gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

Lecture 5

GNSS Measurements and Data Pre-processing

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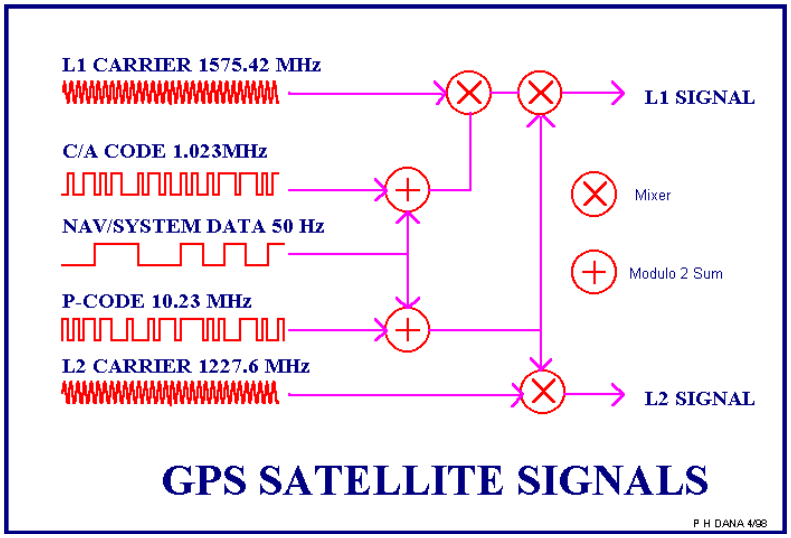
Contents

GPS SIGNAL STRUCTURE

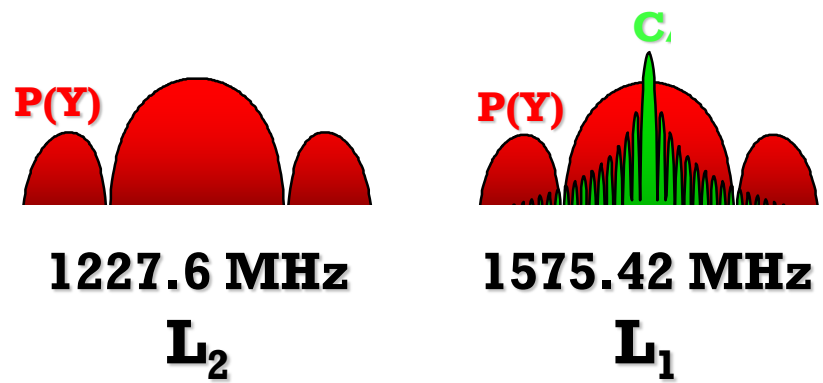
Two carriers in L-band:

- L_1=154·fo=1575.42 MHz
- L_2=120·fo=1227.60 MHz

where fo=10.23 MHz



- C/A-code for civilian users *[$X_C(t)$]*
- P-code only for military and authorized users *[$X_P(t)$]*
- Navigation message with satellite ephemeris and clock corrections *[$D(t)$]*



$$S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_1 t + \phi_{L_1}) + a_C X_C^{(k)}(t)\, D^{(k)}(t) \cos(\omega_1 t + \phi_{L_1})$$

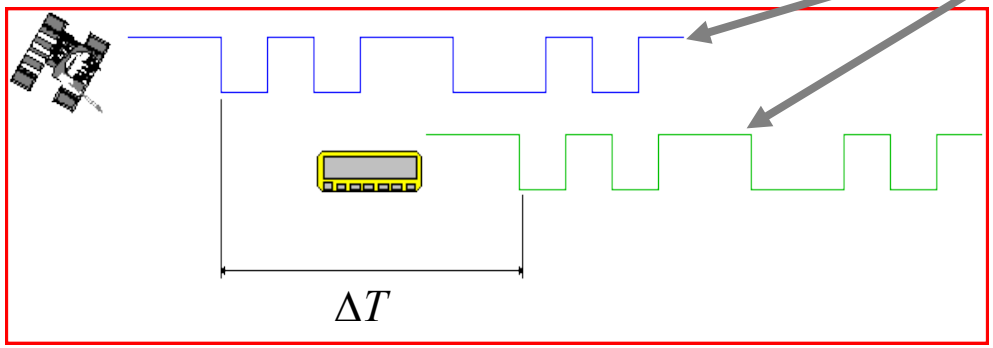
$$S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_2 t + \phi_{L_2})$$

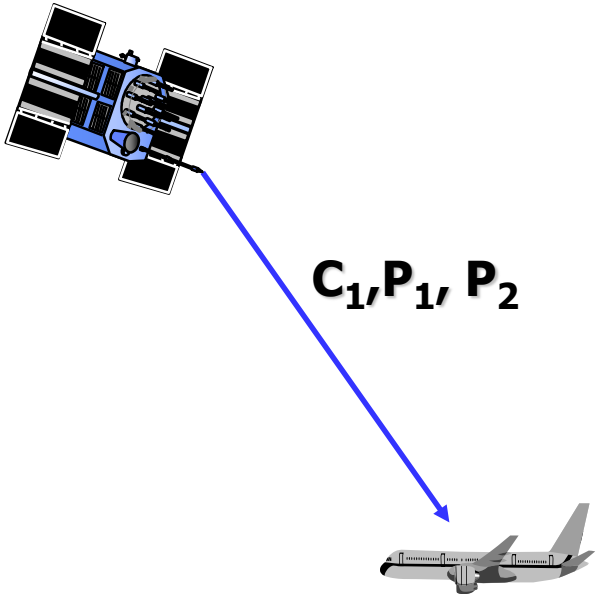
GPS Code Pseudorange Measurements

$$S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_1 t + \varphi_{L_1}) + a_C X_C^{(k)}(t)\, D^{(k)}(t) \cos(\omega_1 t + \varphi_{L_1})$$

$$S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_2 t + \varphi_{L_2})$$

binary code $X_P(t)$





$$\mathrm{P}(T) = c\,\Delta T = c\left[t_{rec}(T) - t^{sat}(T - \Delta T)\right]$$

From hereafter we will call:

- C_1 pseudorange computed from $X_C(t)$ binary code (on frequency 1)
- P_1 pseudorange computed from $X_P(t)$ binary code (on frequency 1)
- P_2 pseudorange computed from $X_P(t)$ binary code (on frequency 2)

GPS Carrier Phase Measurements

$$S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_1 t + \varphi_{L_1}) + a_C X_C^{(k)}(t)\, D^{(k)}(t) \cos(\omega_1 t + \varphi_{L_1})$$

$$S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_2 t + \varphi_{L_2})$$

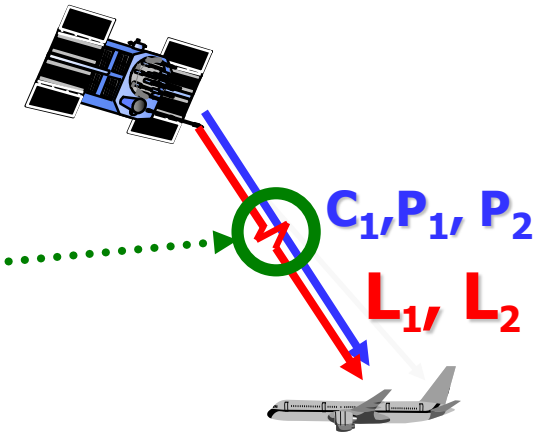
Carrier phase

Carrier beat phase:

$$\phi_L(T) = \phi_{L\,rec}(T) - \phi_L^{sat}(T - \Delta\overline{T})$$

$$= \frac{c}{\lambda}\Delta\overline{T} + N$$

Unknown ambiguity

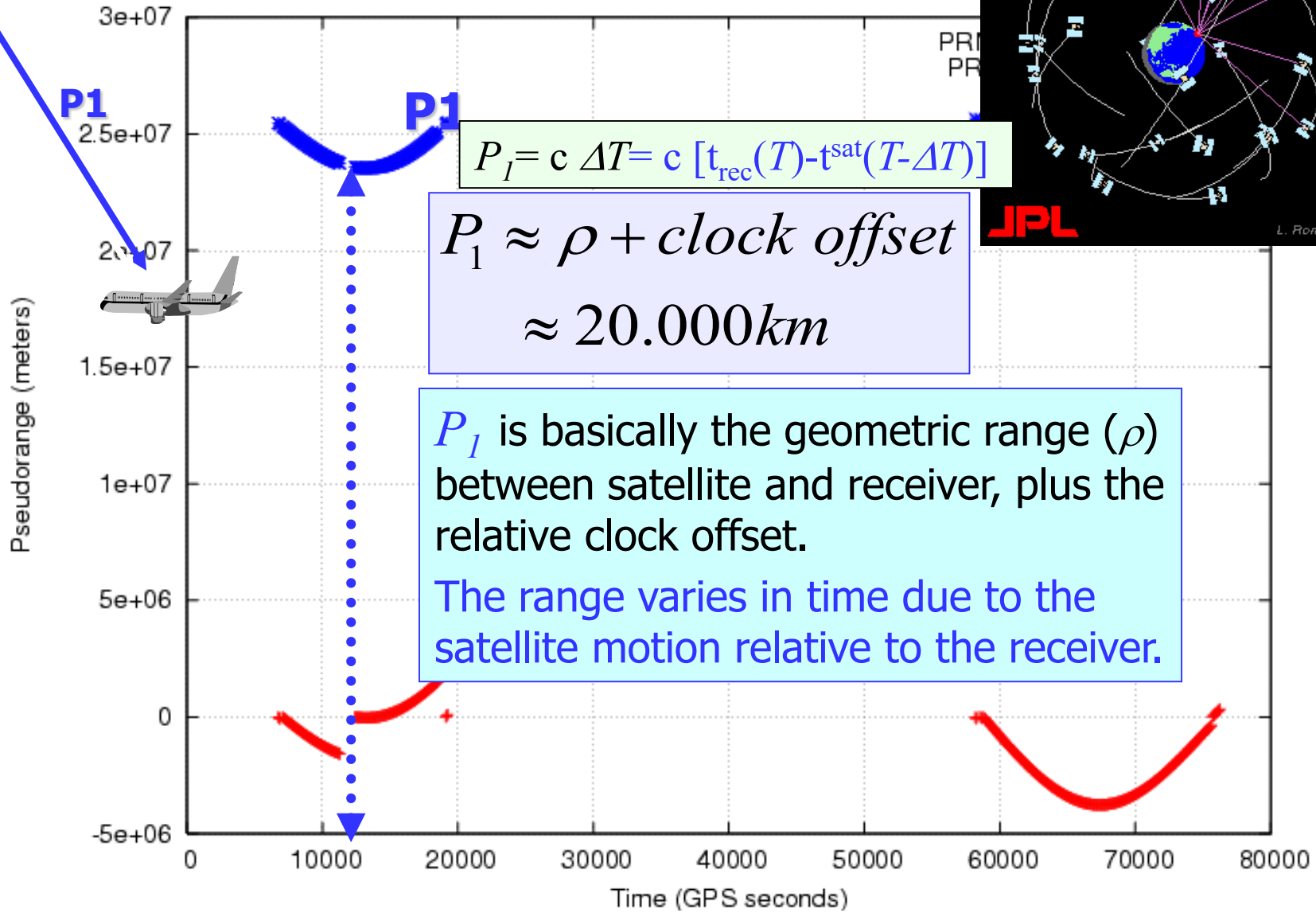


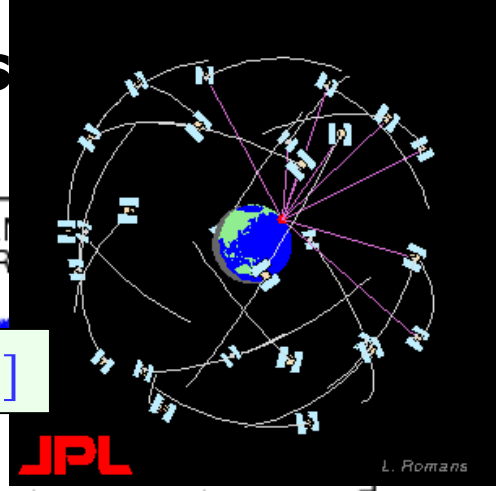
From hereafter we will call:

- $L_1 = \lambda_1 \phi_{L1}$ measur. computed from the carrier phase on frequency 1
- $L_2 = \lambda_2 \phi_{L2}$ measur. computed from the carrier phase on frequency 2

- C_1 pseudorange computed from $X_C(t)$ binary code (on frequency 1)
- P_1 pseudorange computed from $X_P(t)$ binary code (on frequency 1)
- P_2 pseudorange computed from $X_P(t)$ binary code (on frequency 2)

Carrier and Code pseudorange meas





$$P_1 = c\,\Delta T = c\,[t_{rec}(T) - t^{sat}(T-\Delta T)]$$

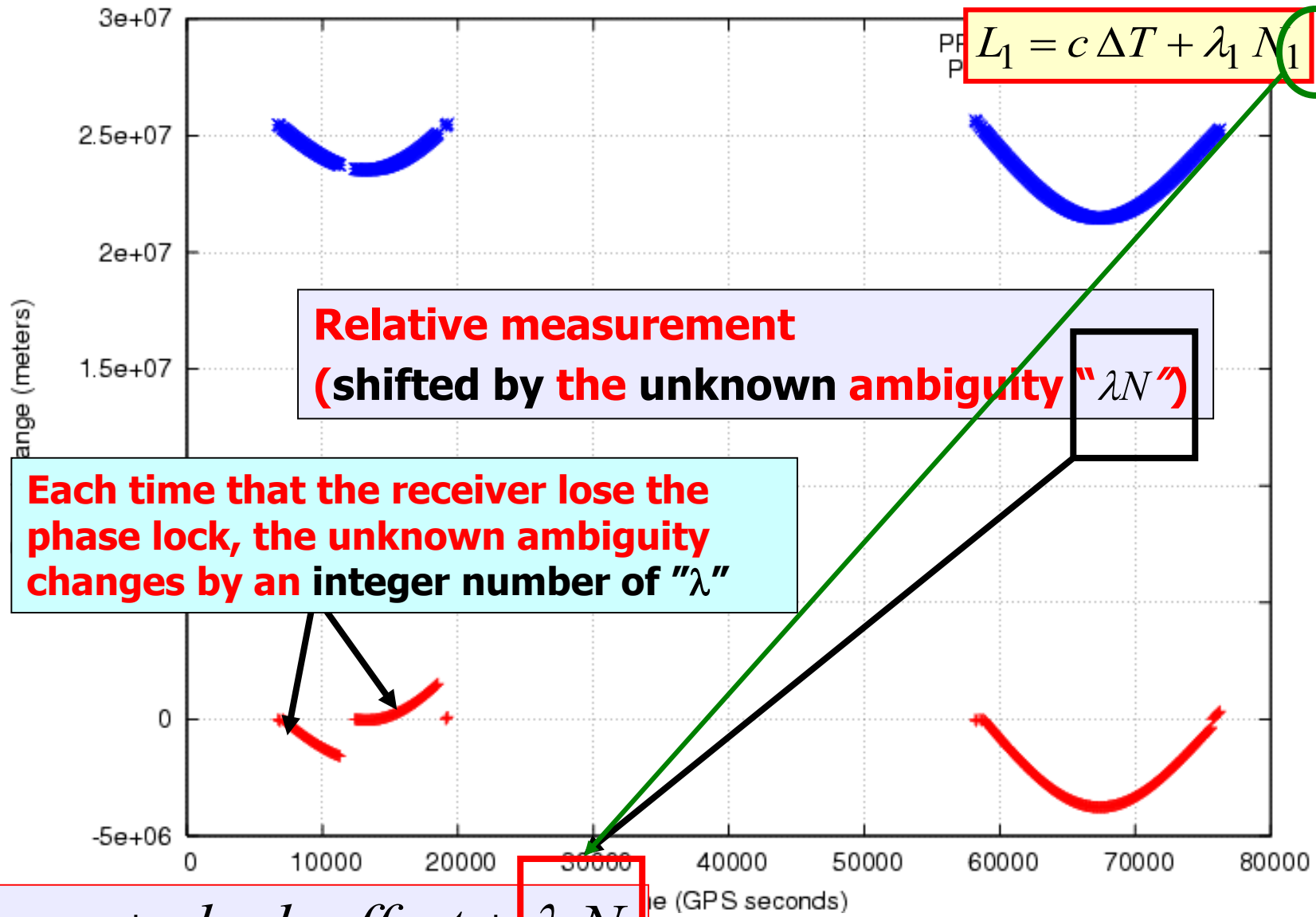
$$P_1 \approx \rho + clock\ offset$$
$$\approx 20.000 km$$

P_1 is basically the geometric range (ρ) between satellite and receiver, plus the relative clock offset.

The range varies in time due to the satellite motion relative to the receiver.

P_1 is an **absolute** measurement **(unambiguous)**

Phase and Code pseudorange measurements



$$L_1 = c\,\Delta T + \lambda_1 N_1$$

Relative measurement
(shifted by the unknown ambiguity "λN")

Each time that the receiver lose the phase lock, the unknown ambiguity changes by an integer number of "λ"

$$L_1 \approx \rho + clock\ offset + \lambda_1 N_1$$

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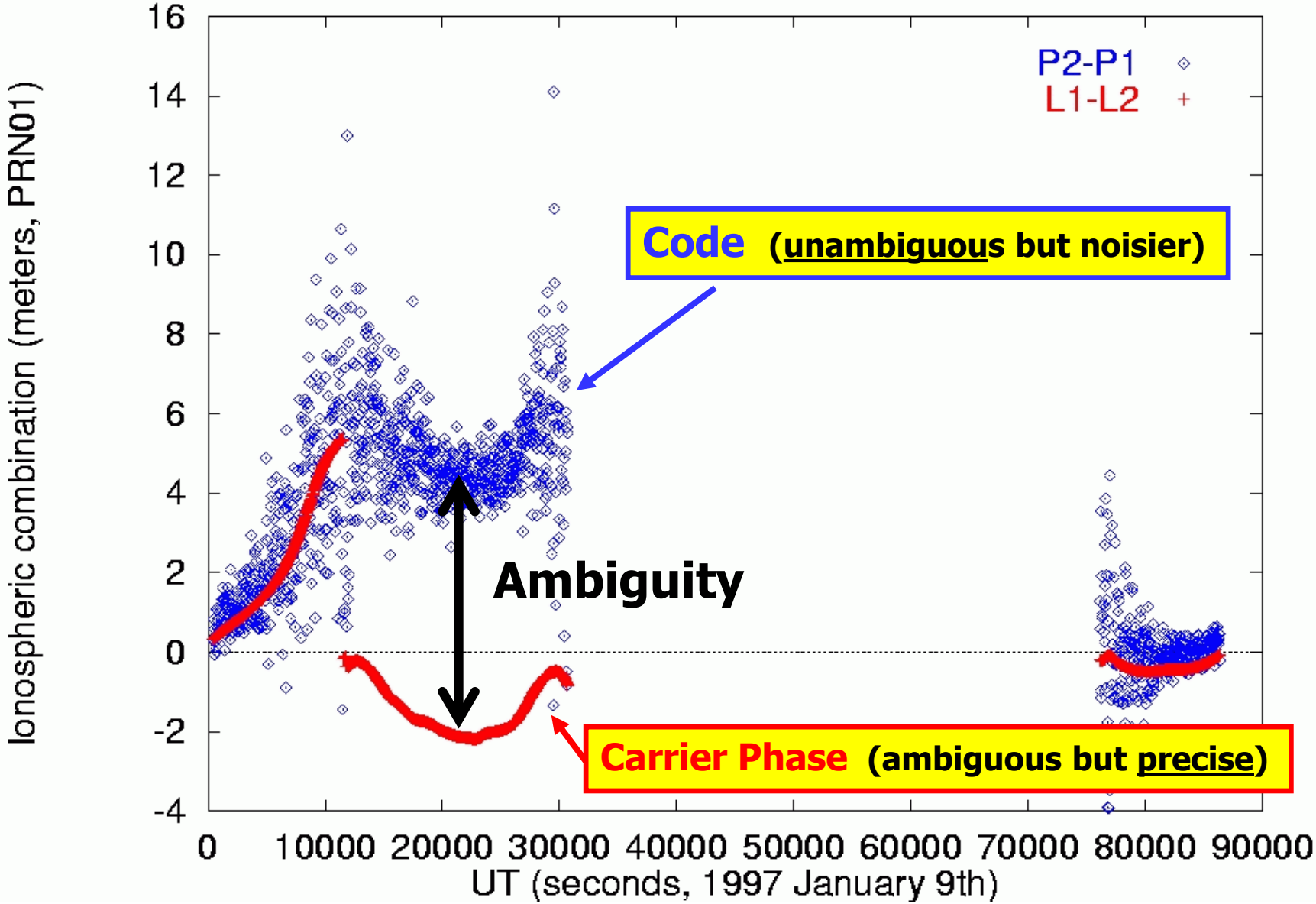
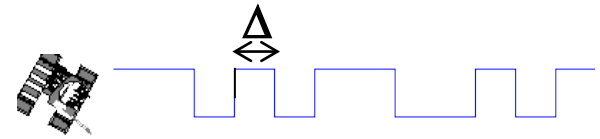
Code and Carrier Phase measurements



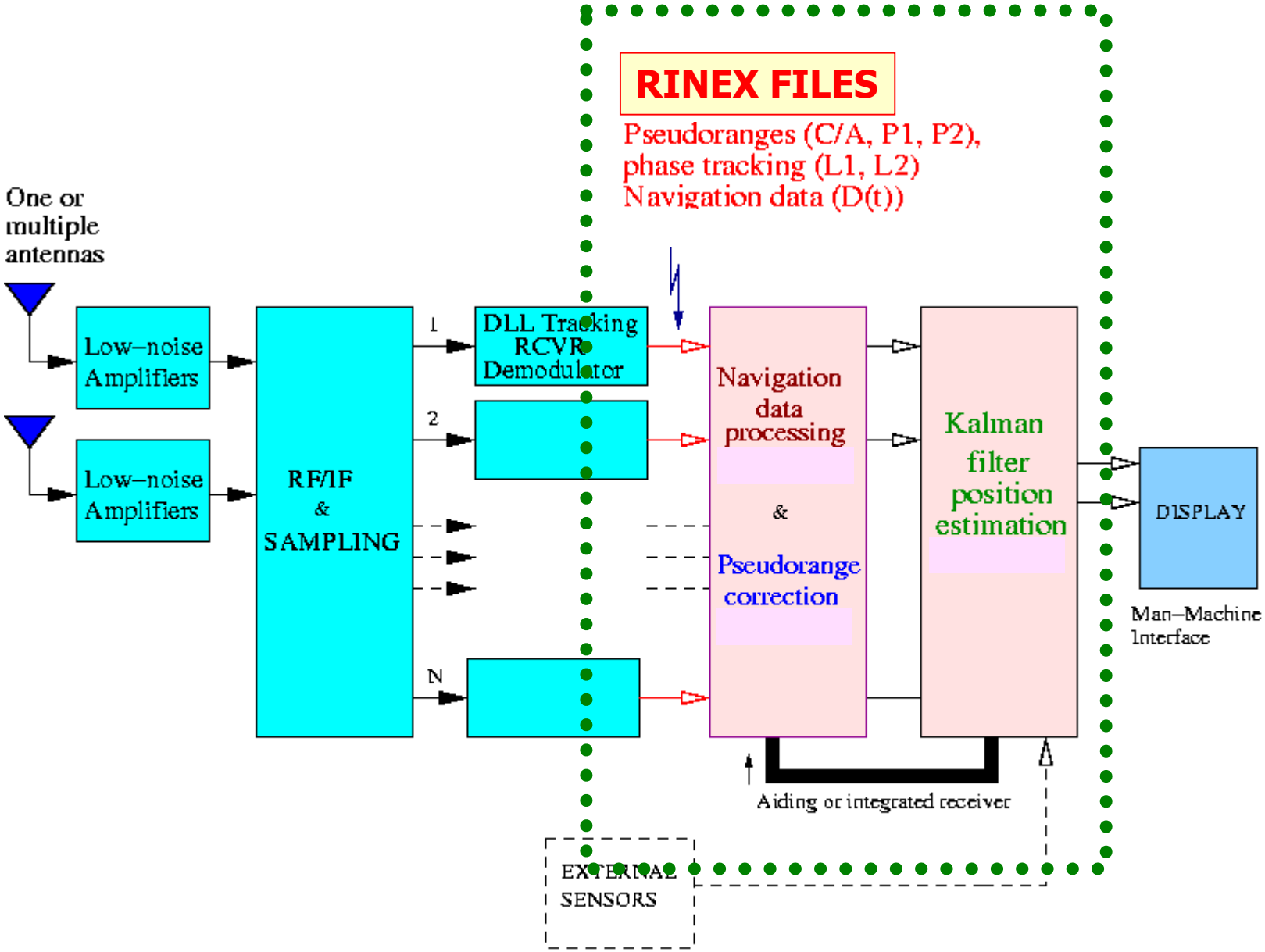
GPS measurements: Code and Carrier Phase

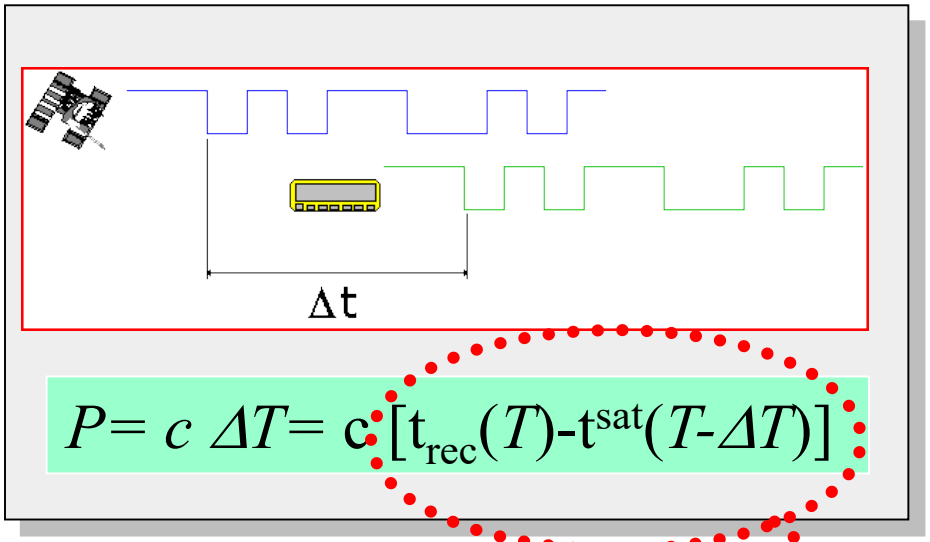


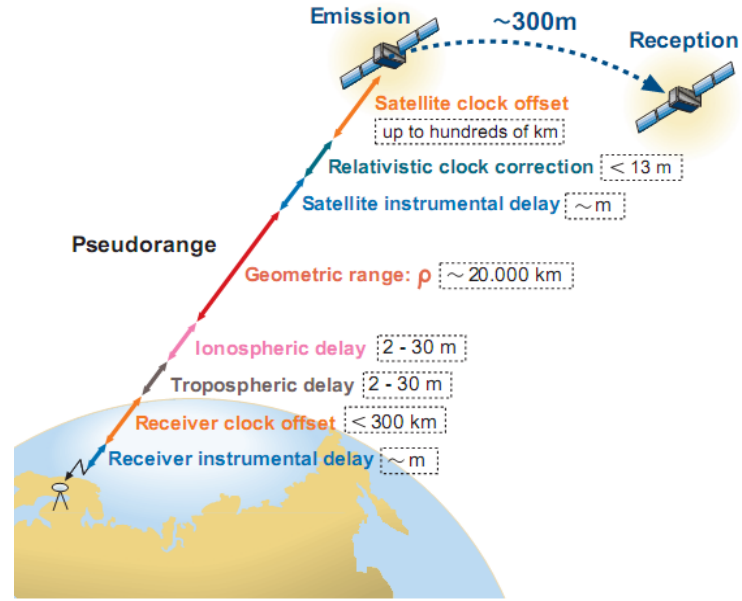
Antispoofing (A/S):
The code P is encrypted to Y.
➔ Only the code C at frequency L1 is available.

	Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
Code measurements			
C_1	300 m	3 m	**Unambiguous** but noisier
P_1 (Y1): encrypted	30 m	30 cm	
P_2 (Y2): encrypted	30 m	30 cm	
Phase measurements			
L_1	19.05 cm	2 mm	**Precise** but ambiguous
L_2	24.45 cm	2 mm	

[*] the codes can be smoothed with the phases in order to reduce noise (i.e, C_1 smoothed with L_1 ➔ 50 cm noise)


RINEX FILES
Pseudoranges (C/A, P1, P2),
phase tracking (L1, L2)
Navigation data (D(t))
One or multiple antennas
Low–noise Amplifiers
Low–noise Amplifiers
RF/IF & SAMPLING
1
2
N
DLL Tracking RCVR Demodulator
Navigation data processing
&
Pseudorange correction
Kalman filter position estimation
DISPLAY
Man–Machine Interface
Aiding or integrated receiver
EXTERNAL SENSORS





$$P_{rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + \sum \delta$$

Geometric range

Clock offsets

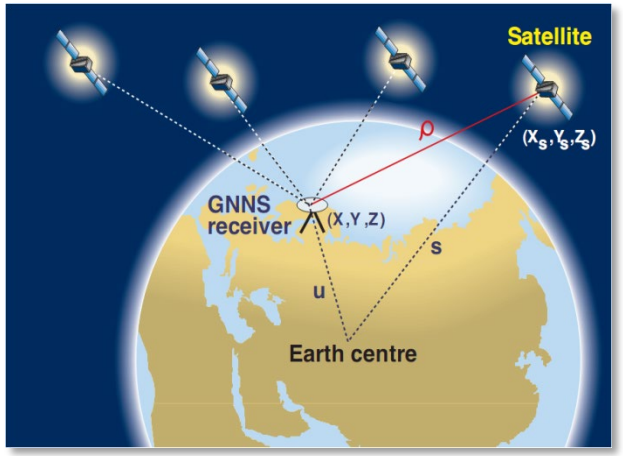
$$\sum \delta = Trop_{rec}^{sat} + Ion_{rec}^{sat} + K_{rec} + K^{sat} + \varepsilon$$

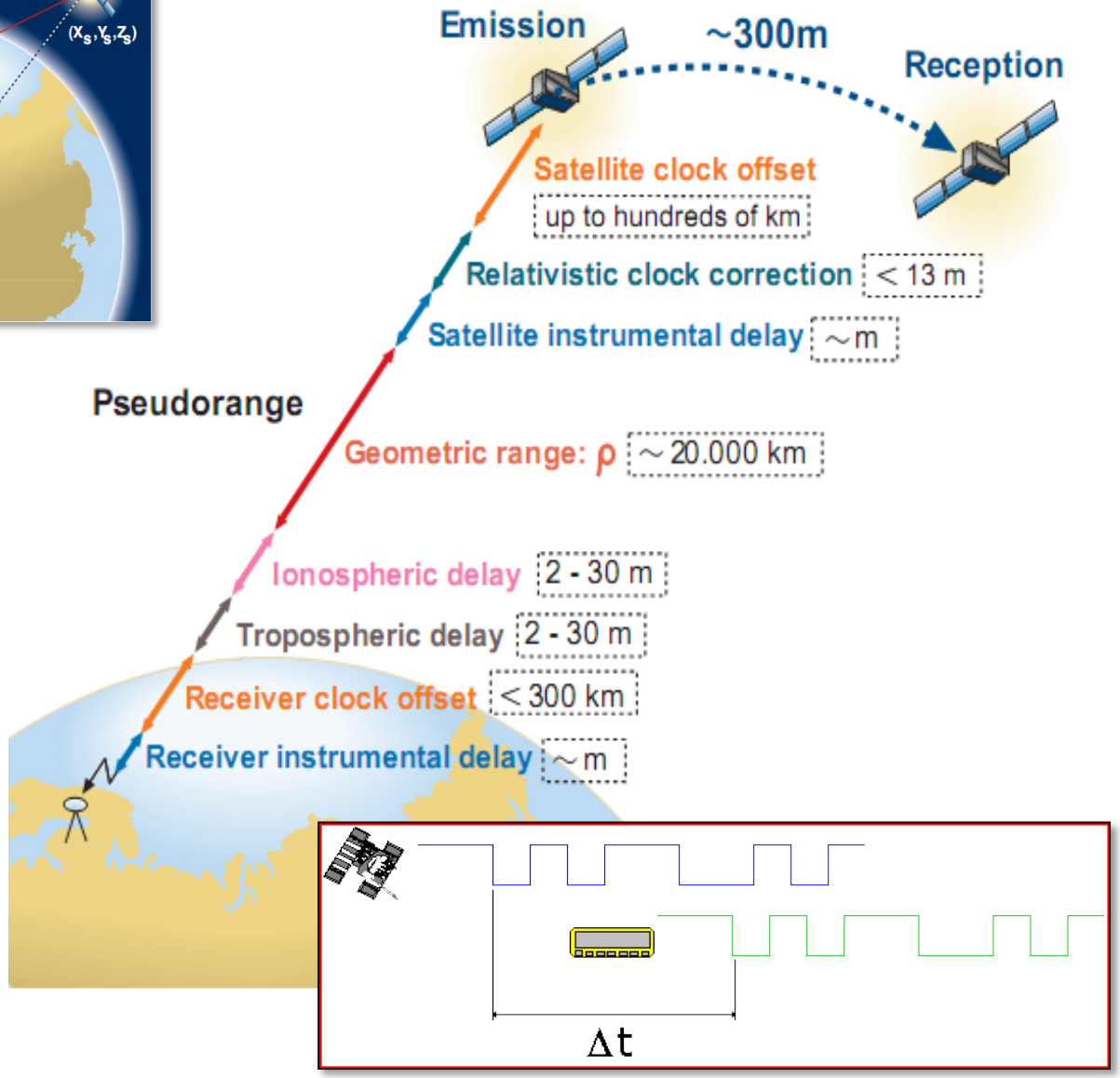
Tropospheric delay

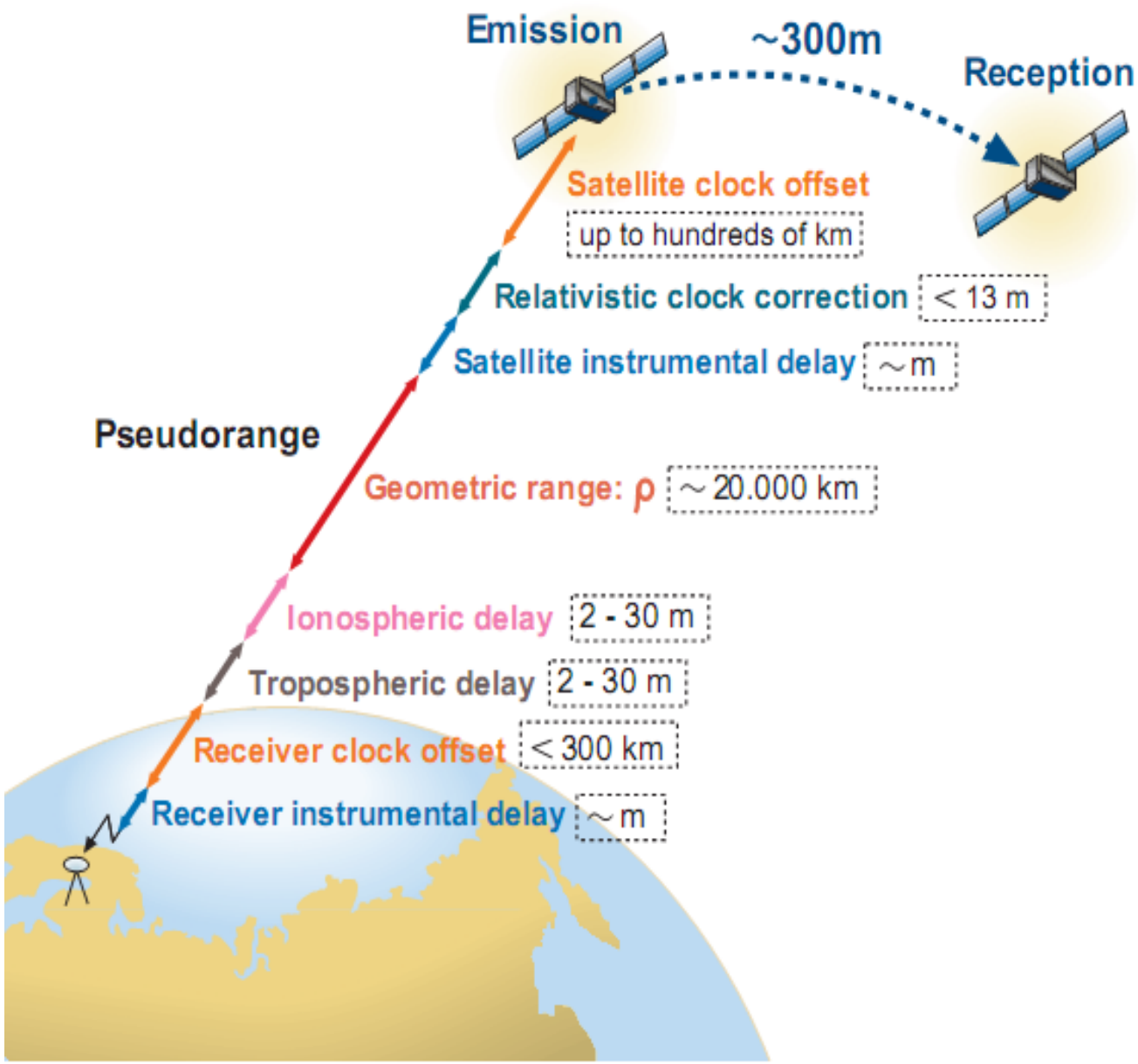
Ionospheric delay

Instrumental delays

noise


Satellite
ρ
(X_s, Y_s, Z_s)
GNNS receiver
(X,Y,Z)
s
u
Earth centre


Emission
~300m
Reception
Satellite clock offset
up to hundreds of km
Relativistic clock correction
< 13 m
Satellite instrumental delay
~m
Pseudorange
Geometric range: ρ
~ 20.000 km
Ionospheric delay
2 - 30 m
Tropospheric delay
2 - 30 m
Receiver clock offset
< 300 km
Receiver instrumental delay
~m
Δt


Emission
~300m
Reception
Satellite clock offset
up to hundreds of km
Relativistic clock correction
< 13 m
Satellite instrumental delay
~m
Pseudorange
Geometric range: ρ
~ 20.000 km
Ionospheric delay
2 - 30 m
Tropospheric delay
2 - 30 m
Receiver clock offset
< 300 km
Receiver instrumental delay
~m

Exercise:

a) Using the file coco0090.97o, generate the "txt" file 95oct18casa.a (with data ordered in columns).

b) Plot code and phase measurements for satellite PRN28 and discuss the results.

Resolution:

a) **gLAB_linux** -input:cfg **meas.cfg** -input:obs **coco0090.97o**

b) See next plots:

An example of program to read the RINEX: gLAB

RINEX file → gLAB → txt file

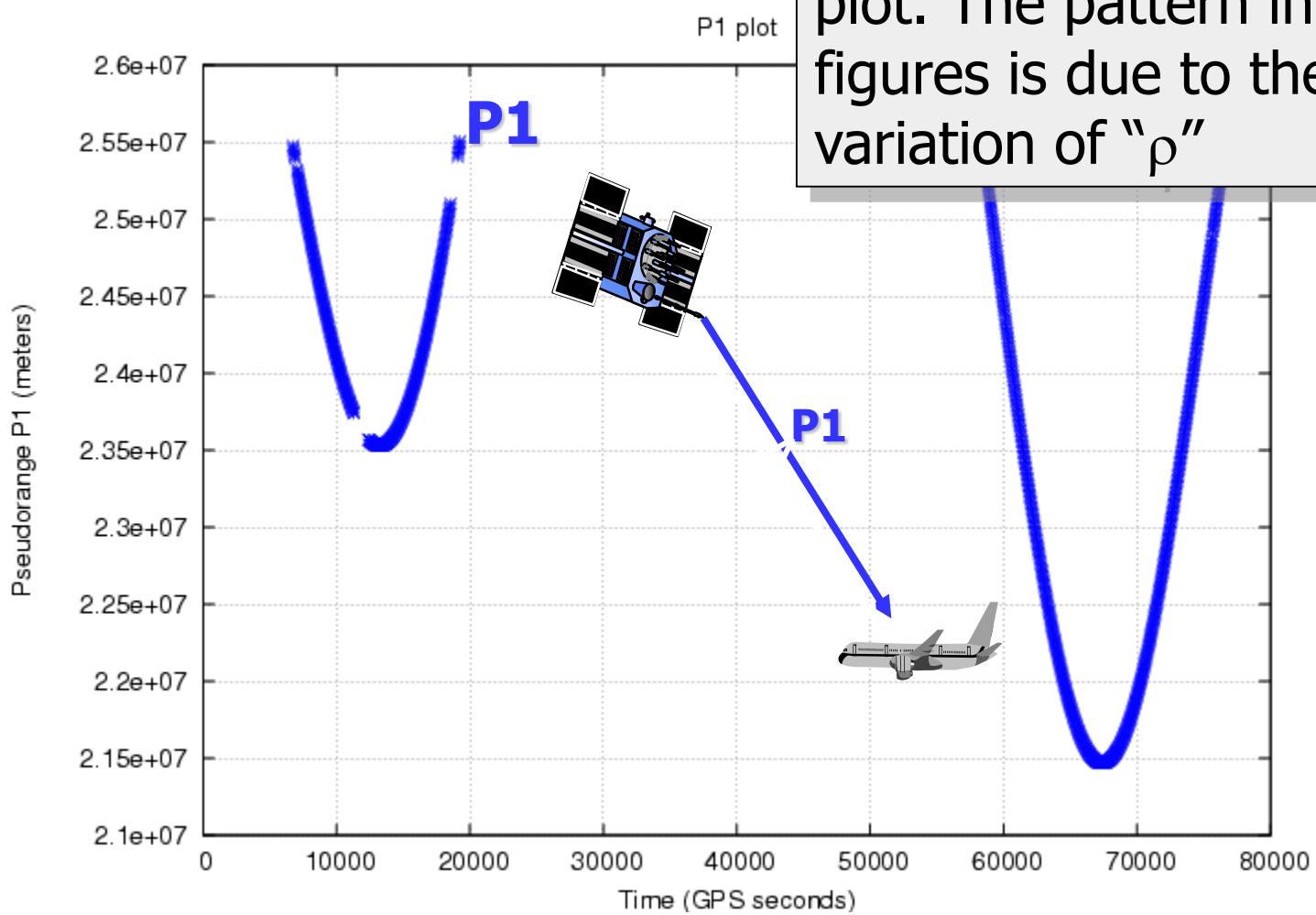
```
     2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX  AUSLIG              10-JAN-97 10:19     PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN)  - COCOS ISLAND      COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
     -0.000000000103          HARDWARE CALIBRATION (S)      COMMENT
     -0.000000054663          CLOCK OFFSET (S)              COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
       0.0040        0.0000        0.0000                   ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:   >500  >100   >50   >10    >5    >0   bad   n/a      COMMENT
 sig:      9     8     7     6     5     4     1     0      COMMENT
   30                                                       INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014  <===== 1
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184  <===== 25
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394  <===== 9
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094  <===== 5
  24610920.702     -924108.174 6   -720085.67045  24610920.0404  <===== 23
  20718775.074   -18605935.474 9 -14498133.97346  20718775.6074  <===== 17
  20842713.610   -19083282.892 9 -14870090.55546  20842713.4814  <===== 6
```

sta	Doy	sec	PRN	L1	L2	C1	P2	
casa	291	0.00	14	3832855.061	3832852.989	20764791.163	20764791.889	0
casa	291	0.00	15	-1932753.473	-1932752.152	23625606.133	23625608.420	0
casa	291	0.00	18	-191430.529	-191430.252	24656587.151	24656589.163	0
casa	291	0.00	22	-2444624.551	-2444621.901	22508513.354	22508514.937	0
casa	291	0.00	25	-2949812.363	-2949808.662	22258999.265	22258999.532	0
casa	291	0.00	29	-1670473.859	-1670472.308	22409115.477	22409115.535	0
casa	291	30.00	14	-3840286.848	-3840284.776	20757359.266	20757360.162	0
casa	291	30.00	15	-1914283.549	-1914282.239	23644076.112	23644078.373	0
casa	291	30.00	18	-207329.153	-207328.868	24640688.394	24640690.535	0
casa	291	30.00	22	-2458225.787	-2458223.122	22494912.015	22494913.648	0
casa	291	30.00	25	2935690.693	2935686.992	22273121.015	22273121.208	0
casa	291	30.00	29	1681115.594	1681114.037	22398473.854	22398474.054	0
casa	291	60.00	14	-3847635.893	-3847633.821	20750010.062	20750010.988	0
casa	291	60.00	15	-1895770.976	-1895769.678	23662588.653	23662591.023	0
casa	291	60.00	18	-223219.005	-223218.710	24624790.015	24624808.000	0
casa	291	60.00	22	-2471750.391	-2471747.715	22481387.456	22481388.934	0
casa	291	60.00	25	-2921510.699	-2921507.003	22287301.001	22287301.347	0
casa	291	60.00	29	-1691735.199	-1691733.640	22387854.082	22387854.429	0
casa	291	90.00	14	-3854900.513	-3854898.440	20742745.403	20742746.301	0
casa	291	90.00	15	-1877216.337	-1877215.051	23681143.251	23681145.556	0
casa	291	90.00	18	-239097.495	-239097.188	24608919.438	24608922.372	0
casa	291	90.00	22	2485199.565	2485196.877	22467938.317	22467939.302	0
casa	291	90.00	25	2907272.799	2907269.105	22301538.849	22301539.273	0
casa	291	90.00	29	-1702332.295	-1702330.732	22377257.010	22377257.225	0
casa	291	120.00	14	-3862079.743	-3862077.674	20735566.138	20735566.939	0
casa	291	120.00	15	-1858620.266	-1858618.992	23699739.479	23699741.540	0
casa	291	120.00	18	-254966.373	-254966.052	24593059.725	24593059.168	0

The RINEX file is converted to a "columnar format" to easily plot its content and to analyze the measurements (the public domain free tool **"gnuplot"** is used in the book to make the plots).

Code measurements

The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ"



$$P_{1sta}^{sat} = \boxed{\rho_{sta}^{sat} + c \cdot (dt_{sta} - dt^{sat})} + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

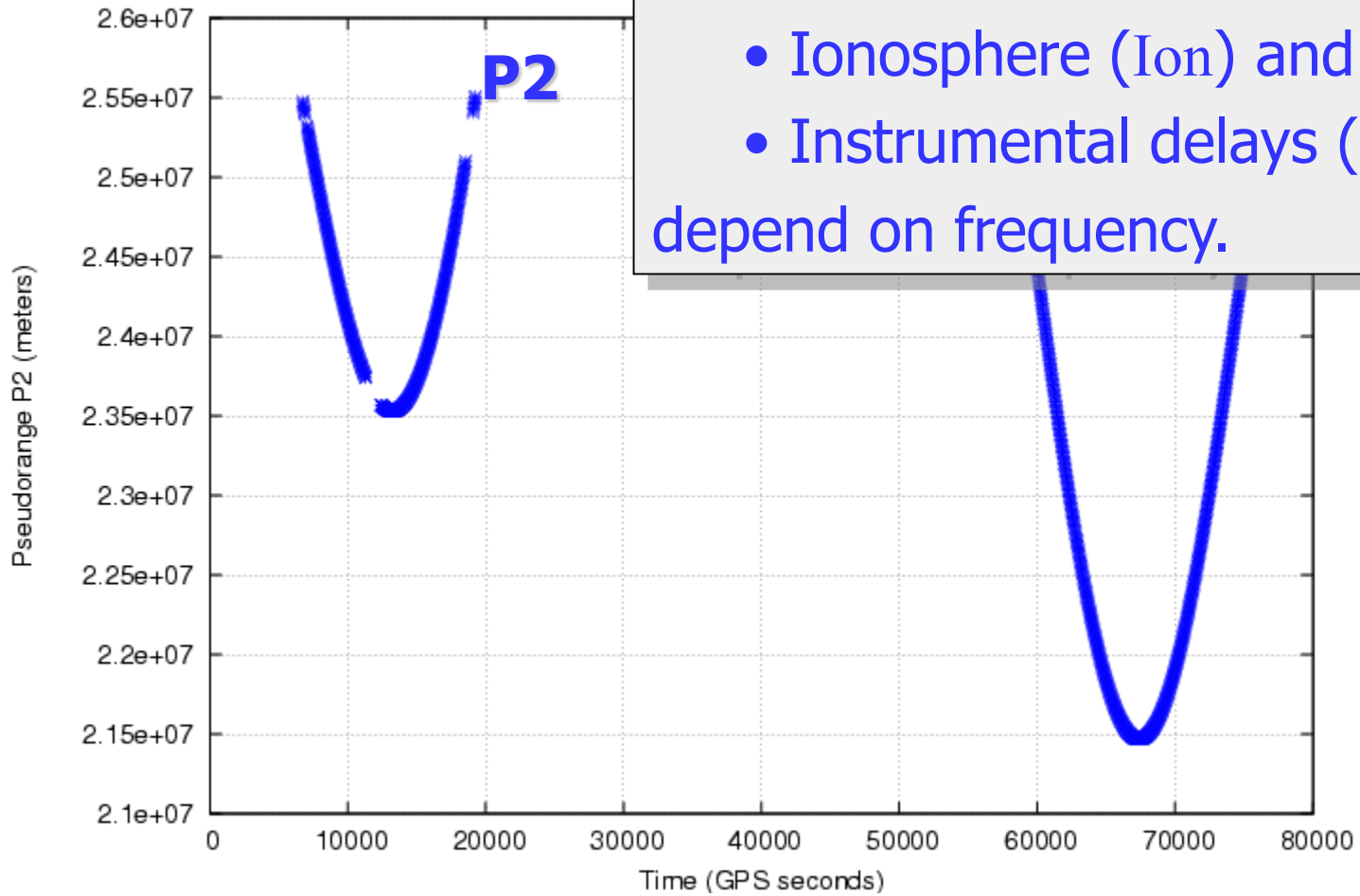
Code measurements

Similar plot for code measurements at f_2.

Notice that

- Ionosphere (Ion) and
- Instrumental delays (K)

depend on frequency.



$$P_{2sta}^{sat} = \boxed{\rho_{sta}^{sat} + c \cdot (dt_{sta} - dt^{sat})} + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_{2}^{sat} + \varepsilon_2$$

Ionosphere delays code and advances phase measurements

ase measurements

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C_{1sta}^{sat};

Code measurements: C1,P1,P2

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_{1}^{sat} + \varepsilon_1$$

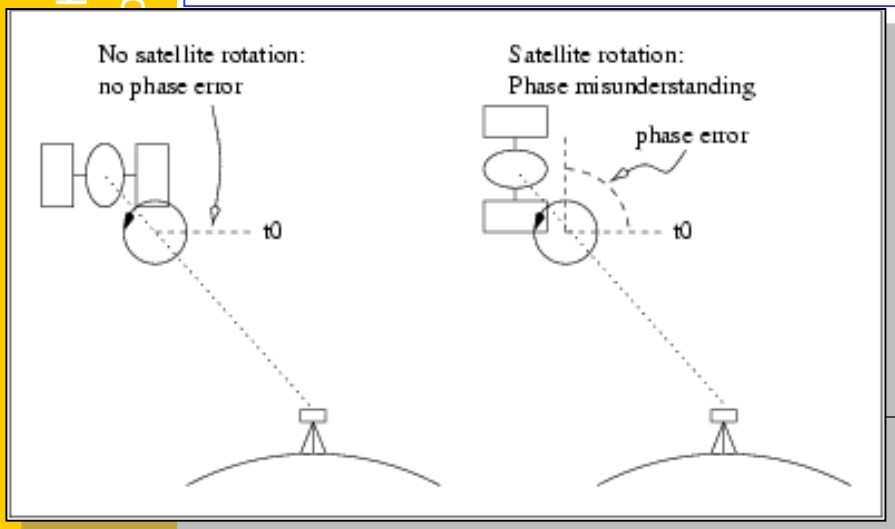
$$P_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_{2}^{sat} + \varepsilon_2$$

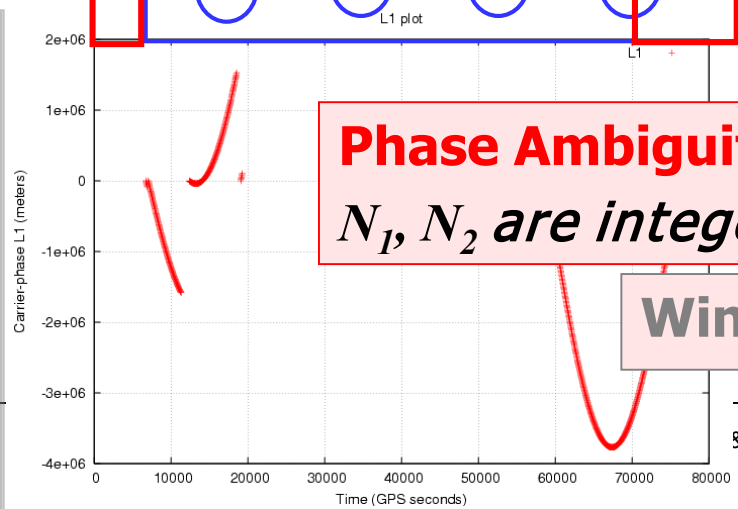
Frequency dependent

Phase measurements: L1,L2

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_{1}^{sat} + \lambda_1 N_{1sta}^{sat} + \lambda_1 w_{sta}^{sat} + \nu_1$$

$$L_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + b_{2sta} + b_{2}^{sat} + \lambda_2 N_{2sta}^{sat} + \lambda_2 w_{sta}^{sat} + \nu_2$$





Phase Ambiguities

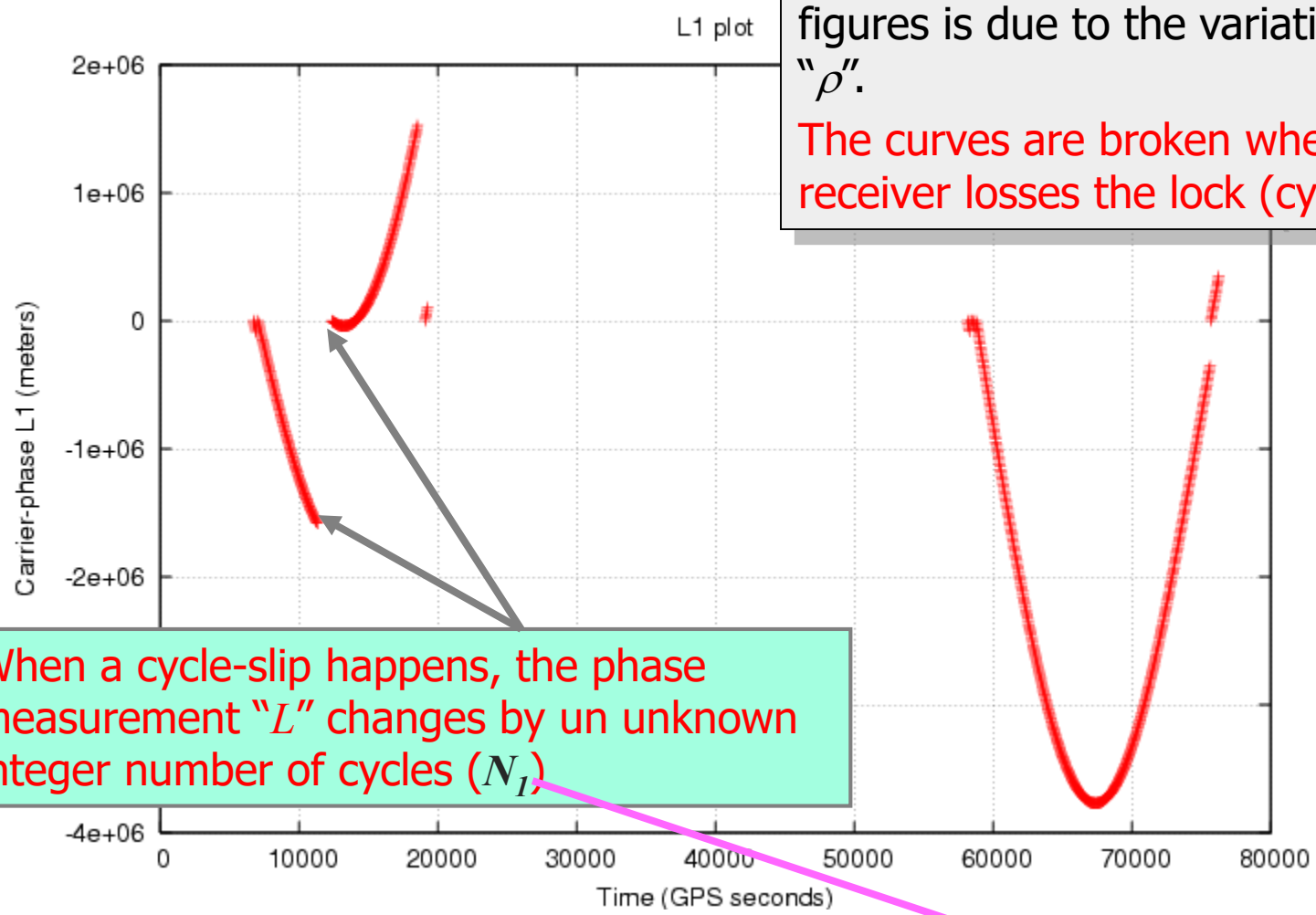
N_1, N_2 are integers

Wind Up

Carrier Phase measurements

The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ".

The curves are broken when the receiver losses the lock (cycle-slip).



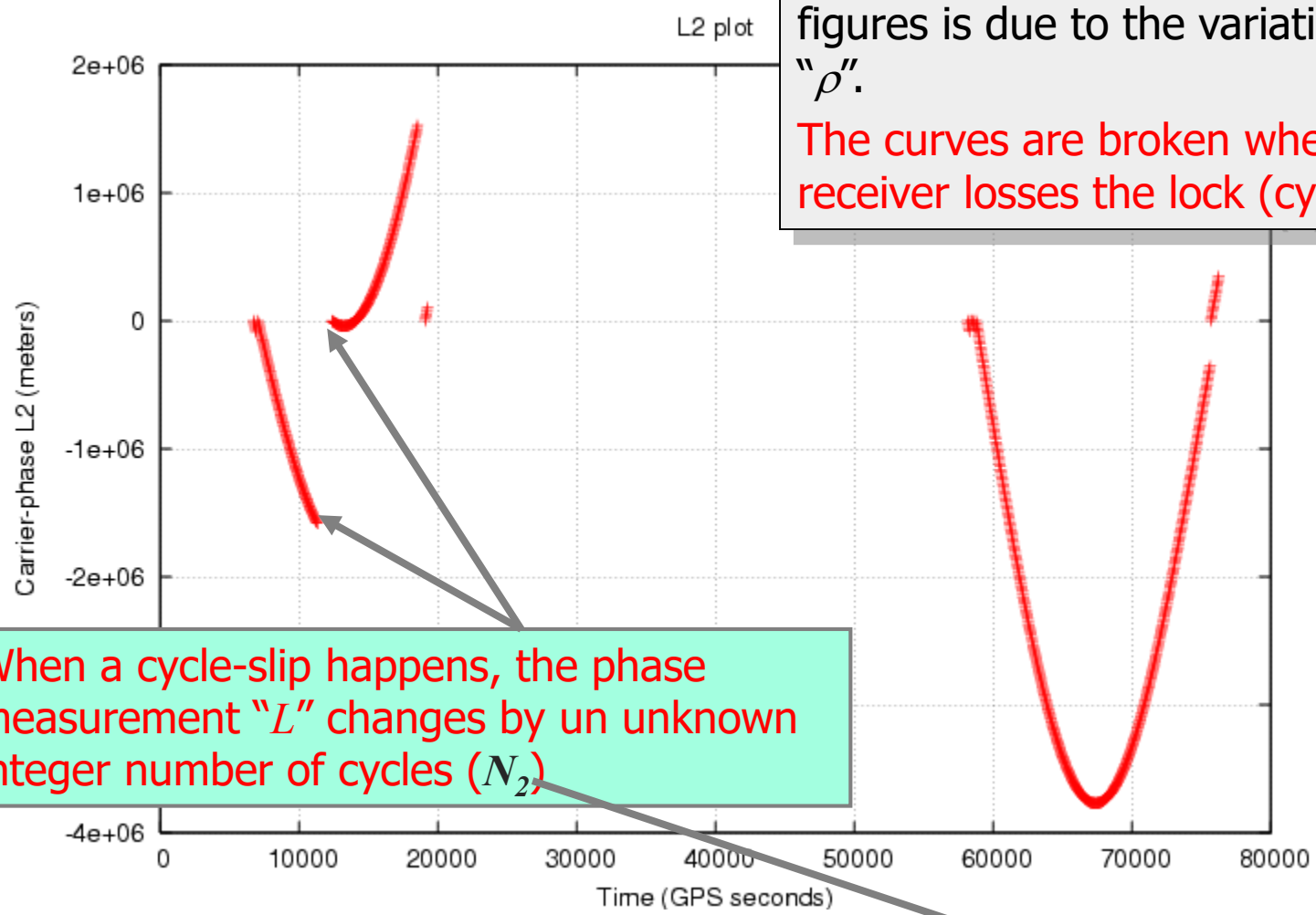
When a cycle-slip happens, the phase measurement "L" changes by un unknown integer number of cycles (N_1)

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_{1sta}^{sat} + \lambda_1 w_{sta}^{sat} + \nu_1$$

Carrier Phase measurements

The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ".

The curves are broken when the receiver losses the lock (cycle-slip).



When a cycle-slip happens, the phase measurement "L" changes by un unknown integer number of cycles (N_2)

$$L_{2sta}^{sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + b_{2sta} + b_2^{sat} + \lambda_2 \boxed{N_{2sta}^{sat}} + \lambda_2 w_{sta}^{sat} + \nu_2$$

Contents

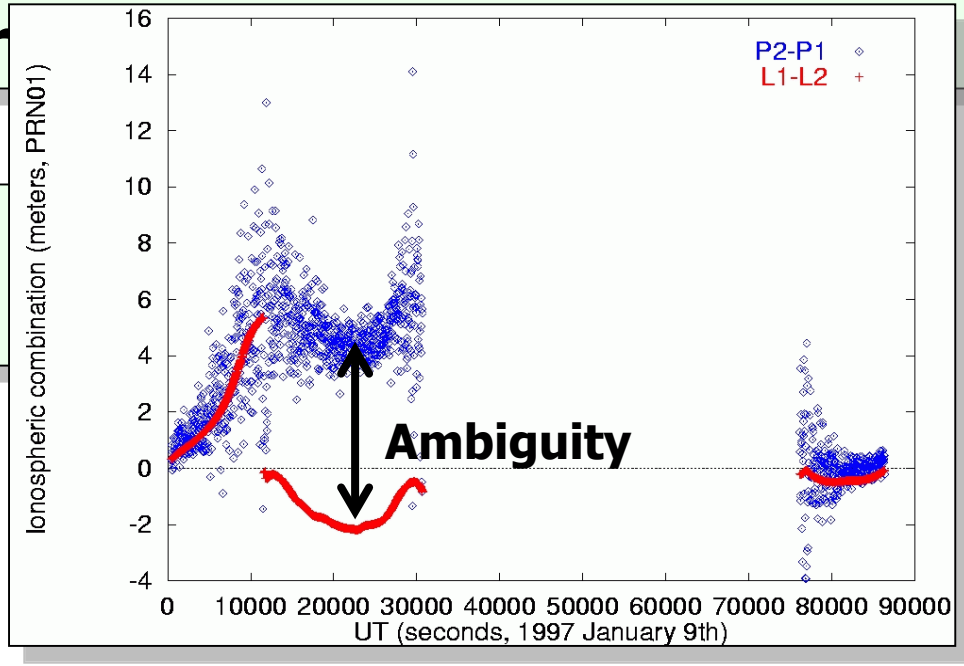
Linear Combinations of measurements:

- Geometry-free (or Ionospheric) combination.
- Ionosphere-Free combination.
- Wide-lane and Narrow-lane combinations.

