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gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

Lecture 1
Introduction to GNSS

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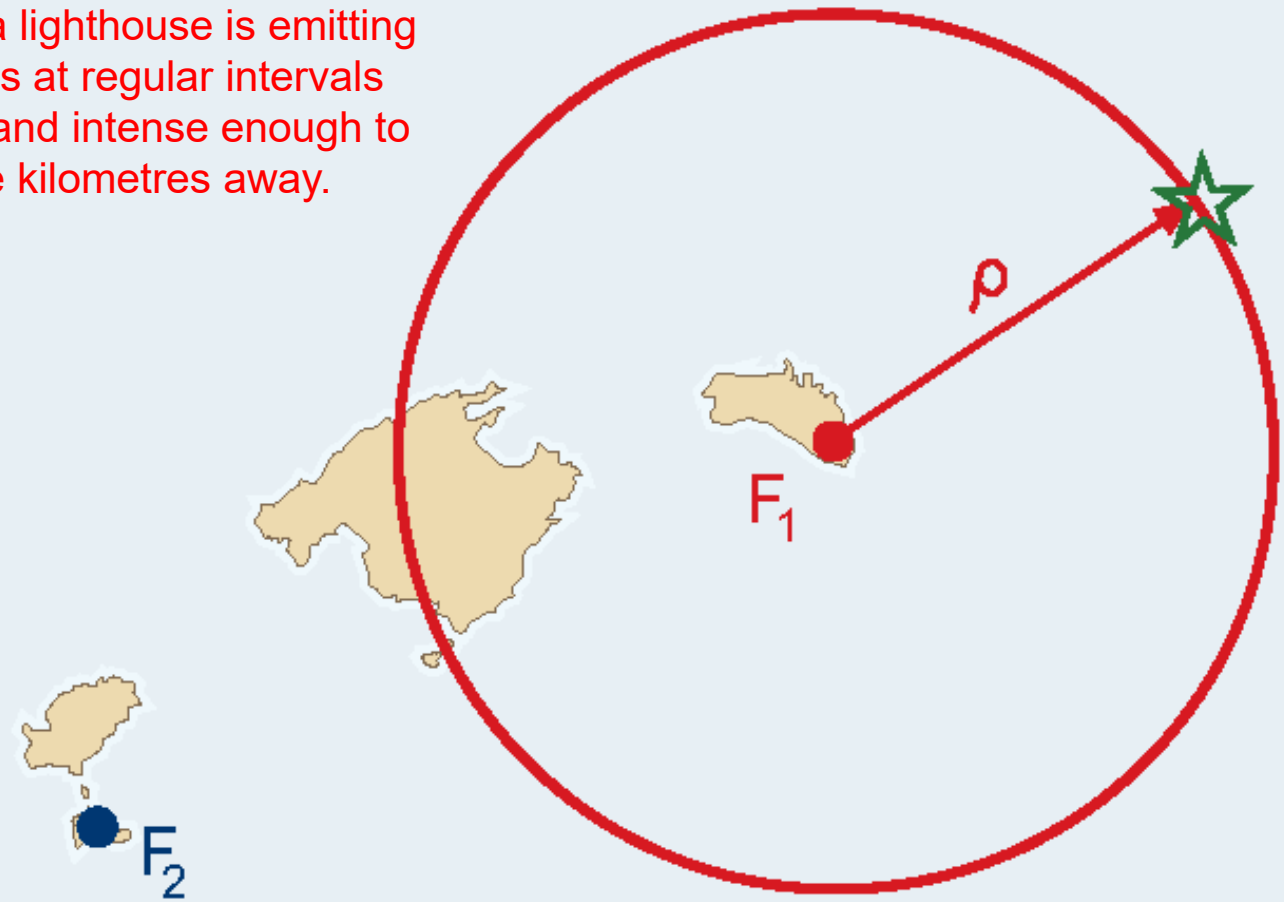
Contents

1.- An intuitive approach to GNSS positioning

● With a single lighthouse, possible solutions lie on a circle of radius ρ

☆ With two lighthouses ★ the possible solutions are reduced to two

Suppose that a lighthouse is emitting acoustic signals at regular intervals of 10 minutes and intense enough to be heard some kilometres away.



Assume that a ship with a clock perfectly synchronised to the one in the lighthouse is receiving these signals in a time that is not an exact multiple of 10 minutes, for example 20 seconds later (t = n*10m + 20s)

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★ the possible solutions are reduced to two

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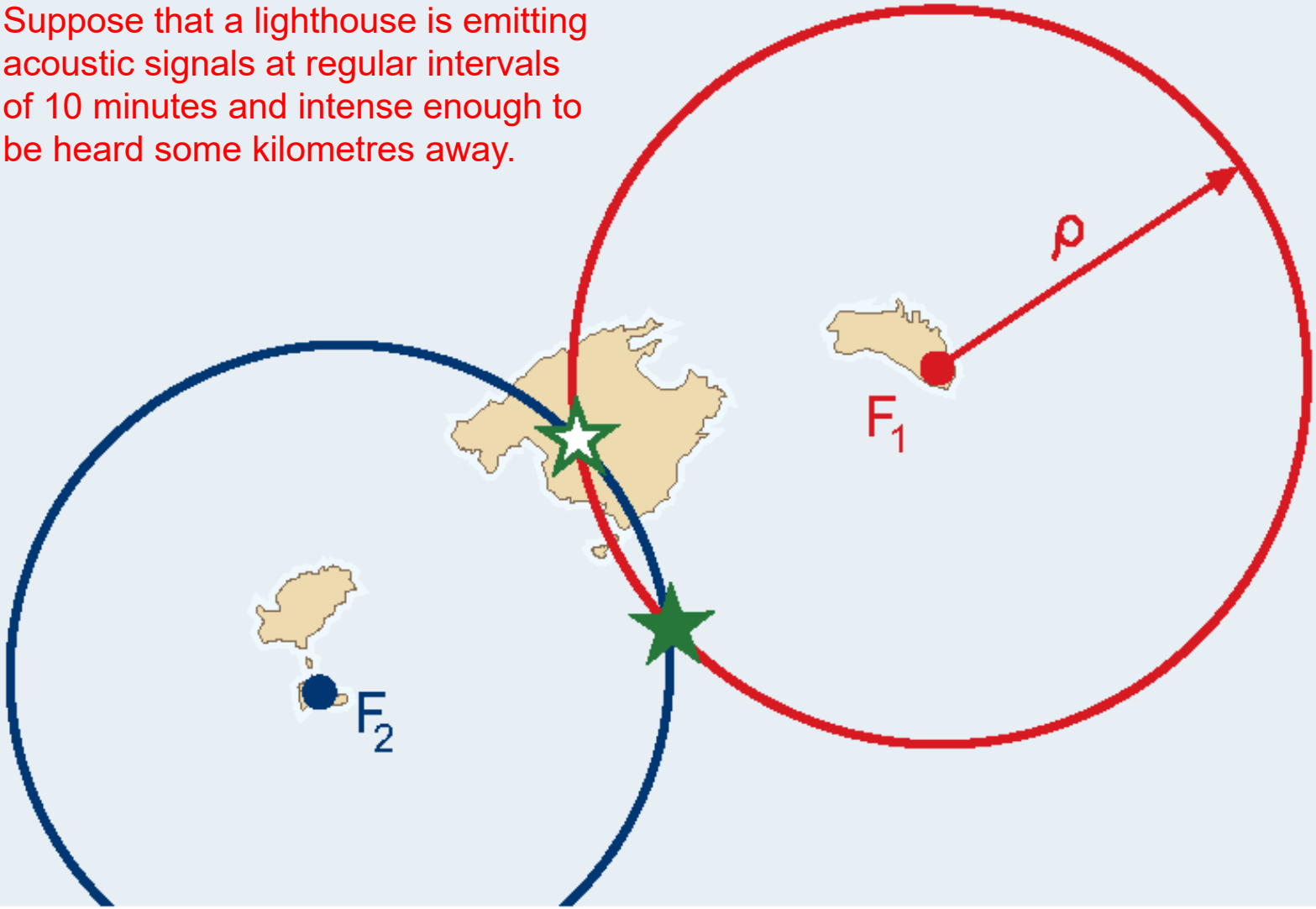

