

Lecture 12

Differential positioning with carrier

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Linear model for DGNSS: Double Differences

1. Differential Code and carrier based positioning
2. Precise relative Positioning
3. The Role of Geometric Diversity

Linear model for Differential Positioning

Single difference $(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$

$$P_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \underbrace{v_{pru}^j}_{\text{Noise terms}}$$

$$L_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + \underbrace{v_{Lru}^j}_{\text{Noise terms}}$$

Double difference

$$(\bullet)_{ru}^{jk} \equiv \nabla \Delta(\bullet)_{ru}^{jk} = (\bullet)_{ru}^k - (\bullet)_{ru}^j = (\bullet)_u^k - (\bullet)_r^k - [(\bullet)_u^j - (\bullet)_r^j]$$

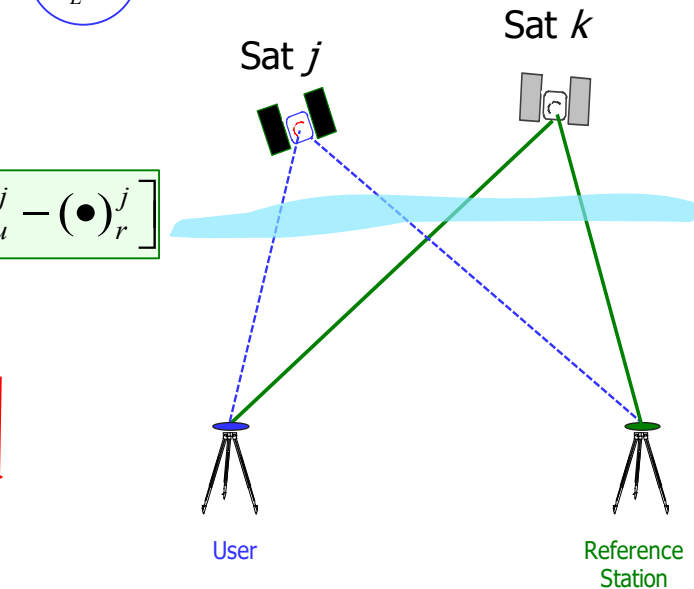
$$P_{ru}^k = \rho_{ru}^k + \cancel{c \delta t_{ru}} + T_{ru}^k + \cancel{I_{ru}^k} + \cancel{K_{ru}} + v_{pru}^k$$

$$P_{ru}^j = \rho_{ru}^j + \cancel{c \delta t_{ru}} + T_{ru}^j + \cancel{I_{ru}^j} + \cancel{K_{ru}} + v_{pru}^j$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + v_{pru}^{jk}$$

The same for carrier:

$$L_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} - I_{ru}^{jk} + \lambda \omega_{ru}^{jk} + \lambda \underbrace{N_{ru}^{jk}}_{\text{Carrier ambiguities}} + v_{Lru}^{jk}$$

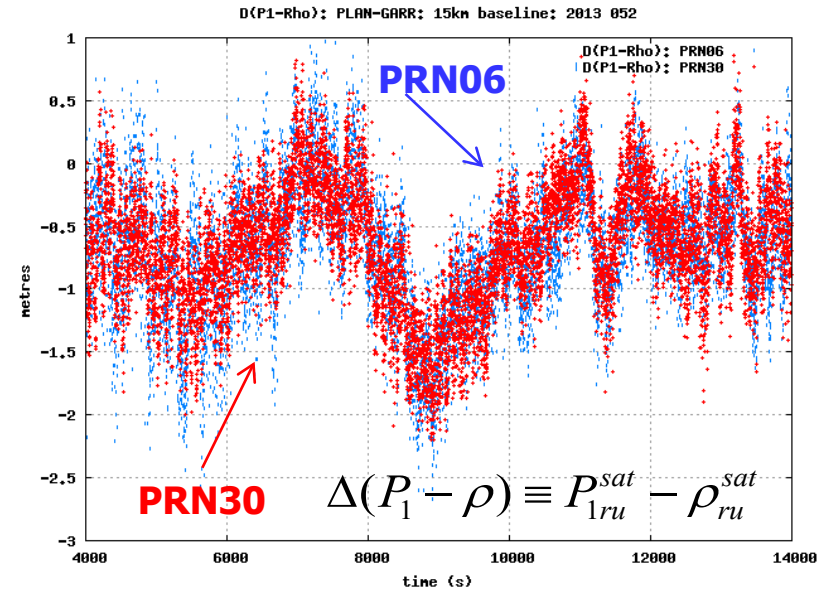
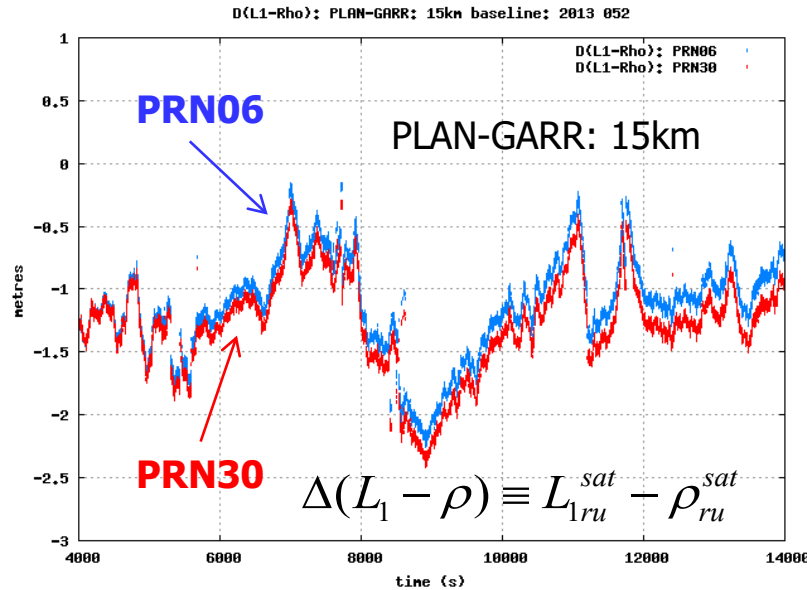


Now are cancelled:

- Receiver clock
- Receiver code instrumental delays
- Receiver carrier instrumental delays

→ Carrier ambiguities N are integer

Single-Difference of measurements (corrected by geometric range!!)



Dif. Wind-up: Very small

$$\Delta(L_1 - \rho) \equiv L_{ru}^j - \rho_{ru}^j = c \delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + v_{Lru}^j$$

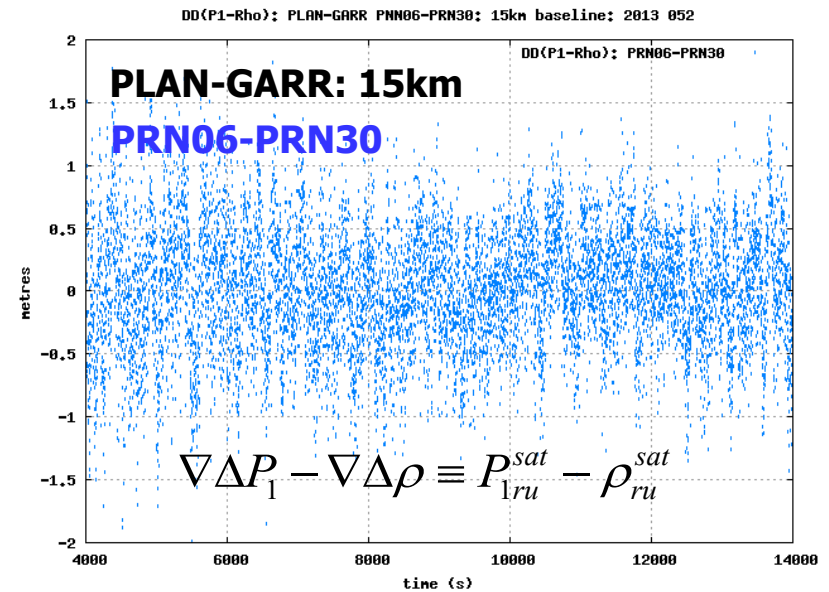
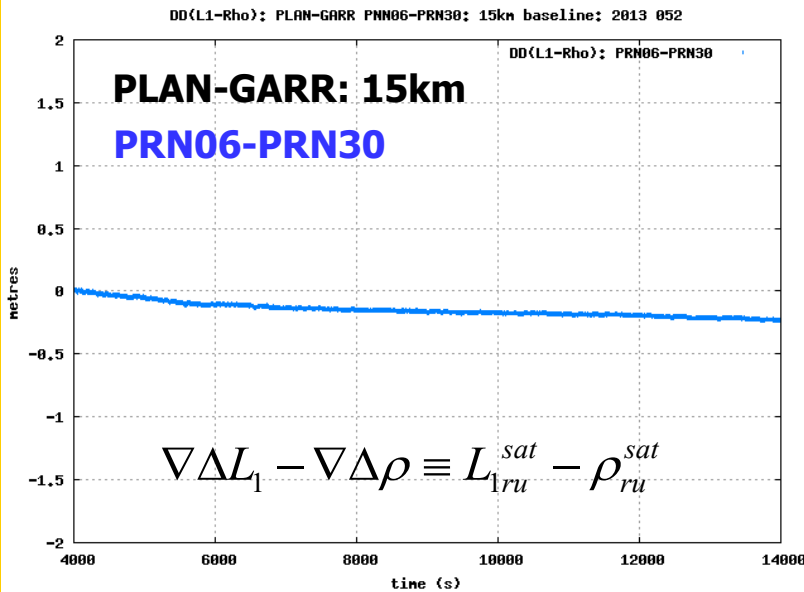
$$\Delta(P_1 - \rho) \equiv P_{ru}^j - \rho_{ru}^j = c \delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + v_{Pru}^j$$

Dif. Receiver clock:
Main variations Common
for all satellites

Dif. Tropo. and Iono. :
Small variations

Dif. Instrumental
delays and carrier
ambiguities:
constant

Double-Difference of measurements (corrected by geometric range!!)



