users will install the gLAB software.



Laboratory 1. UNIX environment tools and skills

```
Jaume@Jaume-PC:~/cygdrive/c/gLAB/win/GNSS_TUTBOOK_MASTER
-rwxr-xr-x 1 Jaume None 4120075 Feb 18 14:30 gLAB.out
-rw-r--r-- 1 Jaume None 5406 Feb 18 12:08 ntpd_tut5.txt
drwxr-xr-x 1 Jaume None 0 Feb 18 11:01 PROG
drwxr-xr-x 1 Jaume None 0 Feb 18 11:01 HTML
drwxr-xr-x 1 Jaume None 0 Feb 18 11:01 FILES
drwxr-xr-x 1 Jaume None 0 Feb 18 11:00 WORK
-rw-r--r-- 1 Jaume None 29721 Nov 22 18:59 ntpd_tut7.txt
-rw-r--r-- 1 Jaume None 16824 Nov 22 18:58 ntpd_tut6.txt
-rw-r--r-- 1 Jaume None 19562 Nov 22 18:58 ntpd_tut3.txt
-rw-r--r-- 1 Jaume None 36272 Nov 22 18:58 ntpd_tut2.txt
-rw-r--r-- 1 Jaume None 2331 Nov 22 18:41 README_start.txt
-rw-r--r-- 1 Jaume None 4552 Nov 22 18:41 README_install.txt
-rw-r--r-- 1 Jaume None 110096
-rw-r--r-- 1 Jaume None 49512
-rw-r--r-- 1 Jaume None 29037
-rw-r--r-- 1 Jaume None 16375 Nov 22 18:59 ntpd_tut6.in
-rw-r--r-- 1 Jaume None 13
-rw-r--r-- 1 Jaume None 35
-rw-r--r-- 1 Jaume None 313
-rw-r--r-- 1 Jaume None 5969
-rw-r--r-- 1 Jaume None 2680
-rw-r--r-- 1 Jaume None 6447
-rw-r--r-- 1 Jaume None 2373
-rw-r--r-- 1 Jaume None 4073
-rw-r--r-- 1 Jaume None 8727
-rw-r--r-- 1 Jaume None 2125
-rw-r--r-- 1 Jaume None 2432
Jaume@Jaume-PC:~/cygdrive/c/gLAB/win/GNSS_TUTBOOK_MASTER
awk '{if ($1=="PREFIX" && $4==300) print $6,$8,$15,$16}' gLAB.out > dat
```

For the selected rows prints columns: 6, 8, 15 and 16

Tutorial 1 UNIX environment, Tools and Skills. GNSS Standard File Formats

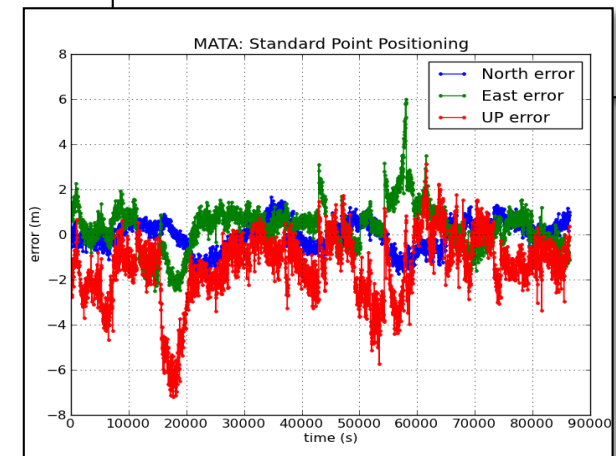
J. Sanz Subirana, J.M. Juan Zornoza, A. Rovira García