1. Geometry-free (or

$P_I = P_2 - P_1 = Iono + ctt$

$L_I = L_1 - L_2 = Iono + ctt + Ambig$



Code measurements: C_1, P_1, P_2

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

$$P_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_2^{sat} + \varepsilon_2$$

Carrier measurements: L1,L2

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_{1sta}^{sat} + \lambda_1 w_{sta}^{sat} + \nu_1$$

$$L_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + b_{2sta} + b_2^{sat} + \lambda_2 N_{2sta}^{sat} + \lambda_2 w_{sta}^{sat} + \nu_2$$

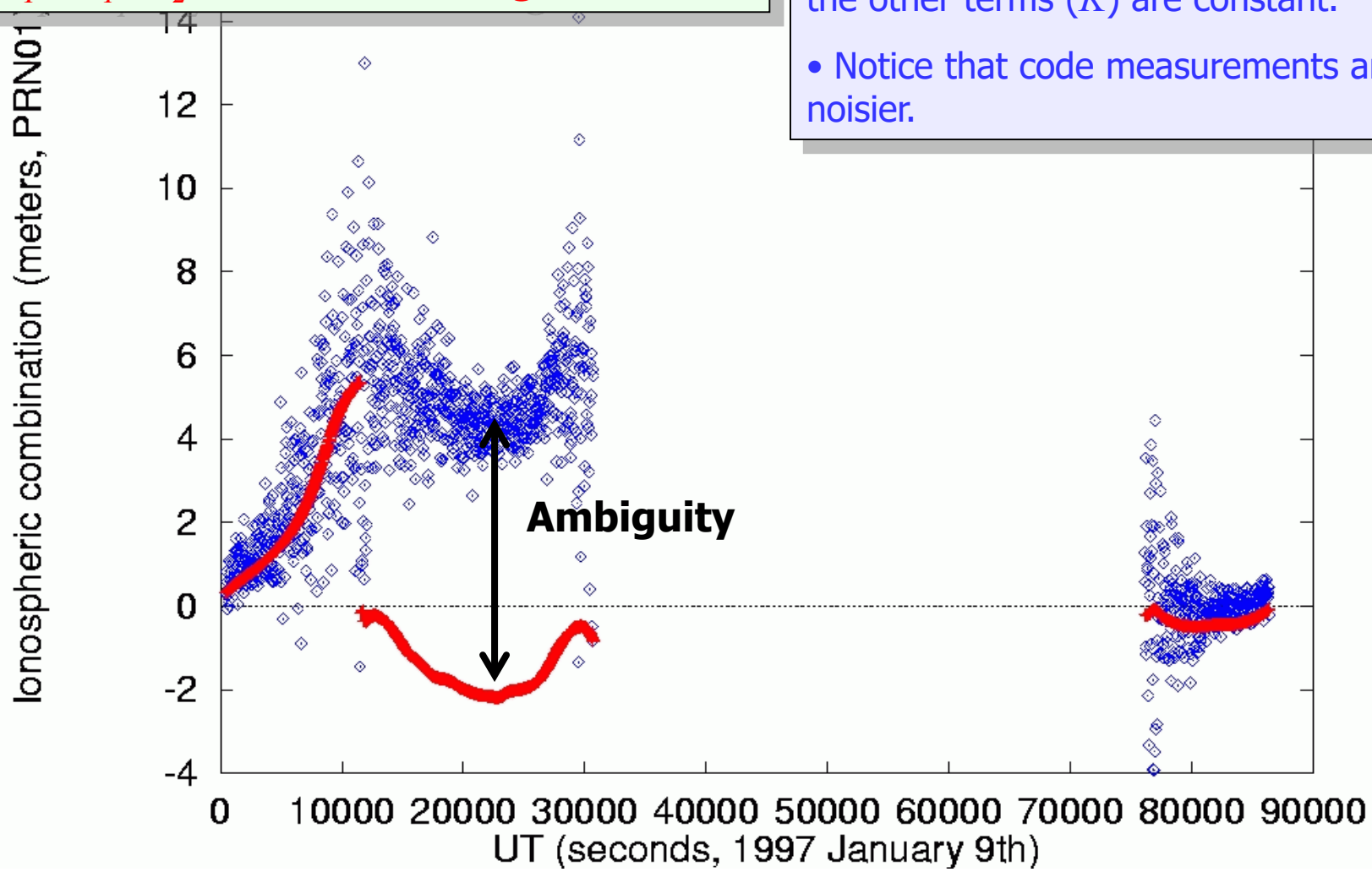
Carrier Ambiguities

1. Geometry-free (or ionospheric) combination

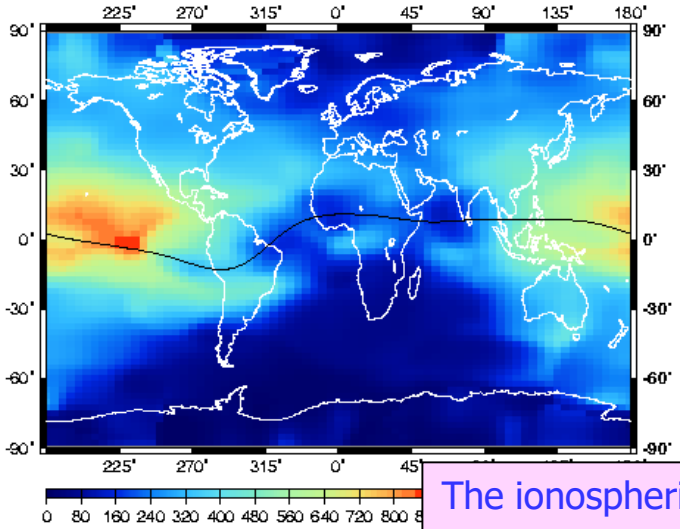
$P_I = P_2 - P_1 = Iono + ctt$

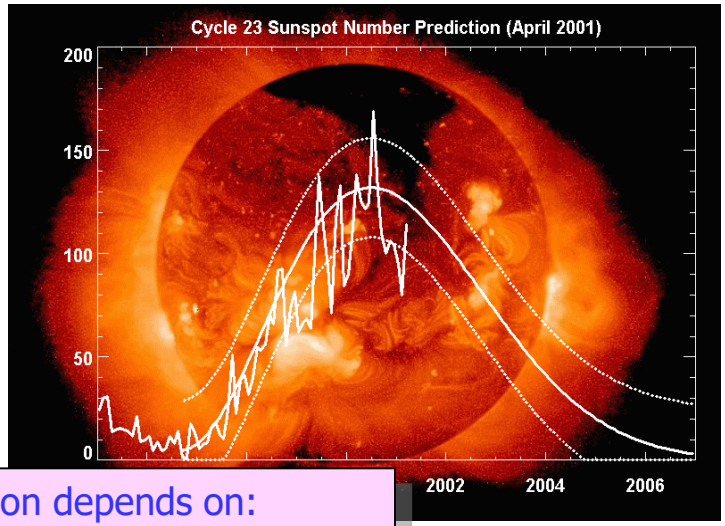
$L_I = L_1 - L_2 = Iono + ctt + Ambig$

- The pattern corresponds to the ionospheric refraction (Ion), because the other terms (K) are constant.
- Notice that code measurements are noisier.



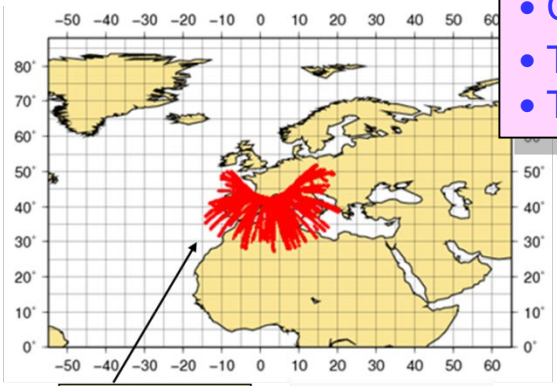
Ionospheric effects



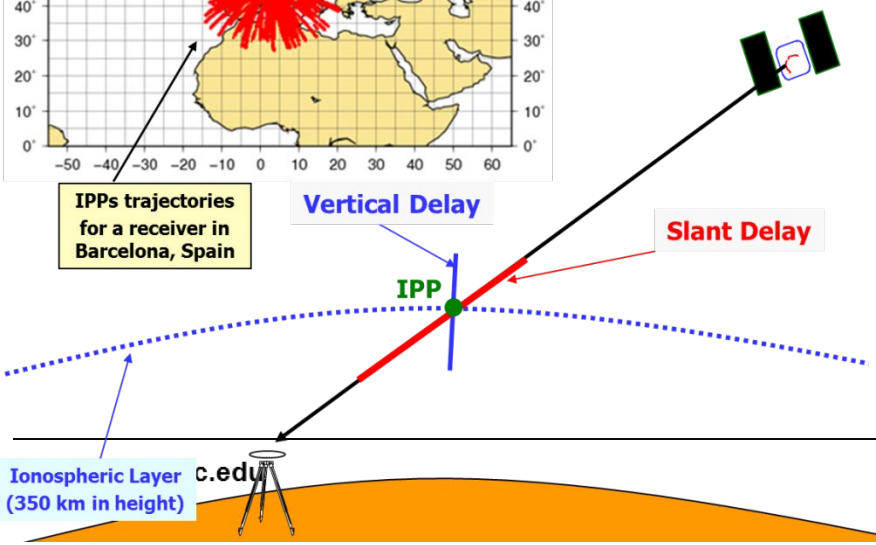


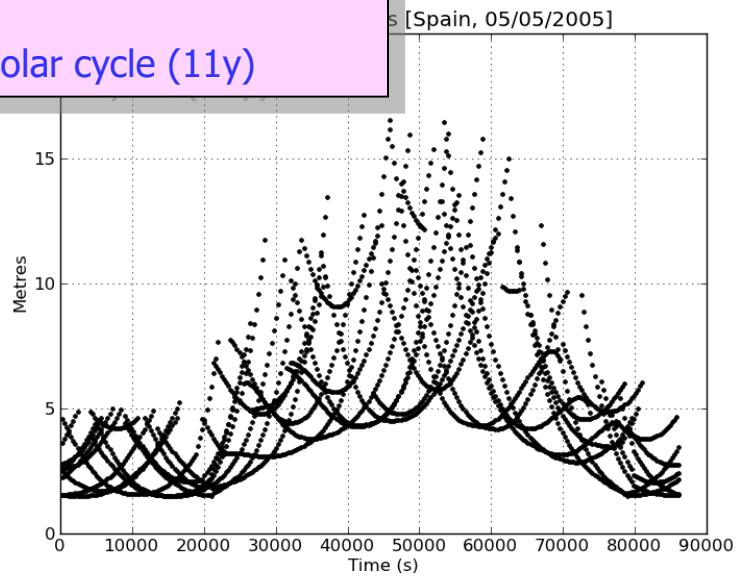
The ionospheric refraction depends on:

- Geographic location
- Time of day
- Time with respect to solar cycle (11y)



IPPs trajectories for a receiver in Barcelona, Spain





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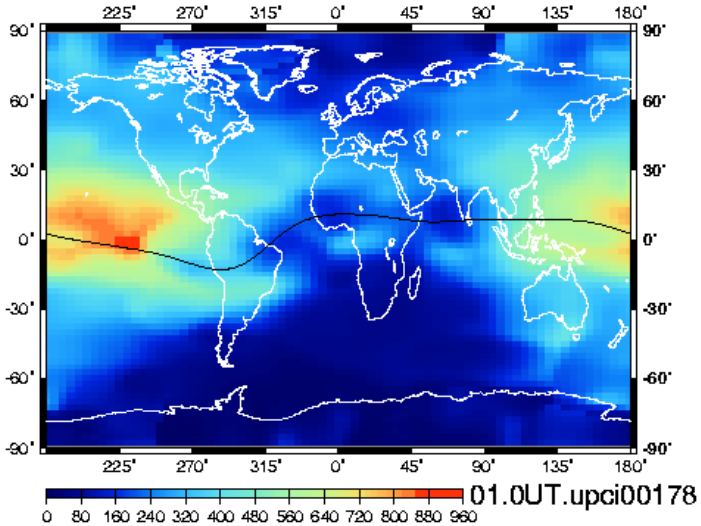
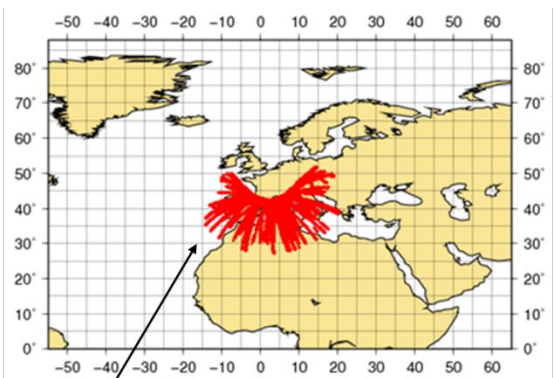
Ionospheric effects



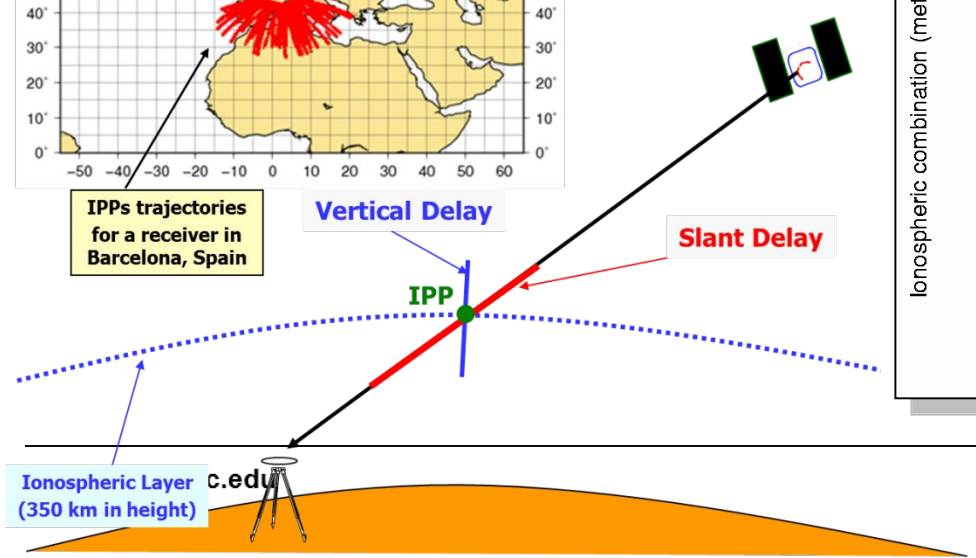
The ionospheric delay (Ion) is proportional to the electron density integrated along the ray path ($STEC$)

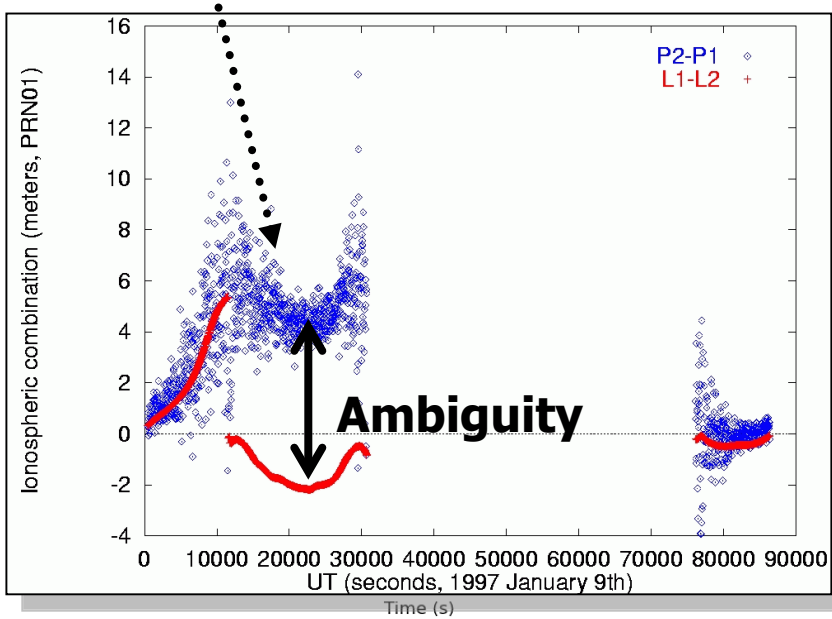
$$Ion = \frac{40.3}{f^2} STEC$$

$$STEC = \int_{\vec{r}[GPStransmitter]}^{\vec{r}[GPSreceiver]} N_e(\vec{r},t)dr$$



IPPs trajectories for a receiver in Barcelona, Spain





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2. Ionosphere-free Combination *(Pc,Lc)*

The ionospheric refraction depends on the inverse of the squared frequency and can be removed up to 99.9% combining *f1* and *f2* signals:

$$Ion = \frac{40.3}{f^2} STEC$$

$$Pc = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2}$$

$$Lc = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2}$$

$$Pc_{sta}^{sat} = \rho_{sta}^{sat} + c \cdot (dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + \varepsilon_c$$

Note: K^{sat} cancels in Pc and K_{sta} included in dt_{sta}

$$Lc_{sta}^{sat} = \rho_{sta}^{sat} + c \cdot (dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + \lambda_N w_{sta}^{sat} + b_{c,sta} + b_c^{sat} + \boxed{\lambda_N (N_{1sta}^{sat} - \tfrac{\lambda_W}{\lambda_2} N_{W\,sta}^{sat})} + \nu_c$$

$$N_W = N_1 - N_2$$

- **The ionospheric refraction has been removed in *Lc* and *Pc***

  λ_N= 10.7 cm, λ_W=86.2cm

Comments:

Two-frequency receivers are needed to apply the ionosphere-free combination.

If a single-frequency receiver is used, a ionospheric model must be applied to remove the ionospheric refraction. The GPS navigation message provides the parameters of the Klobuchar model which accounts for more than 50% (RMS) of the ionospheric delay.

3.- Narrow-lane (P_N) and Wide-lane Combination (L_W)

The wide-lane combination L_W provides a signal with a large wave-length (λ_W=86.2cm ~ 4*λ_1). This makes it very useful for detecting cycle- slips through the **Melbourne-Wübbena** combination: $MW = L_W - P_N$

$$P_N = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2} \qquad L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$$

The same iono. and sign

$$P_{N\,sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{w\,sta}^{sat} + K_{N\,sta} + K_N^{sat} + \varepsilon_N$$

$$L_{W\,sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{w\,sta}^{sat} + b_{w\,sta} + b_w^{sat} + \lambda_w N_{w,sta}^{sat} + \nu_w$$

No wind-up

$$N_W = N_1 - N_2$$

Melbourne-Wübbena combination

$$MW_{sta}^{sat} = L_{W\,sta}^{sat} - P_{N\,sta}^{sat} = b_{w\,sta} + b_w^{sat} - K_{N\,sta} - K_N^{sat} + \lambda_w N_{w,sta}^{sat} + \varepsilon_{MW}$$

Hardware biases **Integer Ambiguity**

The ambiguities N_W are INTEGER Numbers!

Exercises:

1) Consider the wide-lane combination of carrier phase measurements

$$L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$$, where L_W is given in length units (i.e. $L_i = \lambda_i \phi_i$).

Show that the corresponding wavelength is: $\lambda_W = \dfrac{c}{f_1 - f_2}$

Hint:

$L_W = \lambda_W \phi_W$; $\phi_W = \phi_1 - \phi_2$

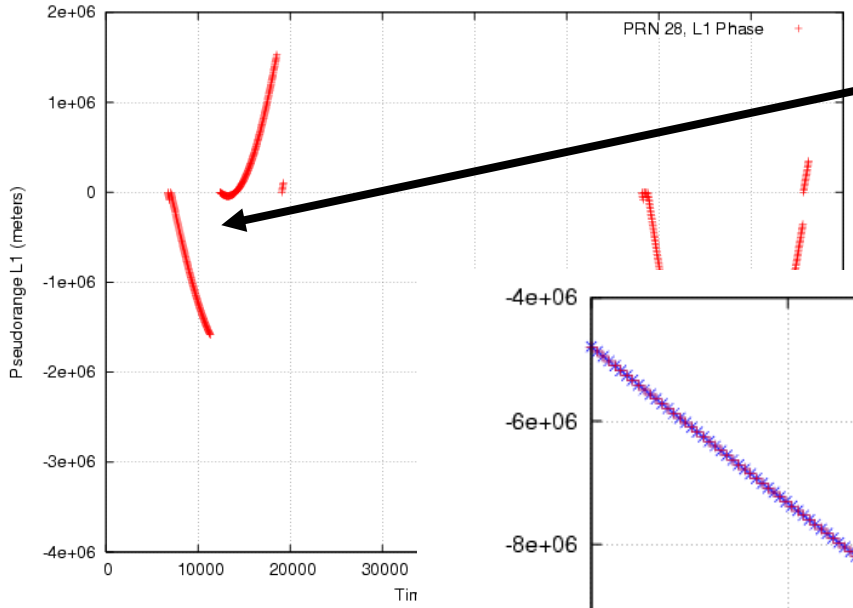
2) Assuming L_1, L_2 uncorrelated measurements with equal noise σ_L, show that:

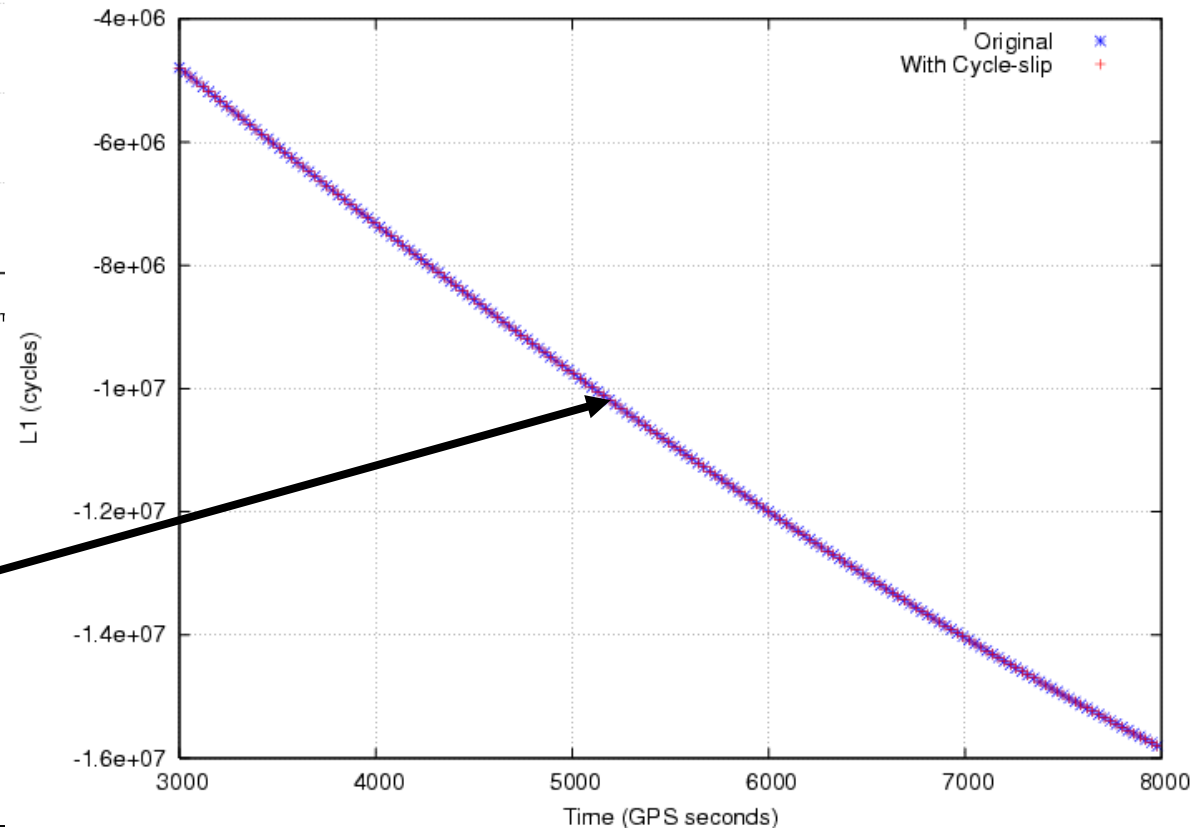
$$\sigma_{L_W} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}}-1}\sigma_L \quad ; \quad \gamma_{12} = \left(\frac{f_1}{f_2}\right)^2$$

Contents

Detecting cycle-slips

This cycle-slip involves millions of cycles ➔ it is easy to detect!!





There is a cycle-slip of only one cycle (~20cm) ➔ How to detect it?

Exercise:

a) Using the file 95oct18casa___r0.rnx, generate the "txt" file 95oct18casa.a (with data ordered in columns).

b) Insert a **cycle-slip** of "one wavelength" **(19cm) in L1** measurement at t=5000 s (and no cycle-slip in L2).

c) Plot the measurements "L1, L1-P1, LC-PC, Lw-PN and L1-L2" and discuss which combination/s should be used to detect the cycle-slip.

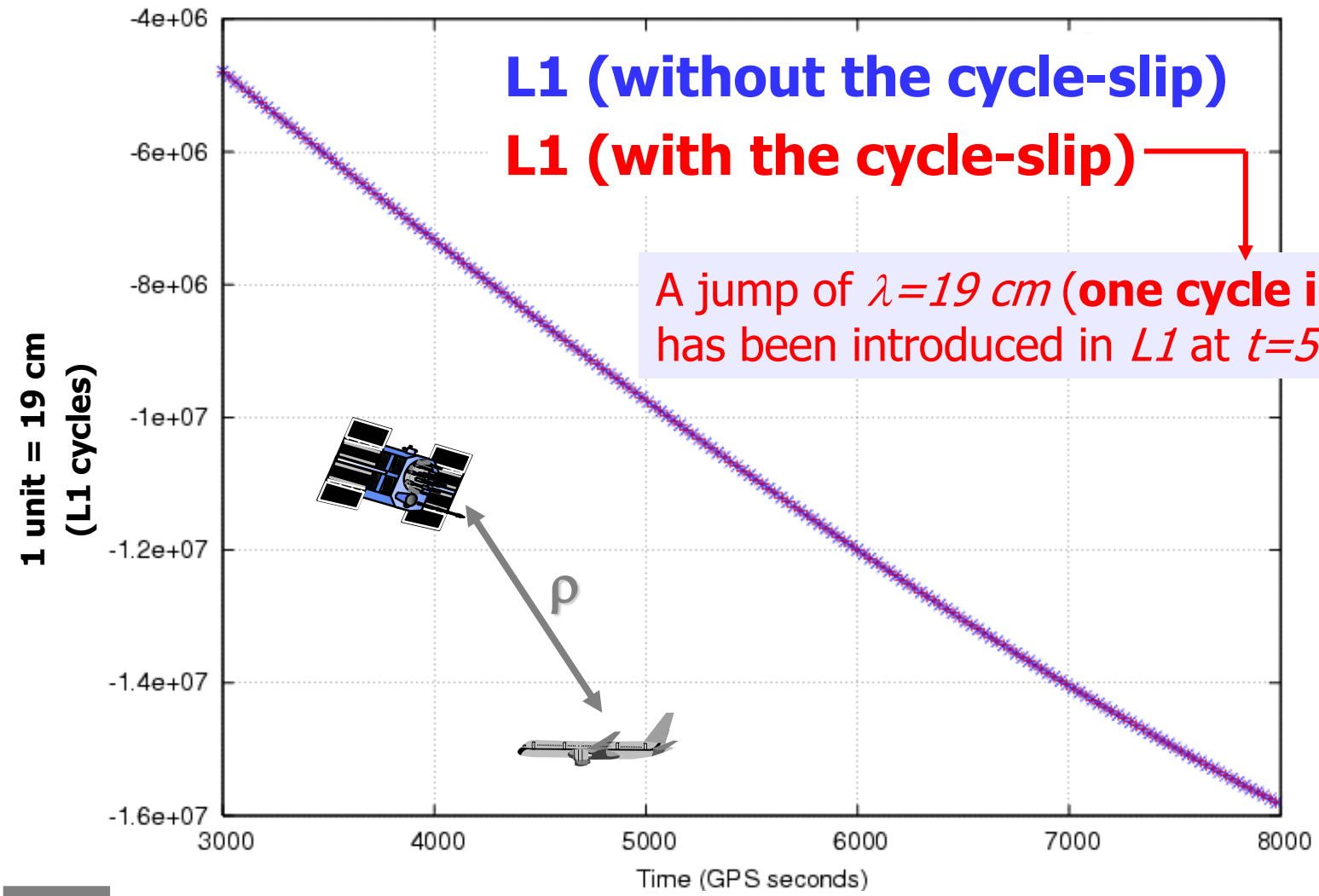
Resolution:

a)
```
gLAB_linux -input:cfg meas.cfg -input:obs 95oct18casa_r0.rnx
```

b)
```
cat 95oct18casa.a | gawk '{if ($4==18)
                                print $3,$5,$6,$7,$8}' > s18.org
cat s18.org | gawk '{if ($1>=5000) $2=$2+0.19;
  printf "%s %f %f %f %f \n", $1,$2,$3,$4,$5}' > s18.cl
```

c) See next plots:

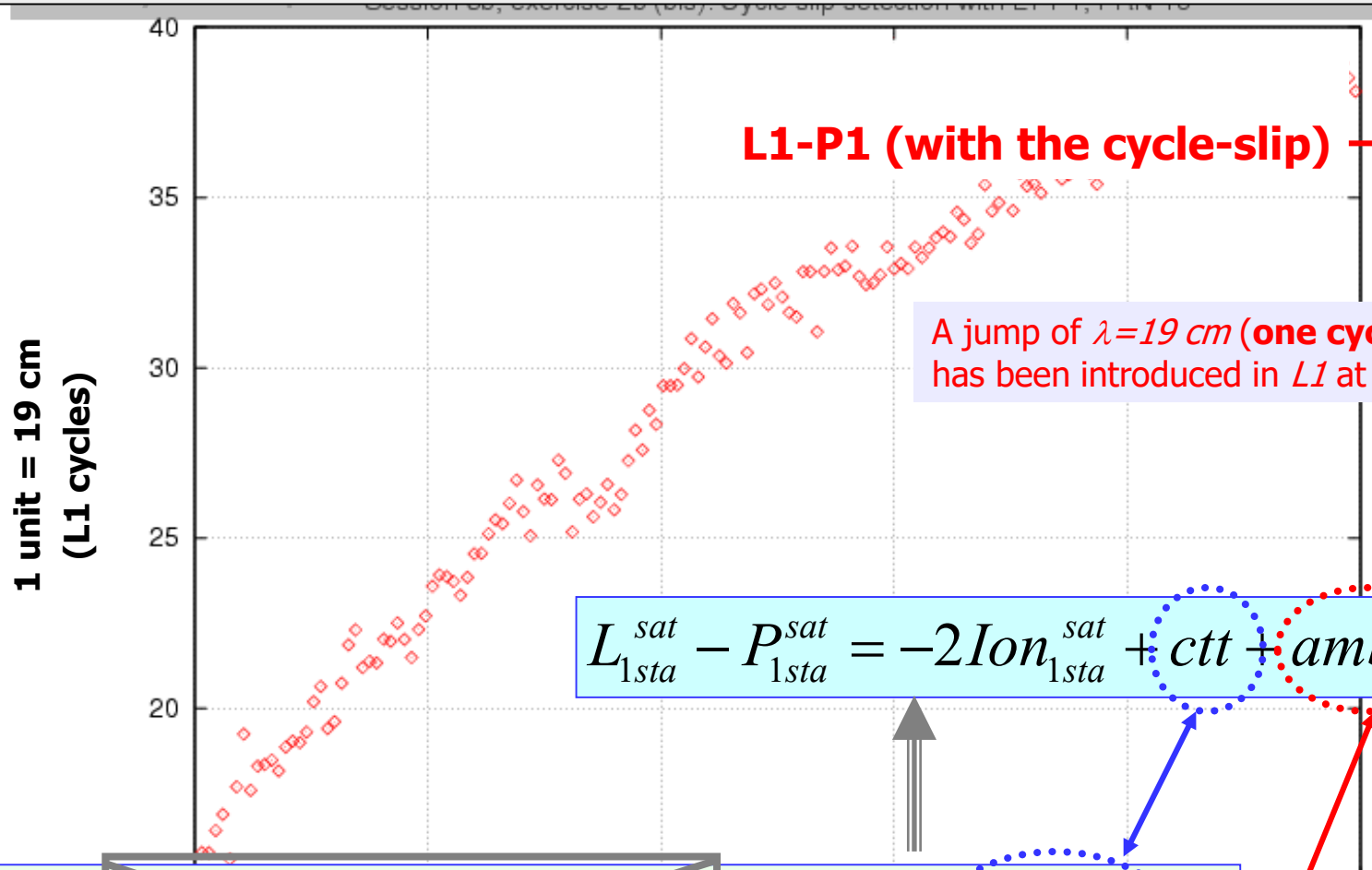
The geometry "ρ" is the dominant term in the plot. The variation of "ρ" in 1 sec may be hundreds of meters, many times greater than the cycle-slip *(19 cm)* ➔ **the variation of ρ shadows the cycle-slip!**



$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_{1sta}^{sat} + \lambda_1 w_{sta}^{sat} + \nu_1$$

The geometry and clock offsets have been removed.