Dif. wind-up: negligible

$$\nabla \Delta (L_1 - \rho) \equiv L_{ru}^{jk} - \rho_{ru}^{jk} = T_{ru}^{jk} - I_{ru}^{jk} + \lambda \omega_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{Lru}^{jk}$$

$$\nabla \Delta (P_1 - \rho) \equiv P_{ru}^{jk} - \rho_{ru}^{jk} = T_{ru}^{jk} + I_{ru}^{jk} + v_{pru}^{jk}$$

**Carrier ambiguities:
constant**

**Dif. Tropo. and Iono. :
Small variations**

Linear model for Differential Positioning

Single difference $(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$

$$P_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + v_{pru}^j$$

$$L_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + v_{Lru}^j$$

where: $\rho_{ru}^j = \rho_{0ru}^j - \hat{\rho}_{0u}^j \cdot \Delta \mathbf{r}_{ru} - \hat{\rho}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{site} + \hat{\rho}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j$

Double difference

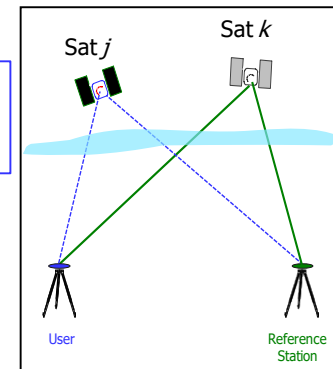
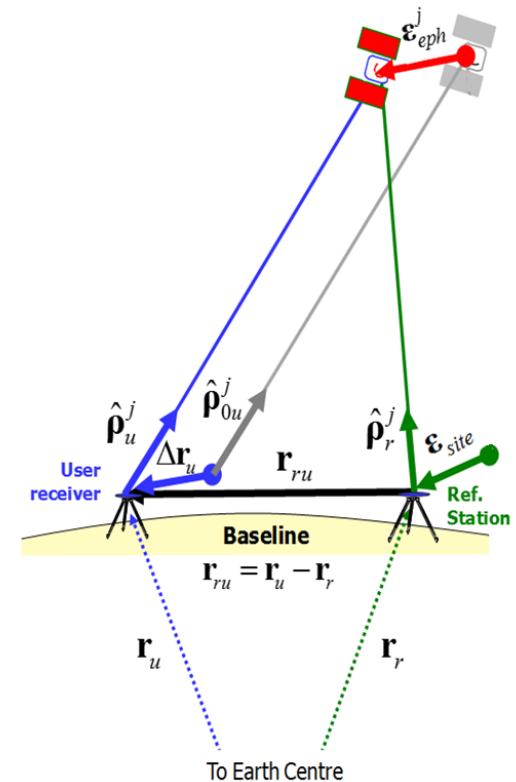
$$(\bullet)_{ru}^{jk} \equiv \nabla \Delta(\bullet)_{ru}^{jk} = (\bullet)_{ru}^k - (\bullet)_{ru}^j = (\bullet)_u^k - (\bullet)_r^k - [(\bullet)_u^j - (\bullet)_r^j]$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + v_{pru}^{jk}$$

$$L_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} - I_{ru}^{jk} + \lambda \omega_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{Lru}^{jk}$$

where: $\rho_{ru}^{jk} = \rho_{0ru}^{jk} - \hat{\rho}_{0u}^{jk} \cdot \Delta \mathbf{r}_{ru} - \hat{\rho}_{0ru}^{jk} \cdot \boldsymbol{\varepsilon}_{site} + \hat{\rho}_{0ru}^k \cdot \boldsymbol{\varepsilon}_{eph}^k - \hat{\rho}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j$

being: $\rho_{0ru}^{jk} \equiv \rho_{0ru}^k - \rho_{0ru}^j$; $\Delta \mathbf{r}_{ru} \equiv \Delta \mathbf{r}_u - \boldsymbol{\varepsilon}_{site}$



Exercise:

Consider the Single Differences of geometric range:

$$\rho_{ru}^j = \rho_{0ru}^j - \hat{\mathbf{p}}_{0u}^j \cdot \Delta \mathbf{r}_{ru} - \hat{\mathbf{p}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{site} + \hat{\mathbf{p}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j$$

where $\rho_{ru}^j = \rho_u^j - \rho_r^j$

Show that the Double Differences are given by:

$$\rho_{ru}^{jk} = \rho_{0ru}^{jk} - \hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_{ru} - \hat{\mathbf{p}}_{0ru}^{jk} \cdot \boldsymbol{\varepsilon}_{site} + \hat{\mathbf{p}}_{0ru}^k \cdot \boldsymbol{\varepsilon}_{eph}^k - \hat{\mathbf{p}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j$$

being: $\rho_{0ru}^{jk} \equiv \rho_{0ru}^k - \rho_{0ru}^j$; $\Delta \mathbf{r}_{ru} \equiv \Delta \mathbf{r}_u - \boldsymbol{\varepsilon}_{site}$

Linear model for Differential Positioning

$$(\bullet)_{ru}^{jk} \equiv \nabla \Delta(\bullet)_{ru}^{jk} = (\bullet)_{ru}^k - (\bullet)_{ru}^j = (\bullet)_u^k - (\bullet)_r^k - [(\bullet)_u^j - (\bullet)_r^j]$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + \cancel{T_{ru}^{jk}} + \cancel{I_{ru}^{jk}} + \cancel{\nu_{Pru}^{jk}}$$

$$L_{ru}^{jk} = \rho_{ru}^{jk} + \cancel{T_{ru}^{jk}} - \cancel{I_{ru}^{jk}} + \cancel{\lambda \omega_{ru}^{jk}} + \lambda N_{ru}^{jk} + \nu_{Lru}^{jk}$$

where: $\rho_{ru}^{jk} = \rho_{0ru}^{jk} - \hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_{ru} - \hat{\mathbf{p}}_{0ru}^{jk} \cdot \boldsymbol{\varepsilon}_{site} + \hat{\mathbf{p}}_{0ru}^k \cdot \boldsymbol{\varepsilon}_{eph}^k - \hat{\mathbf{p}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j$

For short baselines (e.g. up to 10 km) and if the reference station coordinates are accurately known, we can assume:

$$\Delta \mathbf{r}_{ru} \equiv \Delta \mathbf{r}_u - \boldsymbol{\varepsilon}_{site}$$

$$T_{ru}^{jk} \approx 0; I_{ru}^{jk} \approx 0; \omega_{ru}^{jk} \approx 0$$

$$\hat{\mathbf{p}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j \approx 0$$

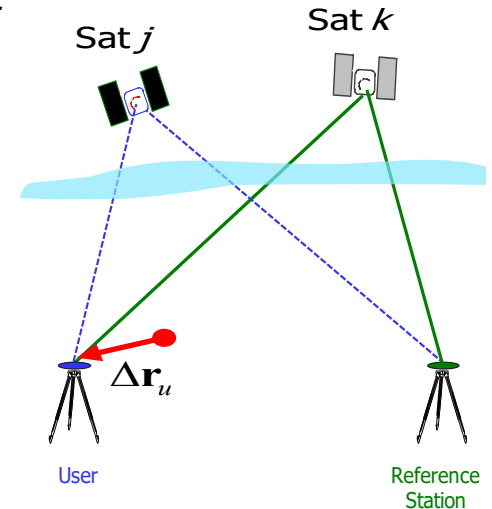
$$\boldsymbol{\varepsilon}_{site} \approx 0 \Rightarrow \Delta \mathbf{r}_{ru} \approx \Delta \mathbf{r}_u$$

Note for baselines up to 10 km
the range error of broadcast
orbits is less than 1 cm
(assuming $\varepsilon_{eph}^j \approx 10$ m).

With these simplifications, we have:

$$P_{ru}^{jk} - \rho_{0ru}^{jk} = -\hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_u + \nu_{Pru}^{jk}$$

$$L_{ru}^{jk} - \rho_{0ru}^{jk} = -\hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_u + \lambda N_{ru}^{jk} + \nu_{Lru}^{jk}$$



Remark: $P_{ru}^{jk} - \rho_{0ru}^{jk} = P_u^{jk} - \rho_{0u}^{jk} - (P_r^{jk} - \rho_{0r}^{jk})$

Differential code and carrier positioning

As with the SD, the left hand side of previous equations can be spitted in two terms: one associated to the reference station and the other to the user. Then, the differential corrections can be computed for code and carrier as:

$$PRC_P^{jk} = \rho_{0r}^{jk} - P_r^{jk} ; \quad PRC_L^{jk} = \rho_{0r}^{jk} - L_r^{jk}$$

- The user applies this differential correction to remove/mitigate common errors:

$$P_u^{jk} - \rho_{0u}^{jk} + PRC_P^{jk} = -\hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_u + \nu_{pru}^{jk}$$

$$L_u^{jk} - \rho_{0u}^{jk} + PRC_L^{jk} = -\hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_u + \lambda N_{ru}^{jk} + \nu_{Lru}^{jk}$$

Where the carrier ambiguities N are integer numbers and must be estimated together with the user solution.

For larger distances, the atmospheric propagation effects (troposphere, ionosphere) must be removed with accurate modelling. Wide area users will require also orbit corrections.

Remark: $P_{ru}^{jk} - \rho_{0ru}^{jk} = P_u^{jk} - \rho_{0u}^{jk} - (P_r^{jk} - \rho_{0r}^{jk})$

Differential code and carrier positioning

The user applies this differential correction to remove/mitigate common errors

$$P_u^{jk} - \rho_{0u}^{jk} + PRC_P^{jk} = -\hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_u + \nu_{Pru}^{jk}$$

$$L_u^{jk} - \rho_{0u}^{jk} + PRC_L^{jk} = -\hat{\mathbf{p}}_{0u}^{jk} \cdot \Delta \mathbf{r}_u + \lambda N_{ru}^{jk} + \nu_{Lru}^{jk}$$

where

$$\hat{\mathbf{p}}_{0u}^{jk} \equiv \hat{\mathbf{p}}_{0u}^k - \hat{\mathbf{p}}_{0u}^j$$

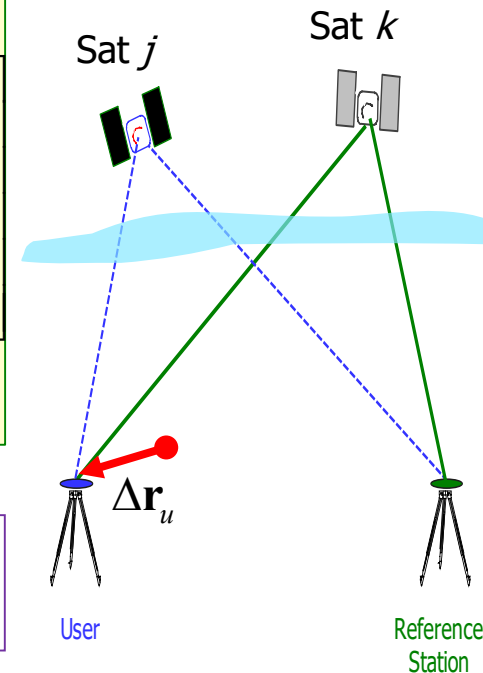
The previous system for navigation equations is written in matrix notation as:

$$\begin{bmatrix} \text{Pref}_P^{R,1} \\ \text{Pref}_L^{R,1} \\ \dots \\ \text{Pref}_P^{R,n-1} \\ \text{Pref}_L^{R,n-1} \end{bmatrix} = \begin{bmatrix} -(\hat{\mathbf{p}}_{0u}^1 - \hat{\mathbf{p}}_{0u}^R)^T & 0 & \dots & 0 \\ -(\hat{\mathbf{p}}_{0u}^1 - \hat{\mathbf{p}}_{0u}^R)^T & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ -(\hat{\mathbf{p}}_{0u}^{n-1} - \hat{\mathbf{p}}_{0u}^R)^T & 0 & \dots & 0 \\ -(\hat{\mathbf{p}}_{0u}^{n-1} - \hat{\mathbf{p}}_{0u}^R)^T & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{r}_u \\ \lambda N_{ru}^{R,1} \\ \dots \\ \lambda N_{ru}^{R,n-1} \end{bmatrix}$$