November 19, 2017

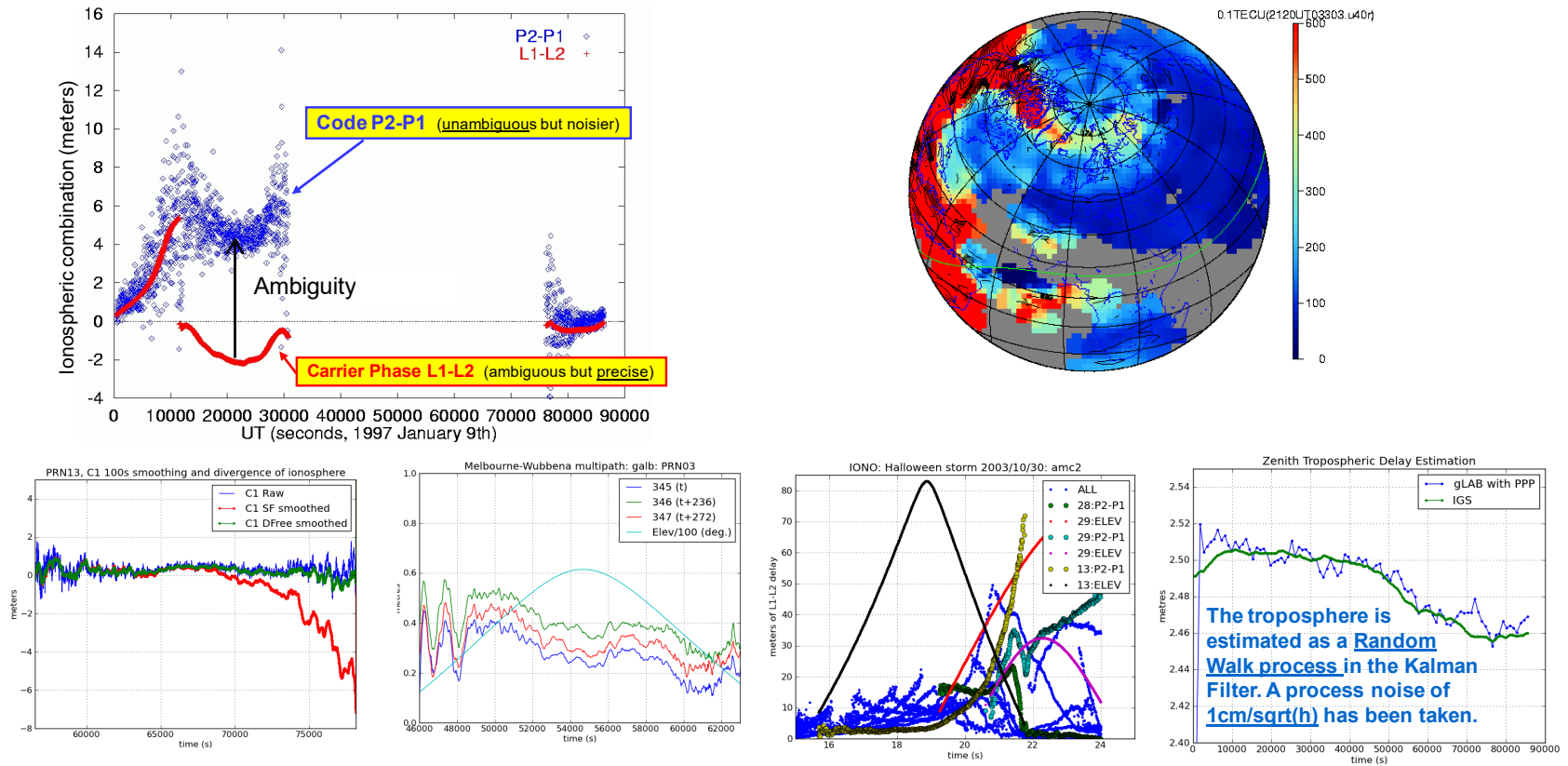
UNIX environment tools and skills (2h)

To introduce a small set of UNIX instructions in order to manage files and directories, as well as some basic elements of awk/gawk programming and the graphical plotting environment graph.py.

The aim is to provide some basic tools needed to develop the practical sessions.



Laboratory 2. Measurement analysis & error budget

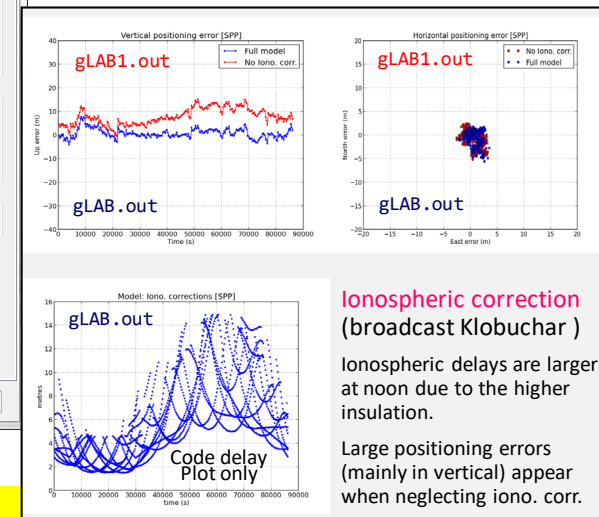
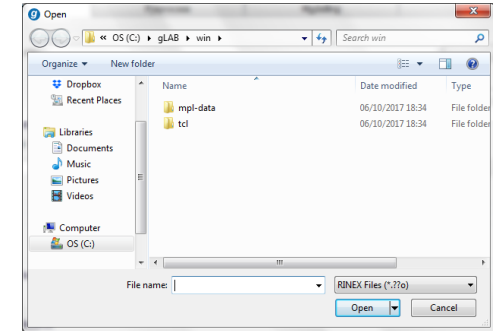


Measurement analysis and error budget (3h)

Exercises on measurements handling and analysis using **gLAB** based on different case studies:

- Ionospheric delay (hallowing storm), troposphere, Multipath, carrier smoothed code...