The trend is due to the Ionosphere. The *P1* code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



A jump of $\lambda = 19\ cm$ (**one cycle in L1**) has been introduced in *L1* at *t=5000s*

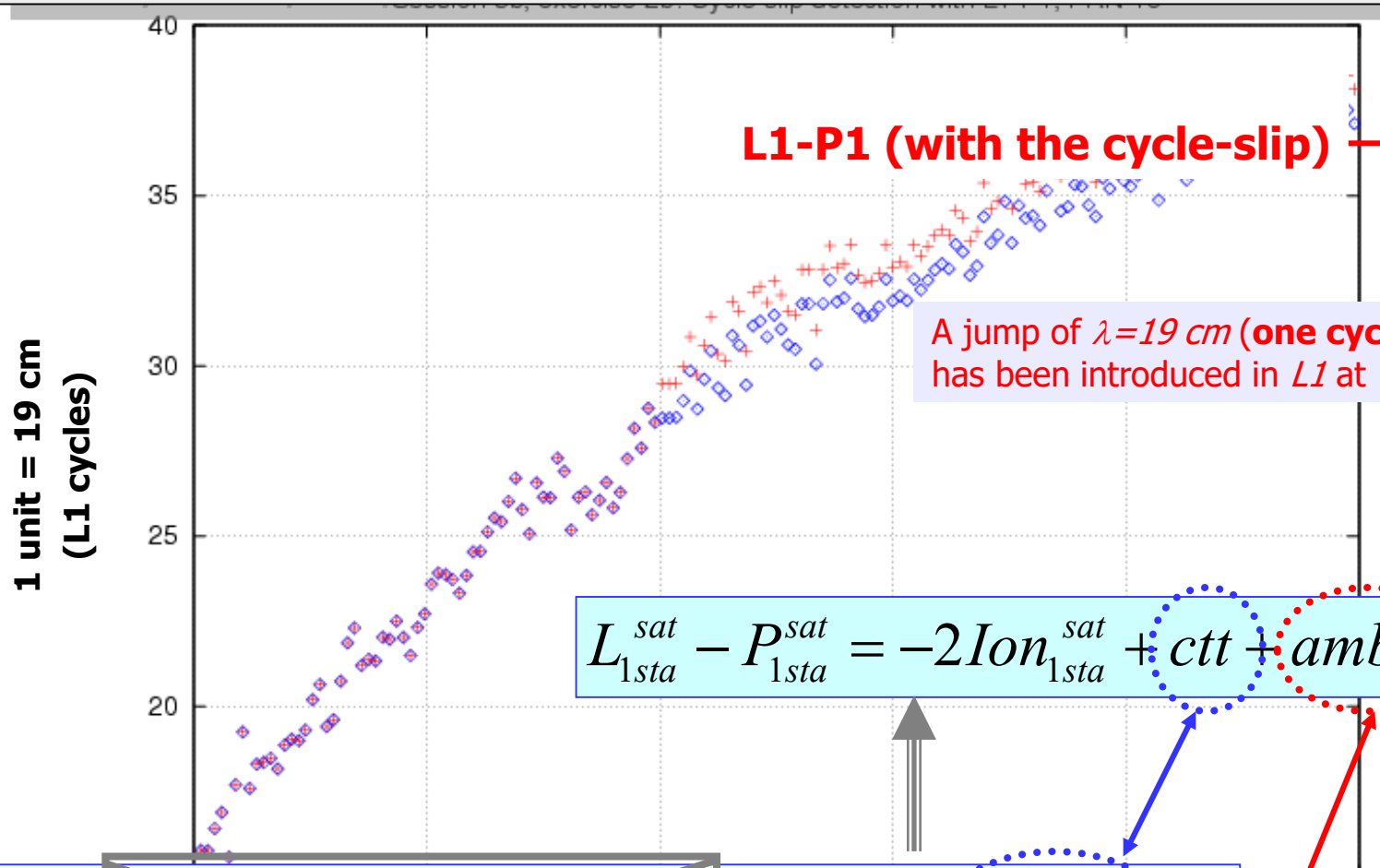
$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + \varepsilon$$

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_{1sta}^{sat} + \lambda_1 w_{sta}^{sat} + \nu_1$$

The geometry and clock offsets have been removed.

The trend is due to the Ionosphere. The *P1* code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



A jump of $\lambda = 19\ cm$ (**one cycle in L1**) has been introduced in *L1* at *t=5000s*

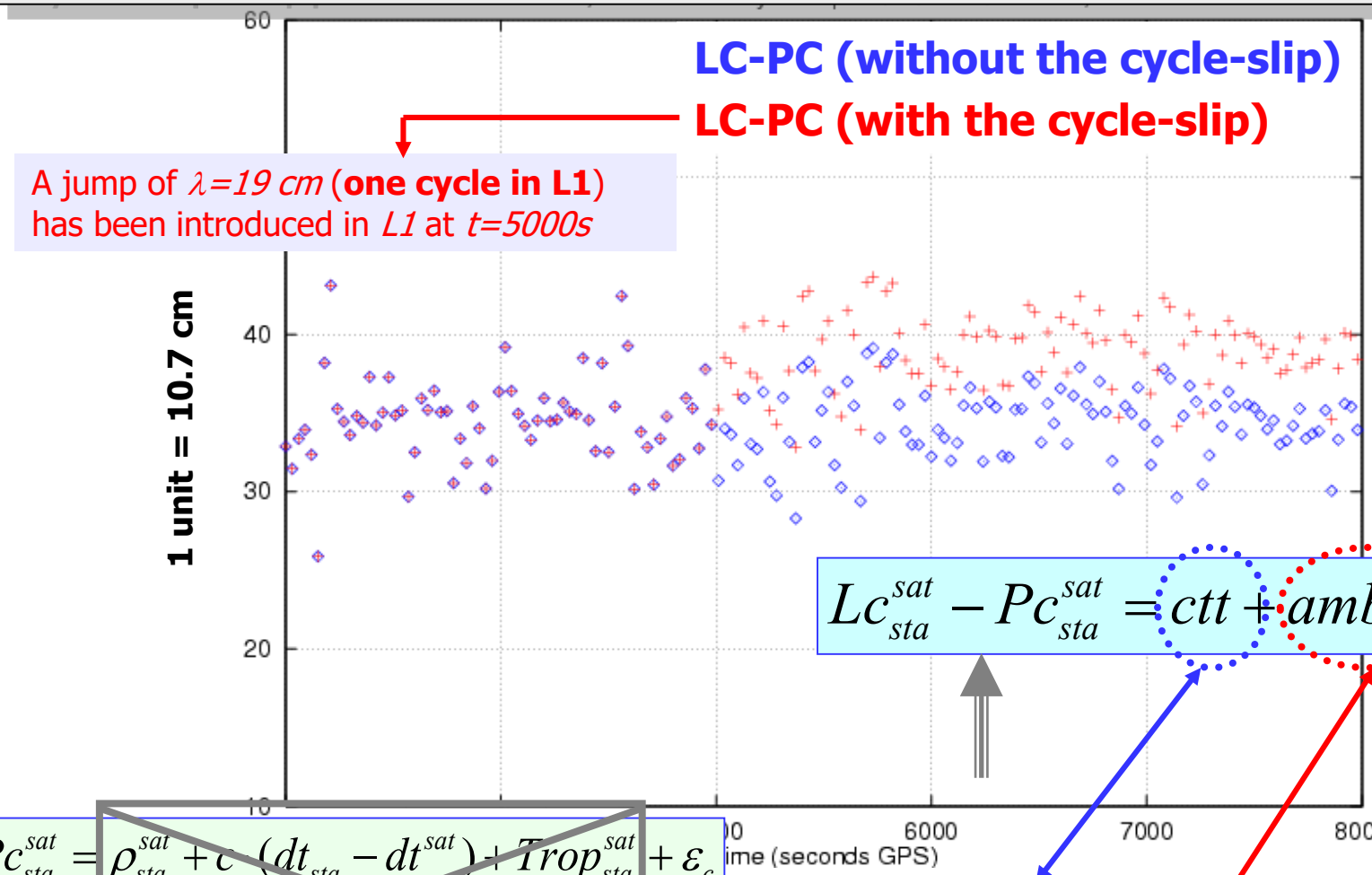
$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + \varepsilon$$

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_{1sta}^{sat} + \lambda_1 w_{sta}^{sat} + \nu_1$$

The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. The P_C code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



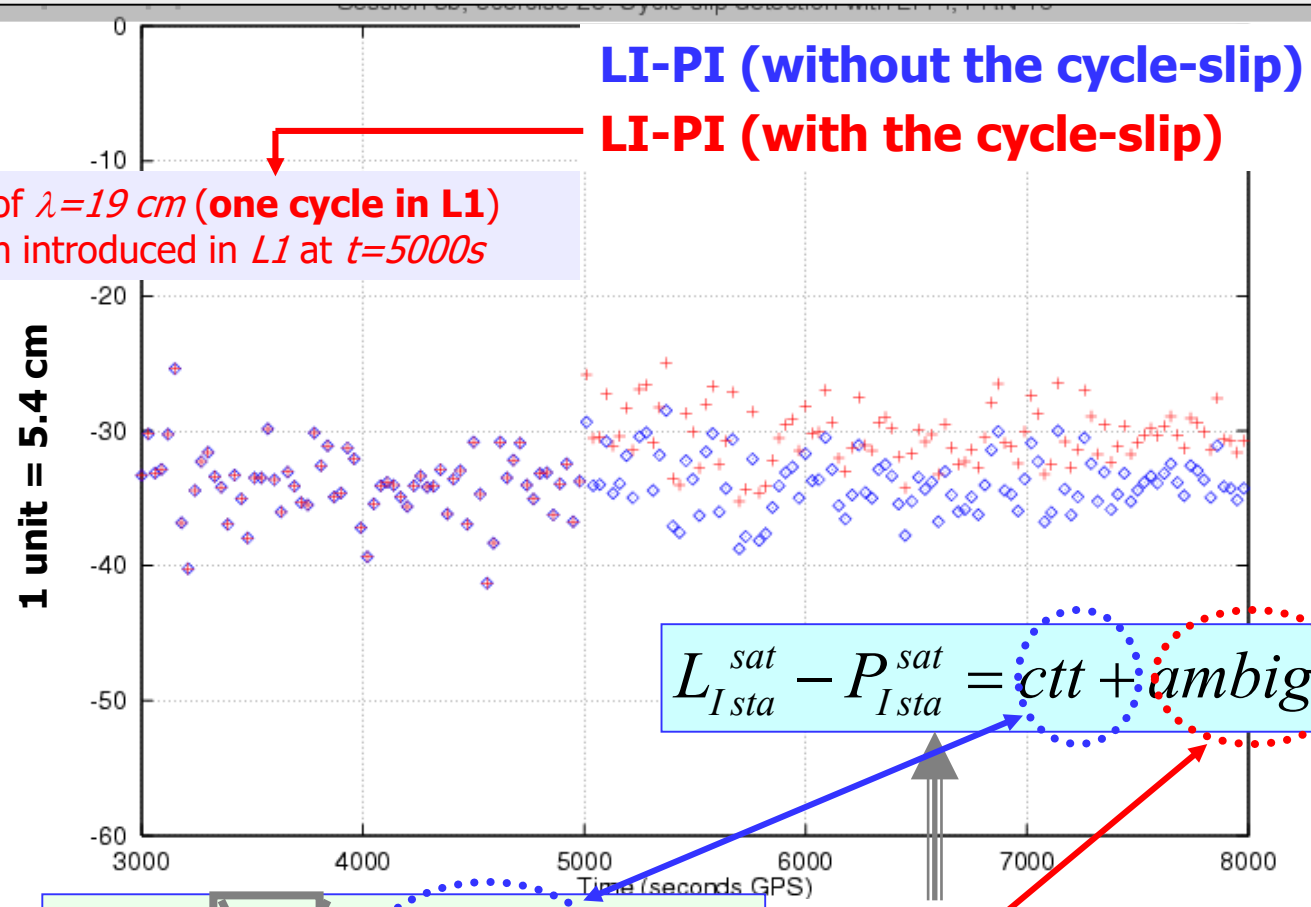
$$Lc_{sta}^{sat} - Pc_{sta}^{sat} = ctt + ambig + \varepsilon$$

$$Pc_{sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + \varepsilon_c$$

$$Lc_{sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + \lambda_N w_{sta}^{sat} + b_{c,sta} + b_c^{sat} + \lambda_N (N_{1sta}^{sat} - \frac{\lambda_W}{\lambda_2} N_{W\,sta}^{sat}) + \nu_c$$

The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. The P_I code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



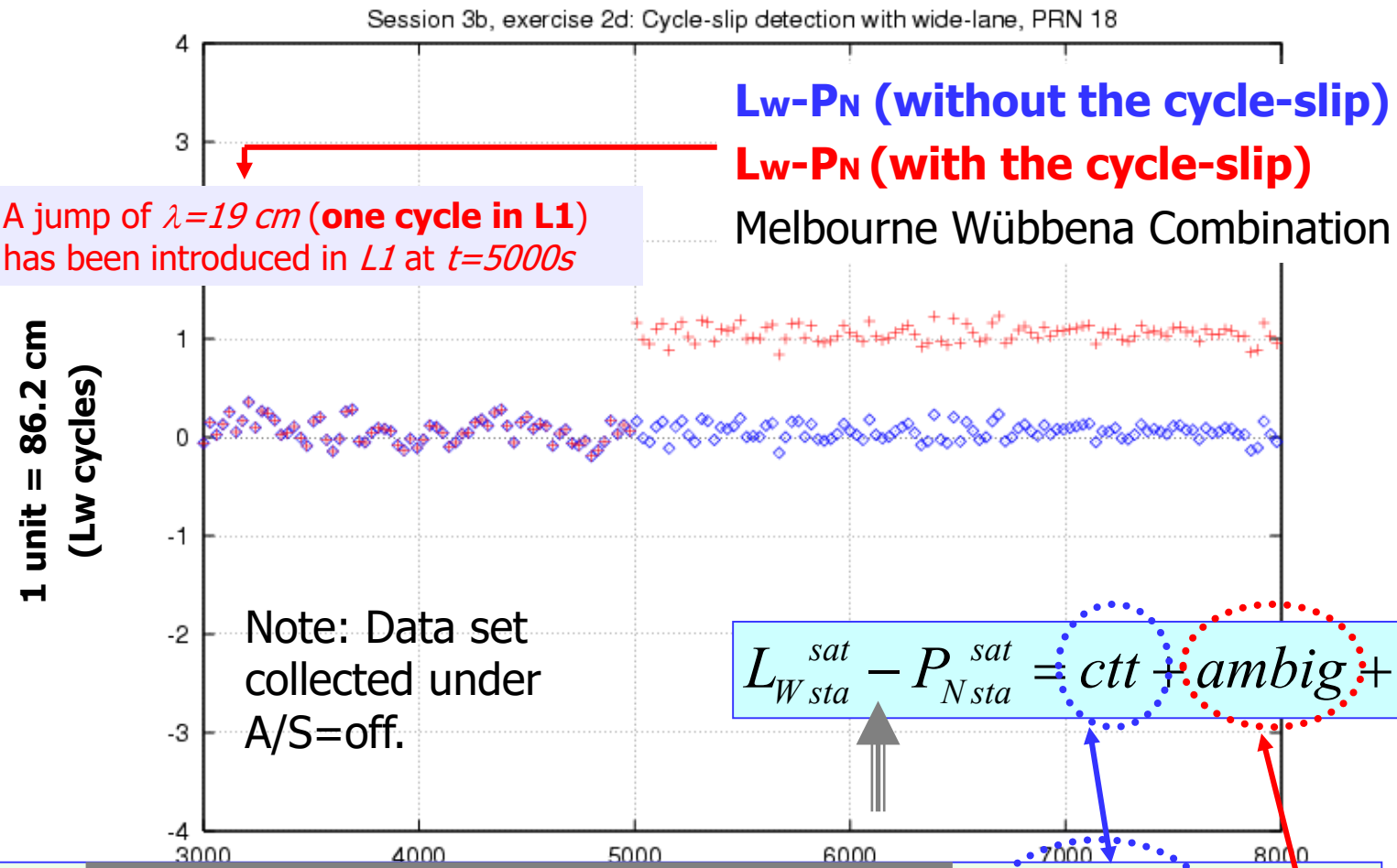
$$L_{I\,sta}^{sat} - P_{I\,sta}^{sat} = ctt + ambig + \varepsilon$$

$$P_{I\,sta}^{sat} = \cancel{Ion_I} + K_{I\,sta} + K_I^{sat} + \varepsilon_I$$

$$L_{I\,sta}^{sat} = \cancel{Ion_I} + b_{I_{sta}} + b_I^{sat} + \lambda_1 N_{1sta}^{sat} - \lambda_2 N_{2sta}^{sat} + (\lambda_1 - \lambda_2) w_{sta}^{sat} + \nu_I$$

The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. The *PN* code noise is under one cycle of *Lw*. Thence, the cycle-slip is clearly detected



$$L_{W\,sta}^{sat} - P_{N\,sta}^{sat} = ctt + ambig + \varepsilon$$

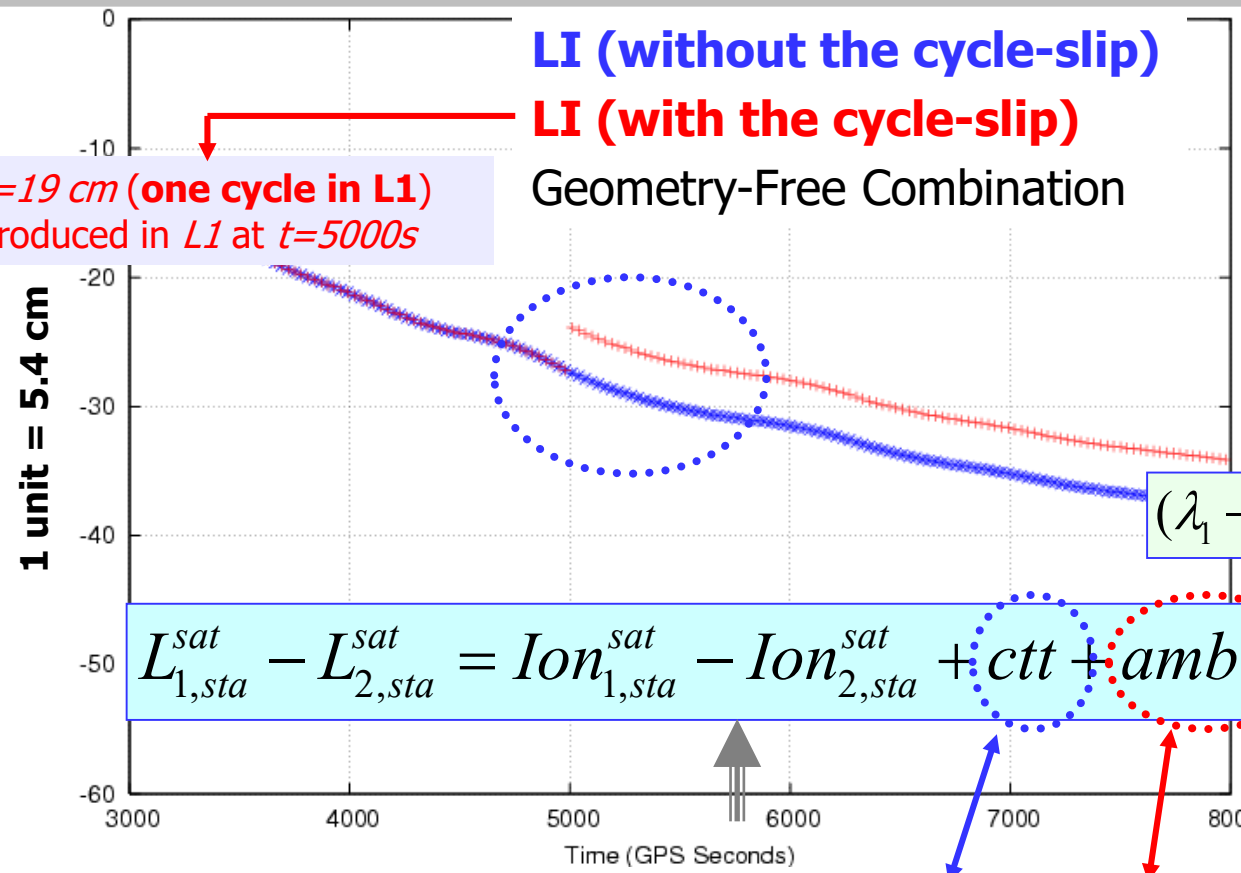
$$P_{N\,sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{w\,sta}^{sat} + K_{w\,sta} + K_{w}^{sat} + \varepsilon_N$$

$$L_{W\,sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{w\,sta}^{sat} + b_{w\,sta} + b_{w}^{sat} + \lambda_{w} N_{w\,sta}^{sat} + \nu_{w}$$

The geometry and clock offsets have been removed.

The trend is due to the Iono. The L_1 carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19\ cm$

Thence, the cycle-slip is detected.



$\varepsilon \sim mm$

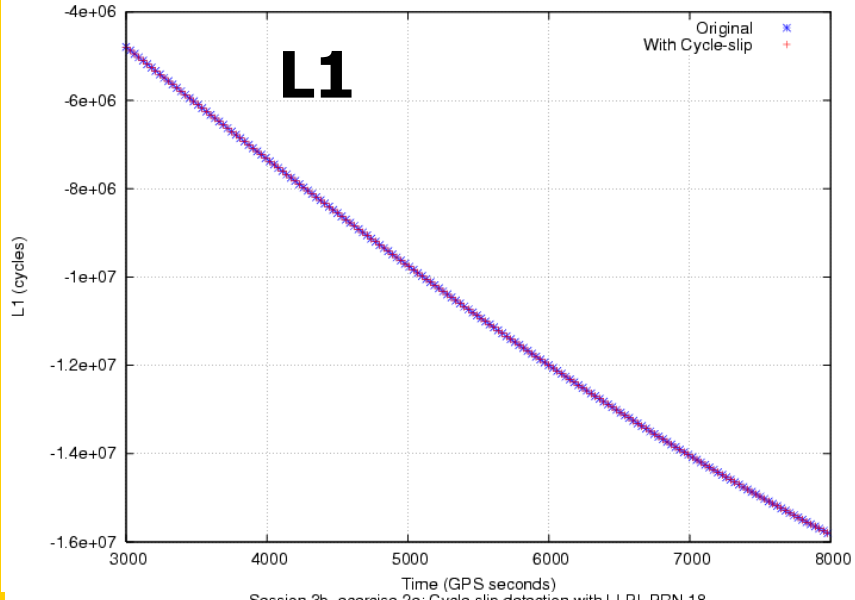
$(\lambda_1 - \lambda_2)w <<$

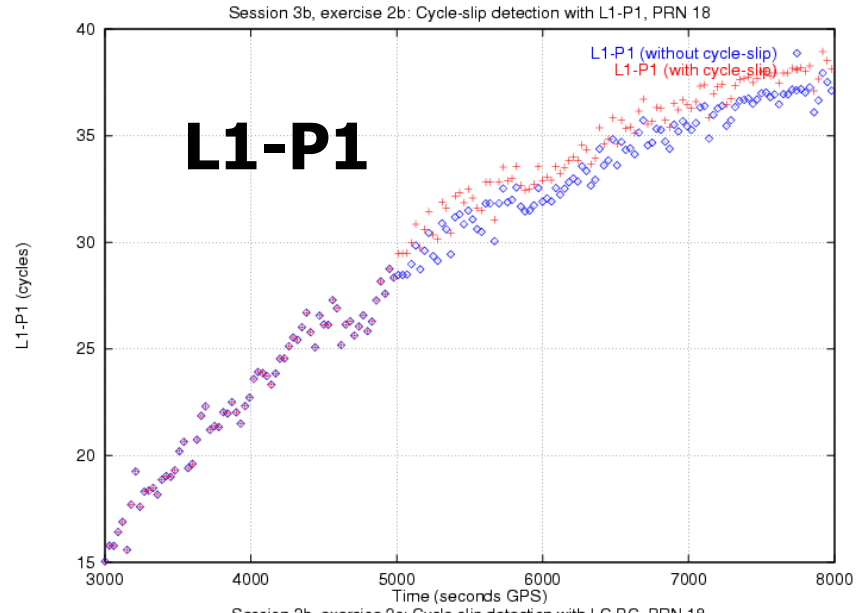
$$L^{sat}_{1,sta} - L^{sat}_{2,sta} = Ion^{sat}_{1,sta} - Ion^{sat}_{2,sta} + ctt + ambig + \varepsilon$$

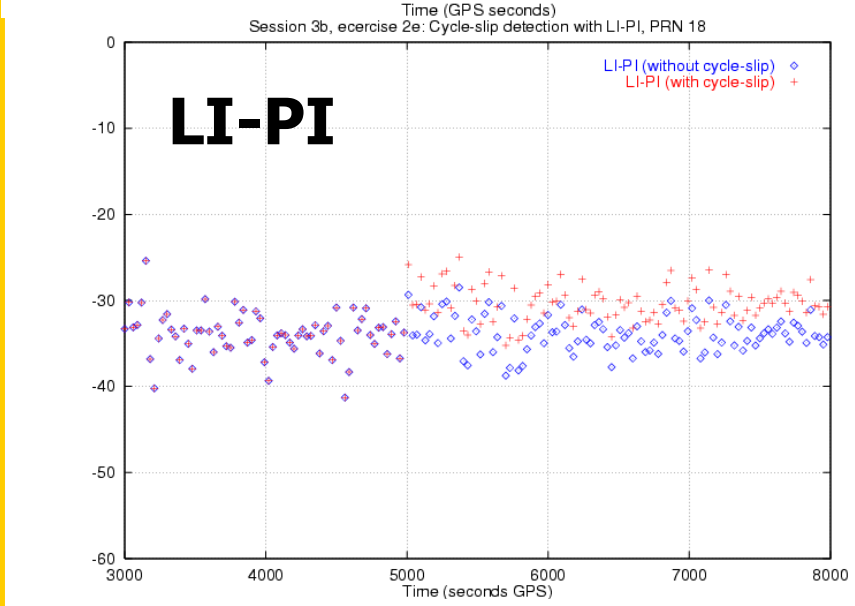
$$L^{sat}_{1sta} = \cancel{\rho^{sat}_{sta} + c\cdot(dt_{sta} - dt^{sat}) + Trop^{sat}_{sta}} - Ion^{sat}_{1sta} + b_{1sta} + b_1^{sat} + \lambda_1 N^{sat}_{1sta} + \lambda_1 w^{sat}_{sta} + \nu_1$$

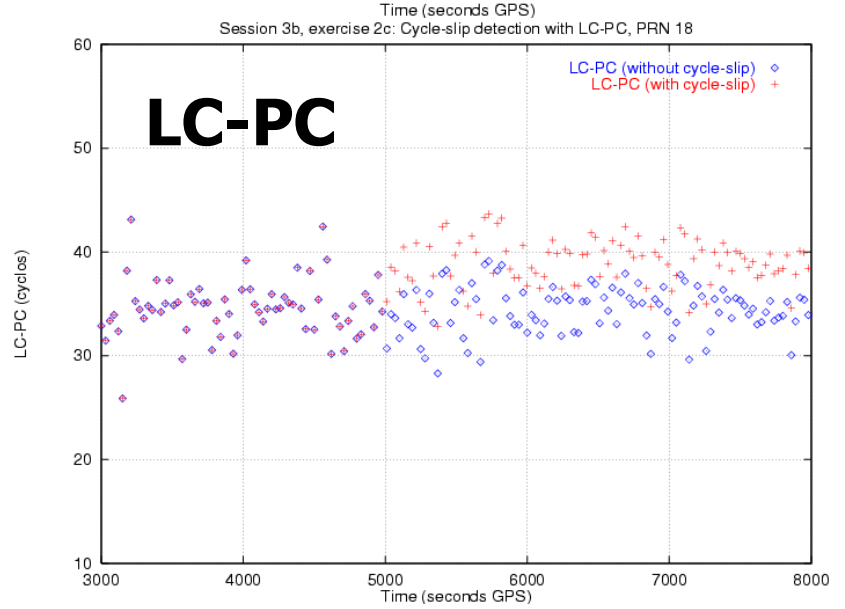
$$L^{sat}_{2sta} = \cancel{\rho^{sat}_{sta} + c\cdot(dt_{sta} - dt^{sat}) + Trop^{sat}_{sta}} - Ion^{sat}_{2sta} + b_{2sta} + b_2^{sat} + \lambda_2 N^{sat}_{2sta} + \lambda_2 w^{sat}_{sta} + \nu_2$$

Summary

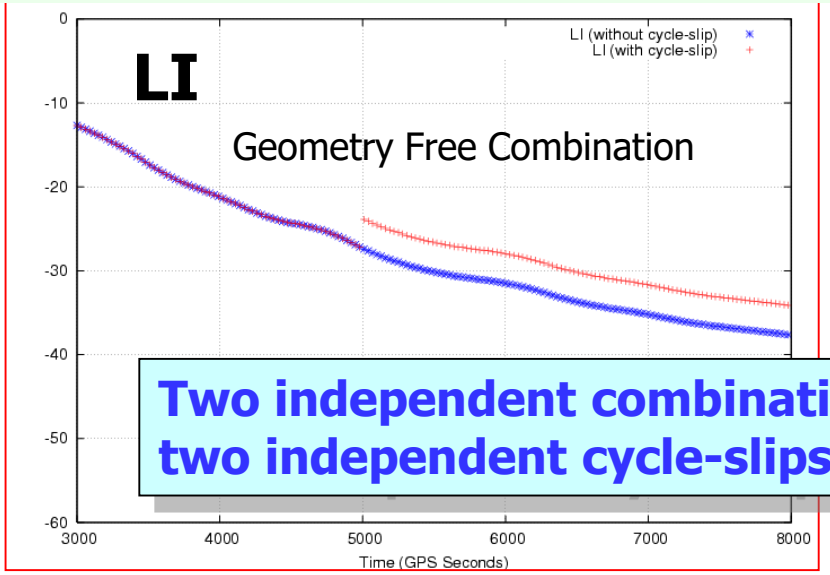

L1
Original
With Cycle-slip
L1 (cycles)
Time (GPS seconds)

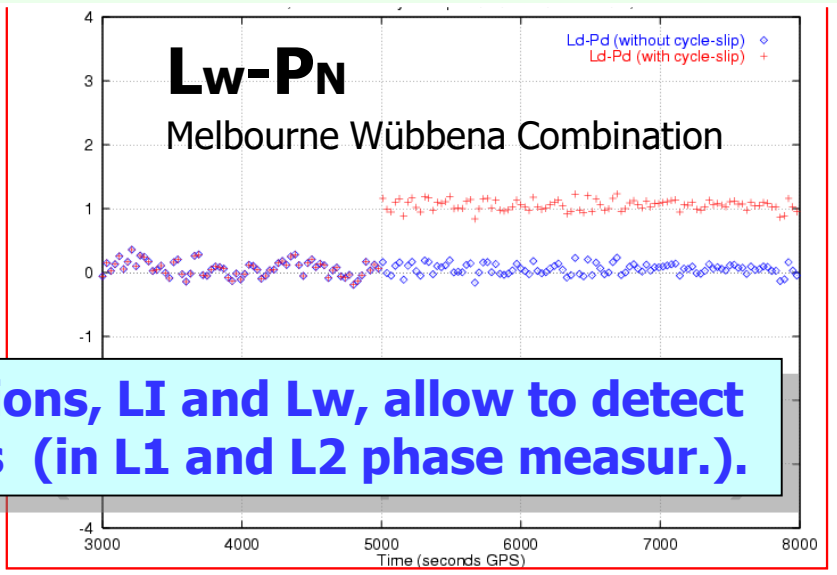

Session 3b, exercise 2b: Cycle-slip detection with L1-P1, PRN 18
L1-P1
L1-P1 (without cycle-slip)
L1-P1 (with cycle-slip)
L1-P1 (cycles)
Time (seconds GPS)


Session 3b, ecercise 2e: Cycle-slip detection with LI-PI, PRN 18
LI-PI
LI-PI (without cycle-slip)
LI-PI (with cycle-slip)
Time (seconds GPS)

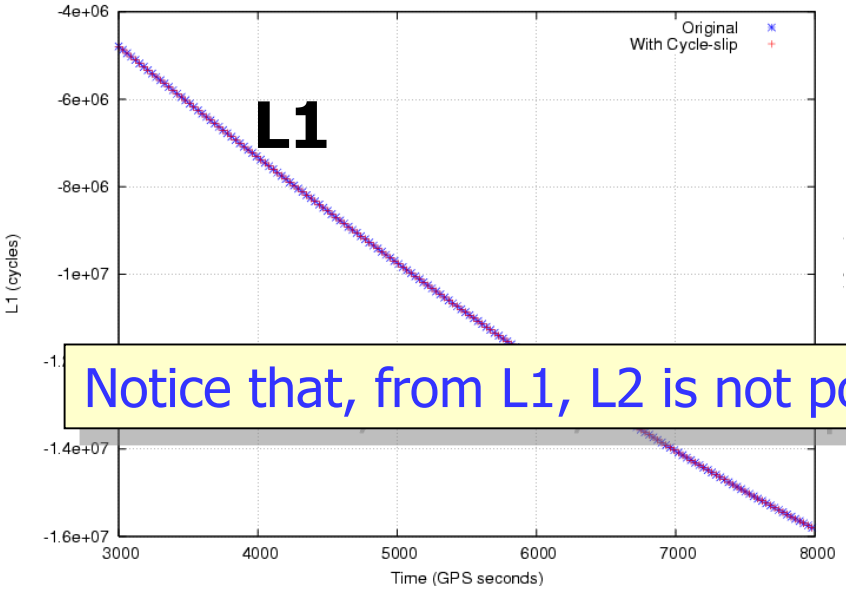

Session 3b, exercise 2c: Cycle-slip detection with LC-PC, PRN 18
LC-PC
LC-PC (without cycle-slip)
LC-PC (with cycle-slip)
LC-PC (cycles)
Time (seconds GPS)

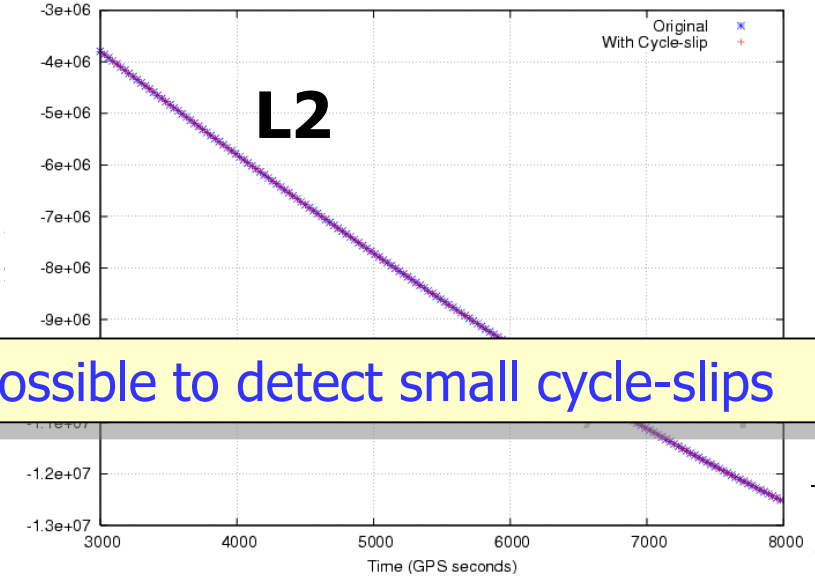
The cycle-slips are detected by the Ionospheric combination (LI=L1-L2) and the Melbourne Wübbena (MW=L_W-P_N)





Two independent combinations, LI and Lw, allow to detect two independent cycle-slips (in L1 and L2 phase measur.).