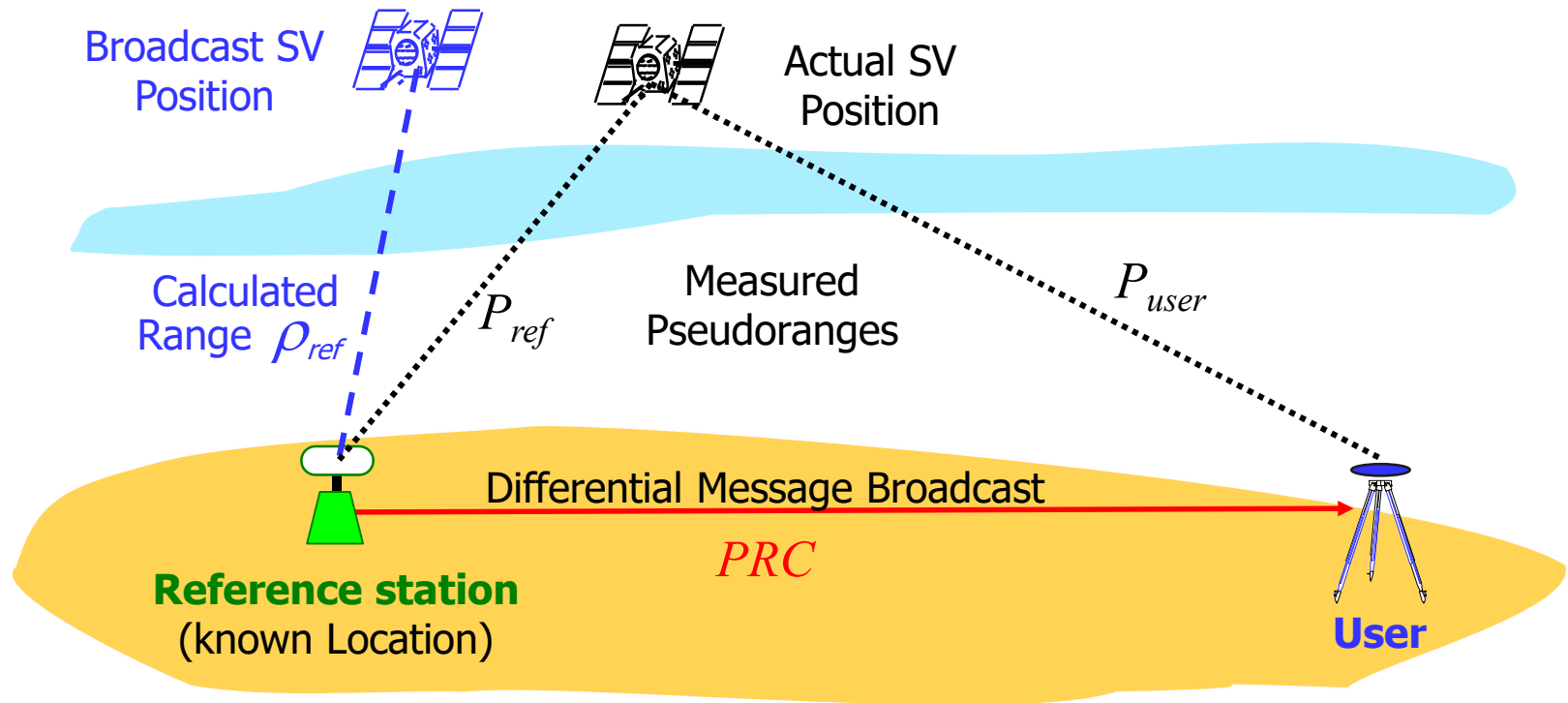
where

$$\begin{aligned} \text{Pref}_P^{R,k} &\equiv P_u^{R,k} - \rho_{0u}^{R,k} + PRC_P^{R,k} \\ \text{Pref}_L^{R,k} &\equiv L_u^{R,k} - \rho_{0u}^{R,k} + PRC_L^{R,k} \end{aligned}$$

Carrier ambiguities



Differential positioning



- The **reference station** with known coordinates, computes differential corrections: $PRC_P^{jk} = \rho_{0r}^{jk} - P_r^{jk}$; $PRC_L^{jk} = \rho_{0r}^{jk} - L_r^{jk}$
- The **user** receiver applies these corrections to its own measurements to remove SIS errors and improve the positioning accuracy.

Correlations among the DD Measurements

We assume uncorrelated measurements (both code and carrier). Then, the covariance matrix is diagonal:

$$\mathbf{P}_P = \sigma_P^2 \mathbf{I} \quad \mathbf{P}_L = \sigma_L^2 \mathbf{I} \quad \text{where, we can assume: } \sigma_P \approx 50\text{cm}; \quad \sigma_L \approx 5\text{mm}$$

Let X represent the code P or the Carrier L measurement.

- The **single difference (SD)** and its covariance matrix can be computed as:

$$\begin{bmatrix} X_{ru}^k \\ X_{ru}^j \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} X_u^k \\ X_r^k \\ X_u^j \\ X_r^j \end{bmatrix};$$

Thence, if the measurements are uncorrelated, so are they in single differences, but the noise is twice!

$$\mathbf{P}_{X_{SD}} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_X^2 & 0 & 0 & 0 \\ 0 & \sigma_X^2 & 0 & 0 \\ 0 & 0 & \sigma_X^2 & 0 \\ 0 & 0 & 0 & \sigma_X^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} = 2\sigma_X^2 \mathbf{I}$$



Correlations among the DD Measurements

- Now, the **double difference (DD)** and its covariance matrix can be computed as:

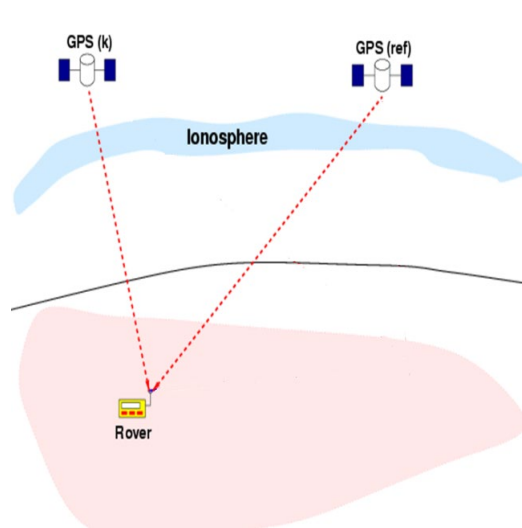
$$\begin{bmatrix} X_{ru}^{jk} \\ X_{ru}^{jl} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} X_{ru}^k \\ X_{ru}^j \\ X_{ru}^l \end{bmatrix} ;$$

$$\mathbf{P}_{X_{DD}} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2\sigma_X^2 & 0 & 0 \\ 0 & 2\sigma_X^2 & 0 \\ 0 & 0 & 2\sigma_X^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \end{bmatrix} = 2\sigma_X^2 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Thence, even if the original measurements are uncorrelated, the double differences are correlated.

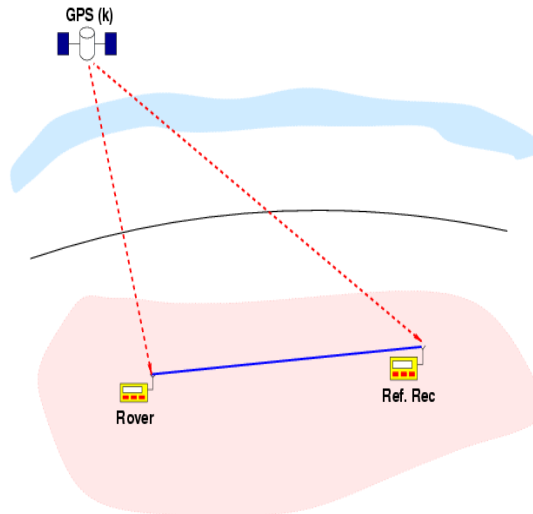
Note: The removal of the relative “User”—“Reference-station” common bias (e.g. relative receiver clock) in DD is **done at the expense of one observation and the introduction of a correlation between measurements**.

Single and double differences of receivers/satellites



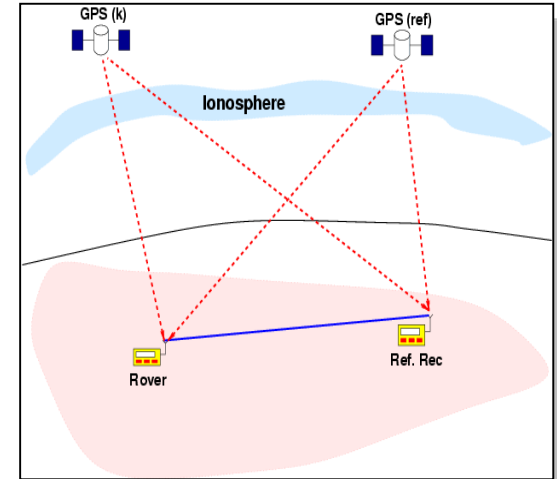
$$\nabla \square \cdot \equiv \square \cdot^k - \square \cdot^R$$

Receiver errors affecting both satellites are removed (e.g. Receiver clock)



$$\Delta \square \cdot \equiv \square \cdot_{rov} - \square \cdot_{ref}$$

SIS errors affecting both receivers are removed (e.g. Satellite clocks,...)

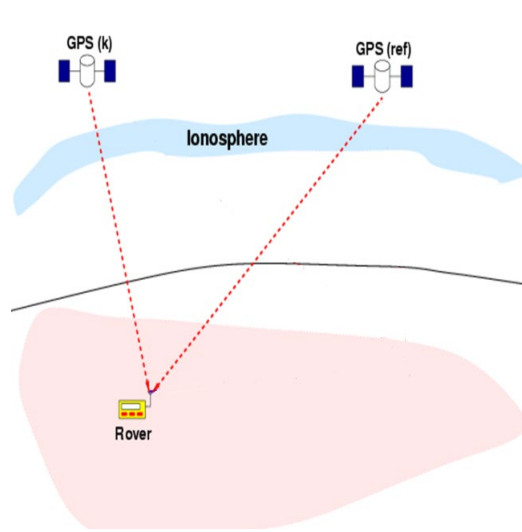


$$\begin{aligned} \Delta \nabla \square &\equiv \Delta \square \cdot^k - \Delta \square \cdot^R = \\ &= \nabla \square \cdot_{rov} - \nabla \square \cdot_{ref} \end{aligned}$$

Receiver errors common for all satellites do not affect positioning (as they are assimilated in the receiver clock estimate). Thence:

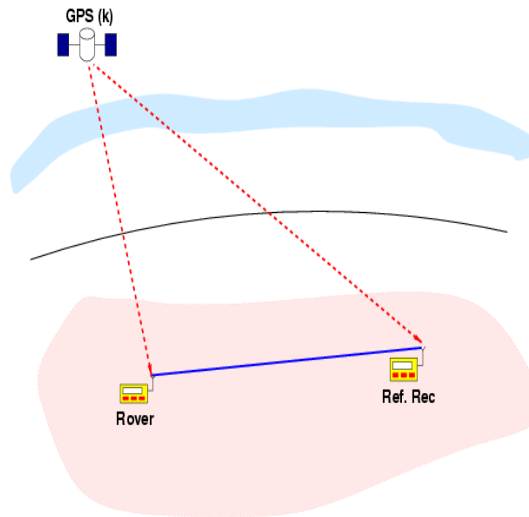
- Only residual errors in single differences between sat. affect absolute posit.
- Only residual errors in double differences between sat. and receivers affect relative positioning.