Laboratory 3. Model components analysis

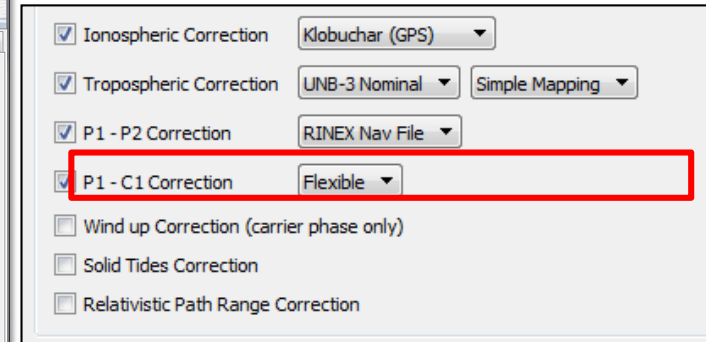
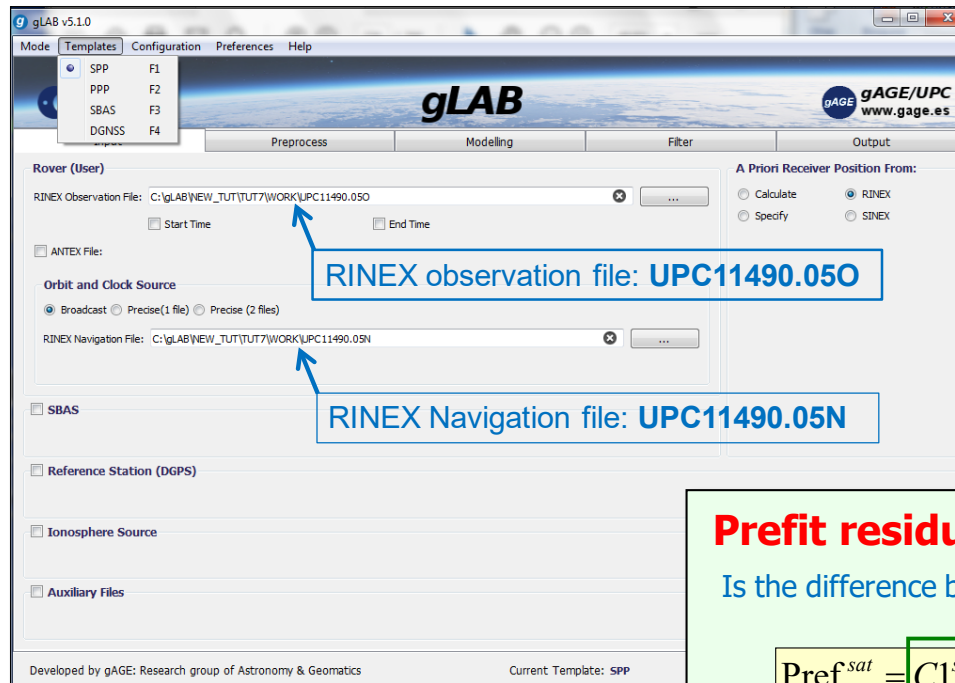


Model components analysis (3h)

Practical lecture on data processing and analysis using **gLAB**.

- Analysis of the model components for Standard and Precise Point Positioning (SPP, PPP)
- To experiment with the concepts seen in the theory.

Laboratory 4. Detailed code measurements modelling



Detailed code measurements modelling (3h)

"Hand modelling" of pseudorange code measurements.

Prefit residual:

Is the difference between measured and modeled pseudorange

$$\text{Pref}_{rec}^{sat} = C1_{rec}^{sat} - C1[\text{mod}]_{rec}^{sat} = \rho_{rec}^{sat} - \rho_{0,rec}^{sat} + c dt_{rec} + K_{1rec} + \varepsilon$$

In the previous example (PRN25 at t= 300 s):

$$\text{Pref} = 22857303.996 - 22857311.347 = -7.35154 \text{ m}$$

From measurement file

Previously calculated

Cross-checking results with gLAB

```
grep PREFIT gLAB.out | grep -v INFO |
gawk '{if ($6==25) print $4,$6,$8}' | head -1
```

Laboratory 5. Solving navigation equations

Receiver solution message. This message provides the estimated receiver position. It is shown in each filter execution.

- Field 1: OUTPUT
- Field 2: Year
- Field 3: Day
- Field 4: Seconds of day
- Field 5: 3D Formal Error: $(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)^{1/2}$
- Field 6: Receiver X position [m]
- Field 7: Receiver Y position [m]
- Field 8: Receiver Z position [m]
- Field 9: Rece
- Field 10: Rece
- Field 11: Rece
- Field 12: Rece
- Field 13: Rece
- Field 14: Rece
- Field 15: Rece
- Field 16: Rece
- Field 17: Rece
- Field 18: Receiver North difference in relation to nominal a priori position [m]
- Field 19: Receiver East difference in relation to nominal a priori position [m]
- Field 20: Receiver Up difference in relation to nominal a priori position [m]
- Field 21: Receiver formal error in North direction [m]
- Field 22: Receiver formal error in East direction [m]
- Field 23: Receiver formal error in Up direction [m]
- Field 24: Geometric Dilution of Precision (GDOP)
- Field 25: Position Dilution of Precision (PDOP)
- Field 26: Time Dilution of Precision (TDOP)
- Field 27: Horizontal Dilution of Precision (HDOP)
- Field 28: Vertical Dilution of Precision (VDOP)
- Field 29: Zenith Tropospheric Delay (including nominal value) [m]
- Field 30: Zenith Tropospheric Delay (excluding nominal value) [m]
- Field 31: Processing mode indicator
- Field 32: Number of satellites used in the navigation solution
- Field 33: Processing mode indicator
- Sample: OUTPUT 2006 200 300 0.0 2.6219 4849203.1236 -360328.5229 4114913.9535 0.7693 0.4145 0.7580 1.9353 0.6998 1.6246 40.429162956 -4.249653155 830.480629026 0.0993 0.4704 1.0522 1.1365 0.6772 2.2637 5.0394 5.5472 6.3482 2.4261 2.2142 2.1982 0.0097 0.4995 6 0