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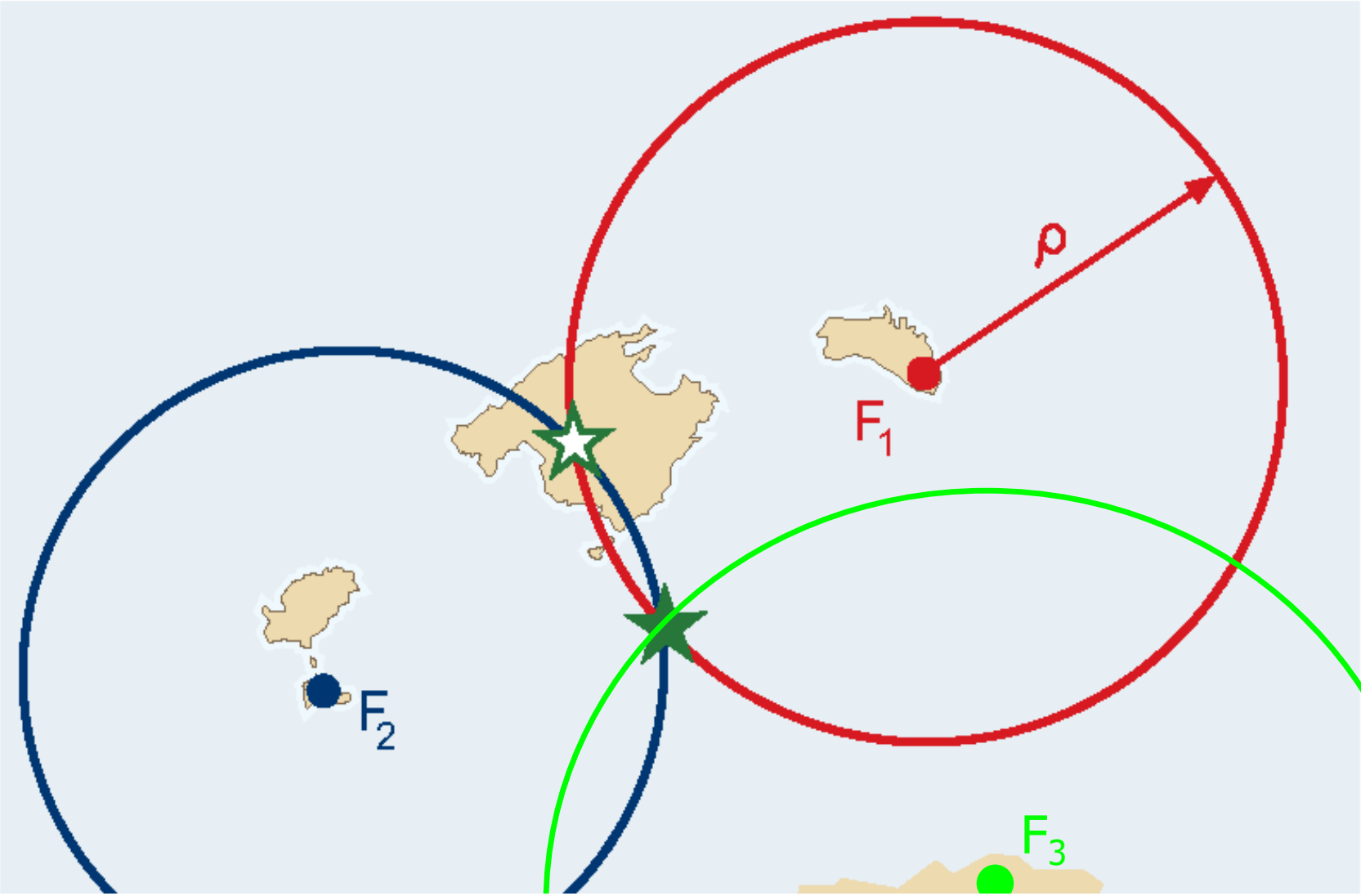