Single and double differences of receivers/satellites



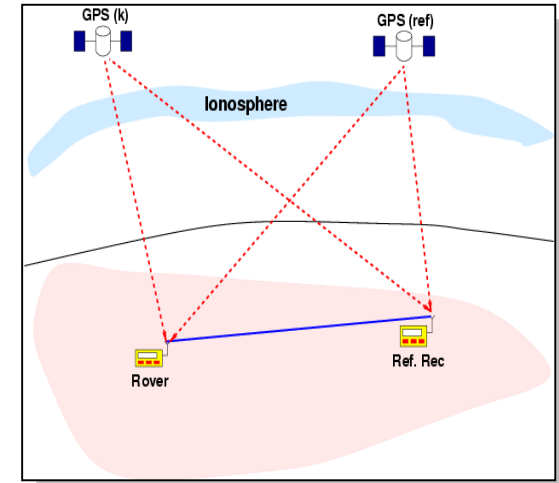
$$\nabla \square \cdot \equiv \square \cdot^k - \square \cdot^R$$

Receiver errors affecting both satellites are removed (e.g. Receiver clock)



$$\Delta \square \cdot \equiv \square \cdot_{rov} - \square \cdot_{ref}$$

SIS errors affecting both receivers are removed (e.g. Satellite clocks,...)



$$\begin{aligned} \Delta \nabla \square &\equiv \Delta \square \cdot^k - \Delta \square \cdot^R = \\ &= \nabla \square \cdot_{rov} - \nabla \square \cdot_{ref} \end{aligned}$$

When comparing SD and DD one might suggest that in the DD formulation there is even further error reduction, positively influencing the results in positioning. This is however not true, since in the SD case the mean value of unmodelled effects is absorbed by the receiver clock. If the DD correlations are taken into account, the positioning results in both cases are the same. **However the DD formulation has the advantage that it allows the direct estimation of the ambiguities.**

Contents

Linear model for DGNSS: Double Differences

1. Differential Code and carrier based positioning
2. Precise relative Positioning
3. The Role of Geometric Diversity

Relative Positioning

The following relationship can be obtained from the figure, where we assume that the baseline is shorter than the distance to the satellite by orders of magnitude:

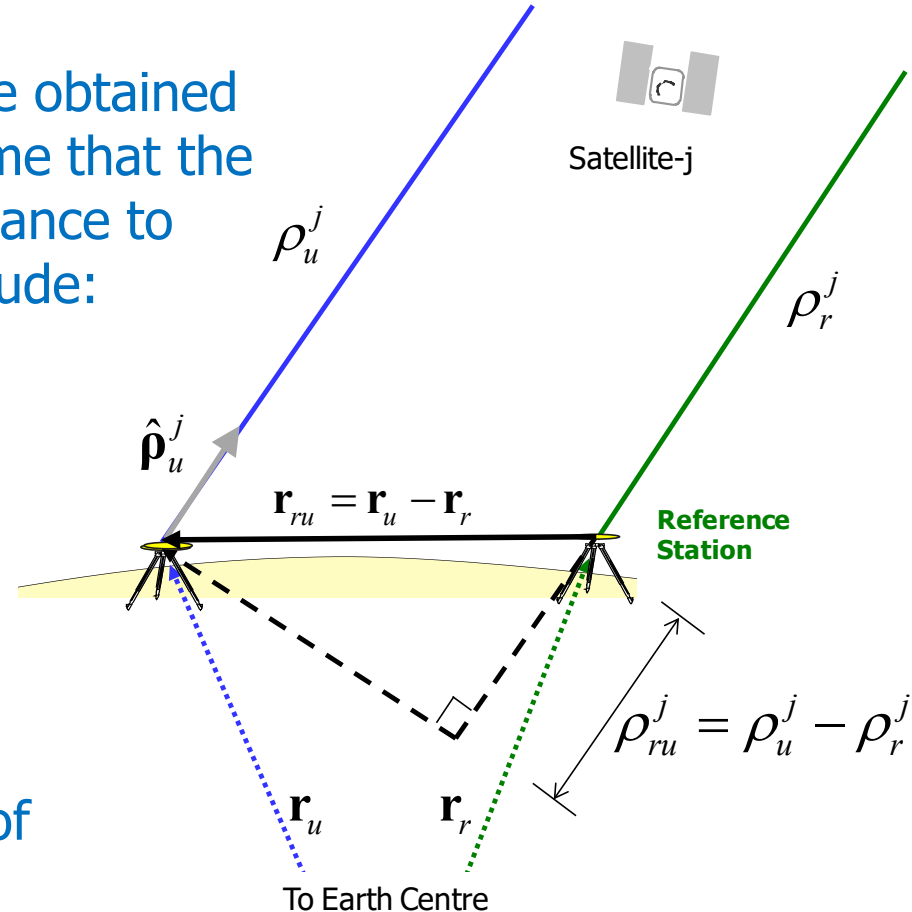
$$\rho_{ru}^j = \rho_u^j - \rho_r^j = -\hat{\mathbf{p}}_u^j \cdot \mathbf{r}_{ru}$$

Applying the same scheme to a second satellite "k"

$$\rho_{ru}^k = \rho_u^k - \rho_r^k = -\hat{\mathbf{p}}_u^k \cdot \mathbf{r}_{ru}$$

Thence, the double differences of ranges are:

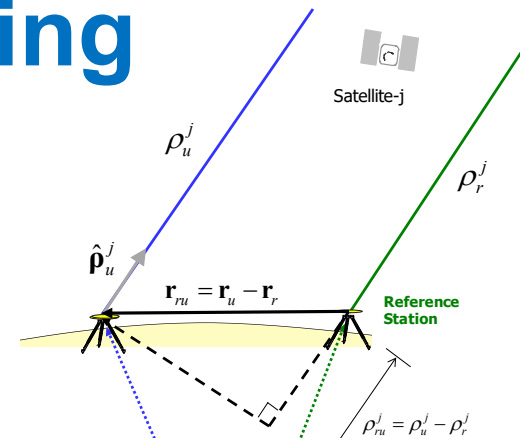
$$\rho_{ru}^{jk} = \rho_{ru}^k - \rho_{ru}^j = -(\hat{\mathbf{p}}_u^k - \hat{\mathbf{p}}_u^j) \cdot \mathbf{r}_{ru} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru}$$



Relative Positioning

Thence, the double differences of ranges are:

$$\rho_{ru}^{jk} = \rho_{ru}^k - \rho_{ru}^j = -(\hat{\mathbf{p}}_u^k - \hat{\mathbf{p}}_u^j) \cdot \mathbf{r}_{ru} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru}$$



As commented before, for short baselines (e.g. less than 10km), we can assume that ephemeris and propagation errors cancel, thence:

$$\begin{aligned} T_{ru}^{jk} &\approx 0; I_{ru}^{jk} \approx 0 \\ \omega_{ru}^{jk} &\approx 0 \end{aligned}$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + v_{pru}^{jk}$$

$$L_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} - I_{ru}^{jk} + \lambda \omega_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{Lru}^{jk}$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + v_{pru}^{jk}$$

$$L_{ru}^{jk} = \rho_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{Lru}^{jk}$$

$$P_{ru}^{jk} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru} + v_{pru}^{jk}$$

$$L_{ru}^{jk} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru} + \lambda N_{ru}^{jk} + v_{Lru}^{jk}$$

Note that these equations allows a direct estimation of the baseline, **without needing an accurate knowledge** of the reference station coordinates.

Relative Positioning

$$P_{ru}^{jk} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru} + \nu_{p\,ru}^{jk}$$

$$L_{ru}^{jk} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru} + \lambda N_{ru}^{jk} + \nu_{L\,ru}^{jk}$$

where

$$\hat{\mathbf{p}}_u^{jk} \equiv \hat{\mathbf{p}}_u^k - \hat{\mathbf{p}}_u^j$$

The previous system for navigation equations is written in matrix notation as:

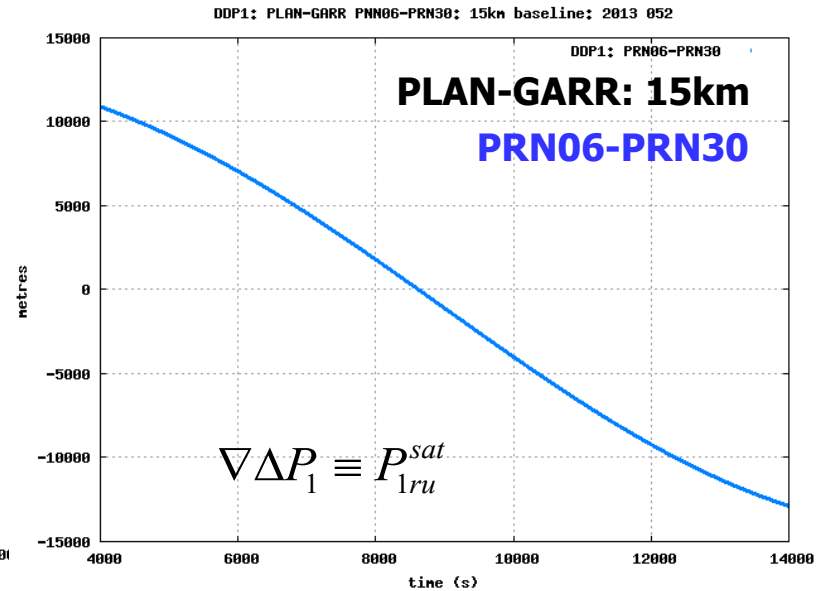
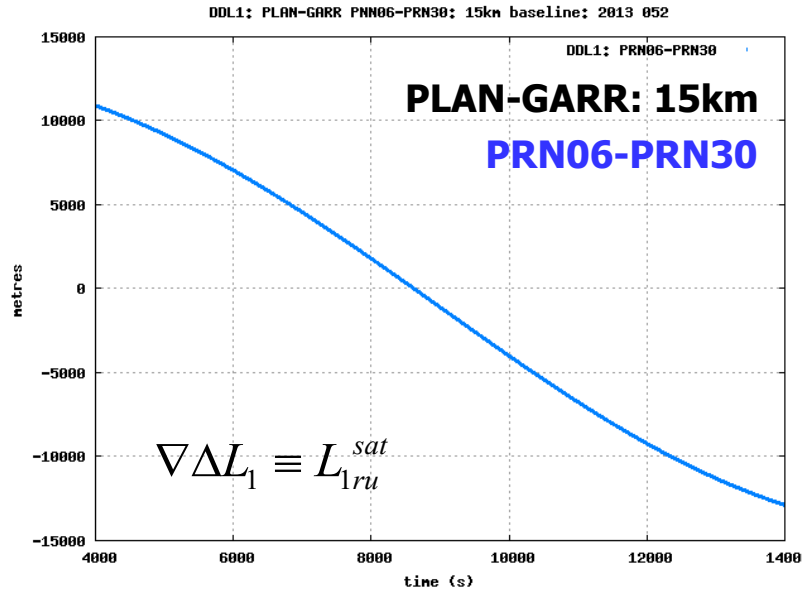
$$\begin{bmatrix} P_{r,u}^{R,1} \\ L_{r,u}^{R,1} \\ \vdots \\ P_{r,u}^{R,n-1} \\ L_{r,u}^{R,n-1} \end{bmatrix} = \begin{bmatrix} -(\hat{\mathbf{p}}_u^1 - \hat{\mathbf{p}}_u^R)^T & 0 & \dots & 0 \\ -(\hat{\mathbf{p}}_u^1 - \hat{\mathbf{p}}_u^R)^T & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -(\hat{\mathbf{p}}_u^{n-1} - \hat{\mathbf{p}}_u^R)^T & 0 & \dots & 0 \\ -(\hat{\mathbf{p}}_u^{n-1} - \hat{\mathbf{p}}_u^R)^T & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_{ru} \\ \lambda N_{ru}^{R,1} \\ \vdots \\ \lambda N_{ru}^{R,n-1} \end{bmatrix}$$