Positioning error, regarding the reference coordinates.

Receiver clock offset

Run gLAB Show Output

```
Jaume@Jaume-PC:/cygdrive/c/gLAB/N...
octave:1> load M.dat
octave:2> y=M(:,1)
y =
-7.3516
-5.3107
-7.4675
-6.8946
-6.5154
-5.2957
-10.3137
-5.6687

octave:3> G=M(:,2:5)
G =
-0.068854 0.632491 -0.771501 1.000000
-0.671994 -0.626096 0.395511 1.000000
-0.887550 0.146645 -0.436749 1.000000
-0.204785 0.798180 -0.566544 1.000000
-0.048353 -0.789036 -0.612441 1.000000
-0.362015 -0.782645 -0.506372 1.000000
-0.527188 -0.150041 -0.836398 1.000000
-0.687410 0.722896 -0.069918 1.000000

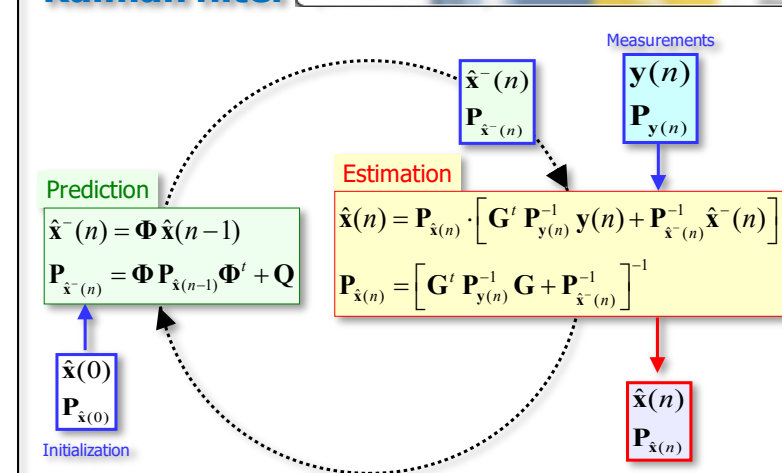
octave:4> x=inv(G'*G)*G'*y
x =
2.659322
-0.068950
3.786600
-4.091713

octave:5> exit
```

Solving navigation equations(3h)

- Practical implementation of GNSS navigation equations.
- Parameter estimation by LS and Kalman filter.