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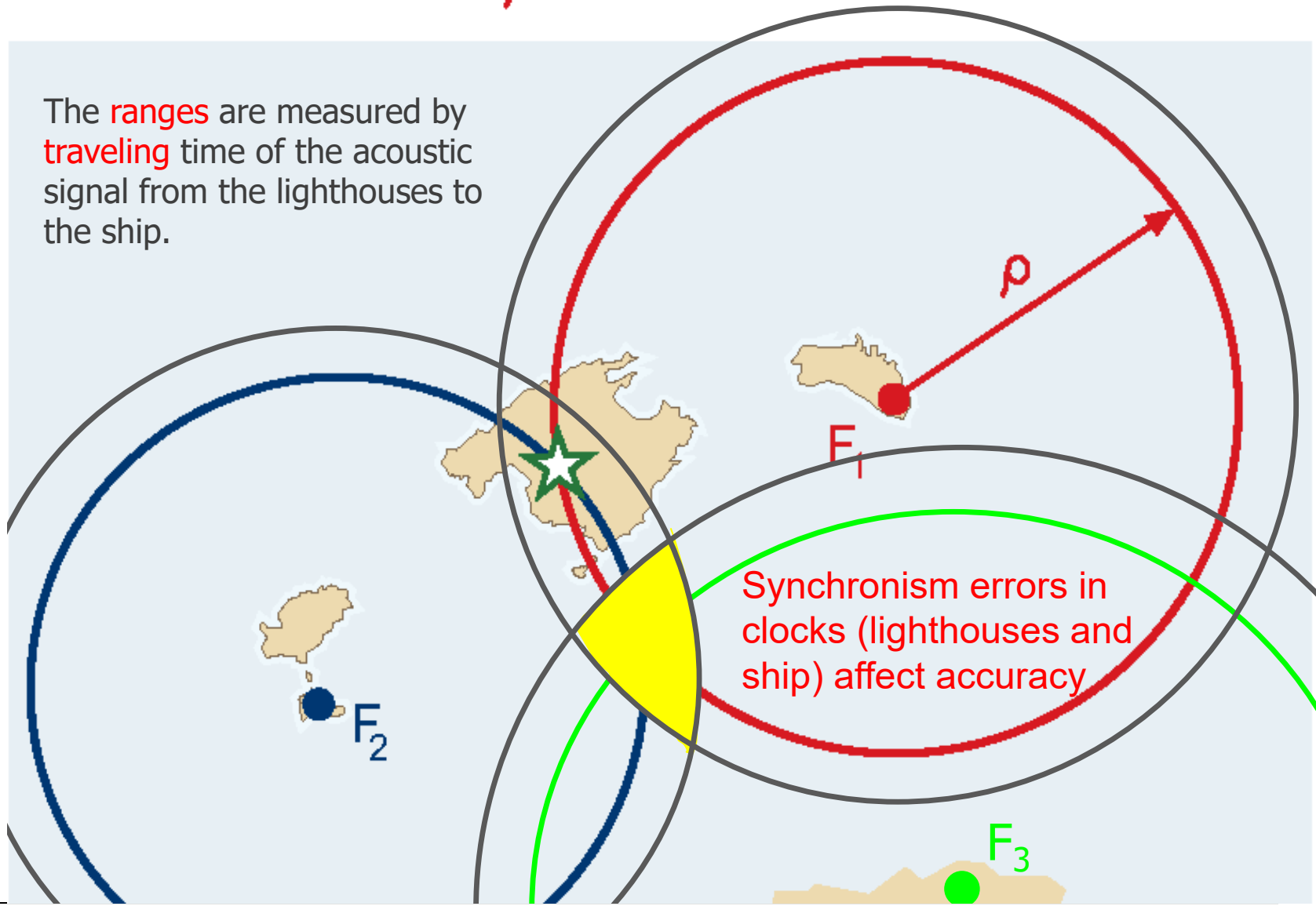
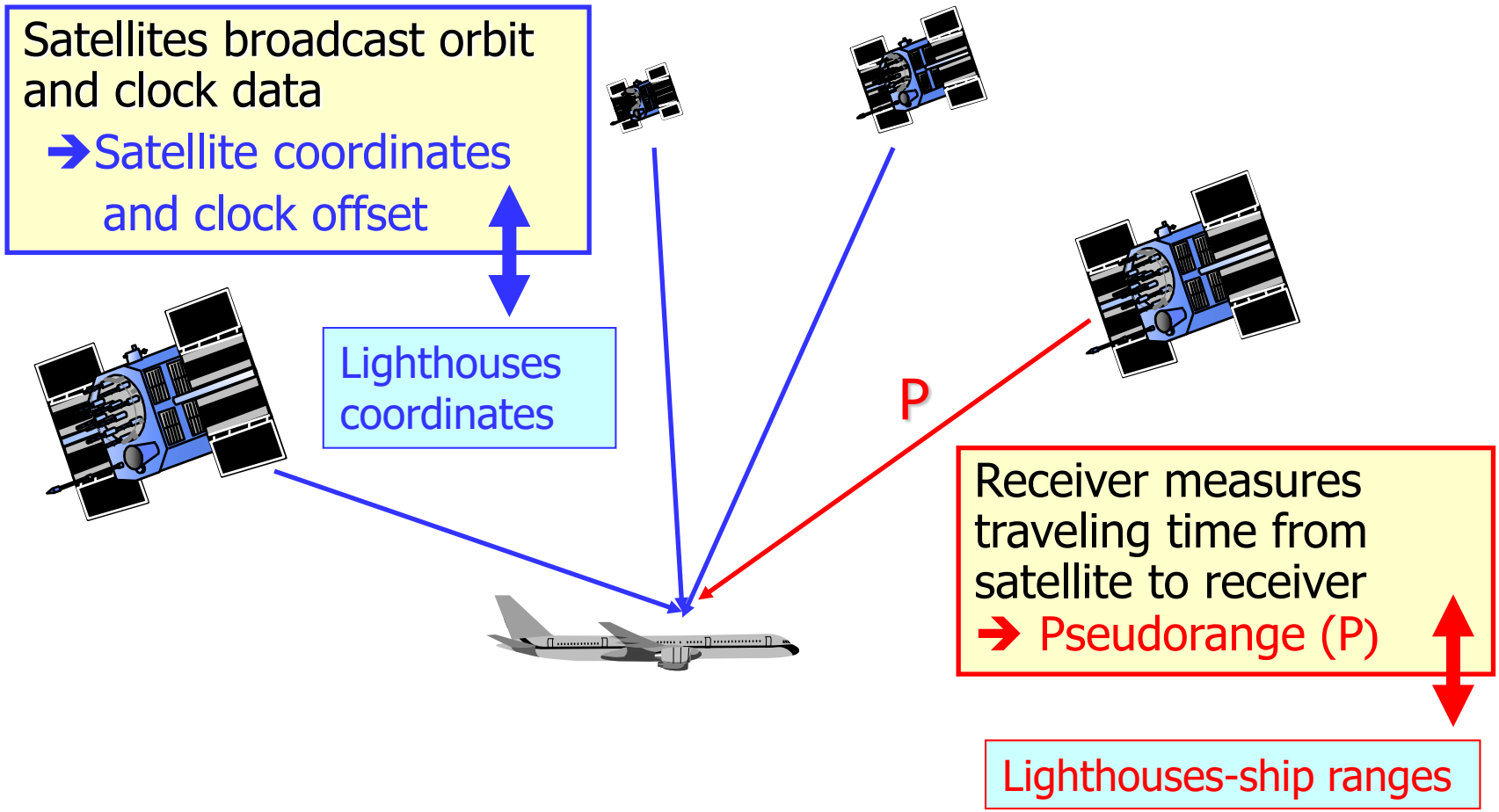
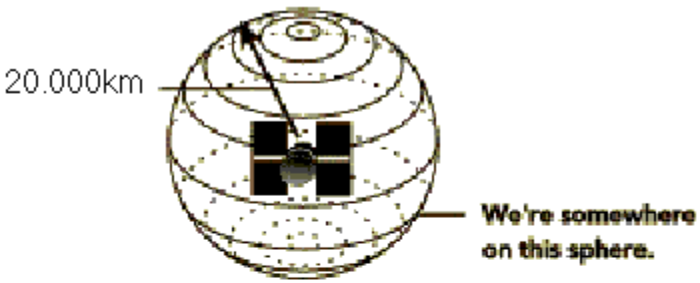
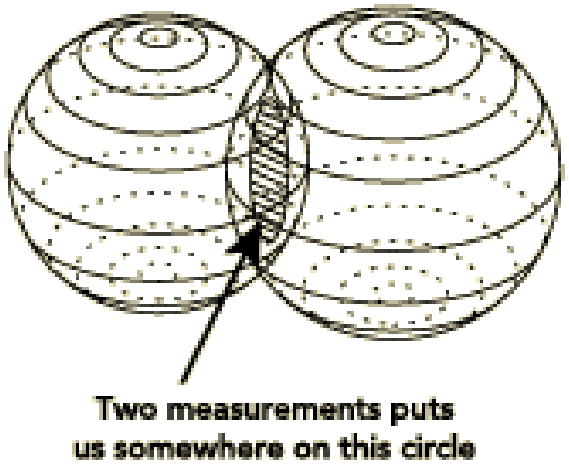


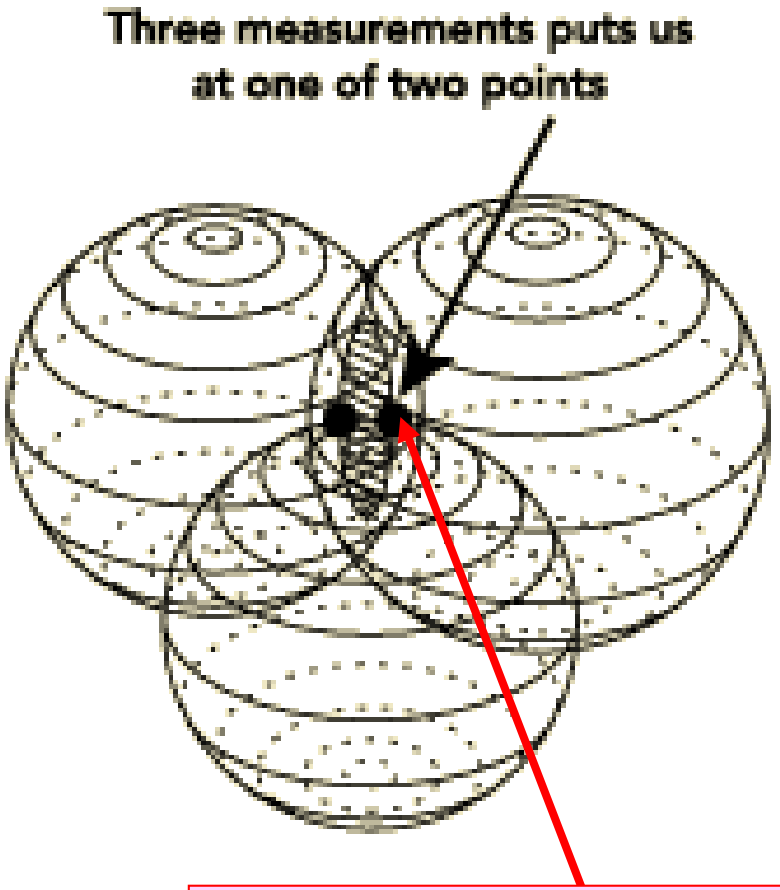
2.- How GNSS Works



Thence, the receiver coordinates are found **solving a geometrical problem**: from satellite coordinates and ranges







One of the solutions is not on the Earth surface.