Notice that, from L1, L2 is not possible to detect small cycle-slips

Summary of Cycle-slip detectors

COMBINATION	MEAS	Combination Noise (σ)	λ	σ/λ
L_1-P_1	$-2 \cdot Ion + K + \lambda_1 \cdot N_1$	$\sigma_{L1-P1} \approx \cdot \sigma_{P1} = 30\ cm$	$\lambda_1 = 19.0\ cm$	1.58
L_c-P_c	$k_c + \lambda_N \cdot R_c$	$\sigma_{LC-PC} \approx \cdot \sigma_{PC} = 2.98\ \sigma_P = 89\ cm$	$\lambda_N = 10.7\ cm$	8.32
L_I-P_I	$k_I + \lambda_1 \cdot N_1 - \lambda_2 \cdot N_2$	$\sigma_{LI-PI} \approx \cdot \sigma_{PI} = \sqrt{2}\ \sigma_P = 42\ cm$	$\lambda_2 - \lambda_1 = 5.4\ cm$	7.78
L_W-P_N	$\lambda_W \cdot N_w$	$\sigma_{LW-PN} \approx \cdot \sigma_{PN} = \sigma_P / \sqrt{2} = 21\ cm$	$\lambda_W = 86.2\ cm$	0.25
L_I	$Ion + k_I + \lambda_1 \cdot N_1 - \lambda_2 \cdot N_2$	$\sigma_{LI} = \sqrt{2}\ \sigma_{L1} = 3\ mm$	$\lambda_2 - \lambda_1 = 5.4\ cm$	0.06

Contents

Cycle-slip detector based on carrier phase data: The Geometry-free combination

Input data: Geometry-free combination of carrier phase measurements

$$L_I = L_1(s;k) - L_2(s;k)$$

Output: [satellite, time, cycle-slip flag].

The detection is based on fitting a second order polynomial over a sliding window of N_I samples.

The predicted value is compared with the observed one to detect cycle-slip.

For each epoch (k)

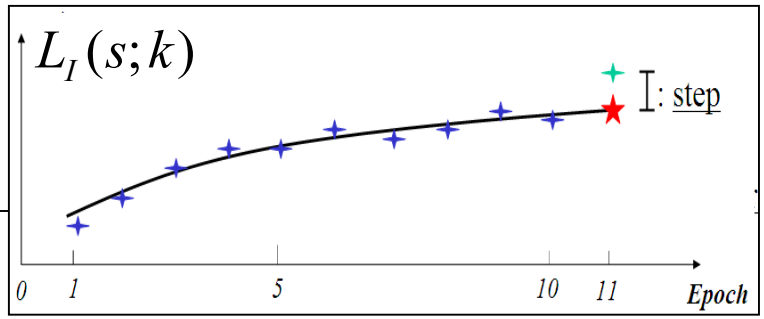
For each tracked satellite (s)

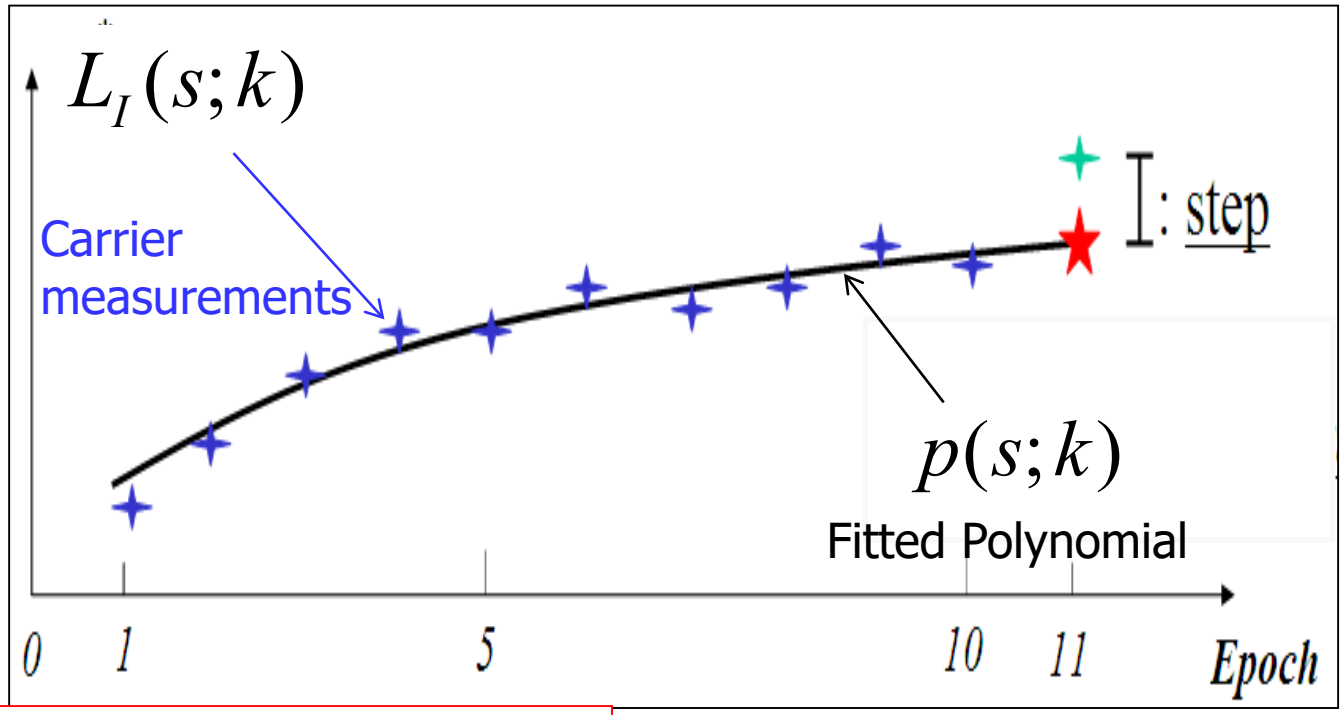
- Declare cycle slip when data gap greater than $tol_{\Delta t}$.[16]
- Fit a second-degree polynomial $p(s;k)$ to the previous values (after the last cycle-slip) $\left[L_I(s;k-N_I),\ldots,L_I(s;k-1)\right]$.
- Compare the measured $L_I(s;k)$ and the predicted value $p(s;k)$ at epoch k. If the discrepancy exceeds a given *threshold*, then declare cycle slip. That is,

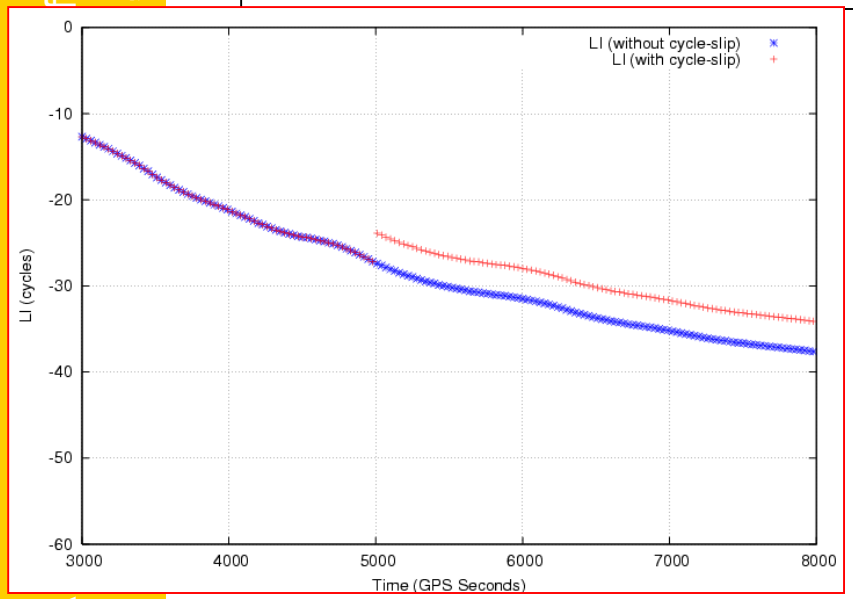
  if $\left|L_I(s;k) - p(s;k)\right| > threshold$ then cycle slip.
- Reset algorithm after cycle slip.

End

End







Under not disturbed ionospheric conditions, the geometry-free combination performs as a **very precise and smooth test signal, driven by the ionospheric refraction.**

Although, for instance, the jump produced by a simultaneous one-cycle slip in both signals is smaller in this combination than in the original signals (λ_2-λ_1=5.4cm), it can provide reliable detection even for small jumps

Cycle-slip detector based on code and carrier phase data: The Melbourne-Wübbena combination

Input data: Melbourne-Wübbena combination $B_W = L_W - P_N = \lambda_W N_W + \varepsilon$

Output: [satellite (PRN), time, cycle-slip flag]

The detection is based on real-time computation of mean (m_{BW}) and sigma (S_{BW}) values of the measurement test data Bw.

For each epoch (k)

For each tracked satellite (s)

- Declare cycle-slip when data hole greater than $tol_{\Delta t}$ (e.g., $60\,s$).
- If no data hole larger than $tol_{\Delta t}$, thence:
- Compare the measurement $B_W(s;k)$ at the epoch k with the mean bias $m_{B_W}(s;k-1)$ computed from the previous values. If the discrepancy is over a $threshod = K_{factor} * S_{B_W}$ (e.g., $K_{factor} = 4$), declare cycle-slip. That is:

  If $|B_W(s;k) - m_{B_W}(s;k-1)| > K_{factor}\, S_{B_W}(s;k-1)$, Thence, cycle-slip.

- Update the mean and sigma values according to the equations:

$$m_{B_W}(s;k) = \frac{k-1}{k} m_{B_W}(s;k-1) + \frac{1}{k} B_W(s;k)$$

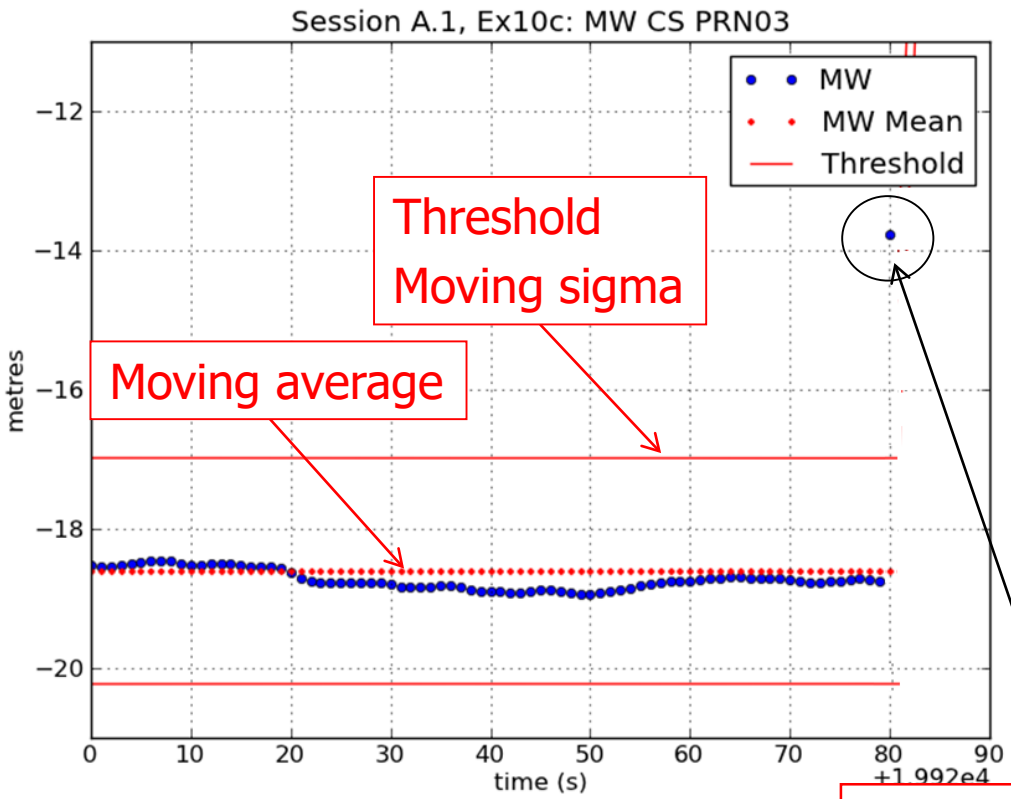
$$S^2_{B_W}(s;k) = \frac{k-1}{k} S^2_{B_W}(s;k-1) + \frac{1}{k}\left(B_W(s;k) - m_{B_W}(s;k-1)\right)^2 \quad (4.24)$$

Note the S_{B_W} is initialised with an a priori $S_0 = \lambda_w/2$.

A cycle-slip is declared when the measurement differs form the mean value by a predefined number of standard deviations (S_{BW})

End

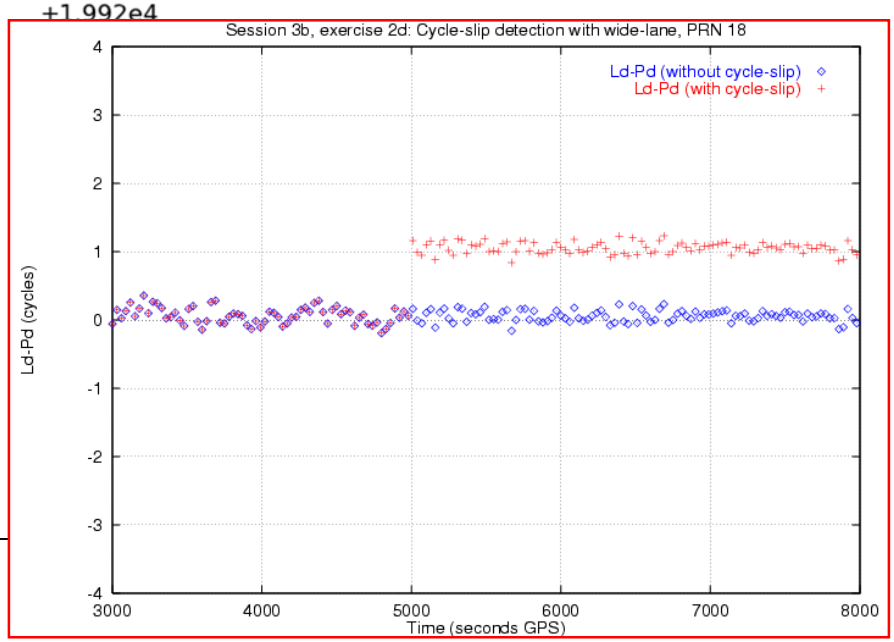
End



The Melbourne-Wübbena combination has a double benefit:

- The enlargement of the ambiguity spacing, thanks to the larger wavelength $\lambda_W = 86.2cm$.
- The noise is reduced by the narrow-lane combination of code measurement

Cycle-slip detection



Nevertheless, in spite of these benefits, the performance is worse than in the previous carrier-phase-only based detector and it is used as a secondary test.

Exercises:

1) Show that $\Delta N_1 = 9$ and $\Delta N_2 = 7$ produces jumps of few millimetres in the geometry-free combination.

2) Show that no jump happens in the geometry-free combination when $\Delta N_1 / \Delta N_2 = 77/60$. In particular when $\Delta N_1 = 77$ and $\Delta N_2 = 60$ the jump in the wide-lane combination is: $17\lambda_W = 15m$

Hint: Consider the following relationships (from [RD-1]):

The effect of a jump in the integer ambiguities in terms of ΔN_1, ΔN_2 and N_W is given next:

$\Delta\Phi_W$, $\Delta\Phi_I$, $\Delta\Phi_C$ variations
$\Delta\Phi_W = \lambda_W \Delta N_W = \lambda_W (\Delta N_1 - \Delta N_2)$
$\Delta\Phi_I = \lambda_1 \Delta N_1 - \lambda_2 \Delta N_2 = (\lambda_2 - \lambda_1)\Delta N_1 + \lambda_2 \Delta N_W$
$\Delta\Phi_C = \lambda_N \left(\frac{\lambda_W}{\lambda_1}\Delta N_1 - \frac{\lambda_W}{\lambda_2}\Delta N_2\right) = \lambda_N \left(\Delta N_1 + \frac{\lambda_W}{\lambda_2}\Delta N_W\right)$

(4.20)

Example of Single frequency Cycle-slip detector

Input data: Code pseudorange (P_1) and carrier phase (L_1) measurements.

Output: [satellite (PRN), time, cycle-slip flag]

For each epoch (k)

For each tracked satellite (s)

- Declare cycle-slip when data hole greater than $tol_{\Delta t}$[24].
- If no data hole larger than $tol_{\Delta t}$, thence:
- Update an array with the last N differences of

$$\boxed{d(s;k) = L_1(s;k) - P_1(s;k)}$$

That is: $[d(s;k-N), \ldots, d(s;k-1)]$

- Compute the mean and sigma discrepancy over the previous N samples $[k-N, \ldots, k-1]$:

$$m_d(s;k-1) = \frac{1}{N}\sum_{i=1}^{N} d(s;k-i)$$
$$m_{d^2}(s;k-1) = \frac{1}{N}\sum_{i=1}^{N} d^2(s;k-i) \qquad (4.27)$$
$$S_d(s;k-1) = \sqrt{m_{d^2}(s;k-1) - m_d^2(s;k-1)}$$

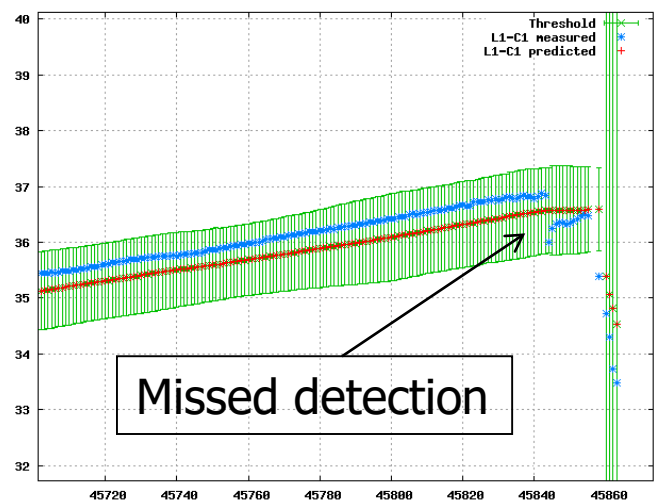
- Compare the difference at the epoch k with the mean value of differences computed over the previous N samples window. If the value is over a $threshod = n_T * S_d$ (e.g., $n_T = 5$), declare cycle-slip[25].
  That is:
  If $|d(s;k) - m_d(s;k-1)| > n_T\, S_d(s;k-1)$,
  Thence, cycle-slip.

End

End

The detection is based on real-time computation of mean and sigma values of the differences ($d=L_1-P_1$) of the code pseudorange and carrier over a sliding window of N samples (e.g. $N=100$).

A cycle-slip is declared when a measurement differs from the mean bias value over a predefined threshold.



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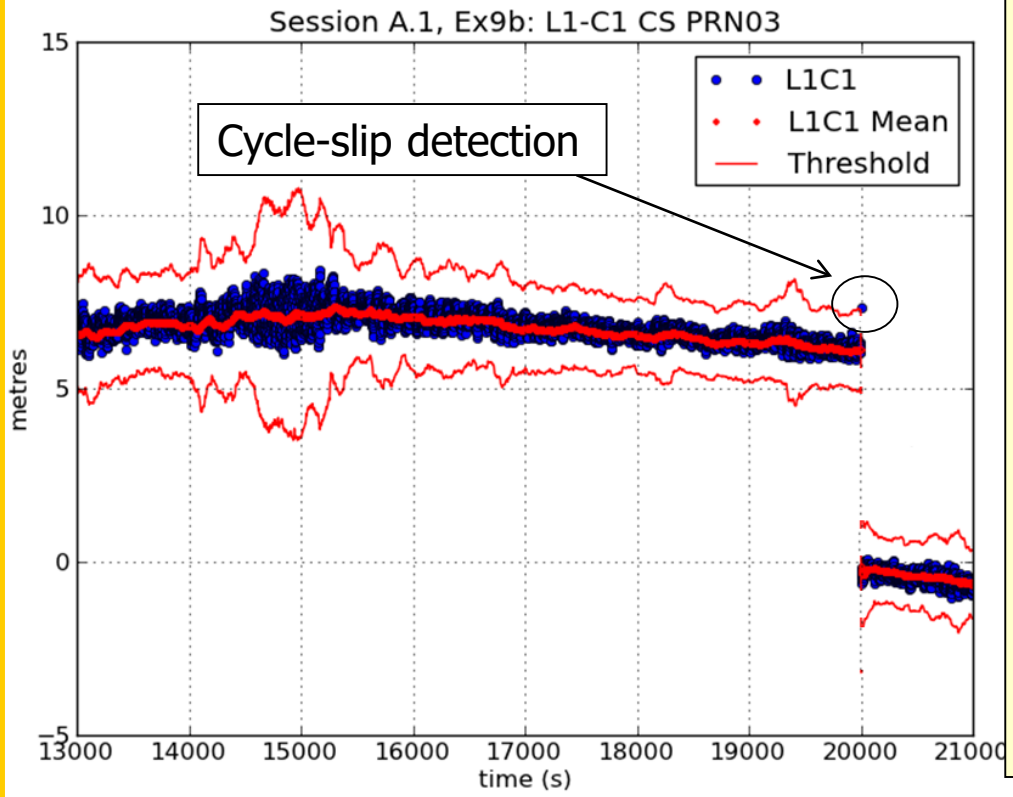
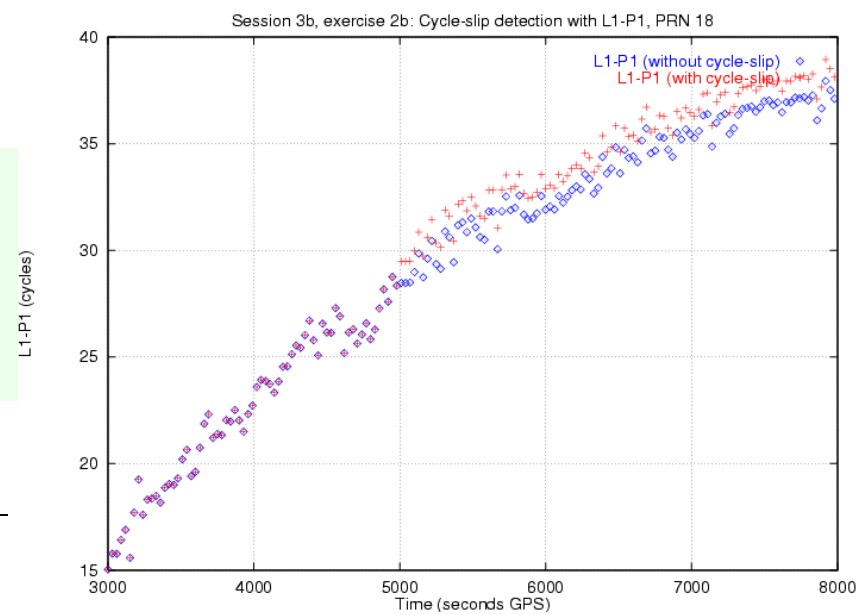


This detector is affected by the code pseudorange noise and multipath as well as the divergence of the ionosphere.

Higher sampling rate improves detection performance, but smalest jumps can still escape from this detector.

On the other hand, a minimum number of samples is needed for filter initialization in order to ensure a reliable value of sigma for the detection threshold

More details, exercises and examples of software code implementation of these detectors can be found in [RD-1] and [RD-2].



Contents

Carrier smoothing of code pseudorange

The noisy (but unambiguous) code pseudorange can be smoothed with the precise (but ambiguous) carrier. A simple algorithm is given next:

Hatch filter:

$$\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right)$$

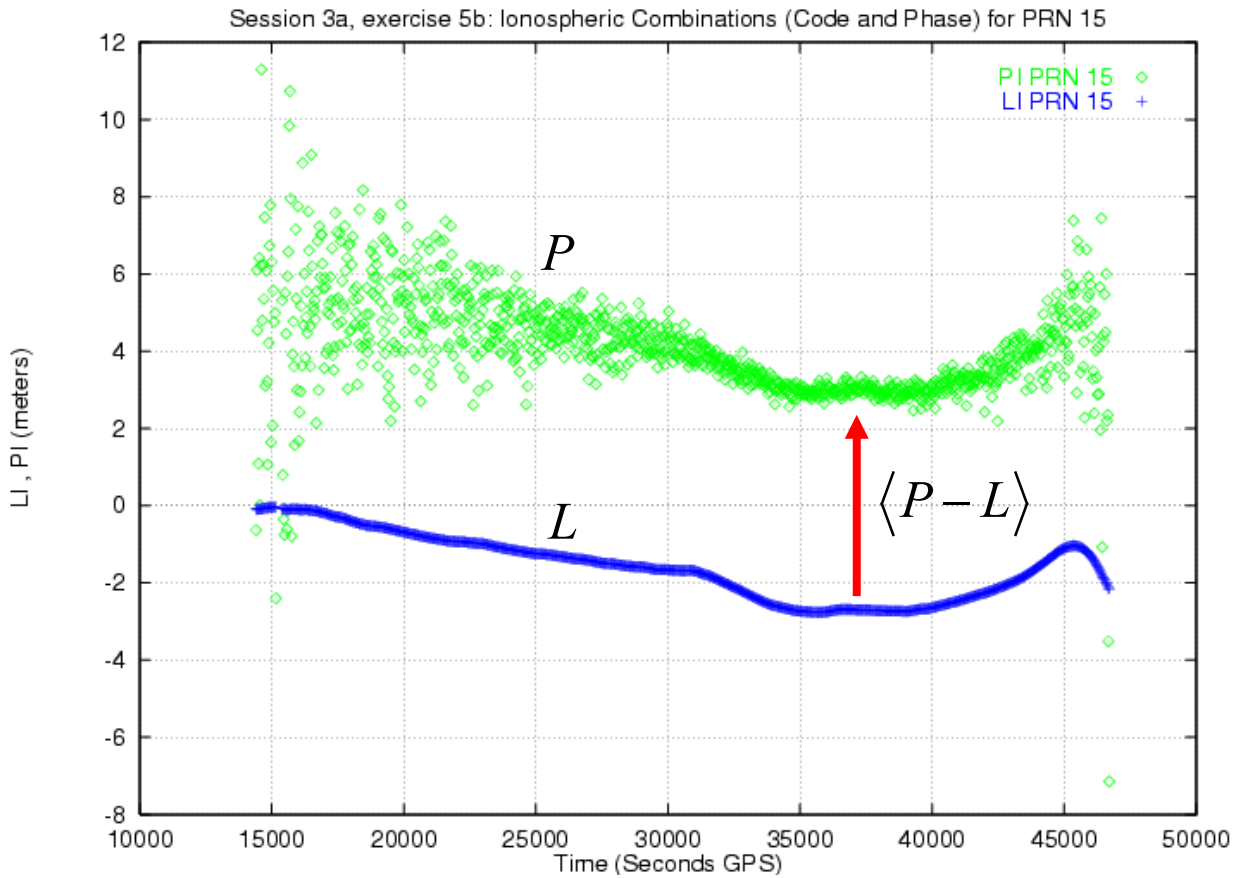
$$\text{where} \quad \hat{P}(1) = P(1) \text{ and}$$

$$n = k; \quad k < N$$

$$n = N; \quad k \geq N$$

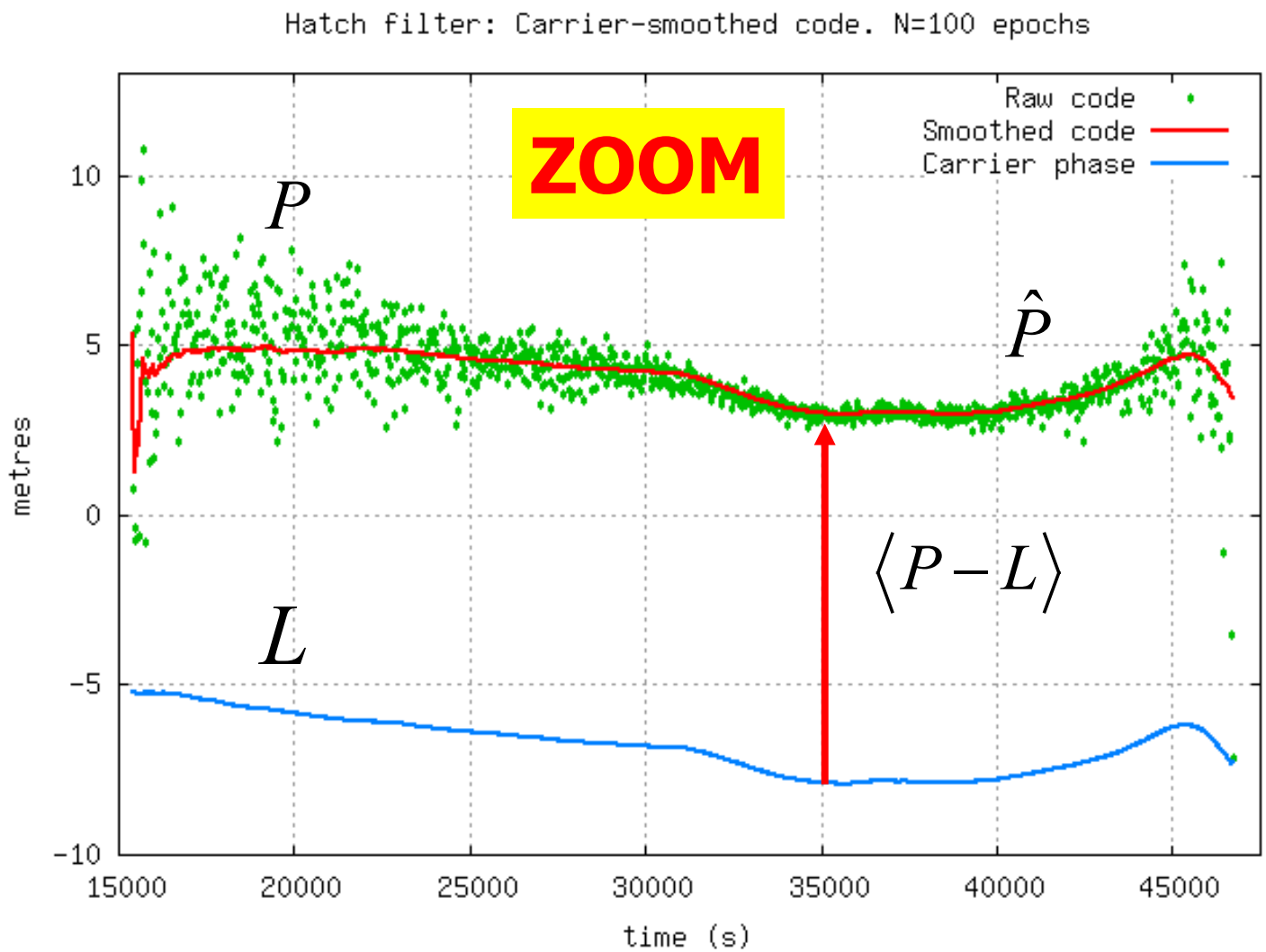
This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

$$\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right) = L(k) + \left\langle P - L \right\rangle_{(k)}$$