Baseline vector

DD Code and Carrier measurements

Carrier ambiguities

Double-Difference of measurements

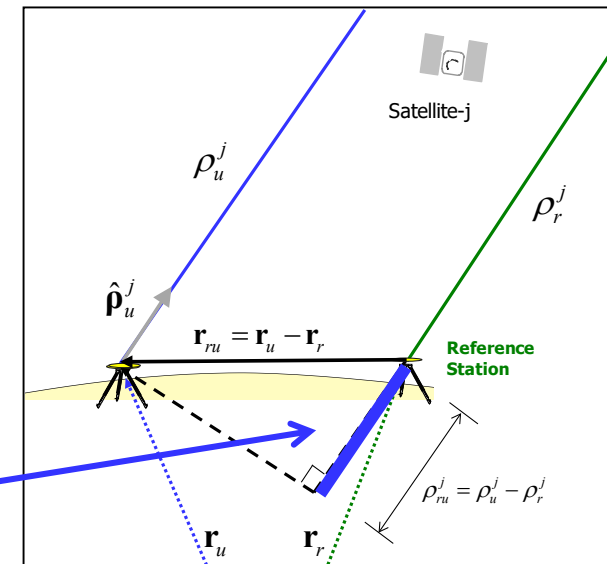


$$\nabla \Delta L_1 \equiv L_{ru}^{jk} = -\rho_{ru}^{jk} + T_{ru}^{jk} - I_{ru}^{jk} + \lambda \omega_{ru}^{jk} + \lambda N_{ru}^{jk} + v_{L_{ru}}^{jk}$$

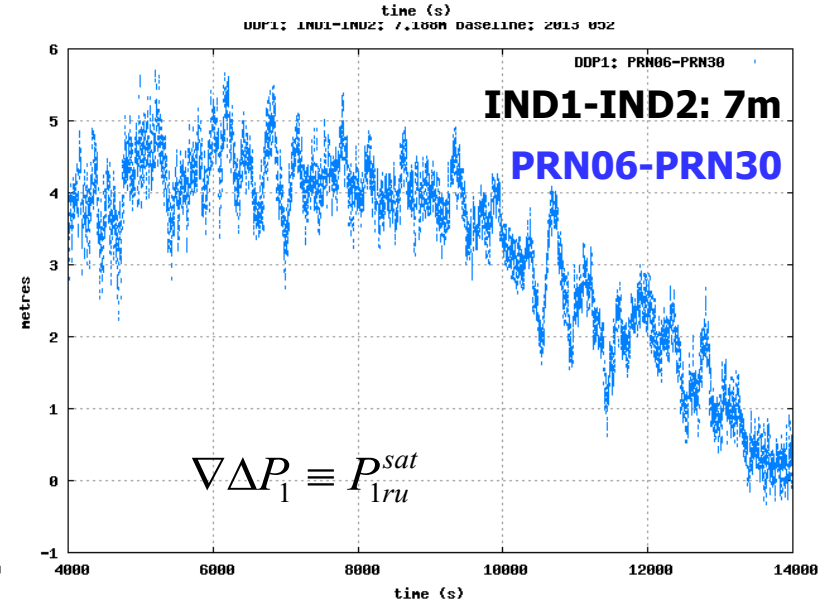
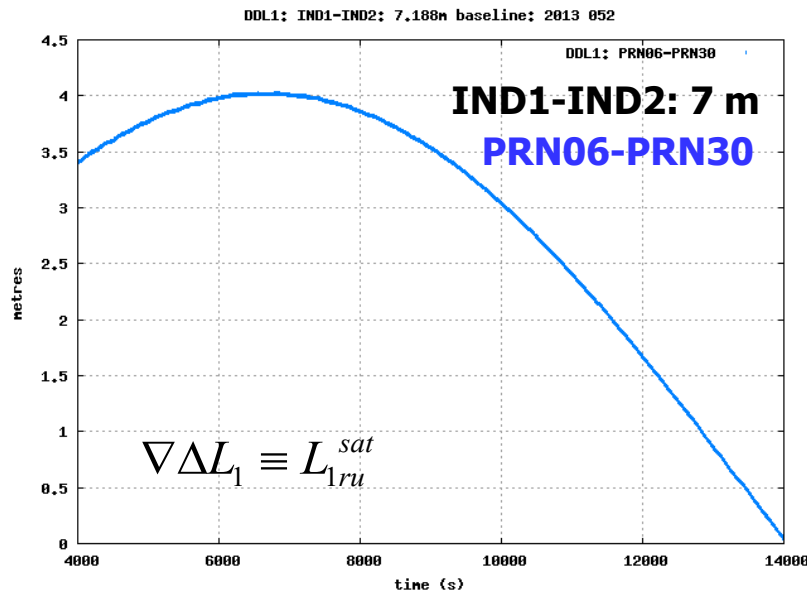
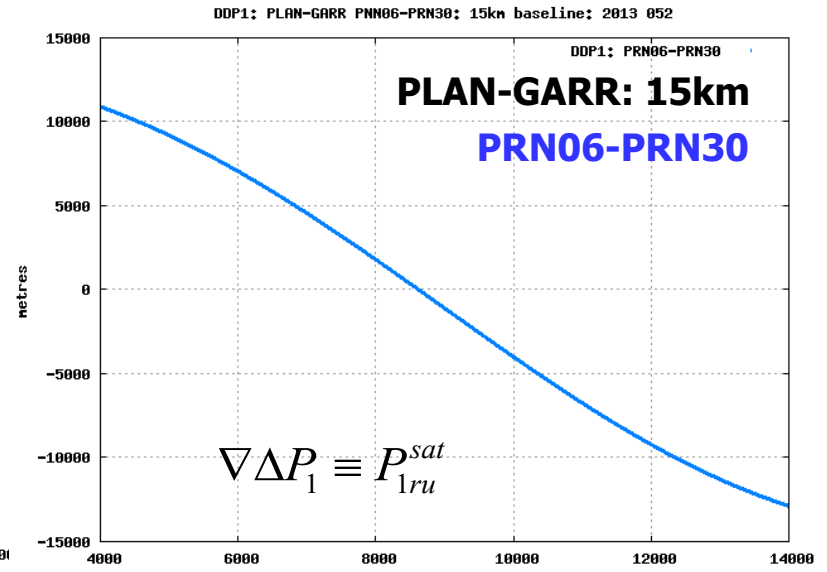
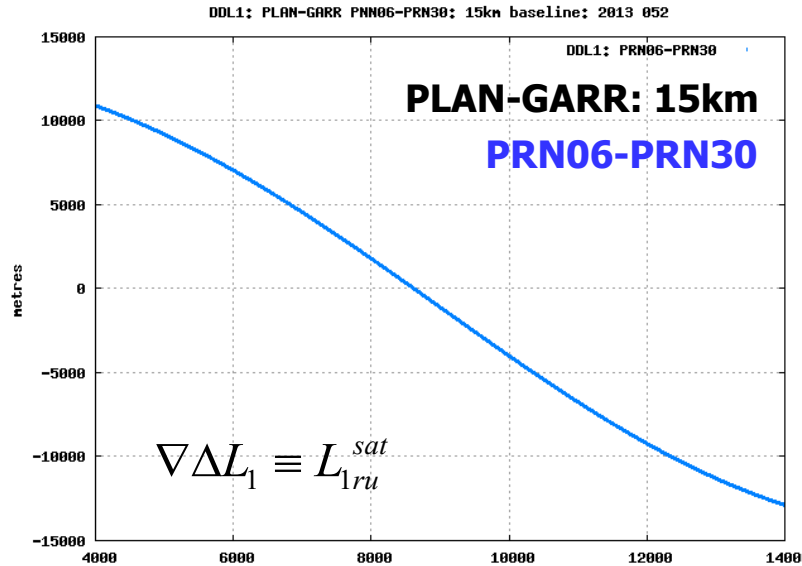
$$\nabla \Delta P_1 \equiv P_{ru}^{jk} = -\rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + v_{P_{ru}}^{jk}$$

Variation of the Baseline
projection over the unit
Line-Of-Sight vector

$$\rho_{ru}^{jk} = -\hat{\mathbf{p}}_u^{jk} \cdot \mathbf{r}_{ru}$$



Double-Difference of measurements

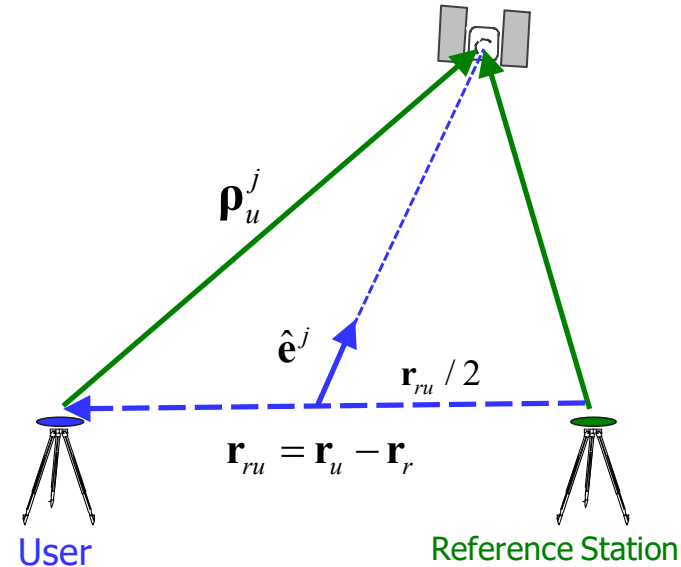


Relative Positioning

Exercise:

Demonstrate the following relationship between the baseline and the differential range [*]:

$$\rho_{ru}^j = \rho_u^j - \rho_r^j = - \left(\frac{2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}}{\|\boldsymbol{\rho}_u^j\| + \|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|} \right) \cdot \mathbf{r}_{ru}$$



Comments:

The previous expression can be written as:

$$\rho_u^j - \rho_r^j = - \left(\omega^j \hat{\mathbf{e}}^j \right) \cdot \mathbf{r}_{ru} \quad \text{with} \quad \omega^j \equiv \frac{\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}{\|\boldsymbol{\rho}_u^j\| + \|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|} \quad \hat{\mathbf{e}}^j = \frac{\boldsymbol{\rho}_u^j + \mathbf{r}_{ru} / 2}{\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru} / 2\|}$$

- Taking $\mathbf{r}_{ru} = 0$ in ω^j and $\hat{\mathbf{e}}^j$ leads to the approximate expression previously found.
- ω^j and $\hat{\mathbf{e}}^j$ depend on the baseline \mathbf{r}_{ru} , which is the vector to estimate. Nevertheless, it is not very sensitive to changes in such baseline and can be computed iteratively, computing the navigation solution starting from $\mathbf{r}_{ru} = 0$.