● With a single lighthouse, possible solutions lie on a circle of radius ρ

☆ With two lighthouses
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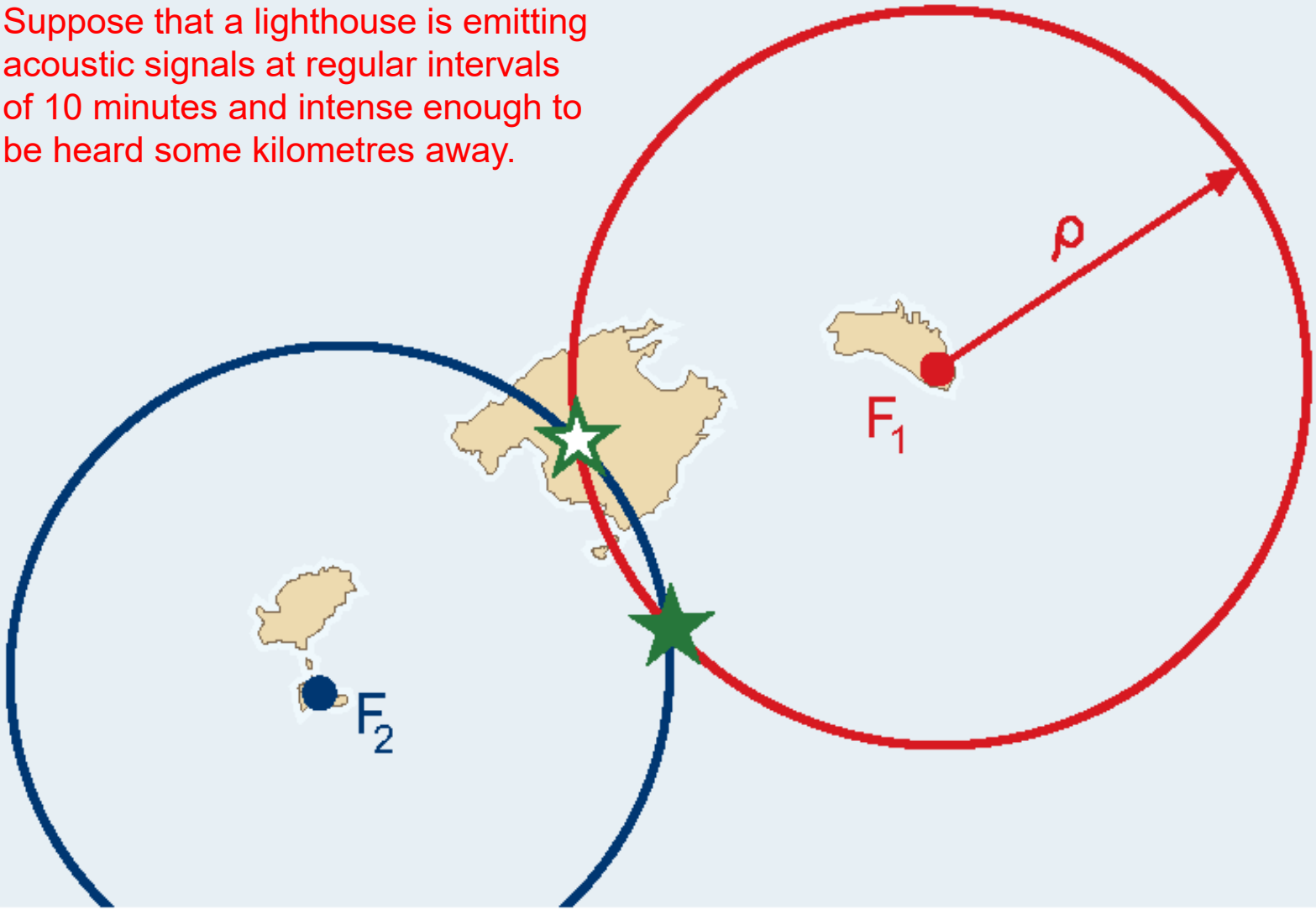
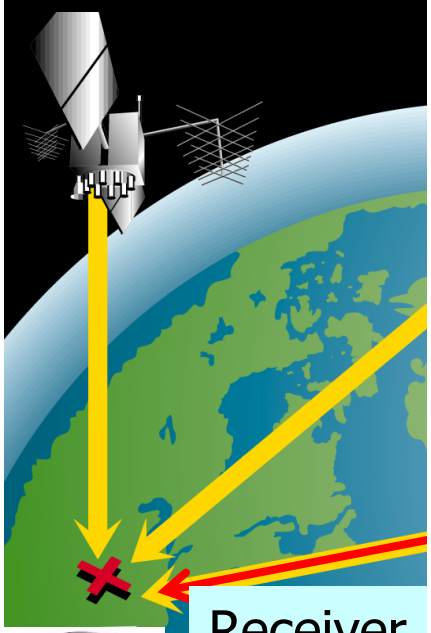
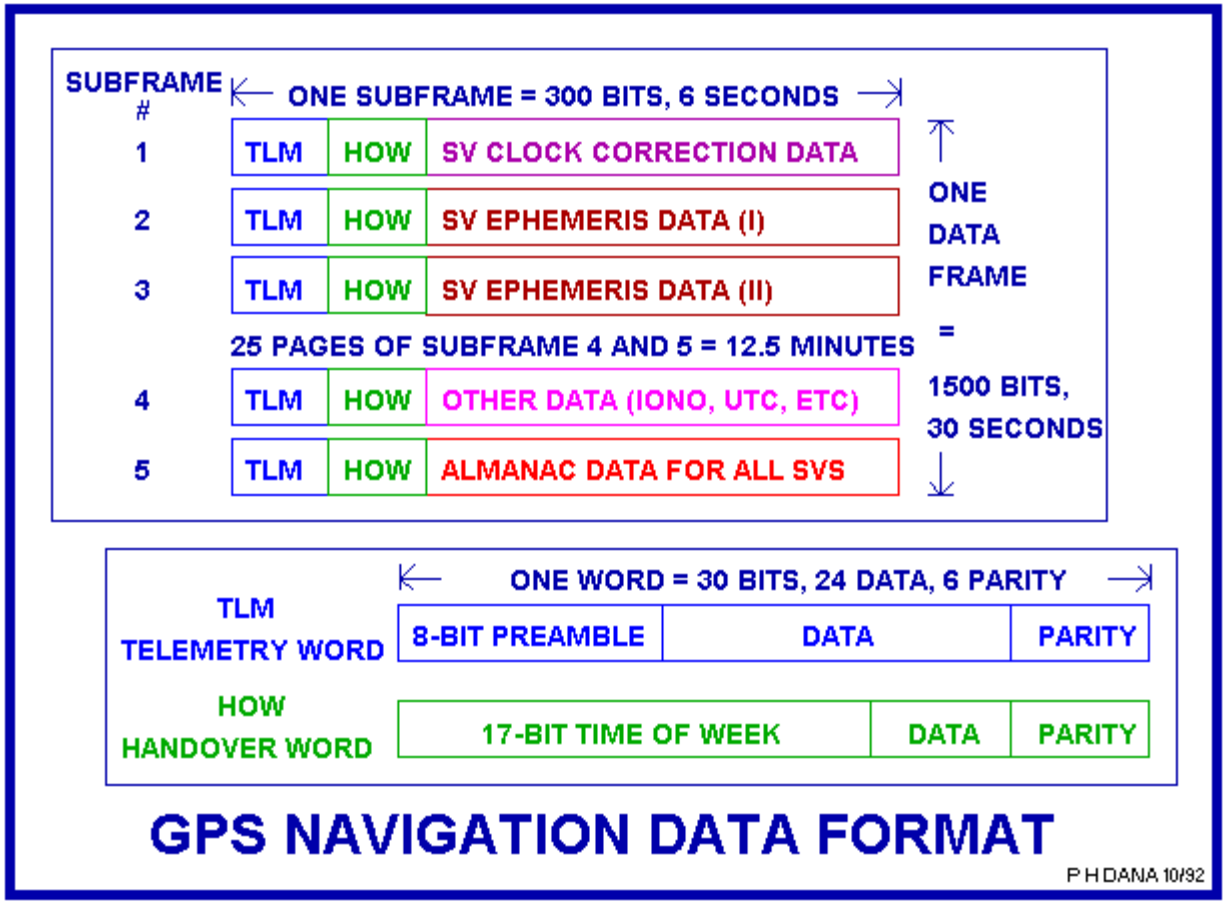
Suppose that a lighthouse is emitting acoustic signals at regular intervals of 10 minutes and intense enough to be heard some kilometres away.