Kalman filter



Laboratory

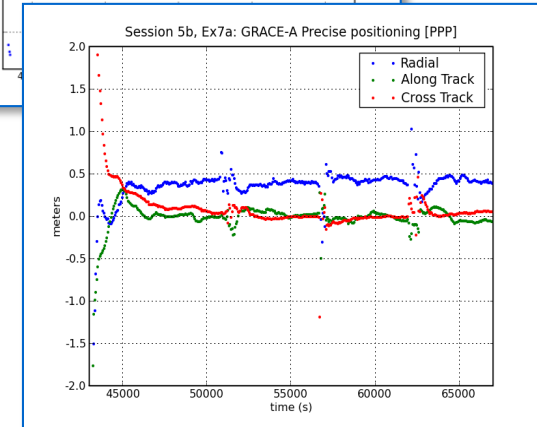
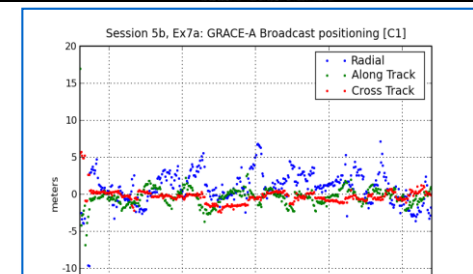
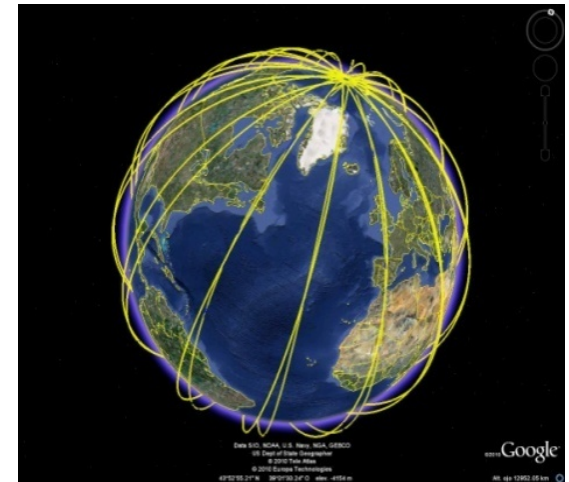
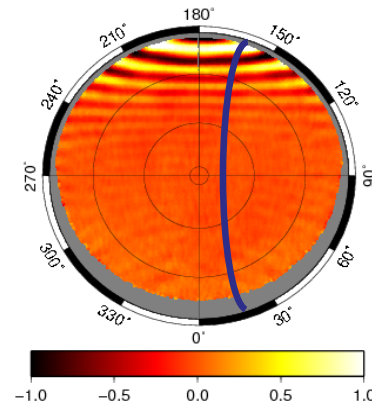
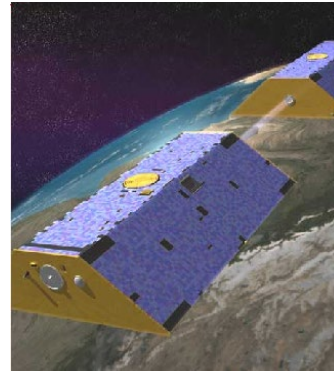
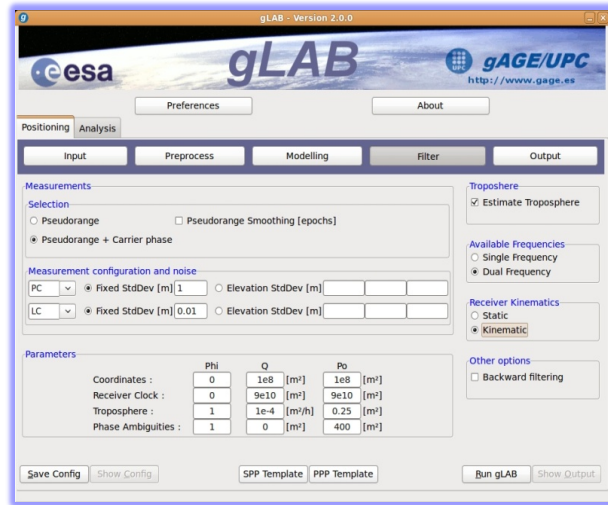
Fundamentals course

- Tutorial 0. Introduction to gLAB tool suite (2h)
- Tutorial 1. UNIX environment tools and skills (2h)
- Tutorial 2. Measurements analysis and error budget (3h)
- Tutorial 3. Model components analysis (3h)
- Tutorial 4. Detailed code measurements modelling (3h)
- Tutorial 5. Solving navigation equations (3h)

Advanced course

- Tutorial 6. Kinematic orbit estimation of a LEO satellite (4h)
- Tutorial 7. Differential positioning with code (4h)
- Tutorial 8. Carrier ambiguity fixing (4h)
- Tutorial 9. Differential positioning with carrier (4h)

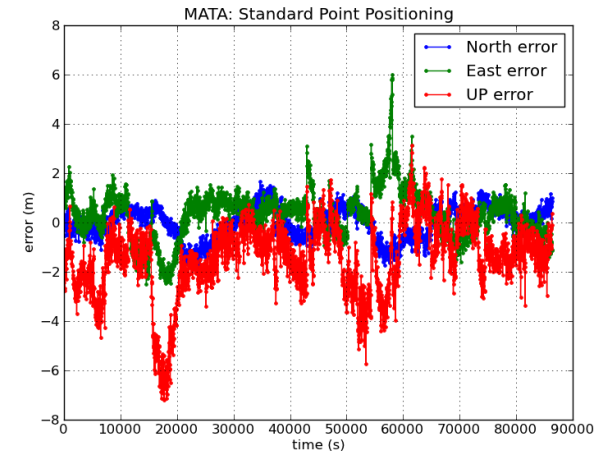
Laboratory 6. Kinematic orbit estimation of a LEO



Kinematic orbit estimation of a LEO (4h)

To perform/investigate the kinematic positioning of a Low Earth Orbit (LEO) satellite, with gLAB, practising concepts learned in the theory.

Laboratory 7. Differential positioning with code



```
cat EBRE.obs | gawk 'BEGIN{g2r=atan2(1,1)/45}{e=$12*g2r;a=$13*g2r;
printf "%8.2f %8.4f %8.4f %8.4f %1i \n",
$4, $14, -cos(e)*sin(a), -cos(e)*cos(a), -sin(e), 1 }' > EBRE.mod
```

$$\begin{bmatrix} \text{DPref}^1 \\ \text{DPref}^2 \\ \dots \\ \text{DPref}^n \end{bmatrix} = \begin{bmatrix} -(\hat{\mathbf{p}}_{\text{creu}}^1)^T & 1 \\ -(\hat{\mathbf{p}}_{\text{creu}}^2)^T & 1 \\ \dots & \dots \\ -(\hat{\mathbf{p}}_{\text{creu}}^n)^T & 1 \end{bmatrix} \mathbf{dx}$$

Differential positioning with code (4h)

Basic exercises on differential positioning with code.
Mitigation of orbit errors and signal propagation effects
on the atmosphere will be analysed with real data.