^ [*] This result is from [RD-7]

Relative Positioning

Solution

Consider the following relations:

$$\mathbf{r}_{ru} = \boldsymbol{\rho}_r^j - \boldsymbol{\rho}_u^j$$

$$\hat{\mathbf{e}}^j = \frac{\boldsymbol{\rho}_u^j + \mathbf{r}_{ru} / 2}{\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru} / 2\|} = \frac{\boldsymbol{\rho}_r^j + \boldsymbol{\rho}_u^j}{\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}$$

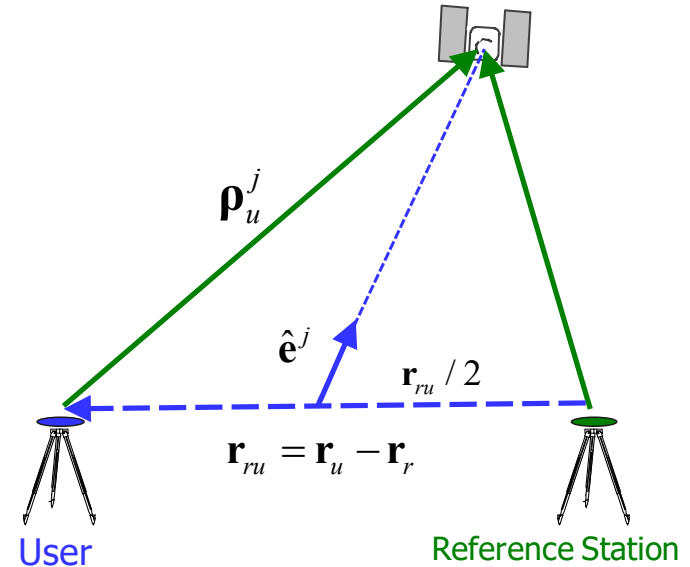
$$\begin{aligned} (\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\| \hat{\mathbf{e}}^j) \cdot \mathbf{r}_{ru} &= \|\boldsymbol{\rho}_r^j\|^2 - \|\boldsymbol{\rho}_u^j\|^2 = \\ &= (\|\boldsymbol{\rho}_r^j\| - \|\boldsymbol{\rho}_u^j\|)(\|\boldsymbol{\rho}_r^j\| + \|\boldsymbol{\rho}_u^j\|) \\ &= (\rho_r^j - \rho_u^j)(\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\| + \|\boldsymbol{\rho}_u^j\|) \end{aligned}$$

Then:

$$\rho_u^j - \rho_r^j = -(\omega^j \hat{\mathbf{e}}^j) \cdot \mathbf{r}_{ru}$$

with: $\omega^j \equiv \frac{\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}{\|\boldsymbol{\rho}_u^j\| + \|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}$

$$\omega^j \hat{\mathbf{e}}^j = \frac{2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}}{\|\boldsymbol{\rho}_u^j\| + \|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}$$



Relative Positioning

In this approach, the reference station broadcast its time-tagged code and carrier measurements, instead of the computed differential corrections.

Thence, the user can form the double differences of its own measurements with those of the reference receiver, satellite by satellite, and estimate its position relative to the reference receiver.

Notice that, the baseline can be estimated without needing an accurate knowledge of reference the station coordinates. Of course, the knowledge of the reference station coordinates would allow the user to compute its absolute coordinates.

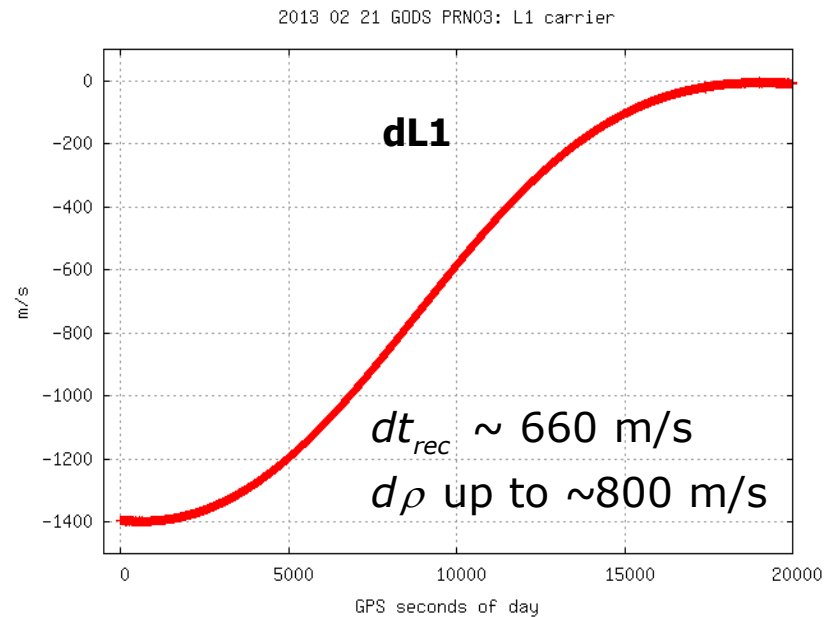
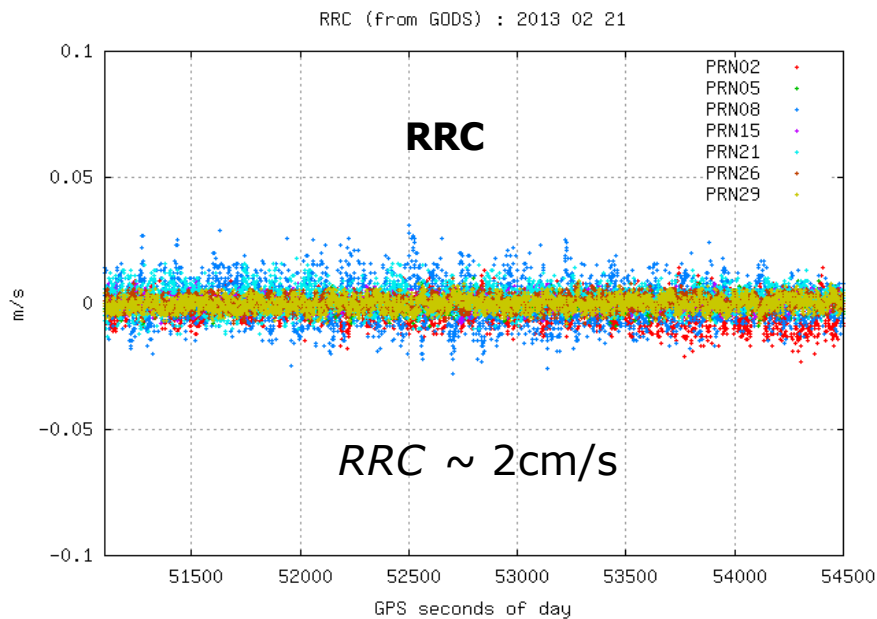
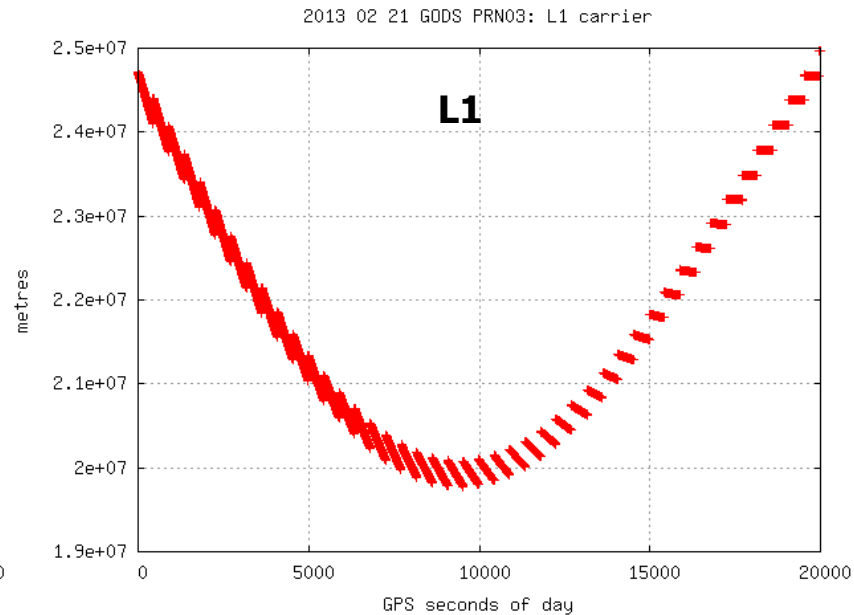
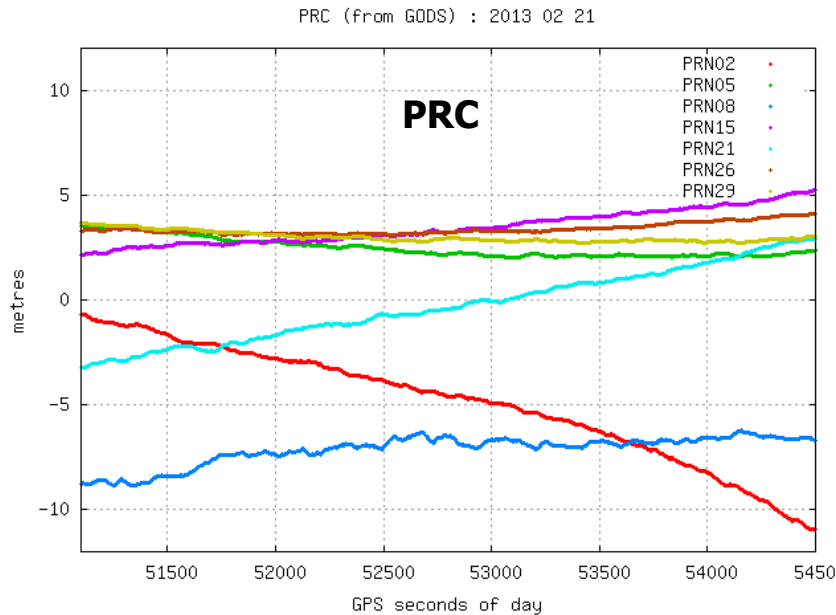