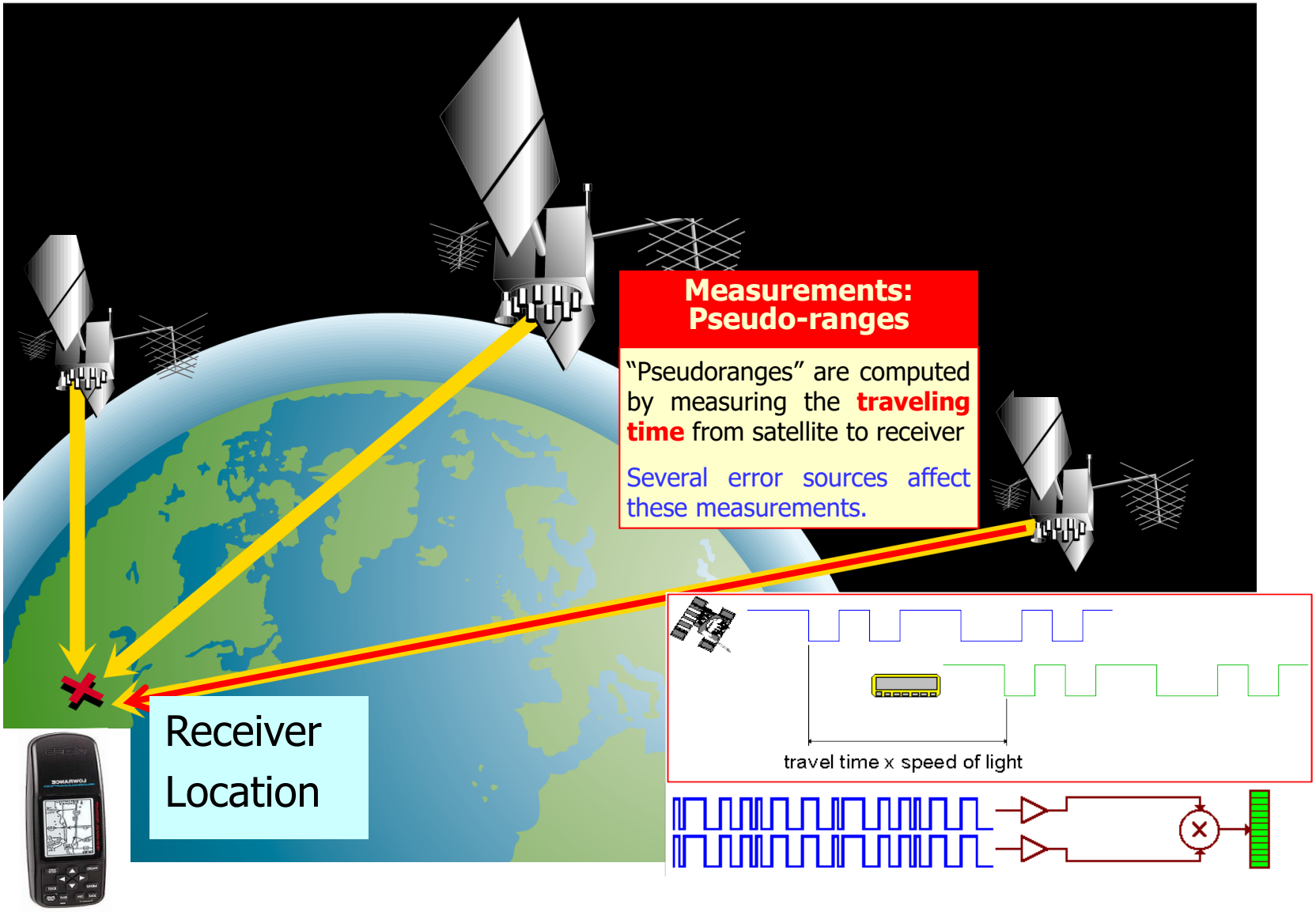


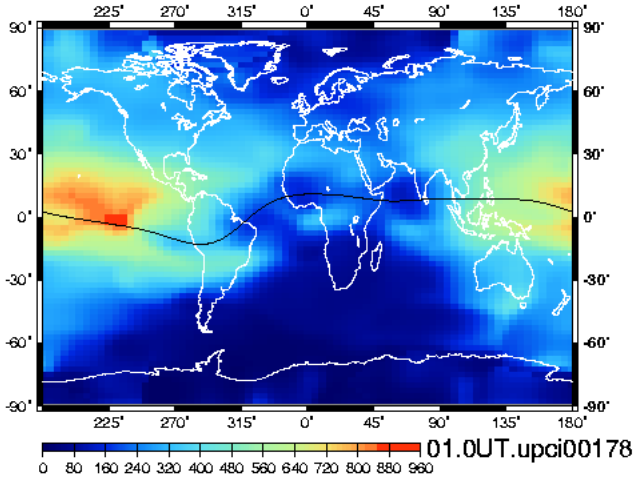
Satellite location

Satellite coordinates and clock offsets are computed from navigation message.









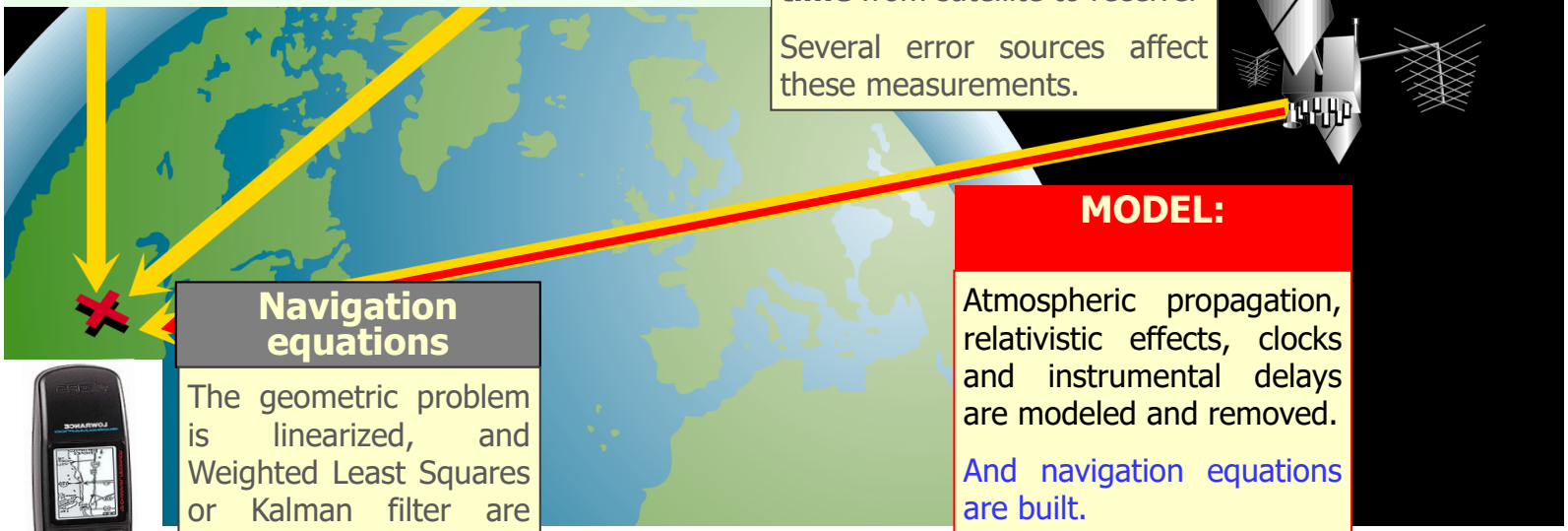
Atmospheric propagation errors: Ionosphere and Troposphere

Measurements: Pseudo-ranges

"Pseudoranges" are computed by measuring the **traveling time** from satellite to receiver

Several error sources affect these measurements.