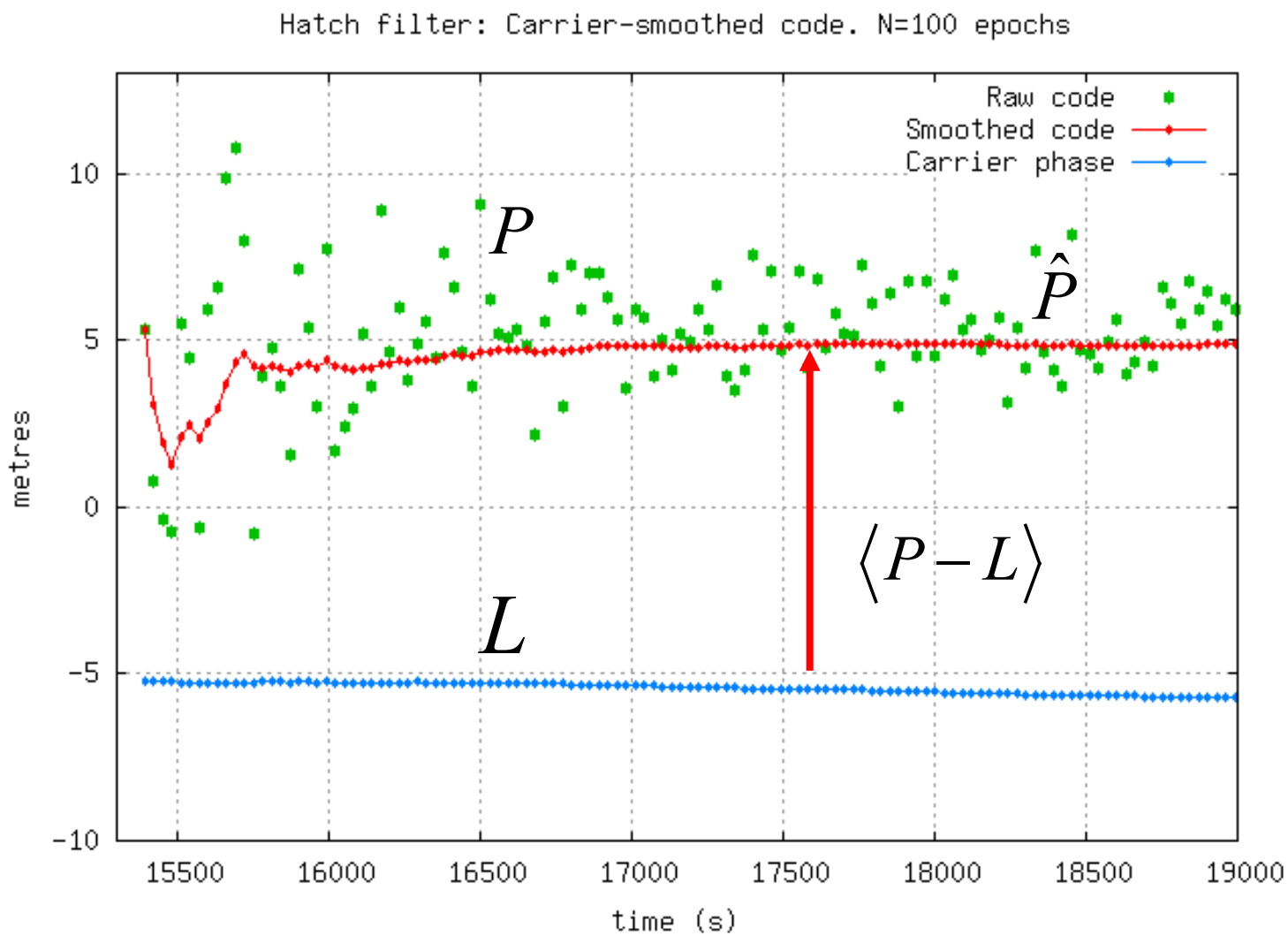


This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

$$\hat{P}(k) = \frac{1}{n}P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right) = L(k) + \boxed{\langle P - L \rangle_{(k)}}$$



$$\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right) = L(k) + \boxed{\langle P - L \rangle_{(k)}}$$



$$\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right) = L(k) + \boxed{\langle P - L \rangle_{(k)}}$$

Code-carrier divergence: SF smoother

Time varying ionosphere induces a bias in the single frequency (SF) smoothed code when it is averaged in the smoothing filter (Hatch filter).

Let:

$$P_1 = \rho + I_1 + \varepsilon_1$$
$$L_1 = \rho - I_1 + B_1 + \varsigma_1$$

Where ρ includes all non dispersive terms (geometric range, clock offsets, troposphere) and I_1 represents the frequency dependent terms (ionosphere and DCBs). B_1 is the carrier ambiguity, which is constant along continuous carrier phase arcs and $\varepsilon_1, \varsigma_1$ account for code and carrier multipath and thermal noise.

thence,

$$P_1 - L_1 = 2I_1 - B_1 + \varepsilon_1 \qquad \Rightarrow 2I_1 : \text{ Code-carrier divergence}$$

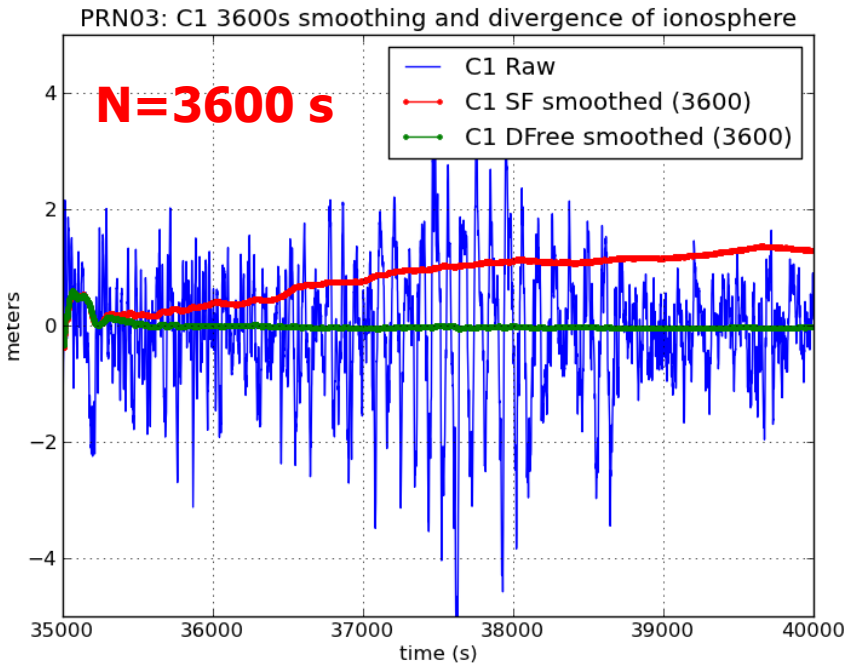
Substituting $P_1 - L_1$ in Hatch filter equation

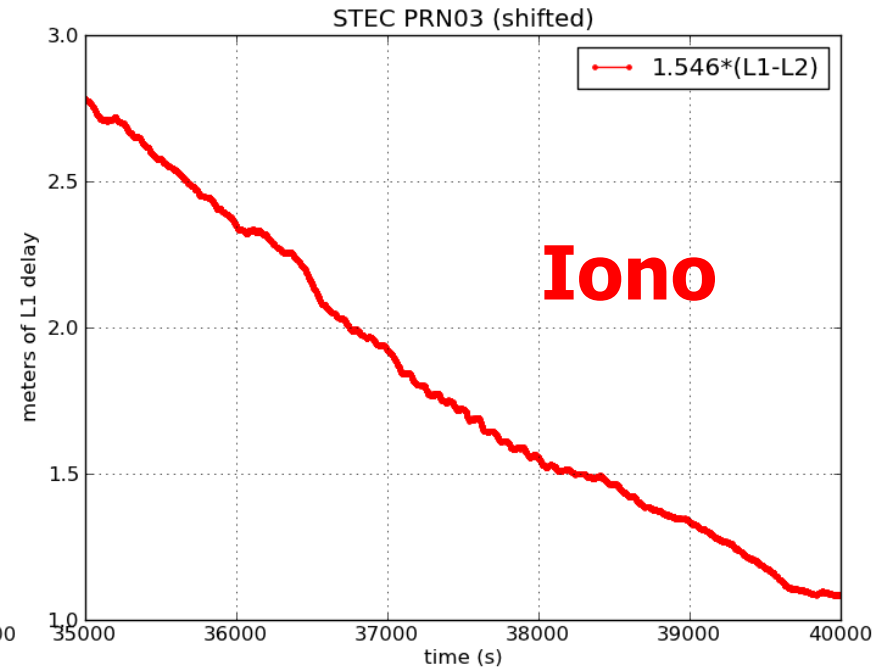
$$\hat{P}(k) = L(k) + \langle P - L \rangle_{(k)} = \rho(k) - I_1(k) + B_1 + \langle 2I_1 - B_1 \rangle_{(k)} =$$
$$= \rho(k) + I_1(k) + 2\underbrace{\boxed{\left(\langle I_1 \rangle_{(k)} - I_1(k)\right)}}_{bias_I}$$

$$\Rightarrow \hat{P}_1 = \rho + I_1 + bias_I + \upsilon_1$$

where, being the ambiguity term B_1 a constant bias, thence $\langle B_1 \rangle_{(k)} \approx B_1$, and cancels in the previous expression.

where υ_1 is the noise term after smoothing.





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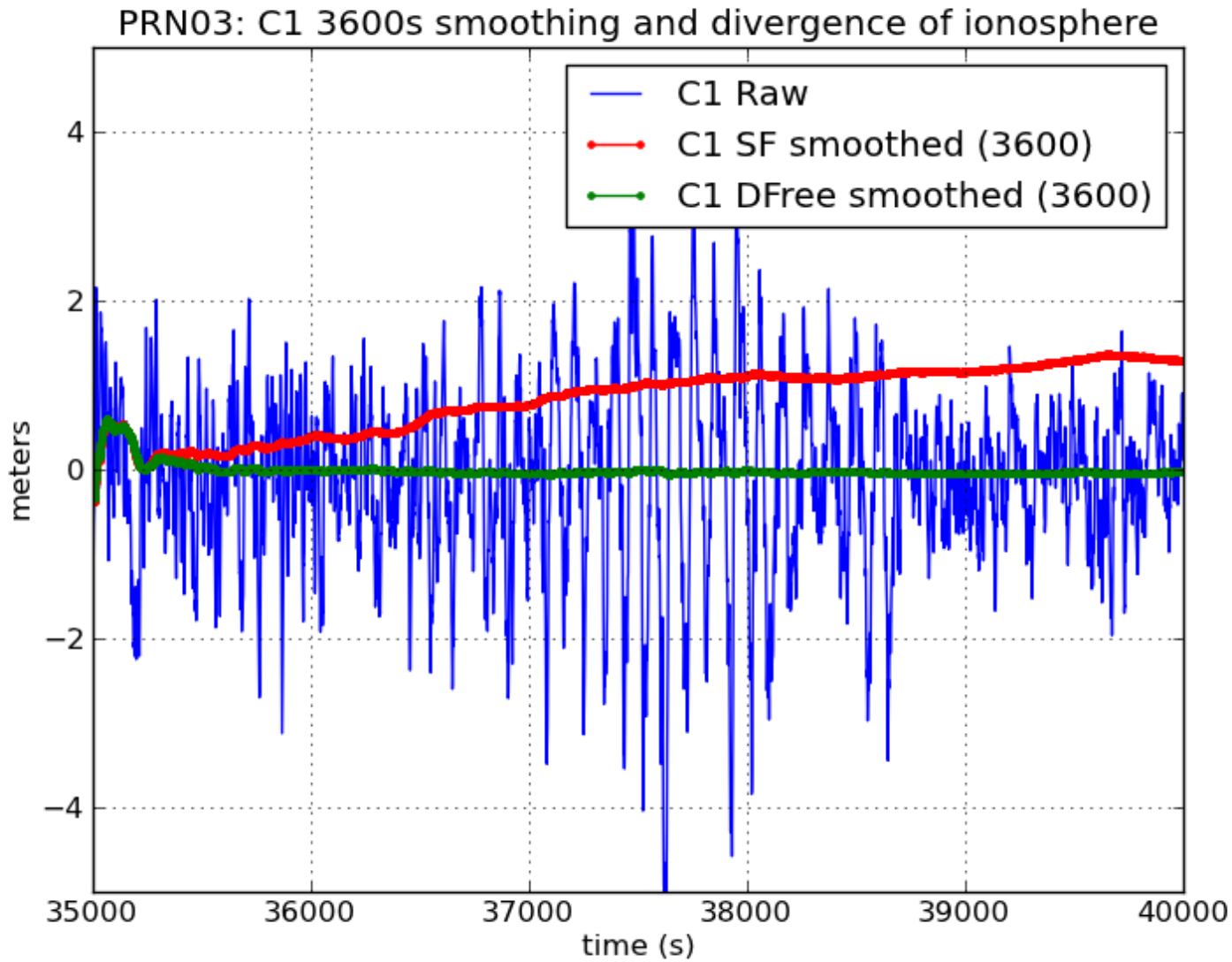
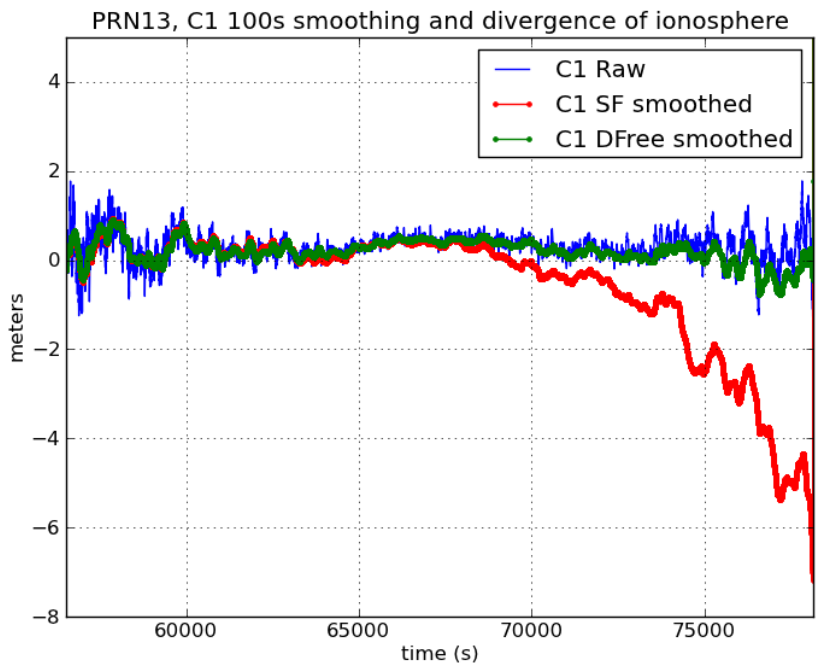
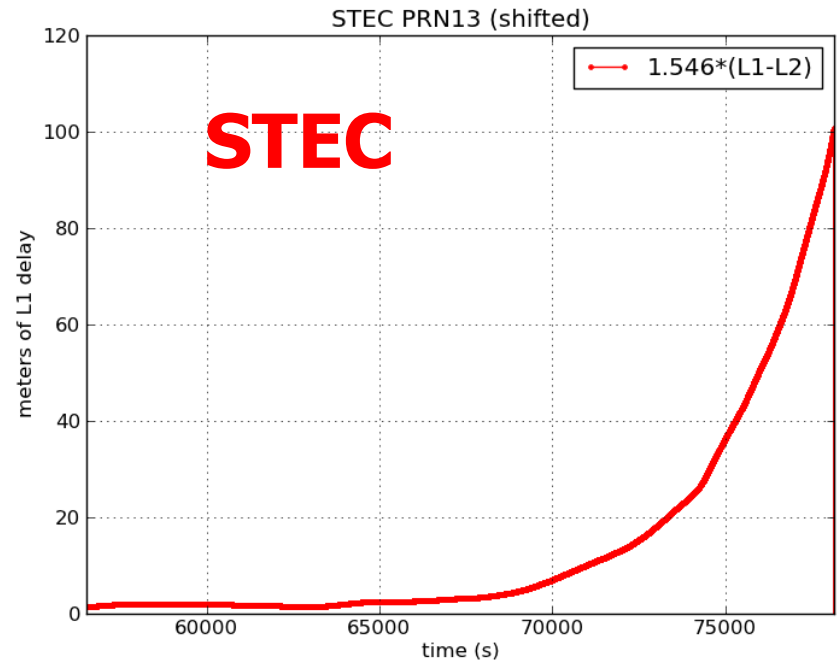

PRN03: C1 3600s smoothing and divergence of ionosphere
C1 Raw
C1 SF smoothed (3600)
C1 DFree smoothed (3600)
meters
4
2
0
−2
−4
35000
36000
37000
38000
39000
40000
time (s)

Halloween storm

Data File: amc23030.03o_1Hz





N=100 (i.e. filter smoothing time constant *τ=100 sec*).

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Halloween storm

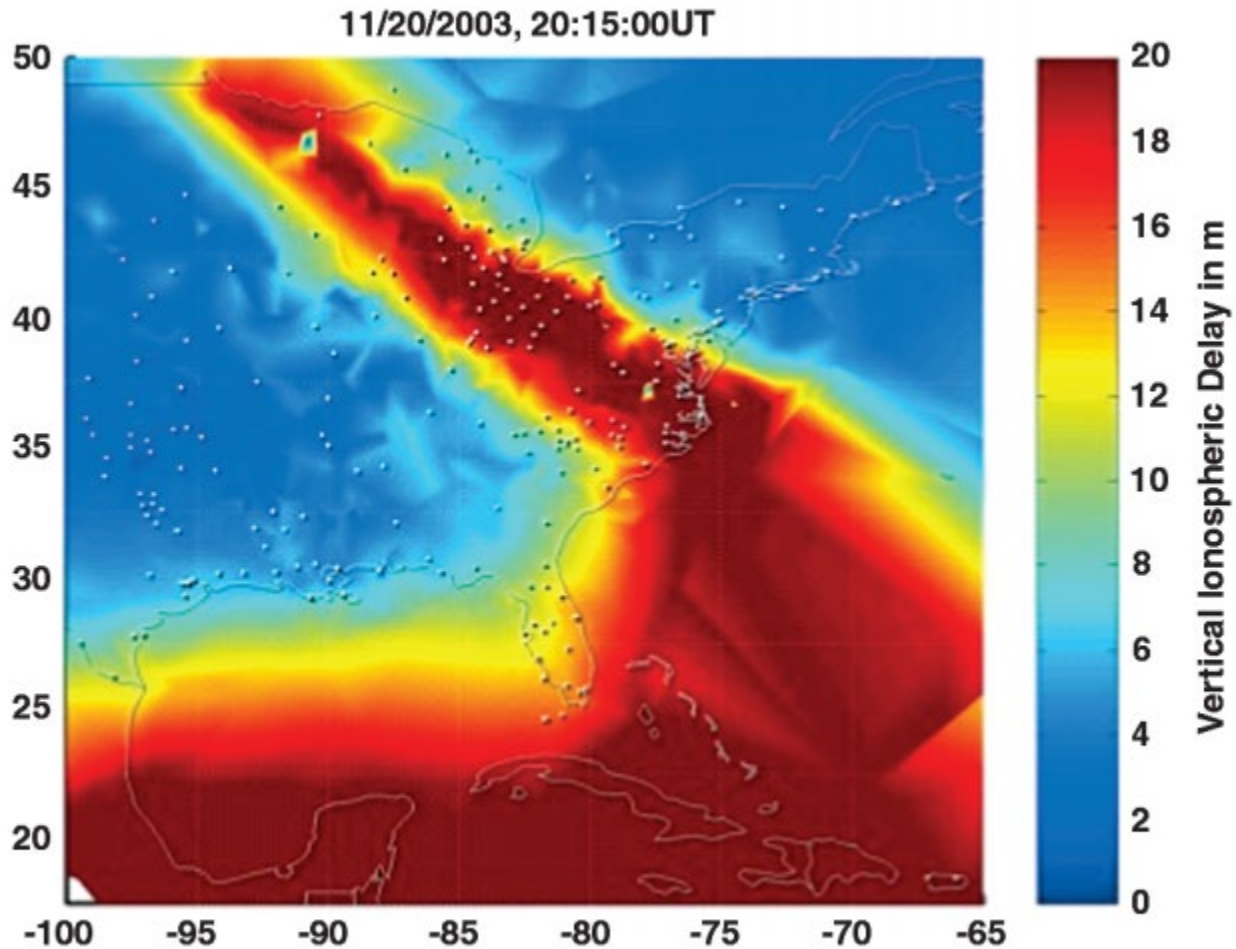


Fig. 2 Map of equivalent vertical ionospheric delay over eastern United States on 20 Nov. 2003 at 20:15 UT.

[*] Ionospheric Threat Parameterization for Local Area Global-Positioning-System-Based Aircraft Landing Systems, Datta-Barua et al, Journal of Aircraft Vol. 47, No. 4, July–August 2010, DOI: 10.2514/1.46719

Carrier-smoothed pseudorange: DFree

Divergence-Free (Dfree) smoother:

With two frequency **carrier** measurements **a combination of carriers** with the same ionospheric delay (the same sign) as the code can be generated:

$$L_{1,DF} = L_1 + 2\bar{\alpha}_1 \left(L_1 - L_2 \right) = \rho + I_1 + B_{1,DF} + \varsigma_{1,DF}$$

$$\bar{\alpha}_1 = \frac{f_2^2}{f_1^2 - f_2^2} = \frac{1}{\gamma - 1} = 1.545$$

$$\gamma = \left(\frac{77}{60} \right)^2$$

With this new combination we have:

$$P_1 = \rho + I_1 + \varepsilon_1$$

$$L_{1,DF} = \rho + I_1 + B_{1,DF} + \varsigma_{1,DF}$$

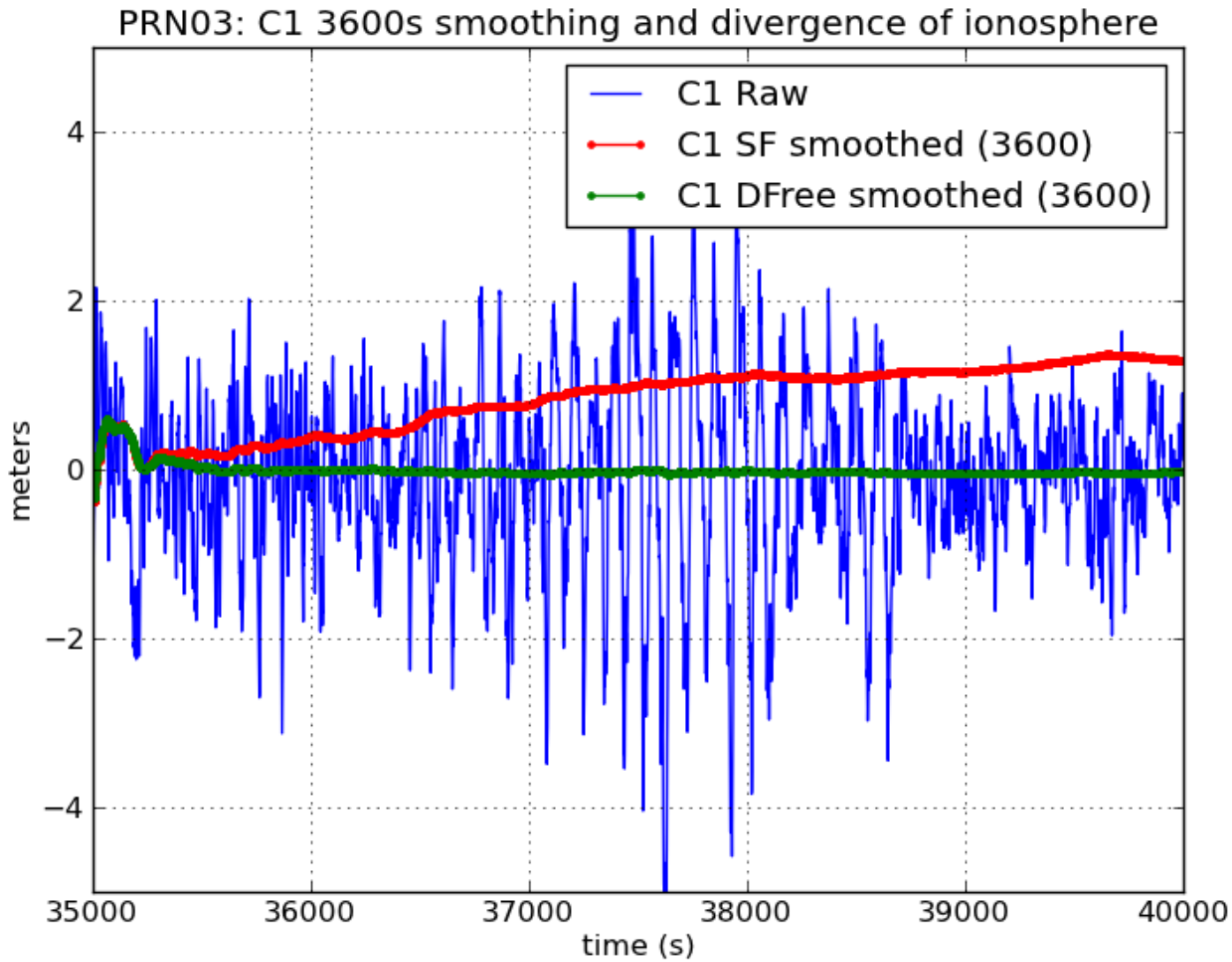
Thence,

$$P_1 - L_{1,DF} = B_{1,DF} + \varepsilon_1$$

$$\Rightarrow \textit{No} \text{ Code-carrier divergence!}$$

$$\Rightarrow \hat{P}_{1,DF} = \rho + I_1 + \upsilon_{12}$$

This smoothed code is immune to temporal gradients (unlike the SF smoother), being the same ionospheric delay as in the original raw code (i.e. I_1). Nevertheless, as it is still affected by the ionosphere, its **spatial decorrelation** must be taken into account in differential positioning.


PRN03: C1 3600s smoothing and divergence of ionosphere
C1 Raw
C1 SF smoothed (3600)
C1 DFree smoothed (3600)
meters
4
2
0
−2
−4
35000
36000
37000
38000
39000
40000
time (s)

Carrier-smoothed pseudorange: IFree

Ionosphere-Free (Ifree) smoother:

Using both code and carrier dual-frequency measurements, it is possible to remove the frequency dependent effects using the ionosphere-free combination of code and carriers (PC and LC). Thence:

$$P_C = \rho + \varepsilon_{P_C}$$
$$L_C = \rho + B_{L_C} + \upsilon_{L_C}$$

$$P_{IFree} \equiv P_C = \frac{\gamma P_1 - P_2}{\gamma - 1} \quad ; \quad L_{IFree} \equiv L_C = \frac{\gamma L_1 - L_2}{\gamma - 1} \qquad \gamma = \left(\frac{77}{60}\right)^2$$

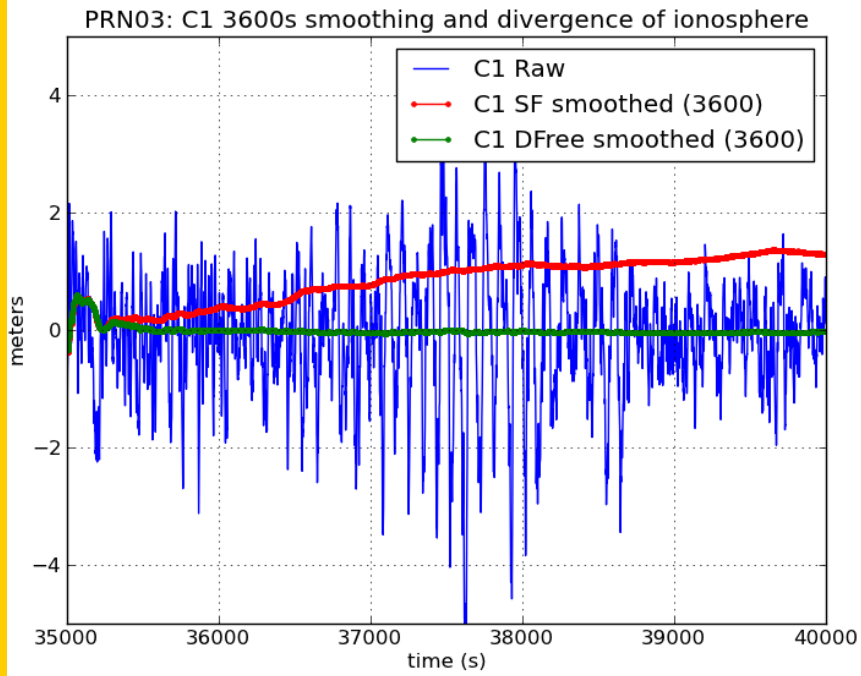
Thence,

$$P_C - L_C = B_C + \varepsilon_{P_C} \Rightarrow \boxed{\hat{P}_{IFree} \equiv \hat{P}_C = \rho + \upsilon_{IFree}}$$

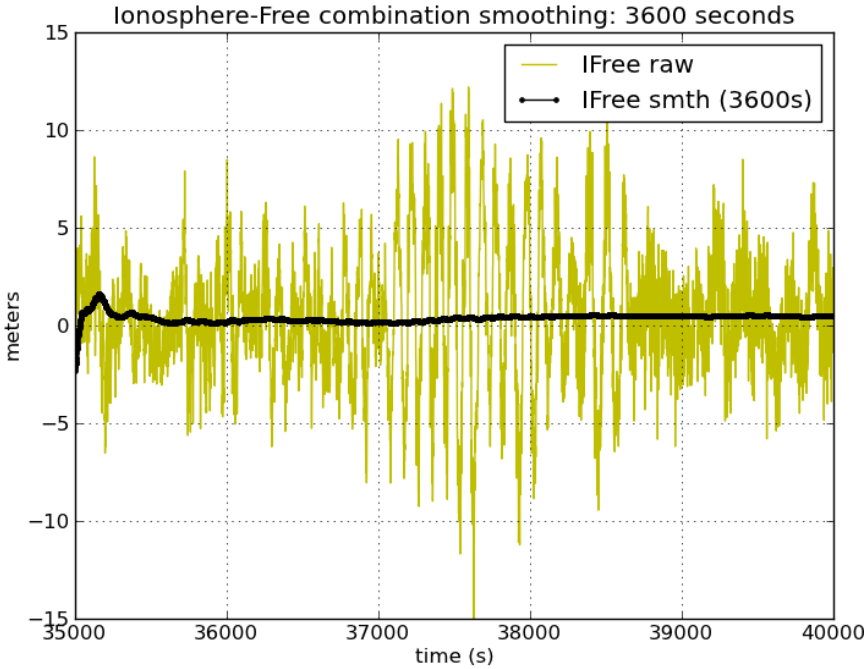
$$\sigma_{Pc} = \frac{\sqrt{\gamma^2 + 1}}{\gamma - 1}\sigma_{P_1} \approx 3\sigma_{P_1}$$

This smoothed is based on the ionosphere-free combination of measurements, and therefore it is unaffected by either the spatial and temporal inospheric gradients, but has the disadvantage that **the noise is amplified by a factor 3** (using the legacy GPS signals).