Relative Positioning

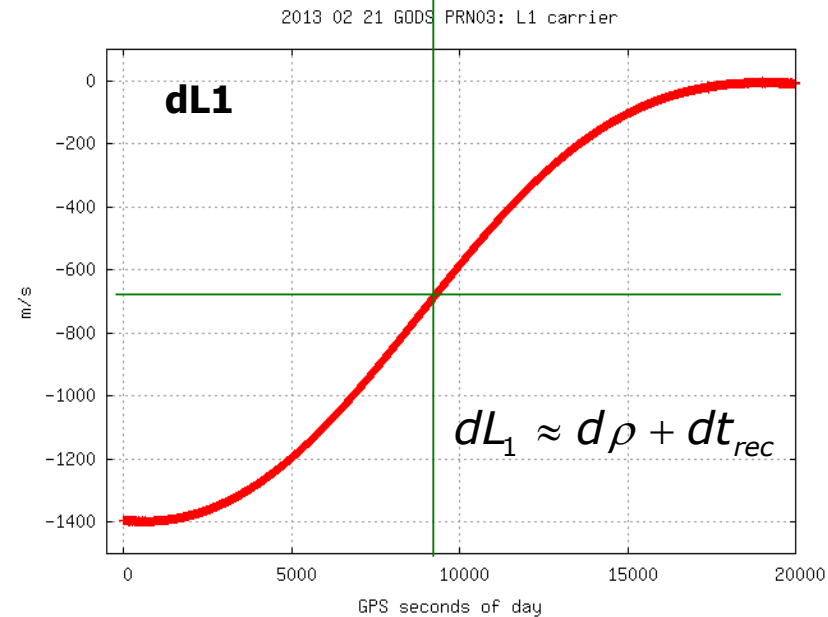
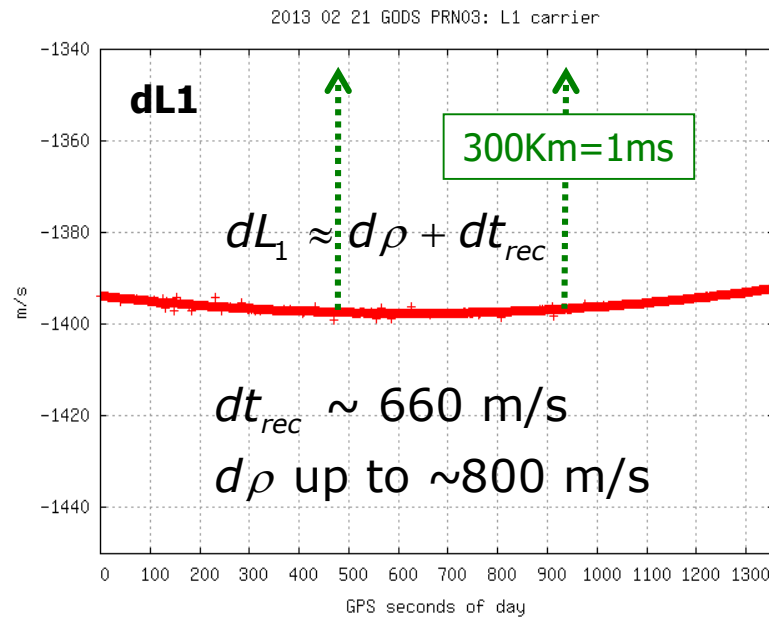
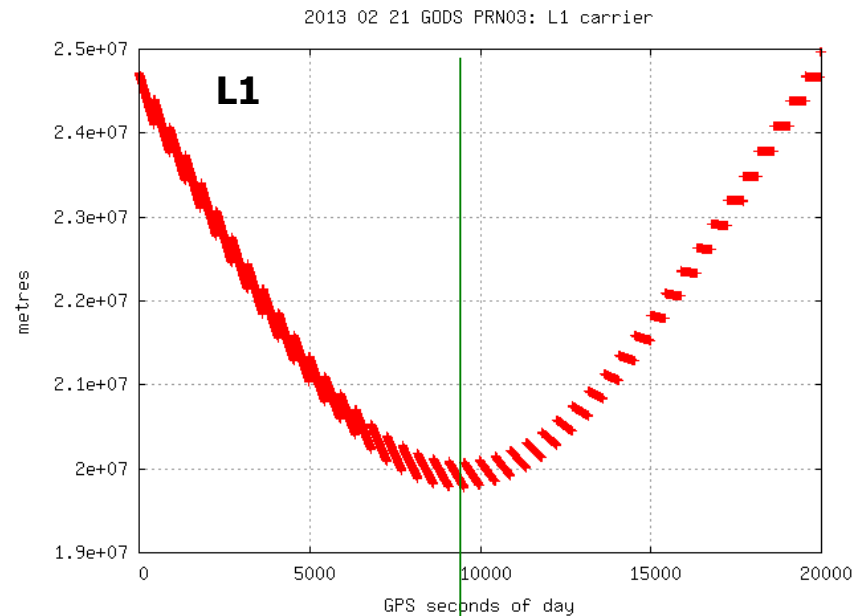
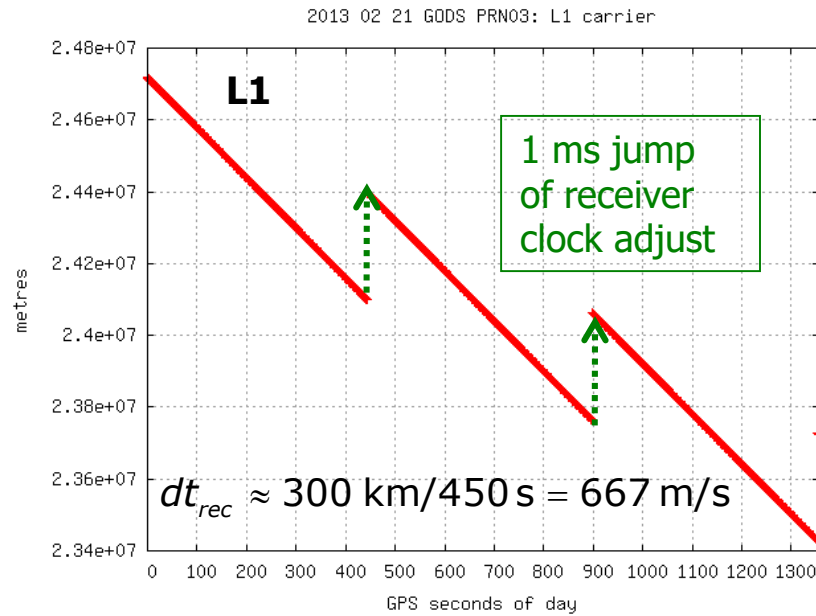
Time synchronization issues:

There is an important and subtitle difference between the previous approach of relative positioning (which does not need to know the reference station coordinates) and the differential positioning approach based on the knowledge of the reference station coordinates.

- The differential corrections vary slowly, and its useful life can be up to several minutes with $S/A=\text{off}$.
- But, the measurements change much faster. The range rate can be up to 800m/s and, thence, a synchronization error of 1millisecond can lead up to more than 1/2 meter of error.
- As commented before, real-time implementation entails also latencies, that can be up to 2 seconds, thence, a extrapolation technique must be applied to the measurements to reduce error due to latency and epoch mismatch (to $<1\text{cm}$ if ambiguities are intended to be fixed).



Receiver: JAVAD TRE_G3TH DELTA3.3.12

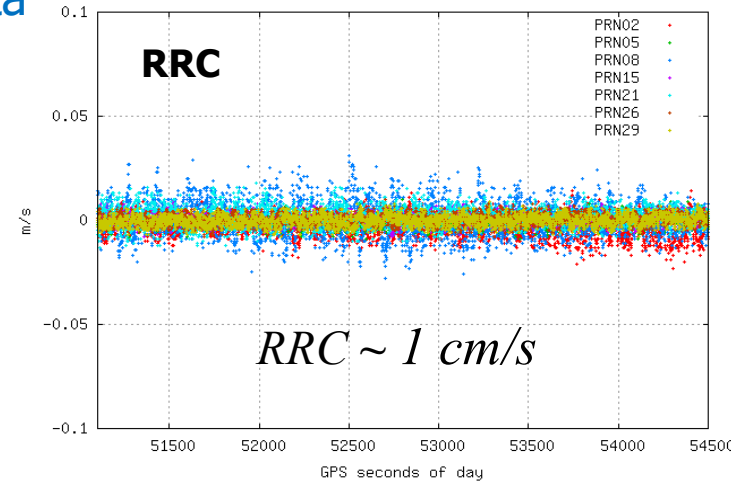


COMMENTS

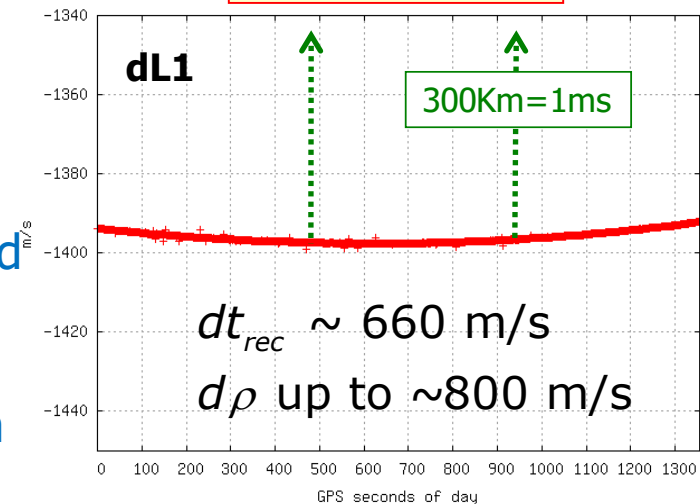
Real-Time implementation entails delays in data transmission, which can reach up to 1 or 2 s.