Relativistic effects



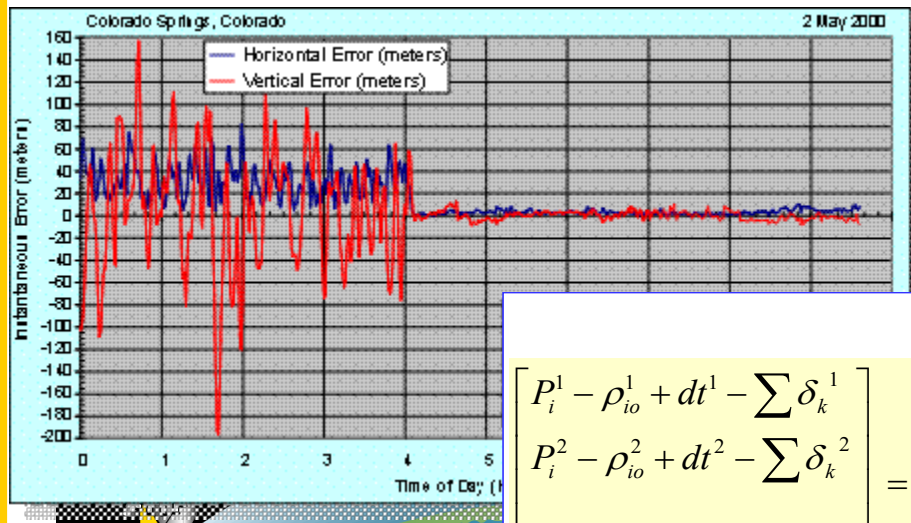
Navigation equations

The geometric problem is linearized, and Weighted Least Squares or Kalman filter are used to compute the navigation solution.

MODEL:

Atmospheric propagation, relativistic effects, clocks and instrumental delays are modeled and removed.

And navigation equations are built.



$$\begin{bmatrix} P_i^1 - \rho_{io}^1 + dt^1 - \sum \delta_k{}^1 \\ P_i^2 - \rho_{io}^2 + dt^2 - \sum \delta_k{}^2 \\ \dots\dots\dots \\ P_i^n - \rho_{io}^n + dt^n - \sum \delta_k{}^n \end{bmatrix} = \begin{bmatrix} \frac{x_{io} - x^1}{\rho_{io}^1} & \frac{y_{io} - y^1}{\rho_{io}^1} & \frac{z_{io} - z^1}{\rho_{io}^1} & 1 \\ \frac{x_{io} - x^2}{\rho_{io}^2} & \frac{y_{io} - y^2}{\rho_{io}^2} & \frac{z_{io} - z^2}{\rho_{io}^2} & 1 \\ & \dots\dots\dots & & \\ \frac{x_{io} - x^n}{\rho_{io}^n} & \frac{y_{io} - y^n}{\rho_{io}^n} & \frac{z_{io} - z^n}{\rho_{io}^n} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ cdt_i \end{bmatrix}$$

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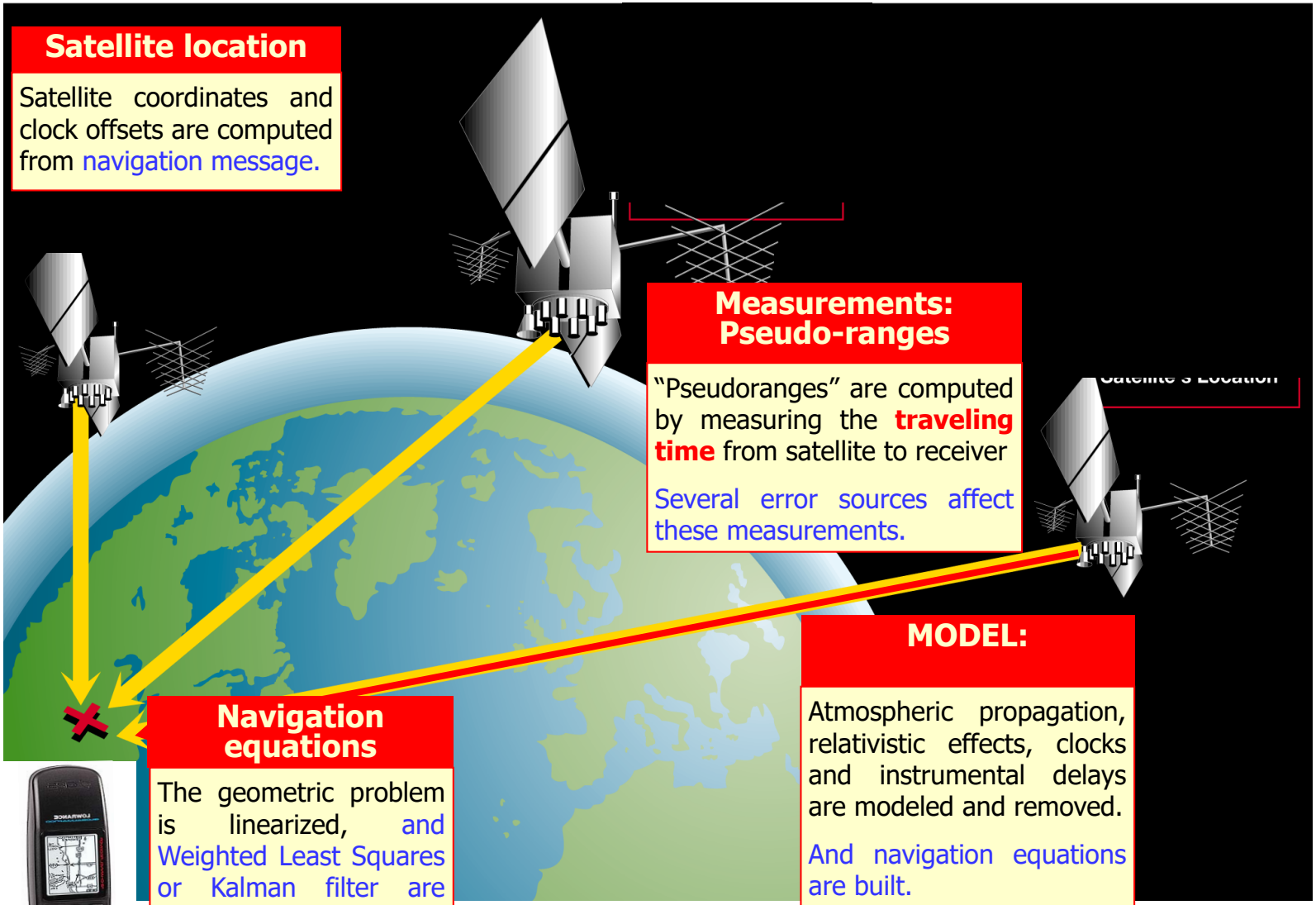