Laboratory 8. Carrier ambiguity fixing

2. Applying the LAMBDA method to FIX the ambiguities.

The following procedure can be applied (justify the computations done)

Compare the different results found.

octave

```
c=299792458;
f0=10.23e+6;
f1=154*f0;
lambda1=c/f1
a=x(4:12)/lambda1;
Q=P(4:12,4:12);
```

Decorrelation and integer LS search solution

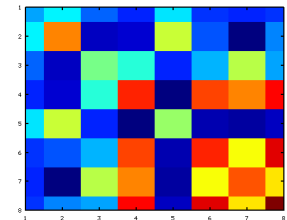
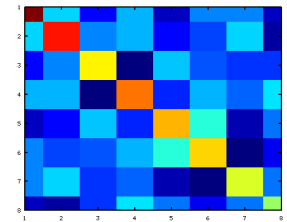
```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
[azfixed,sqnorm] = lsearch (az,Lz,Dz,2);
afixed=iZ*azfixed;
sqnorm(2)/sqnorm(1)
ans = 2.47022808203678
afixed(:,1)'
-19333 130765338 -1759080 -1498083 130765319
130765324 130765334 122888028 130765333
```

Rounding the floated solution directly

```
round(a)' -19334 130765336 -1759081
-1498083 130765320 130765323
130765334 122888029 130765334
```

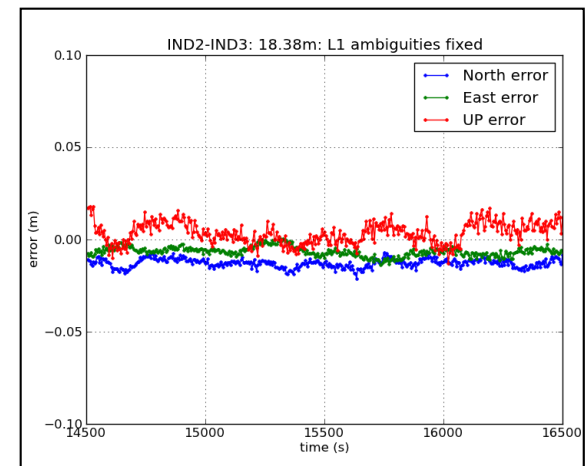
Rounding the decorrelated floated solution

```
afix=iZ*round(az)
-19333 130765338 -1759080 -1498083 130765319
130765324 130765334 122888028 130765333
```



Carrier ambiguity fixing (4h)

The "one at a time" method and LAMBDA method are tested.
Centimetre level of accuracy will be achieved after fixing ambiguities.

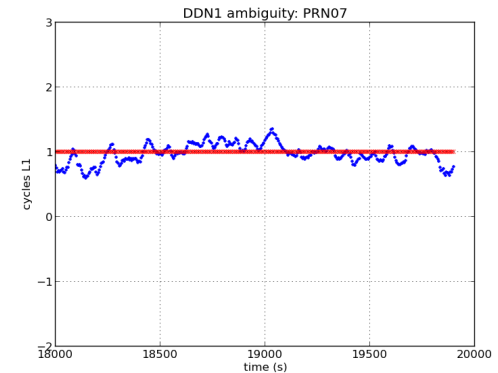
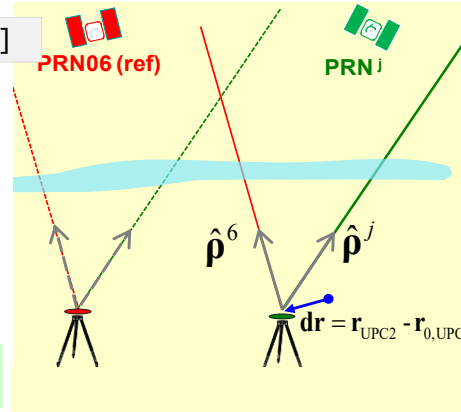


Laboratory 9. Differential Positioning with carrier

$$[DDL1 - DDRho - \lambda_1 DDN1] = [Los_k - Los_06] * [dr]$$

$$\begin{bmatrix} DDL_1^{6,03} - DD\rho^{6,03} - \lambda_1 DDN_1 \\ DDL_1^{6,07} - DD\rho^{6,07} - \lambda_1 DDN_1 \\ \dots \\ DDL_1^{6,24} - DD\rho^{6,30} - \lambda_1 DDN_1 \end{bmatrix} = \begin{bmatrix} -(\hat{\rho}^3 - \hat{\rho}^6)^T \\ -(\hat{\rho}^7 - \hat{\rho}^6)^T \\ \dots \\ -(\hat{\rho}^{24} - \hat{\rho}^6)^T \end{bmatrix} dr$$