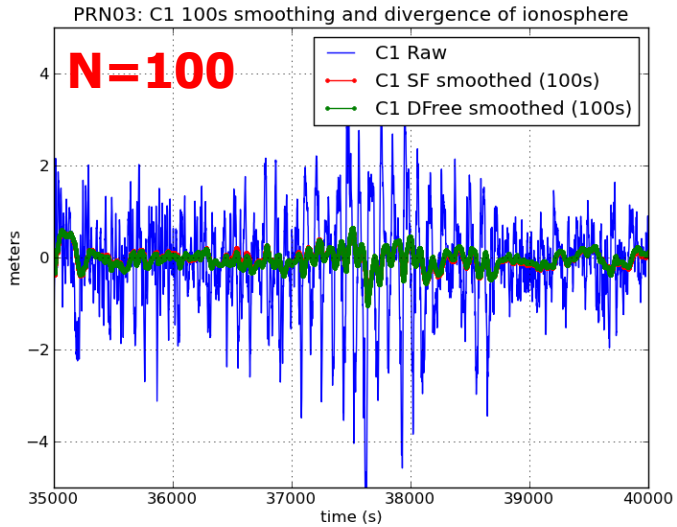
Vertical range: [-5 : 5]



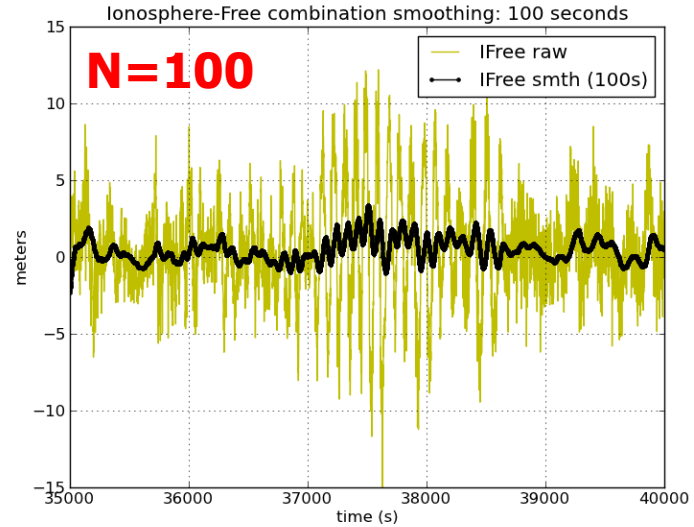
Vertical range: [-15:15]

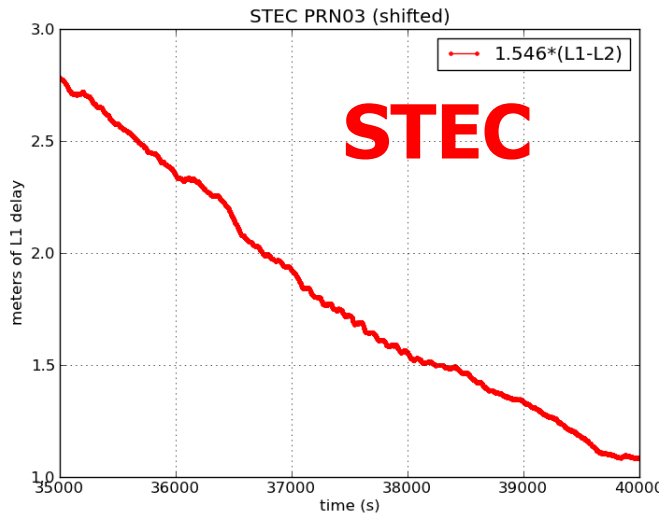


C1, L1



PC, LC





Exercise:

Justify that the ionosphere-free combination (**PC**) is (obviously) not affected by the code-carrier divergence, but it is 3 times noisier.

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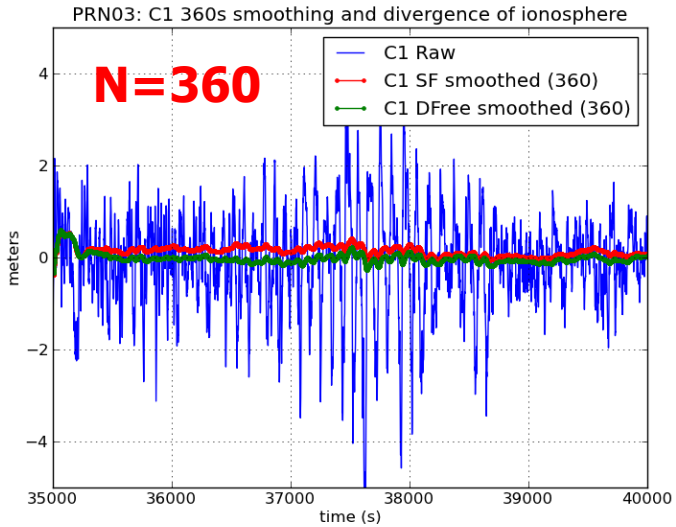
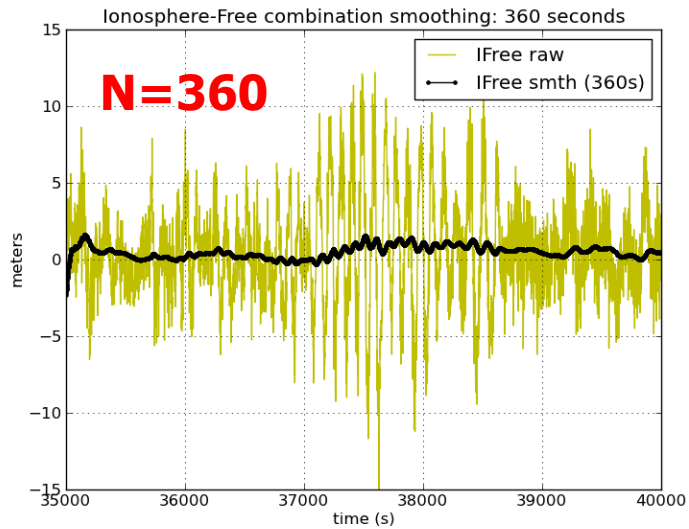
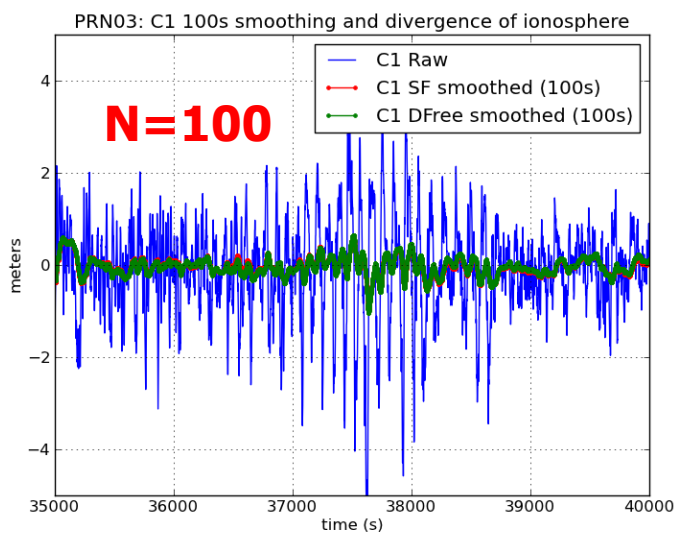
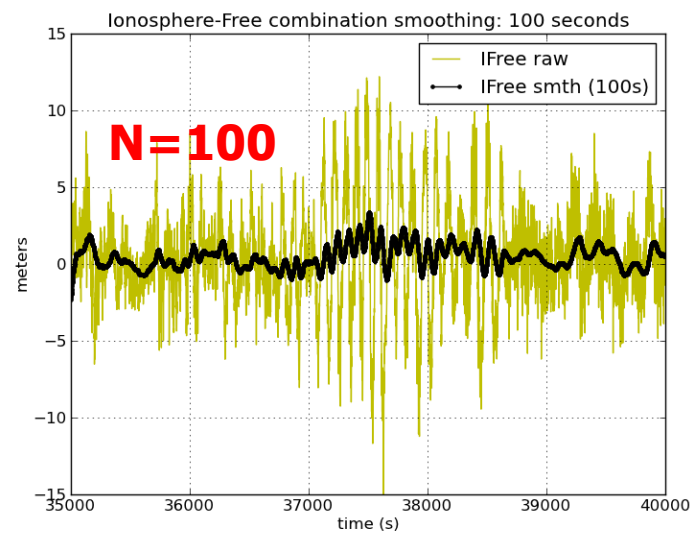
C1, L1

PC, LC









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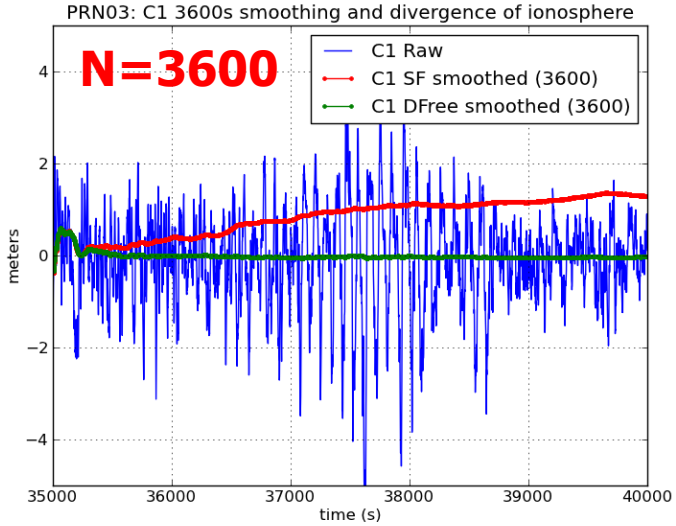
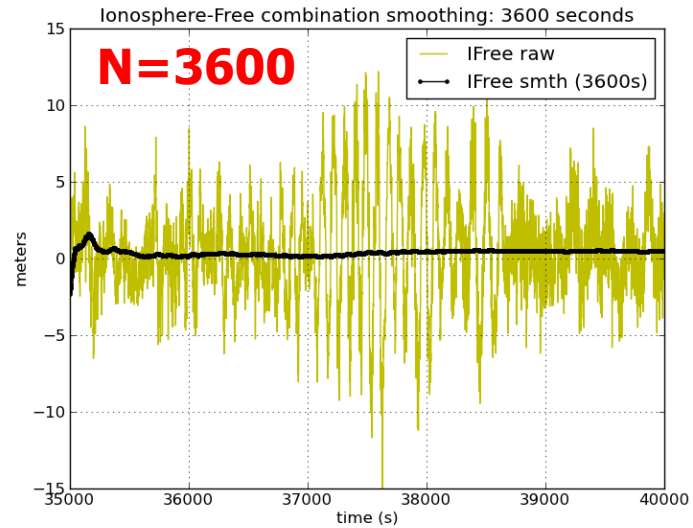
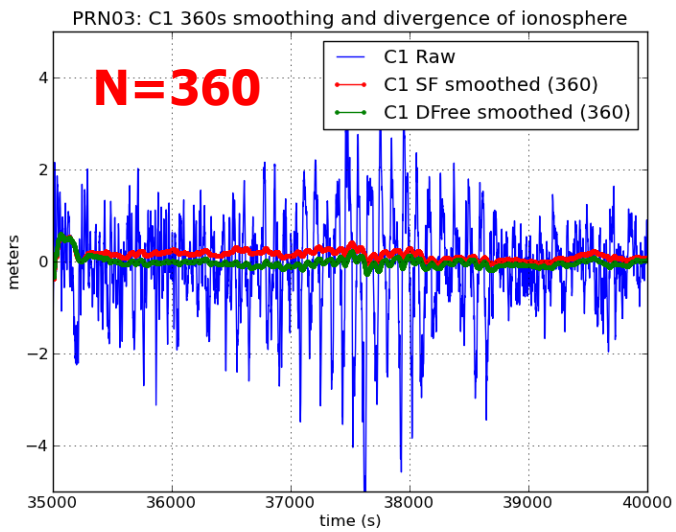
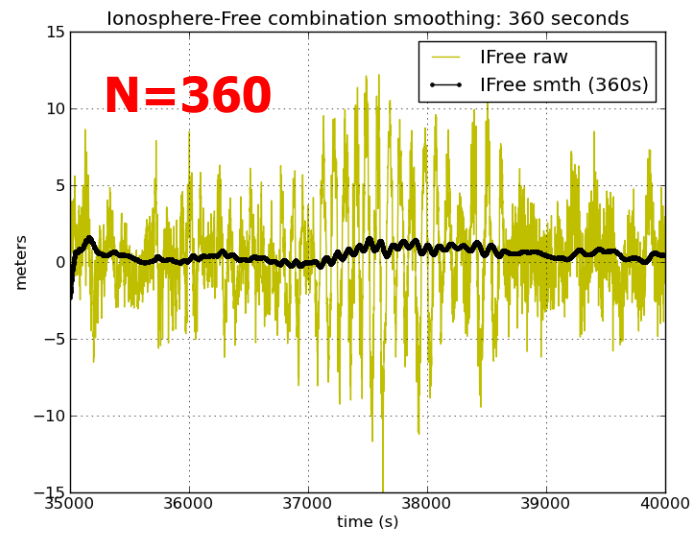
C1, L1

PC, LC









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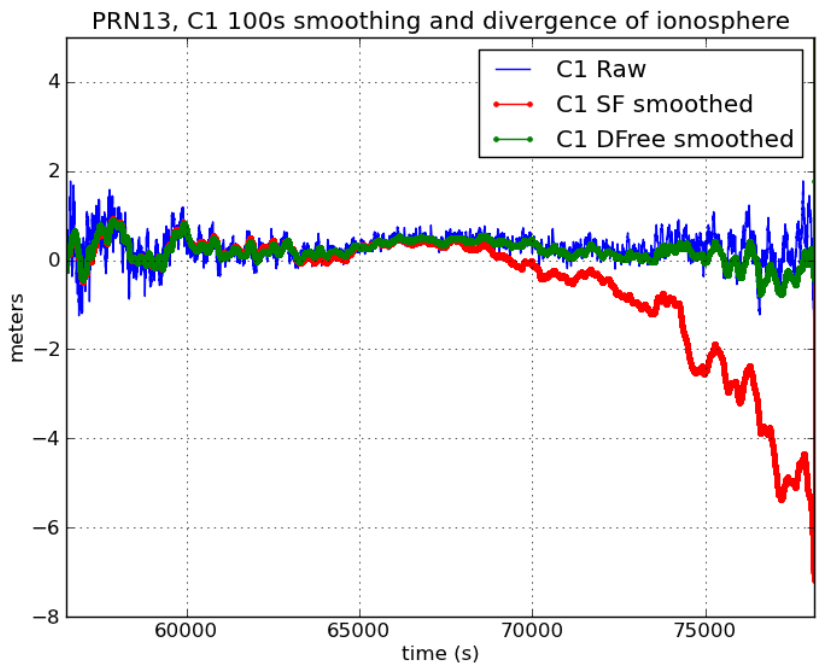
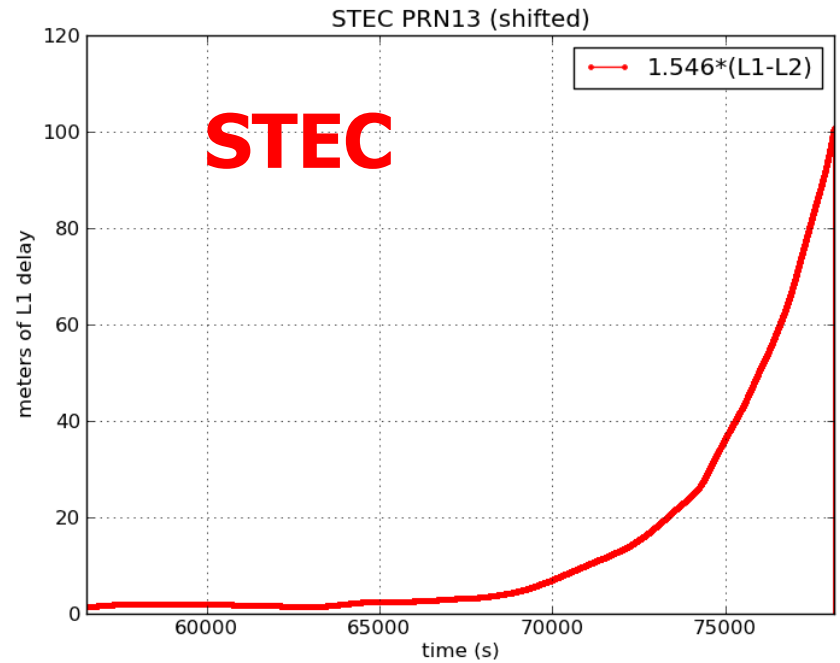
Halloween storm

Data File: amc23030.03o_1Hz





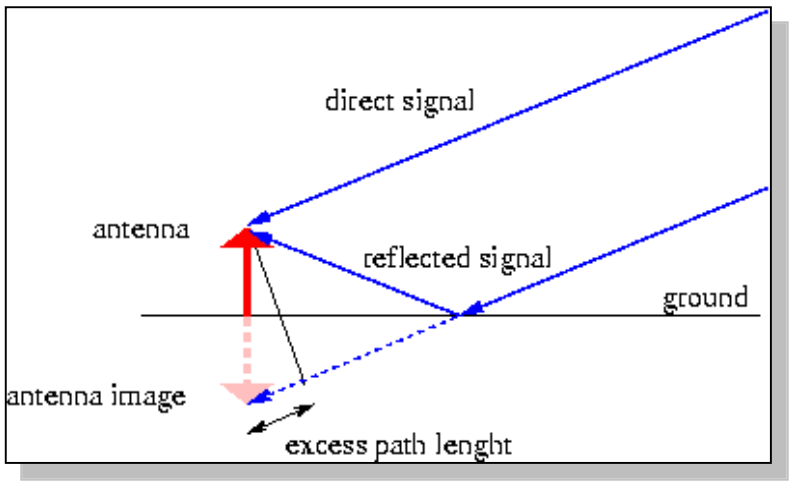
N=100 (i.e. filter smoothing time constant *τ=100 sec*).

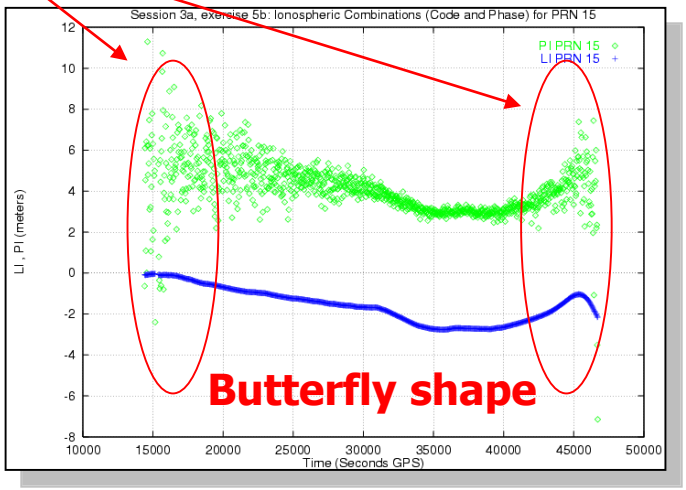
Contents

Multipath

One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.

It affects both code and carrier phase measurements, and it is more important at low elevation angles.



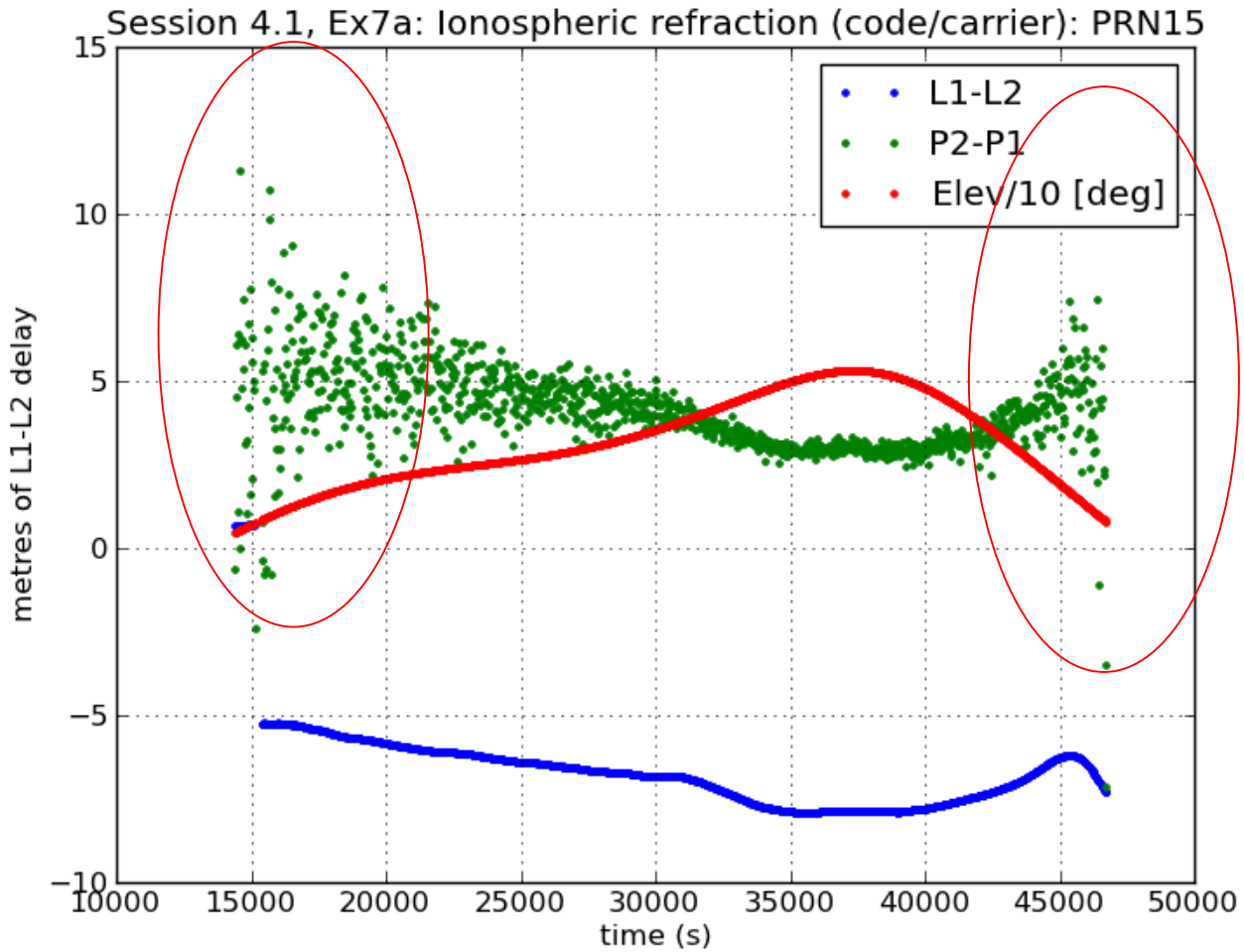


Code: up to 1.5 chip-length ➔ up to 450m for C1 [theoretically]
Typically: less than 2-3 m.

Phase: up to $\lambda/4$ ➔ up to 5 cm for L1 and L2 [theoretically]
Typically: less than 1 cm

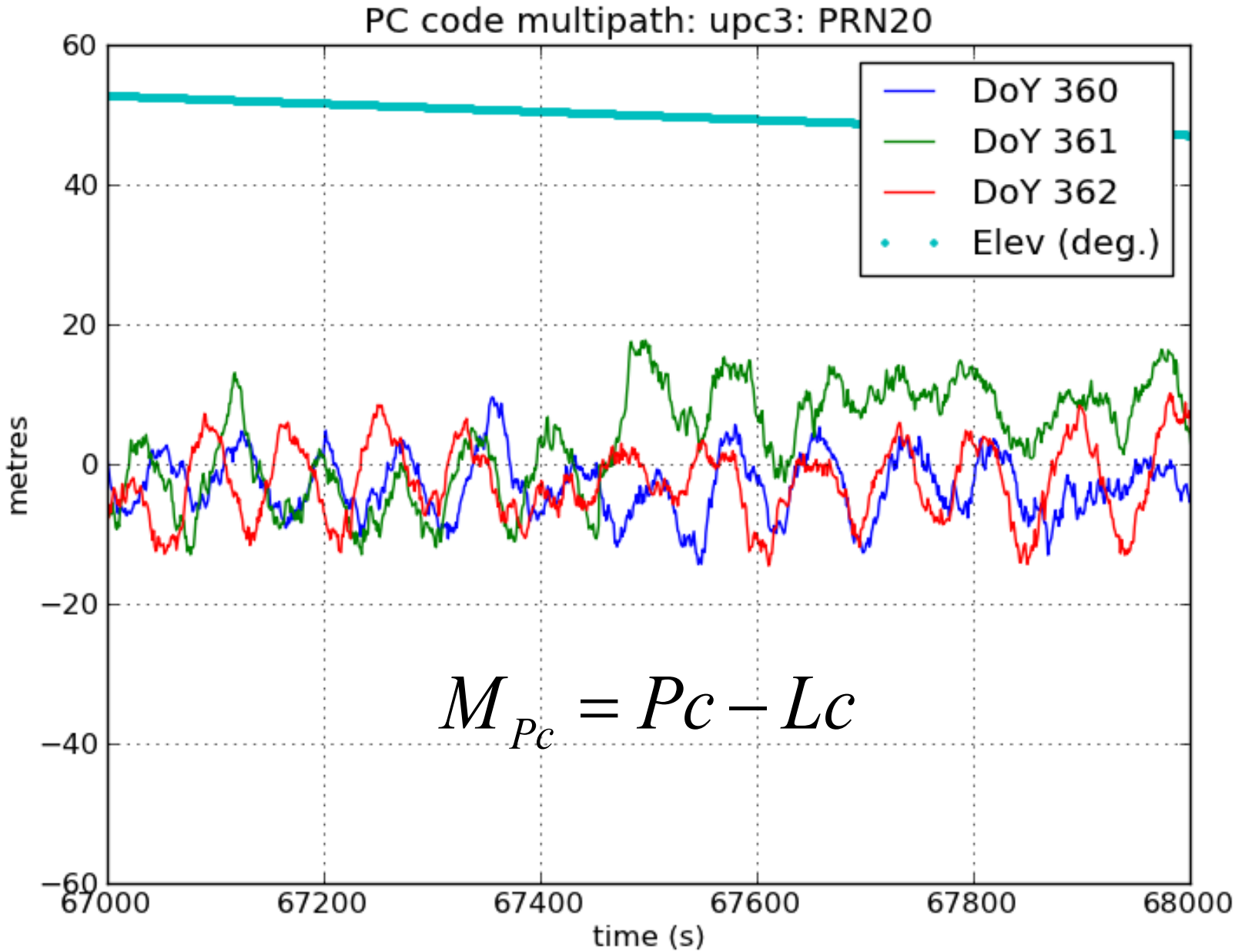
Exercise

Plot code and phase geometry-free combination for satellite PRN 15 of file 97jan09coco_r0.rnx and discuss the results.



Butterfly shape:

High multipath for low elevation rays (when satellite rises and sets)