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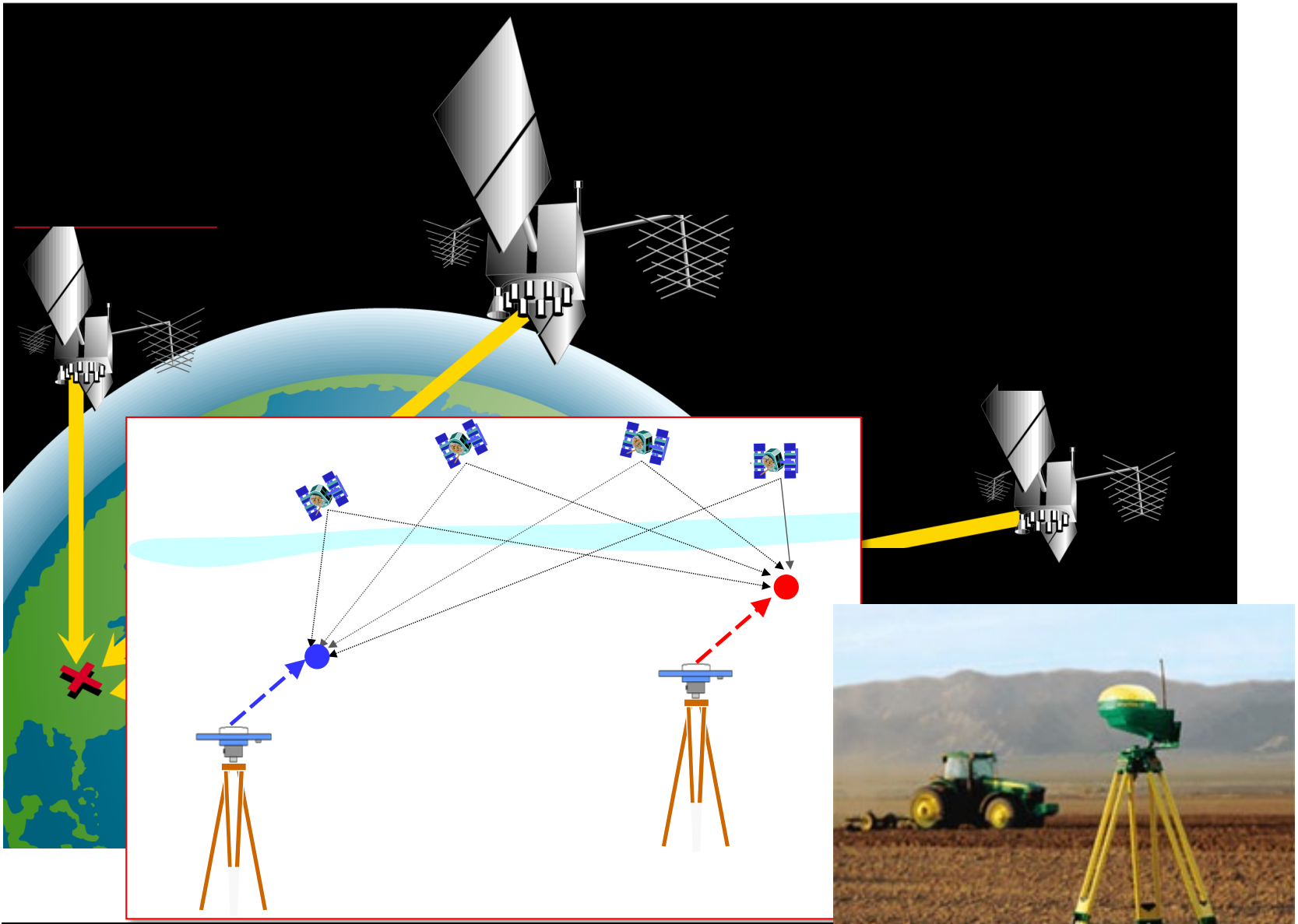

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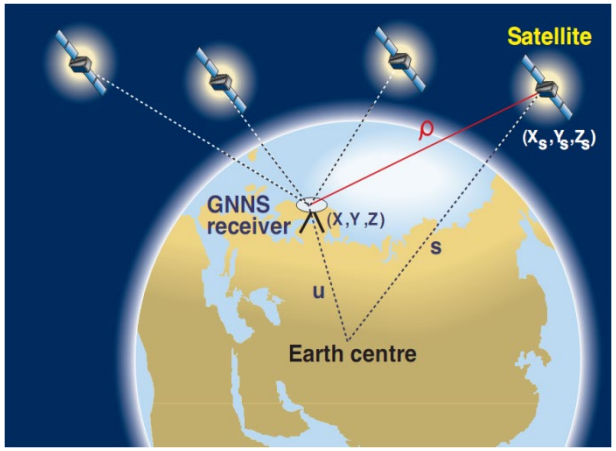
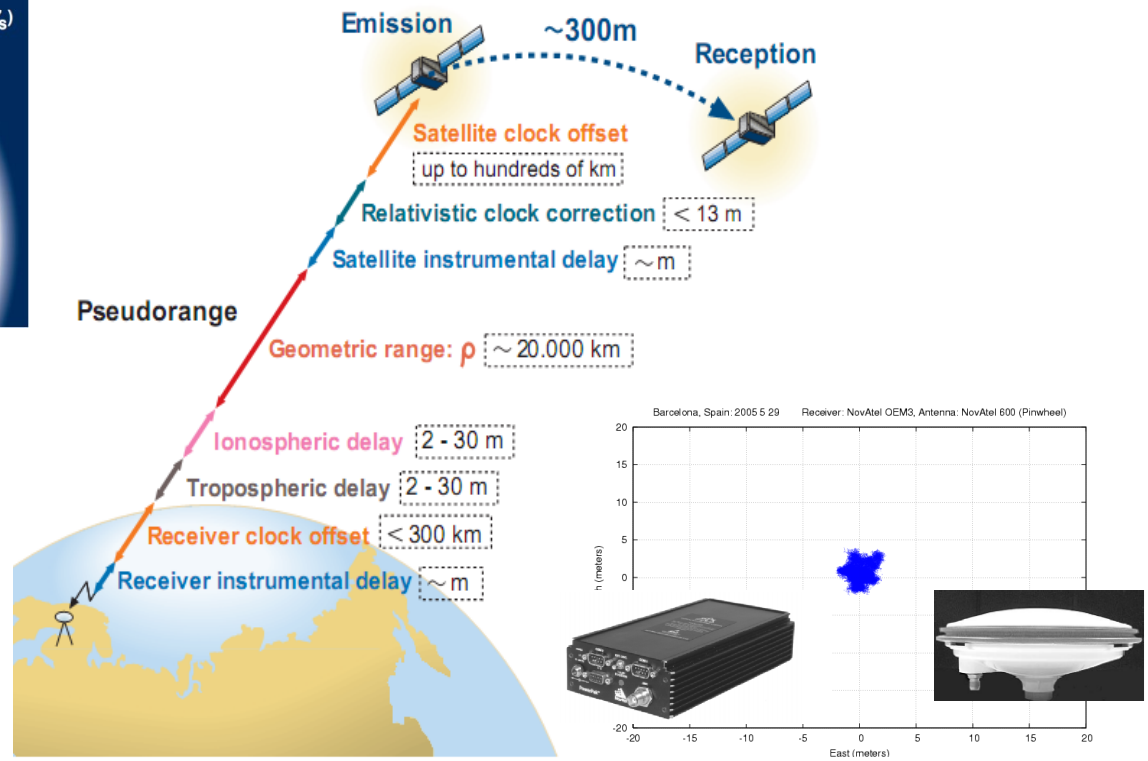
3.- Additional Comments

Measurements Modelling

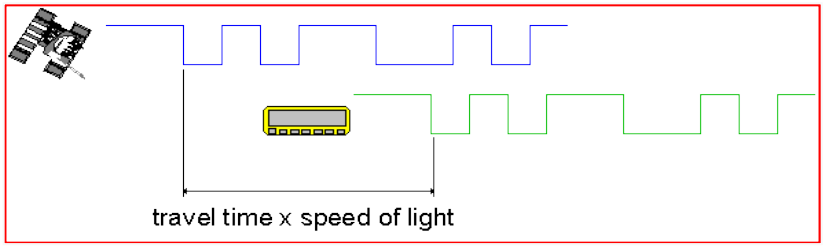


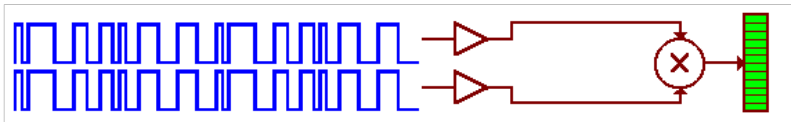


Code Pseudorange Modelling:

Navigation accuracy depends of precise modelling and measurements used (code, carrier).