$$\hat{\rho}^j \equiv [\cos(El_j) \sin(Az_j), \cos(El_j) \cos(Az_j), \sin(El_j)]$$



$$[DDL1 - \lambda_1 DDN1] = [Los_k - Los_06] * [baseline]$$

```
cat DD_UPC1_UPC2_06_ALL.fixL1L2 | gawk 'BEGIN{g2r=atan2(1,1)/45}
{e1=$14*g2r;a1=$15*g2r;e2=$16*g2r;a2=$17*g2r;
printf "%s %14.4f %8.4f %8.4f %8.4f \n",
```

```
$6, $8-$18, -cos(e2)*sin(a2)+cos(e1)*sin(a1),
-cos(e2)*cos(a2)+cos(e1)*cos(a1), -sin(e2)+sin(e1))' > M.dat
```

$$\begin{bmatrix} DDL_1^{6,03} - \lambda_1 DDN_1^{6,03} \\ DDL_1^{6,07} - \lambda_1 DDN_1^{6,07} \\ \dots \\ DDL_1^{6,24} - \lambda_1 DDN_1^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\rho}^3 - \hat{\rho}^6)^T \\ -(\hat{\rho}^7 - \hat{\rho}^6)^T \\ \dots \\ -(\hat{\rho}^{24} - \hat{\rho}^6)^T \end{bmatrix} r$$

←

$$[DDL1 - \lambda_1 DDN1] \quad [Los_k - Los_06]$$

-3.3762	0.3398	-0.1028	0.0714
-7.1131	0.1725	0.5972	0.0691
4.3881	-0.6374	0.0227	0.2725

Differential positioning with carrier (4h)

Basic exercises on differential positioning with carrier. The concepts introduced in the theory on differential carrier positioning are experimented in the laboratory.

The screenshot shows the gAGE website interface. The main header includes the gAGE logo and the text "gAGE : Research Group of Astronomy and Geomatics". The navigation bar contains links for "About gAGE/UPC", "About gAGE-NAV, S.L.", and "Contact Us".

The "Professional Training" section is highlighted, containing the following text:

This website provides you information about the Professional Training of the UPC -Barcelona Tech.

The e-Knot professional training program is an ambitious program aimed at **providing GNSS training to employees of industry, research centres and institutions**. It is based on the provision of twelve 3-day free-of charge tutorials over 3 years (2 in 2015, 5 in 2016 and 5 in 2017).

The instructors of the e-KnoT professional training program are GNSS experts from the Ecole Nationale de l'Aviation Civile (France), Politecnico di Torino (Italy), Universitat Politècnica de Catalunya (Spain), Astri Polska (Poland)

The e-KnoT Professional Training Leaflet 2016 can be downloaded from the following link: e-Knot_Leaflet.pdf

Under the "Professional Training" sub-section, a list of events is shown:

- Professional Training
 - Warsaw2015
 - Barcelona2016
 - Prague2016
 - Madrid2016
 - Warsaw2017
 - Barcelona2017
 - GNSS_COURSE**

A red box highlights the "GNSS_COURSE" link, with a red arrow pointing to it from a yellow box containing the login credentials:

User: gnss
Pass: course

Below the main content, there is a "Learning Material" section with links to "Library" and "Software Tools".

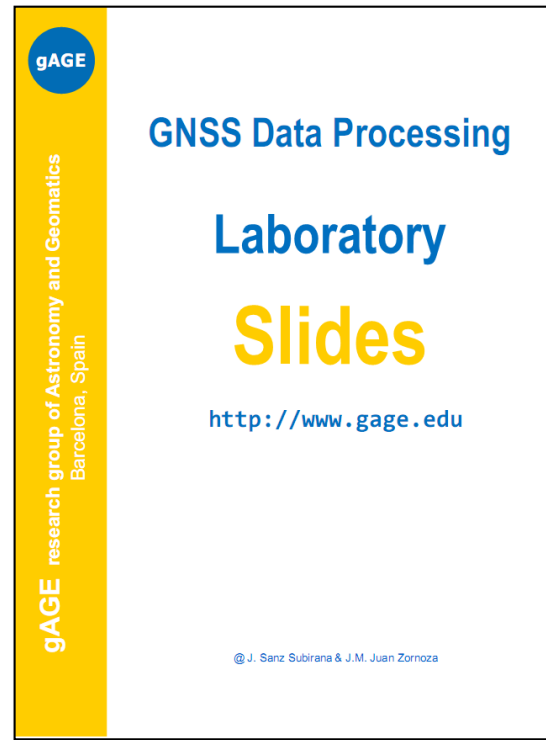
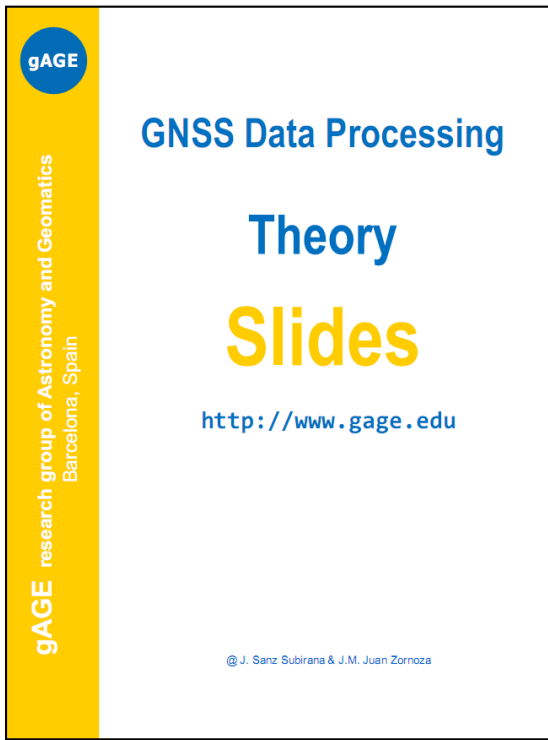
The "Projects" section lists "gAGE/UPC" and "gAGE-NAV, S.L.". The "Patents" section lists "Fast-PPP", "WARTK", and "Iono. Model".