$$M_{Pc} = Pc - Lc$$

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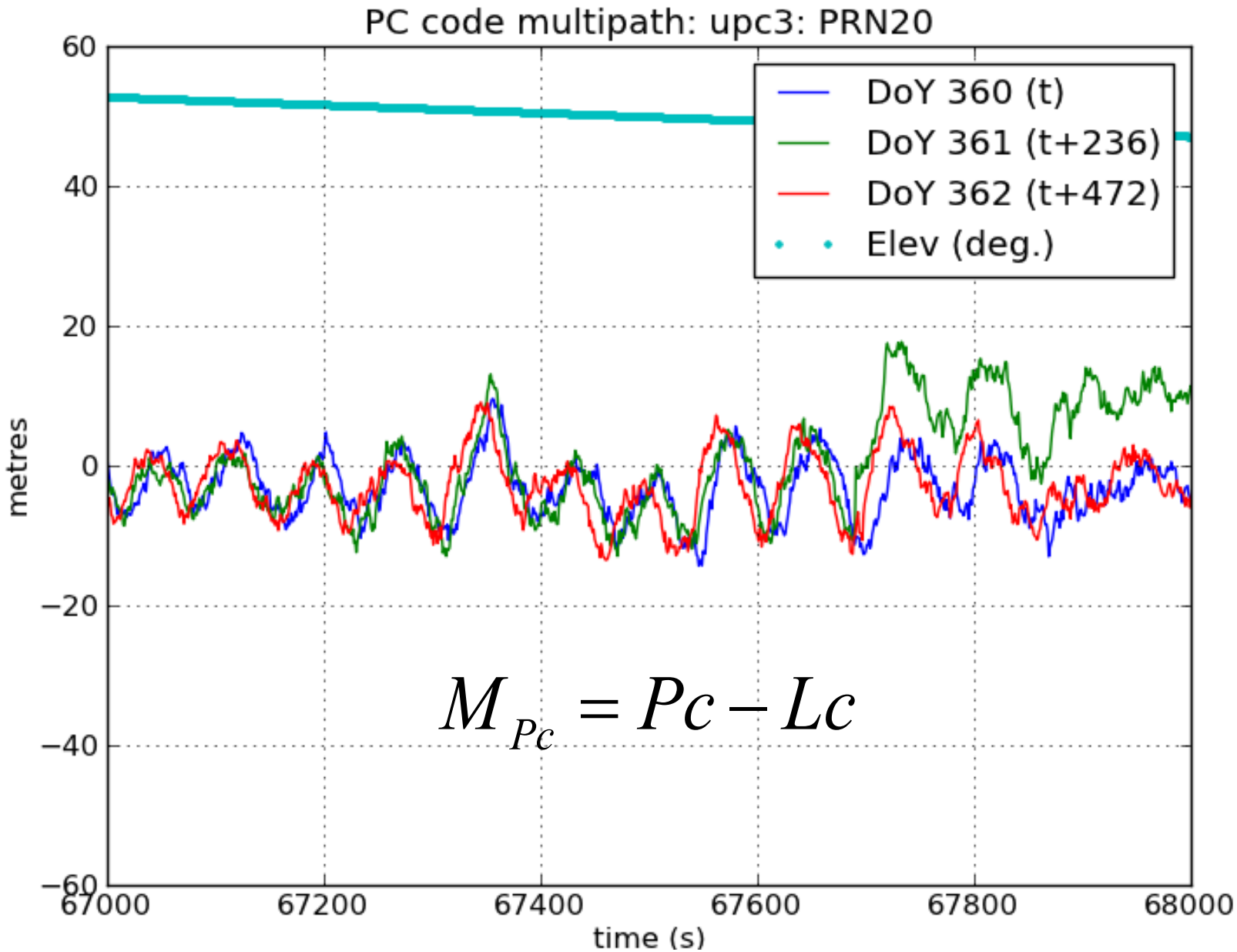
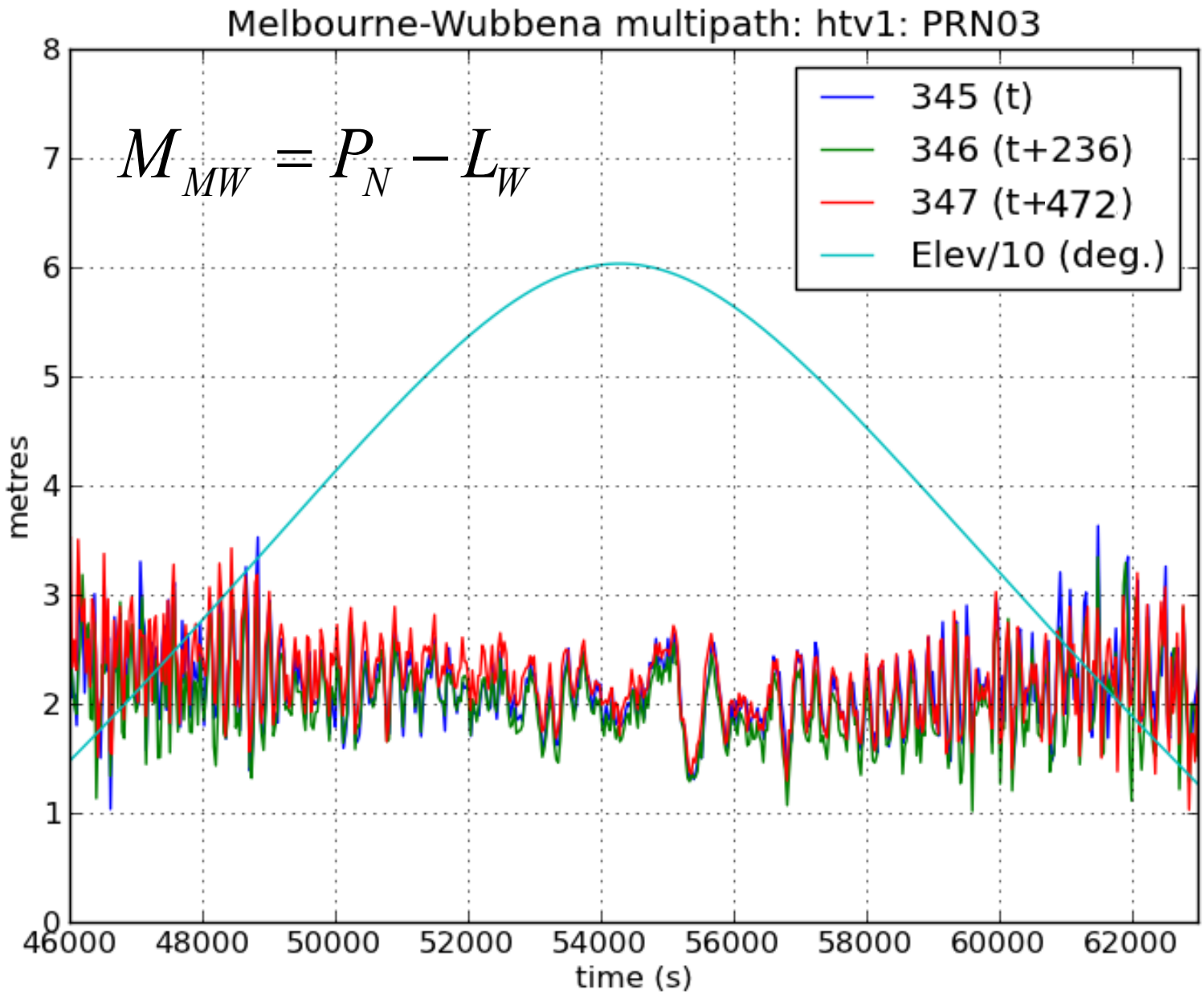
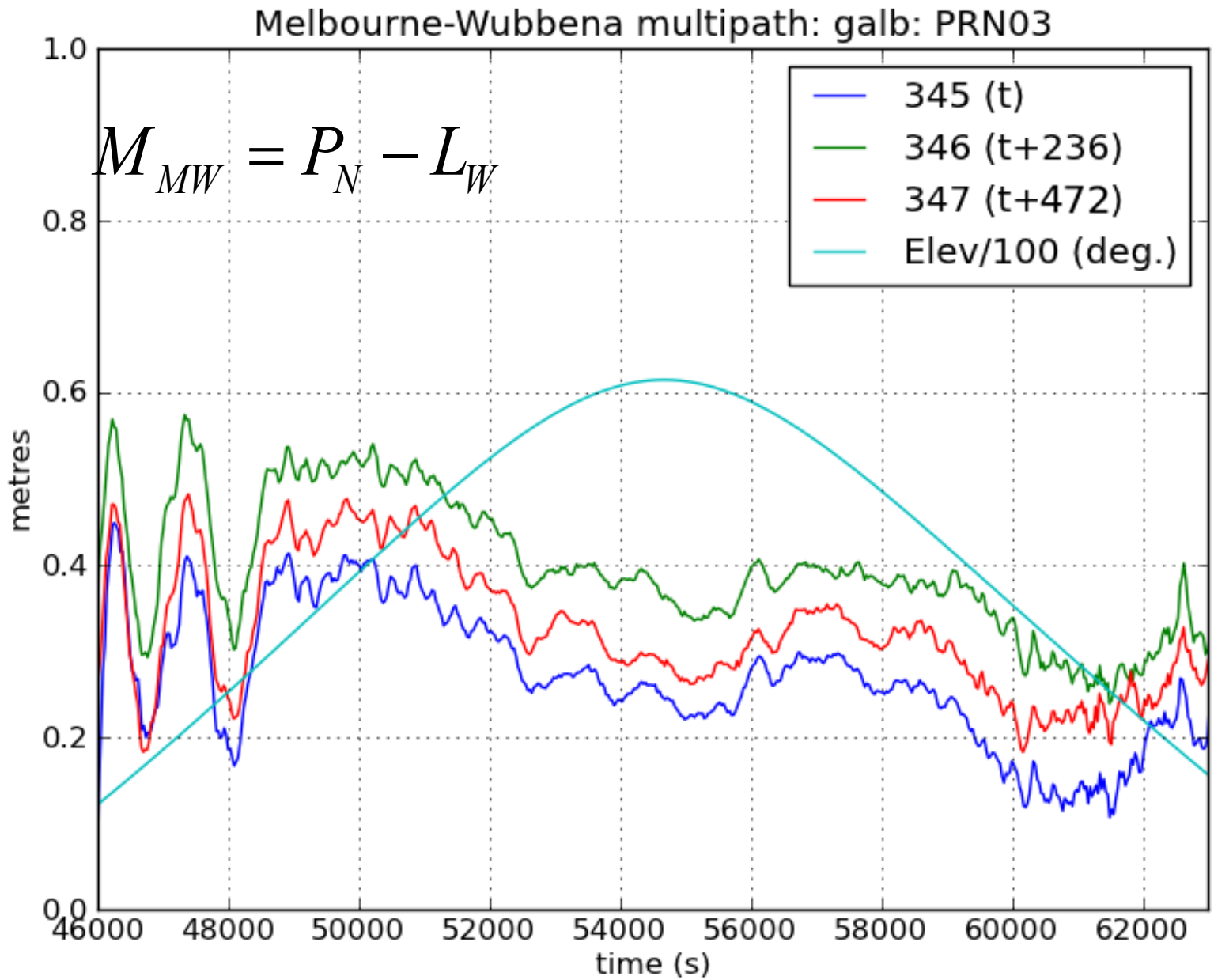
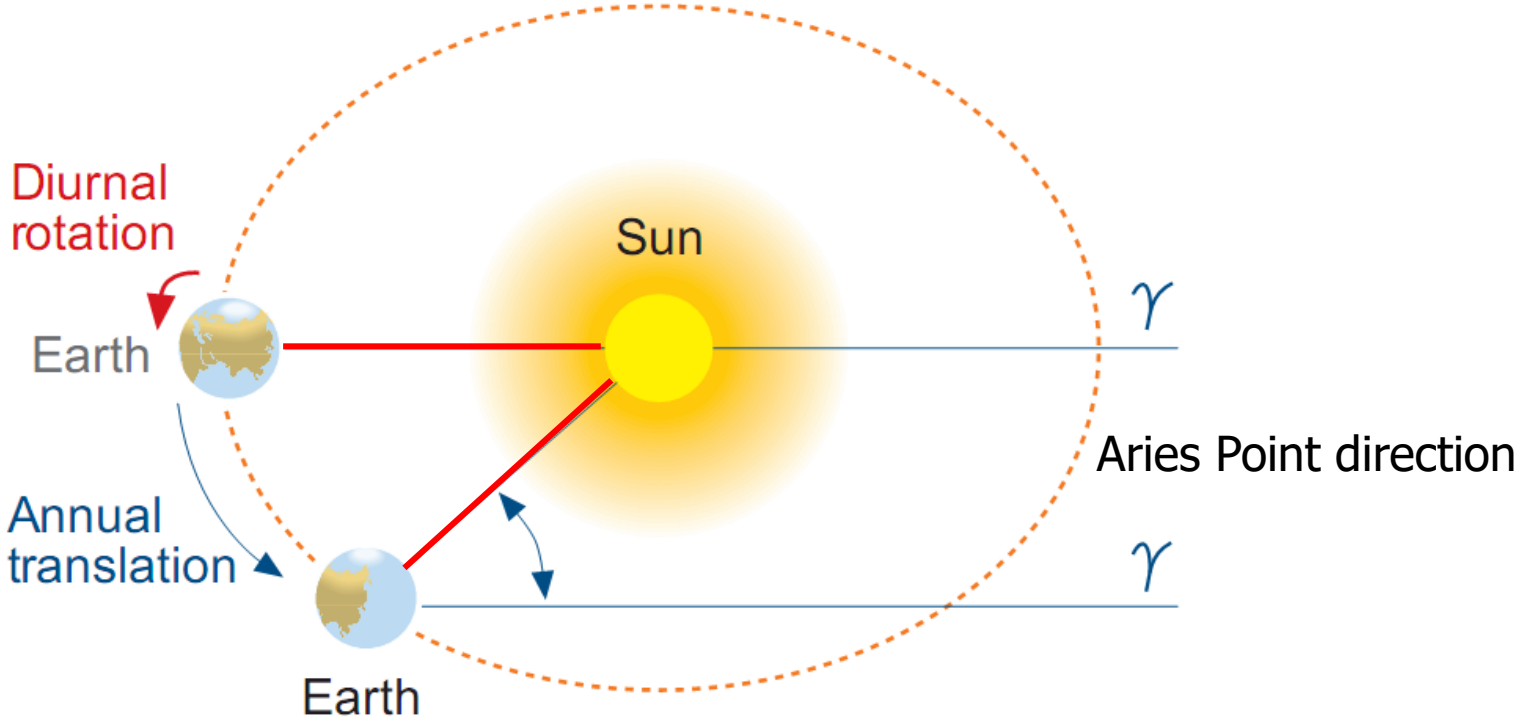
BarcelonaTECH, Spain



$$M_{Pc} = Pc - Lc$$


Melbourne-Wubbena multipath: htv1: PRN03
$M_{MW} = P_N - L_W$
345 (t)
346 (t+236)
347 (t+472)
Elev/10 (deg.)
metres
time (s)
0
1
2
3
4
5
6
7
8
46000
48000
50000
52000
54000
56000
58000
60000
62000


Melbourne-Wubbena multipath: galb: PRN03
$M_{MW} = P_N - L_W$
345 (t)
346 (t+236)
347 (t+472)
Elev/100 (deg.)
metres
time (s)
1.0
0.8
0.6
0.4
0.2
0.0
46000
48000
50000
52000
54000
56000
58000
60000
62000

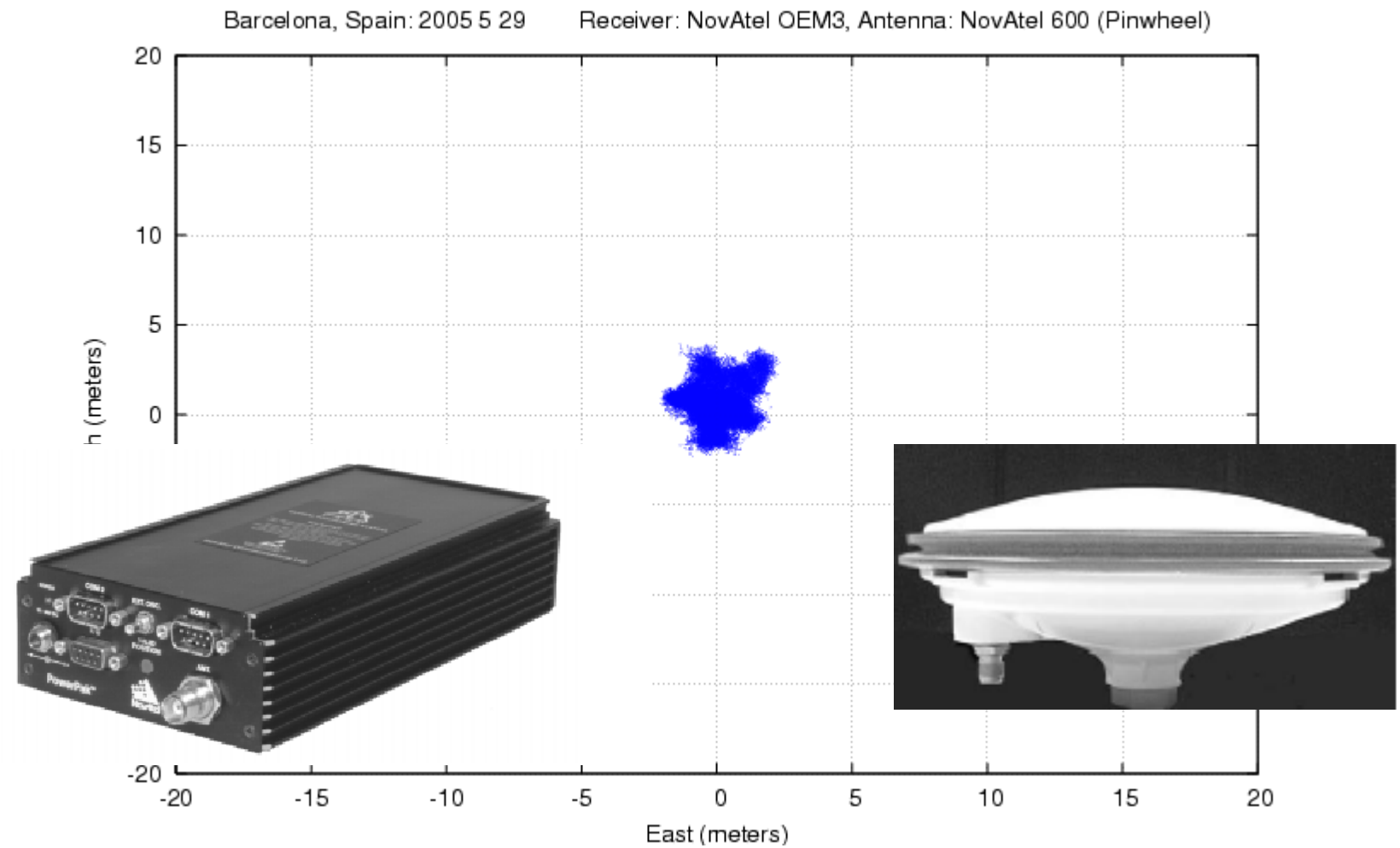


After one year, the directions of the Sun and Aries coincide again, but the number of laps relative to the Sun (solar days) is one less than those relative to Aries (sidereal days).

$$\frac{24\text{h}}{365.2422} = 3^{\text{m}}56^{\text{s}}$$

Thus, a **sidereal day is shorter** than a solar day for about $3^{\text{m}}\,56^{\text{s}}$

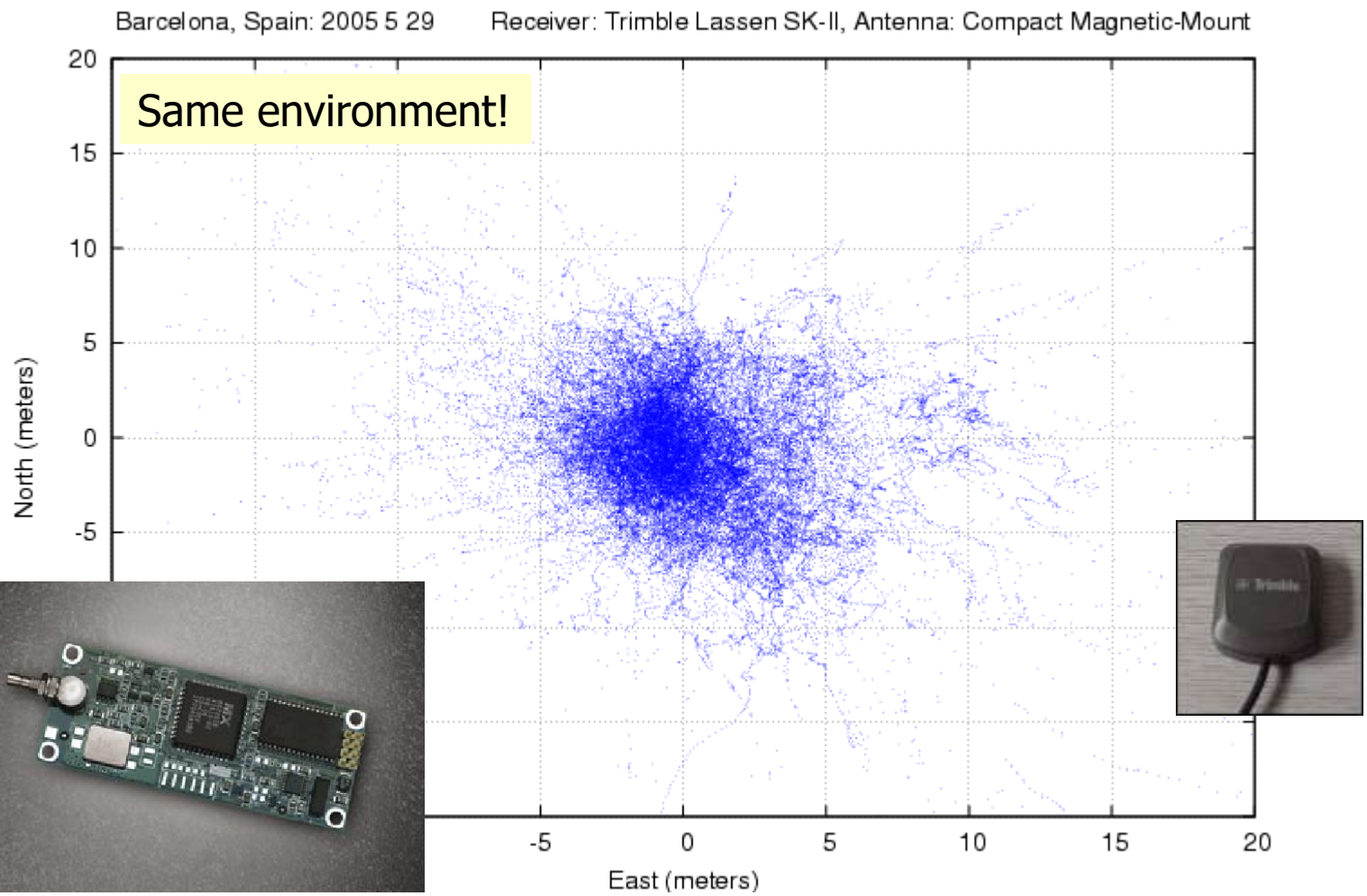
Receiver noise and multipath



GPS standalone (C1 code)

10,000 €

Receiver noise and multipath



GPS standalone (C1 code)

100 €

References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

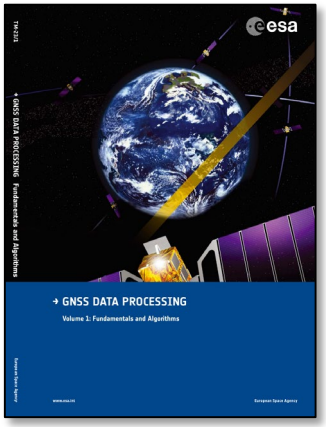
[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

Thank you

Backup

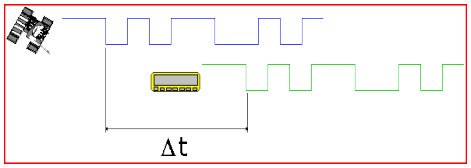


Rewriting GNSS equations:

Combinations of Measurements Written in a Closed Form

See more details in
J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

$$R_{P_f} = c\,[t_{rcv}(T_2) - t^{sat}(T_1)]$$



Notation in book

P ➔ R_{Pf} or R

L ➔ Φ_{Lf} or Φ

$$R_{P_f} = \rho + c(dt_{rcv} - dt^{sat}) + Tr + \alpha_f\, STEC + K_{P_f,rcv} - K^{sat}_{P_f} + \mathcal{M}_{P_f} + \varepsilon_{P_f}$$

$$\Phi_{L_f} = \rho + c(dt_{rcv} - dt^{sat}) + Tr - \alpha_f\, STEC + k_{L_f,rcv} - k^{sat}_{L_f} + \lambda_{L_f} N_{L_f} + \lambda_{L_f} w + m_{L_f} + \epsilon_{L_f}$$

4.1.1.1 Clock Redefinition and Differential Code Biases

By defining a new clock δt as

$$c\,\delta t = c\,dt + K_{C_{12}}, \quad \text{where} \quad K_{C_{12}} = \frac{f_1^2 K_1 - f_2^2 K_2}{f_1^2 - f_2^2}$$

it is not difficult to find that

$$c\,dt + K_1 = c\,\delta t + \tilde{\alpha}_1 (K_2 - K_1)$$
$$c\,dt + K_2 = c\,\delta t + \tilde{\alpha}_2 (K_2 - K_1)$$

with

$$\boxed{1\,\text{TECU} = 10^{16}\text{e}^-/\text{m}^2}$$

$$\tilde{\alpha}_i \equiv \frac{\alpha_i}{\alpha_2 - \alpha_1} \quad (i = 1, 2) \quad \text{and} \quad \alpha_i = \frac{40.3}{f_i^2} 10^{16}\, \text{m}_{\text{delay}(\text{signal } \Phi_{f_i})}/\text{TECU}$$

Ionosphere-free comb.

$$R_C = \frac{f_1^2\, R_1 - f_2^2\, R_2}{f_1^2 - f_2^2}$$

$$\Phi_C = \frac{f_1^2\, \Phi_1 - f_2^2\, \Phi_2}{f_1^2 - f_2^2}$$

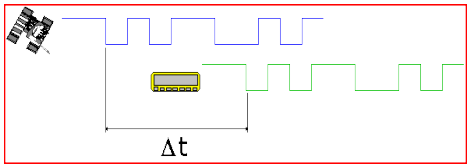
$$K_i \equiv K_{i,rcv} - K_i^{sat}$$

$$k_i \equiv k_{i,rcv} - k_i^{sat}$$

$$K_{21} = K_2 - K_1$$

Both, Ionosphere and Code-Instrumental-Delay, will cancel in the Iono.-Free combination!

$$R_{P_f} = c\,[t_{rcv}(T_2) - t^{sat}(T_1)]$$



Notation in book

P ➔ R_{Pf} or R

L ➔ Φ_{Lf} or Φ

$$R_{P_f} = \rho + c(dt_{rcv} - dt^{sat}) + Tr + \alpha_f\, STEC + K_{P_f,rcv} - K^{sat}_{P_f} + \mathcal{M}_{P_f} + \varepsilon_{P_f}$$

$$\Phi_{L_f} = \rho + c(dt_{rcv} - dt^{sat}) + Tr - \alpha_f\, STEC + k_{L_f,rcv} - k^{sat}_{L_f} + \lambda_{L_f} N_{L_f} + \lambda_{L_f} w + m_{L_f} + \epsilon_{L_f}$$

$$I \equiv (\alpha_2 - \alpha_1)\,\mathrm{STEC} = \frac{40.3(f_1^2 - f_2^2)}{f_1^2 f_2^2} 10^{16}\,\mathrm{STEC}$$

$$\mathrm{STEC} = \int N_e\, dl$$

$$R_i = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + \tilde{\alpha}_i(I + K_{21}) + \mathcal{M}_i + \varepsilon_i$$

$$\Phi_i = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr - \tilde{\alpha}_i(I + K_{21}) + b_i + \lambda_i N_i + \lambda_i w + m_i + \epsilon_i$$

with

$$\tilde{\alpha}_i \equiv \frac{\alpha_i}{\alpha_2 - \alpha_1} \quad (i = 1, 2) \;\text{ and }\; \alpha_i = \frac{40.3}{f_i^2} 10^{16}\, \mathrm{m}_{\mathrm{delay}(\mathrm{signal}\ \Phi_{f_i})}/\mathrm{TECU}$$

$$K_i \equiv K_{i,rcv} - K_i^{sat}$$

$$k_i \equiv k_{i,rcv} - k_i^{sat}$$

$$K_{21} = K_2 - K_1$$

$$1\,\mathrm{TECU} = 10^{16}\mathrm{e}^-/\mathrm{m}^2$$

Input measurements R_i and Φ_i $(i = 1, 2)$:

$$R_i = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + \tilde{\alpha}_i(I + K_{21}) + \mathcal{M}_i + \varepsilon_i$$
$$\Phi_i = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr - \tilde{\alpha}_i(I + K_{21}) + B_i + \lambda_i w + m_i + \epsilon_i$$

where the ambiguity B_i is given by

$$B_i = b_i + \lambda_i N_i, \;\; \lambda_i = c/f_i, \;\; \tilde{\alpha}_1 = 1/(\gamma_{12} - 1), \;\; \tilde{\alpha}_2 = \gamma_{12}\tilde{\alpha}_1 = 1 + \tilde{\alpha}_1,$$
$$\gamma_{12} = (f_1/f_2)^2$$

with the bias b_i a real number and N_i an integer ambiguity.
Note that $K_{21} = K_{21,rcv} - K_{21}^{sat}$, $b_i = b_{i,rcv} - b_i^{sat}$.

Both, Iono. and Code-Instrum.-delay, cancel in the Iono-Free combination.

Ionosphere-free combination:

$$R_C = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + \mathcal{M}_C + \varepsilon_C$$
$$\Phi_C = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + B_C + \lambda_N w + m_C + \epsilon_C$$

where the bias B_C is given by

$$B_C = b_C + \lambda_N \left(N_1 + (\lambda_W/\lambda_2)N_W\right)$$

The frequency dependent terms Iono. and Code-Instrum.-Delay $\boxed{I + K_{21}}$ appear always together with a frequency dependent coef.

The frequency dependent terms Iono. and Code-Instrum.-Delay $I + K_{21}$ appear always together with a frequency dependent coef.

Geometry-free combination:

$$R_I = I + K_{21} + \mathcal{M}_I + \varepsilon_I$$
$$\Phi_I = I + K_{21} + B_I + (\lambda_1 - \lambda_2)w + m_I + \epsilon_I$$

where the bias B_I is given by

$$B_I = b_I + \lambda_1 N_1 - \lambda_2 N_2$$

Wide-lane (phase) and narrow-lane (code) combinations:

$$R_N = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + \tilde{\alpha}_W (I + K_{21}) + \mathcal{M}_N + \varepsilon_N$$
$$\Phi_W = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + \tilde{\alpha}_W (I + K_{21}) + B_W + m_W + \epsilon_W$$

where the bias B_W is given by

$$B_W = b_W + \lambda_W N_W$$

$$N_W \equiv N_1 - N_2,$$
$$\lambda_W \equiv c/(f_1 - f_2), \quad \lambda_N \equiv c/(f_1 + f_2),$$
$$\tilde{\alpha}_W \equiv \sqrt{\tilde{\alpha}_1 \tilde{\alpha}_2} = f_1 f_2/(f_1^2 - f_2^2) = \sqrt{\gamma_{12}}/(\gamma_{12} - 1), \quad \gamma_{12} = (f_1/f_2)^2,$$
$$b_W \equiv (f_1 b_1 - f_2 b_2)/(f_1 - f_2), \quad b_C \equiv (f_1^2 b_1 - f_2^2 b_2)/(f_1^2 - f_2^2),$$
$$b_I \equiv b_1 - b_2, \quad b_W - b_C = \tilde{\alpha}_W b_I,$$

Other combinations involving code and phase measurements:

The Melbourne–Wübbena combination

$$\Phi_W - R_N = b_W + \lambda_W N_W + \mathcal{M}_{MW} + \varepsilon_{MW}$$

The GRAPHIC (Group and Phase Ionospheric Calibration) combination

$$\tfrac{1}{2}(R_i + \Phi_i) = \rho + c(\delta t_{rcv} - \delta t^{sat}) + Tr + \tfrac{1}{2}B_i + \tfrac{1}{2}\lambda_i w + \mathcal{M}_G + \varepsilon_G$$

Definitions and relationships (where $(\cdot)_X \equiv (\cdot)_{X_{12}}$) :

$$N_W \equiv N_1 - N_2,$$
$$\lambda_W \equiv c/(f_1 - f_2), \quad \lambda_N \equiv c/(f_1 + f_2),$$
$$\tilde{\alpha}_W \equiv \sqrt{\tilde{\alpha}_1\,\tilde{\alpha}_2} = f_1 f_2/(f_1^2 - f_2^2) = \sqrt{\gamma_{12}}/(\gamma_{12} - 1), \quad \gamma_{12} = (f_1/f_2)^2,$$

$$b_W \equiv (f_1 b_1 - f_2 b_2)/(f_1 - f_2), \quad b_C \equiv (f_1^2 b_1 - f_2^2 b_2)/(f_1^2 - f_2^2),$$
$$b_I \equiv b_1 - b_2, \quad b_W - b_C = \tilde{\alpha}_W\, b_I,$$

the same expressions for B_X as for b_X. (4.19)

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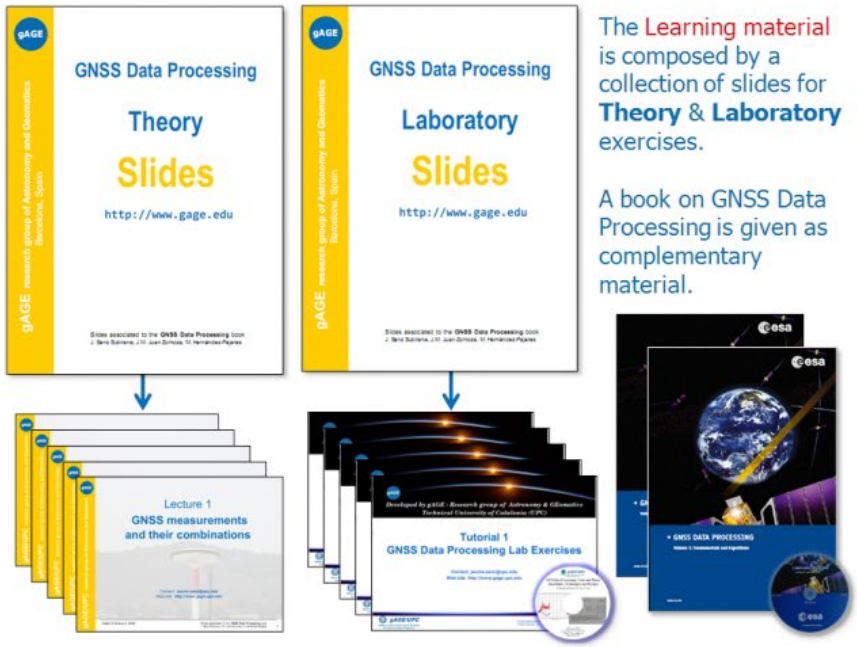
Tue, 09/03/2013 - 15:39 — jaume.sanz



The Learning material is composed by a collection of slides for **Theory** & **Laboratory** exercises.

A book on GNSS Data Processing is given as complementary material.

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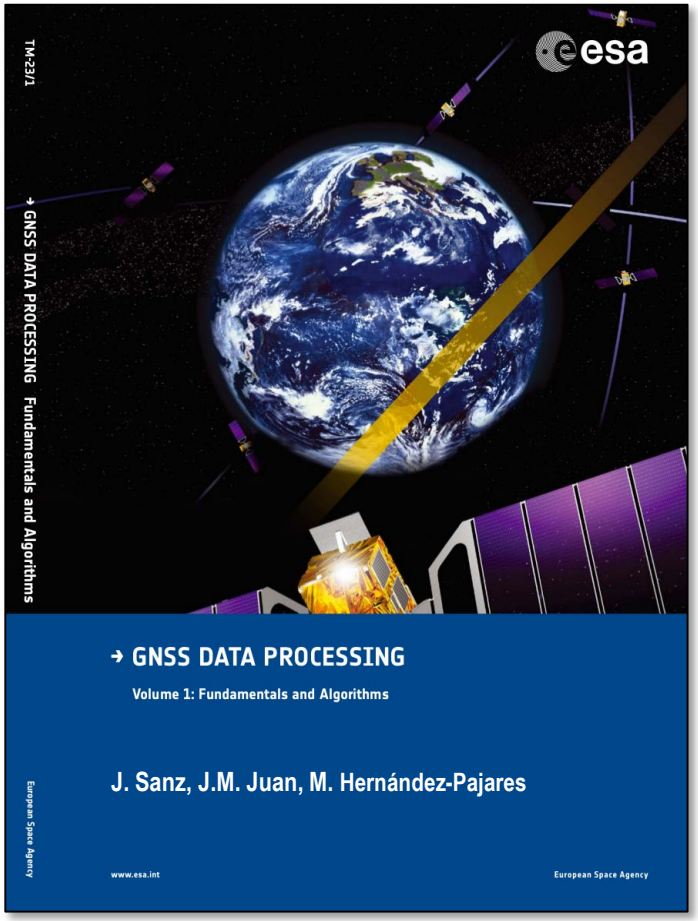

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GNSS Data Processing, Vol. 2: Laboratory exercises.