- Differential corrections vary slowly and its useful life is of several minutes (S/A=off)
- But, the measurements change much faster:
 - The range rate $d\rho/dt$ can be up to 800m/s and, therefore, the range can change by more than half a meter in 1 millisecond. Moreover the receiver clock offset can be up to 1 millisecond (depending on the receiver configuration).
- Thence, the reference station measurements must be :
 - **Synchronized** to reduce station clock mismatch: station clock can be estimated to within $1\mu s \rightarrow \varepsilon_{dt_{sta}} < 1mm$
 - **Extrapolated** to reduce error due to latency: carrier can be extrapolated with error $< 1cm$.

$$RRC = \Delta PRC / \Delta t$$



$$dL_1 \approx d\rho + dt_{rec}$$



Contents

Linear model for DGNSS: Double Differences

1. Differential Code and carrier based positioning
2. Precise relative Positioning
3. The Role of Geometric Diversity

The Role of Geometric Diversity: **Triple differences**

Let us consider again the problem of relative positioning for short baselines. We have previously found the following equation for DD carrier measurements, assuming short baselines (e.g. < 10km)

This is from [RD-3]

$$L_{ru}^{jk} = -\left(\hat{\mathbf{p}}_u^k - \hat{\mathbf{p}}_u^j\right) \cdot \mathbf{r}_{ru} + \lambda N_{ru}^{jk} + \nu_{L ru}^{jk}$$

As the ambiguities are constant along continuous carrier phase arcs, an option could be to take differences on time. Thence, **if the user and reference receiver are stationary** we can write the **"triple differences"** as:

$$\delta L_{ru}^{jk} = -\left(\delta \hat{\mathbf{p}}_u^k - \delta \hat{\mathbf{p}}_u^j\right) \cdot \mathbf{r}_{ru} + \delta \nu_{L ru}^{jk} \Rightarrow \begin{bmatrix} \delta L_{ru}^{12} \\ \delta L_{ru}^{13} \\ \dots \\ \delta L_{ru}^{1K} \end{bmatrix} = \begin{bmatrix} -\left(\delta \hat{\mathbf{p}}_u^2 - \delta \hat{\mathbf{p}}_u^1\right)^T \\ -\left(\delta \hat{\mathbf{p}}_u^3 - \delta \hat{\mathbf{p}}_u^1\right)^T \\ \dots \\ -\left(\delta \hat{\mathbf{p}}_u^K - \delta \hat{\mathbf{p}}_u^1\right)^T \end{bmatrix} \mathbf{r}_{ru} + \tilde{\mathbf{v}}$$

where:

$$\delta L_{ru}^{jk} \equiv L_{ru}^{jk}(t_2) - L_{ru}^{jk}(t_1)$$

For simplicity, we assign ($j=1$) to the reference satellite

The Role of Geometric Diversity: **Triple differences**

This is from [RD-3]

$$\delta L_{ru}^{jk} = -(\delta \hat{\mathbf{p}}_u^k - \delta \hat{\mathbf{p}}_u^j) \cdot \mathbf{r}_{ru} + \delta \nu_{Lru}^{jk} \Rightarrow \begin{bmatrix} \delta L_{ru}^{12} \\ \delta L_{ru}^{13} \\ \dots \\ \delta L_{ru}^{1K} \end{bmatrix} = \begin{bmatrix} -(\delta \hat{\mathbf{p}}_u^2 - \delta \hat{\mathbf{p}}_u^1)^T \\ -(\delta \hat{\mathbf{p}}_u^3 - \delta \hat{\mathbf{p}}_u^1)^T \\ \dots \\ -(\delta \hat{\mathbf{p}}_u^K - \delta \hat{\mathbf{p}}_u^1)^T \end{bmatrix} \mathbf{r}_{ru} + \tilde{\mathbf{v}}$$

Now, we have a “clean” equations system involving only the baseline vector to estimate. But the geometry is very weak (the associated DOP will be large number) and the position estimates will be in general worse than those from double differences.

Estimation of position and change in position: the role of Geometric Diversity

This is
from [RD-3]

Let us now consider a simple model for the estimation of the relative position vector from SD carrier measurements, assuming short baselines (e.g. <10km):

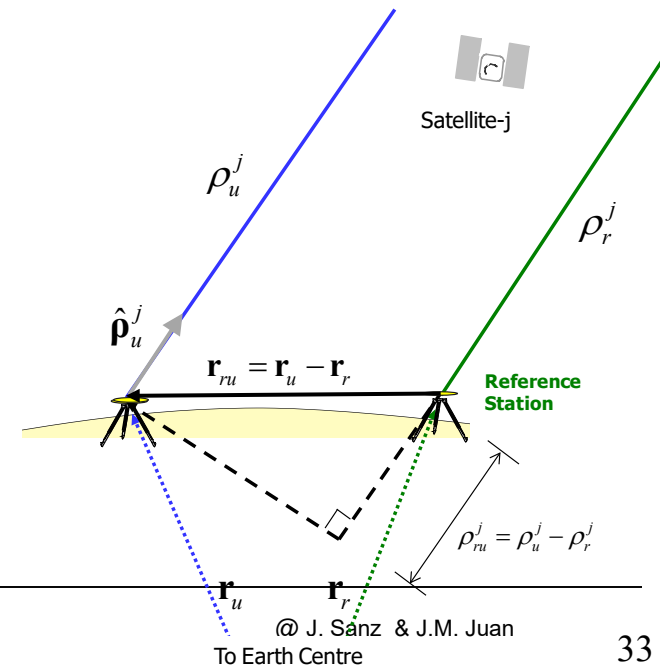
$$L_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + \lambda N_{ru}^j + b_{ru} + v_{L_{ru}}^j$$

$$\rho_{ru}^j = \rho_u^j - \rho_r^j = -\hat{\mathbf{p}}_u^j \cdot \mathbf{r}_{ru}$$

$$L_{ru}^j = -\hat{\mathbf{p}}_u^j \cdot \mathbf{r}_{ru} + d_{ru} + \lambda N_{ru}^j + v_{L_{ru}}^j$$

where $d_{ru} \equiv c \delta t_{ru} + b_{ru}$

$$\begin{bmatrix} L_{ru}^1 \\ L_{ru}^2 \\ \dots \\ L_{ru}^K \end{bmatrix} = \begin{bmatrix} (-\hat{\mathbf{p}}_u^1)^T & 1 \\ (-\hat{\mathbf{p}}_u^2)^T & 1 \\ \dots & \dots \\ (-\hat{\mathbf{p}}_u^K)^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_{ru} \\ d_{ru} \end{bmatrix} + \begin{bmatrix} \lambda N_{ru}^1 \\ \lambda N_{ru}^2 \\ \dots \\ \lambda N_{ru}^K \end{bmatrix} + \mathbf{v}$$



Estimation of position and change in position: the role of Geometric Diversity

This is
from [RD-3]

Previous system can be arranged as:

$$\begin{bmatrix} L_{ru}^1 \\ L_{ru}^2 \\ \dots \\ L_{ru}^K \end{bmatrix} = \begin{bmatrix} (-\hat{\mathbf{p}}_u^1)^T & 1 \\ (-\hat{\mathbf{p}}_u^2)^T & 1 \\ \dots & \dots \\ (-\hat{\mathbf{p}}_u^K)^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_{ru} \\ d_{ru} \end{bmatrix} + \begin{bmatrix} \lambda N_{ru}^1 \\ \lambda N_{ru}^2 \\ \dots \\ \lambda N_{ru}^K \end{bmatrix} + \mathbf{v}$$

\mathbf{L}_{ru}

\mathbf{G}

$$\delta \mathbf{r}_{ru}(t_1) = \mathbf{r}_{ru}(t_1) - \mathbf{r}_{ru}(t_0)$$

$$-\mathbf{G}(t_1) + \mathbf{G}(t_1)$$

Considering now differences between two epochs t_0 and t_1 , and assuming no cycle-slips:

$$\mathbf{L}_{ru}(t_1) - \mathbf{L}_{ru}(t_0) = \mathbf{G}(t_1) \begin{bmatrix} \mathbf{r}_{ru}(t_1) \\ d_{ru}(t_1) \end{bmatrix} - \mathbf{G}(t_0) \begin{bmatrix} \mathbf{r}_{ru}(t_0) \\ d_{ru}(t_0) \end{bmatrix} + \tilde{\mathbf{v}}$$

$$\mathbf{L}_{ru}(t_1) - \mathbf{L}_{ru}(t_0) = \mathbf{G}(t_1) \begin{bmatrix} \delta \mathbf{r}_{ru}(t_1) \\ \delta d_{ru}(t_1) \end{bmatrix} + (\mathbf{G}(t_1) - \mathbf{G}(t_0)) \begin{bmatrix} \mathbf{r}_{ru}(t_0) \\ d_{ru}(t_0) \end{bmatrix} + \tilde{\mathbf{v}}$$

$$\mathbf{G}(t_1) - \mathbf{G}(t_0) = \begin{bmatrix} -(\hat{\mathbf{p}}_u^1(t_1) - \hat{\mathbf{p}}_u^1(t_0))^T & 0 \\ -(\hat{\mathbf{p}}_u^2(t_1) - \hat{\mathbf{p}}_u^2(t_0))^T & 0 \\ \dots & \dots \\ -(\hat{\mathbf{p}}_u^K(t_1) - \hat{\mathbf{p}}_u^K(t_0))^T & 0 \end{bmatrix}$$

Estimation of changes in baseline vector and clock bias is tied to the geometry matrix at time t_1 . This can be well determined.

Estimation of absolute value of baseline vector is tied to the change in geometry matrix at time t_1 . This would be poor determined if such change is not significant.

$d_{ru}(t_0)$ cannot be estimated at all!

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- [RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

Thank you

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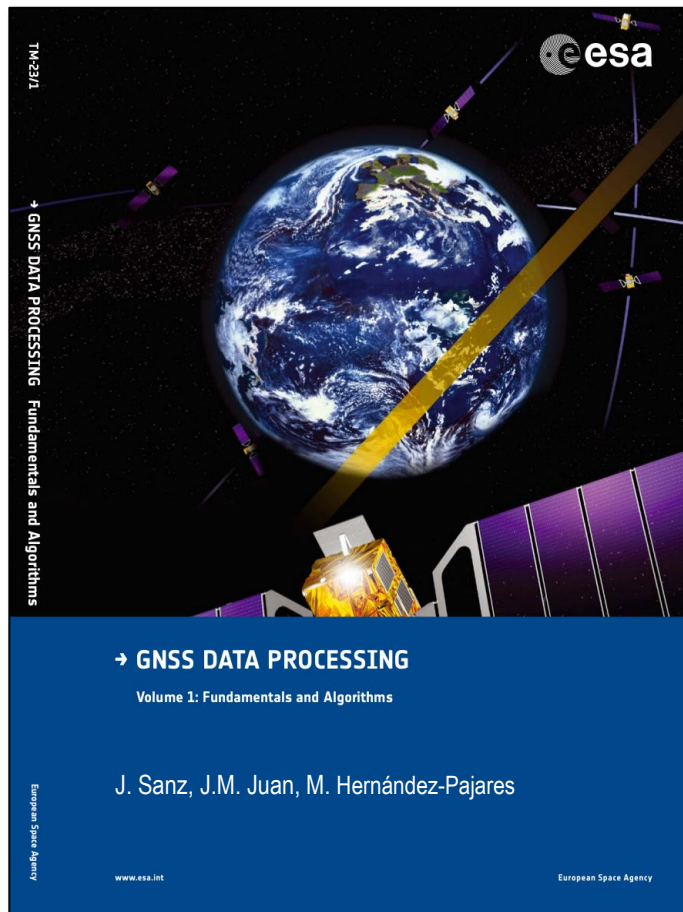
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