An inset window shows a file directory listing:

Name	Last modified	Size	Description
Parent Directory	-	-	-
GNSS BOOK/	20-Mar-2017 19:46	-	-
SLIDES/	31-Oct-2017 14:24	-	-
SOFTWARE/	31-Oct-2017 14:06	-	-
Schedule BCN17_v0.1.pdf	31-Oct-2017 14:24	70K	-

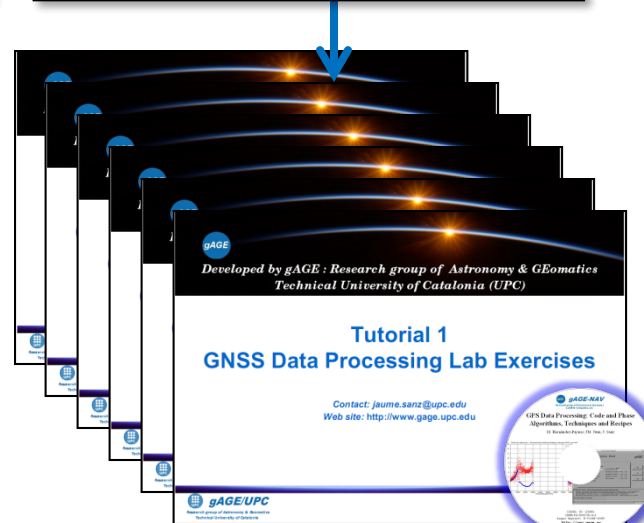
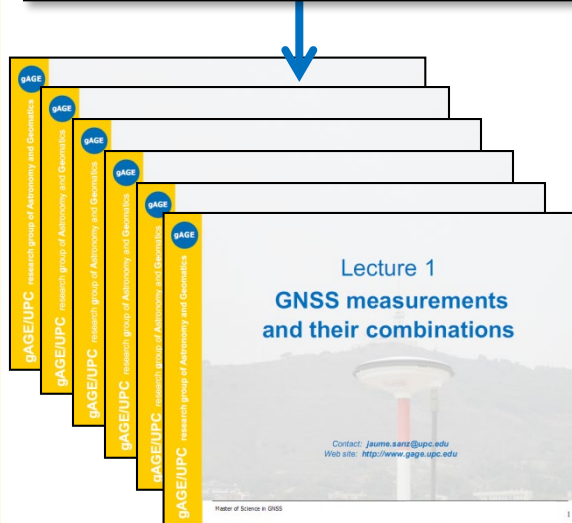
The "Shortcuts" section on the right includes links to "GNSS Data Processing Book", "GNSS Course and associated Tutorials", "GNSS Webinars", "gLAB Tool Suite", "Useful GNSS links", "Masters on GNSS", "PhD Training", and "Professional Training". The "Professional Training" sub-section lists the same events as the main content, with "GNSS_COURSE" highlighted by a red box.

The "User login" section at the bottom right contains a form with the label "Username: *" and an input field.



The **Learning material** is composed by a collection of slides for Theory & Laboratory exercises.

A book on GNSS Data Processing is given as complementary material.



Additional References

- [RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.
- [RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.
- [RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.
- [RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.
- [RD-5] Gang Xie, Optimal on-airport monitoring of the integrity of GPS-based landing systems, PhD Dissertation, 2004.
- [RD-6] Sandra Verhagen and Bofeng L., LAMBDA software package. MATLAB implementation, Version 3.0. Mathematical Geodesy and Positioning, Delft University of Technology.
- [RD-7] ESA/JRC International Summer School on GNSS 2015. Presentations Booklet. Barcelona, Spain. August 31st to September 10th 2015.

We hope you enjoy with this learning material, as we enjoyed compiling it for our GNSS community!!!



Please, do not hesitate in giving us your comments, suggestions and anything you feel useful to help improving this material.

We appreciate!!!

Thank you!