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$





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Navigation Solution

For each satellite in view

Iono+Tropo+TGD...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}}\Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}}\Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}}\Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + \sum \delta_k$$

where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

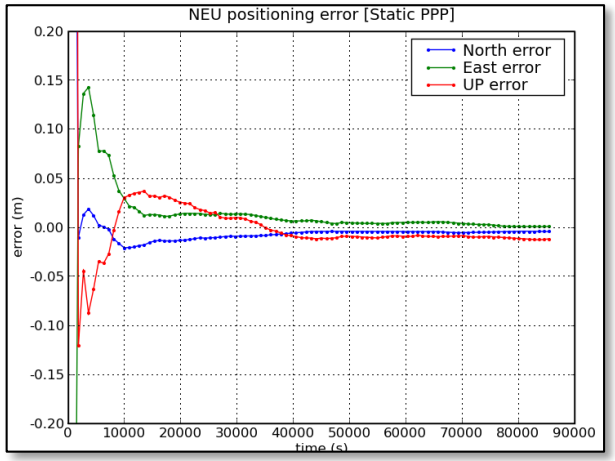
Prefit-residuals (Prefit)

$$C1_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt^{sat} - \sum \delta_k = \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}}\Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}}\Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}}\Delta z_{rec} + c\,dt_{rec}$$

measurement | computed | unknown

Navigation Equations solution:

- The geometrical problem linearised about an approximate receiver coordinates *(x0, y0, z0)*.
- Least Squares or Kalman filter are used to solve the navigation equations.



Kalman filter (see kalman.f)

Measurements

$\mathbf{y}(n)$, $\mathbf{P}_{\mathbf{y}(n)}$

$\hat{\mathbf{x}}^-(n)$, $\mathbf{P}_{\hat{\mathbf{x}}^-(n)}$

Prediction

$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi}\,\hat{\mathbf{x}}(n-1)$$

$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \mathbf{\Phi}\,\mathbf{P}_{\hat{\mathbf{x}}(n-1)}\mathbf{\Phi}^t + \mathbf{Q}$$

Estimation

$$\hat{\mathbf{x}}(n) = \mathbf{P}_{\hat{\mathbf{x}}(n)} \cdot \left[\mathbf{G}^t\,\mathbf{P}_{\mathbf{y}(n)}^{-1}\,\mathbf{y}(n) + \mathbf{P}_{\hat{\mathbf{x}}^-(n)}^{-1}\hat{\mathbf{x}}^-(n)\right]$$

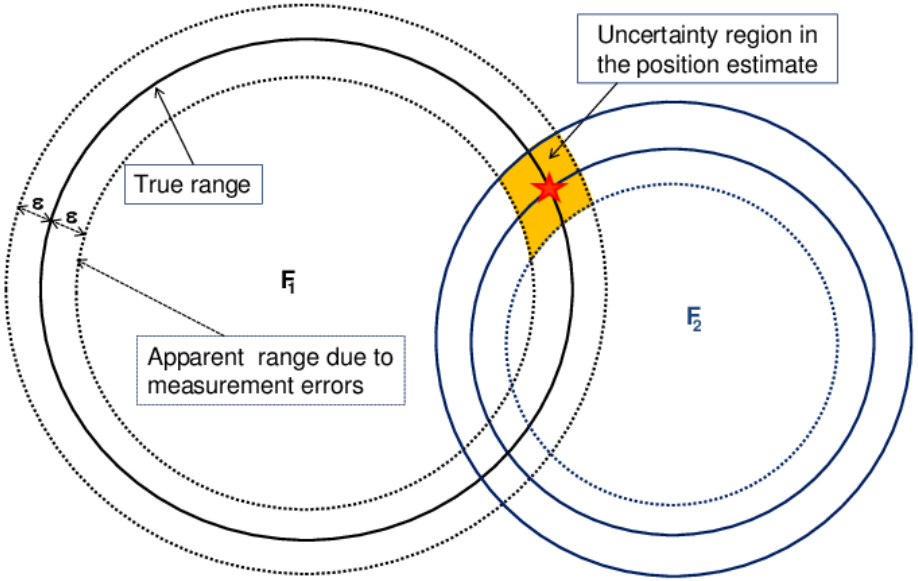
$$\mathbf{P}_{\hat{\mathbf{x}}(n)} = \left[\mathbf{G}^t\,\mathbf{P}_{\mathbf{y}(n)}^{-1}\,\mathbf{G} + \mathbf{P}_{\hat{\mathbf{x}}^-(n)}^{-1}\right]^{-1}$$

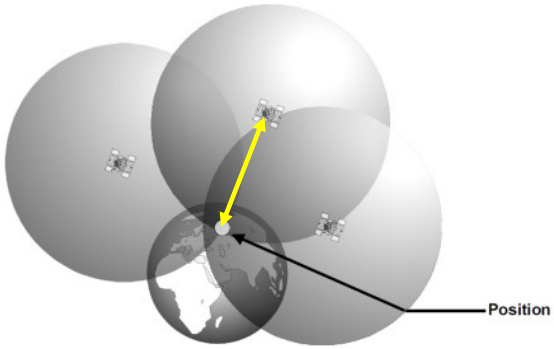
$\hat{\mathbf{x}}(n)$, $\mathbf{P}_{\hat{\mathbf{x}}(n)}$

$\hat{\mathbf{x}}(0)$, $\mathbf{P}_{\hat{\mathbf{x}}(0)}$

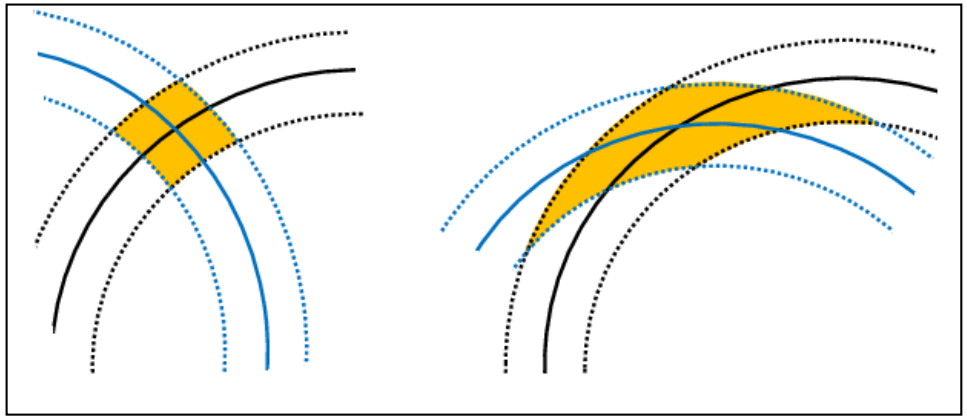
Initialization

Dilution Of Precision (DOP)





Satellite Geometry affects the error



References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

Thank you!