

Lecture 6

Satellite orbits and clocks

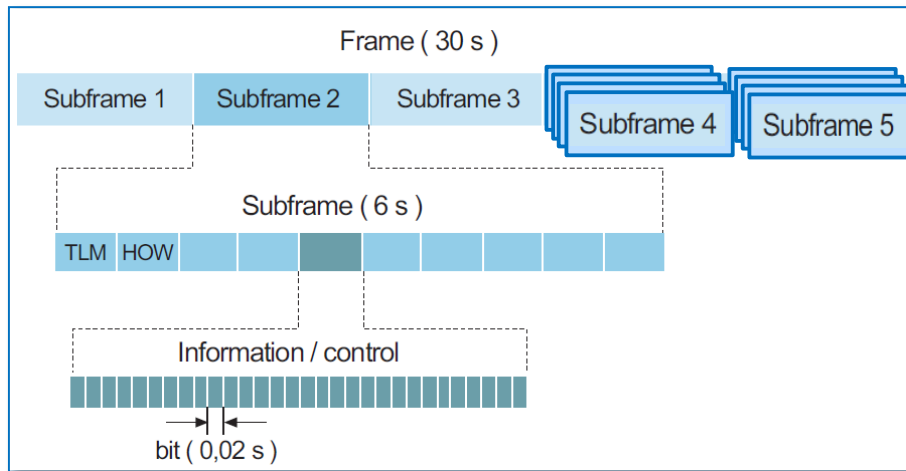
Professors: Dr. J. Sanz Subirana, Dr. J.M. Juan Zornoza
and Dr. Adrià Rovira García

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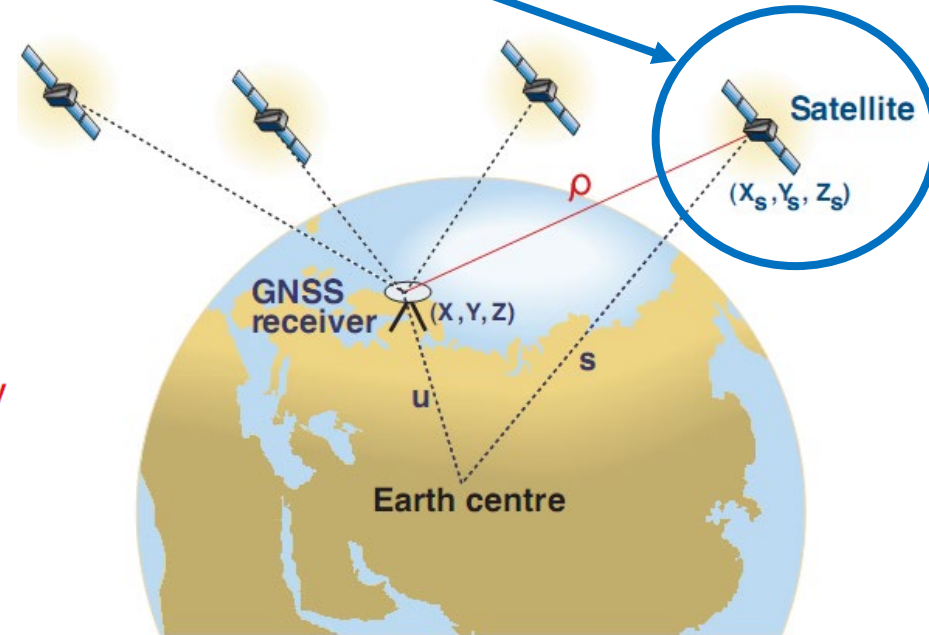
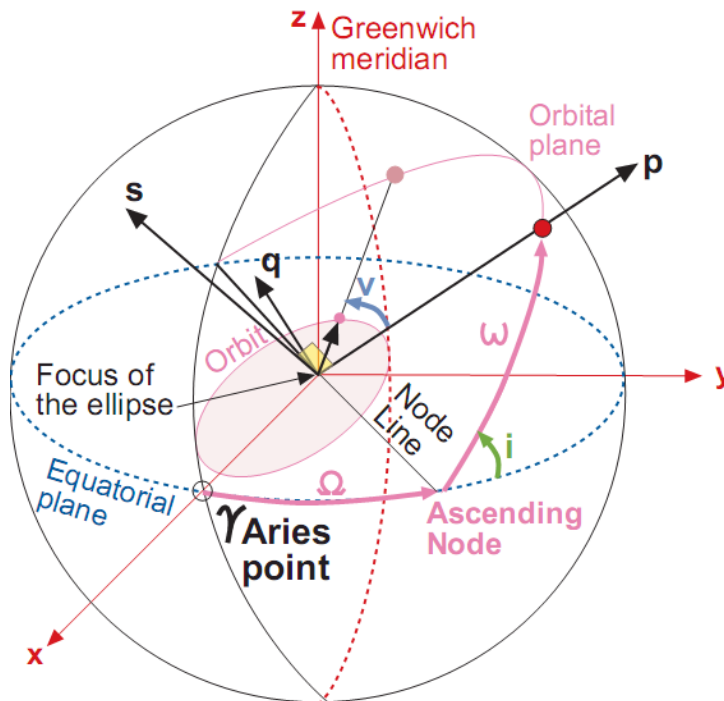
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2. Perturbed Keplerian orbits: Osculating orbit.
3. GPS satellite coordinates computation and accuracy
 - 3.1. From Broadcast Navigation Message.
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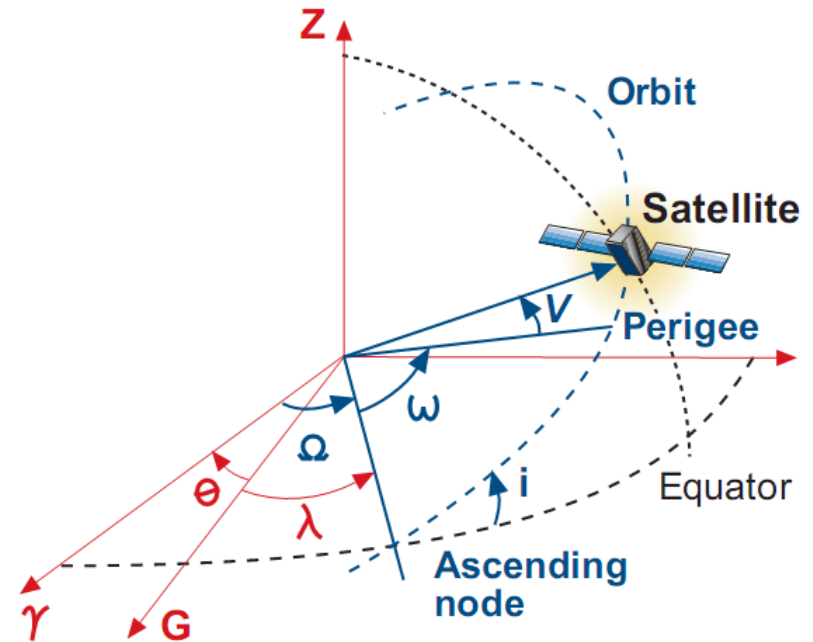
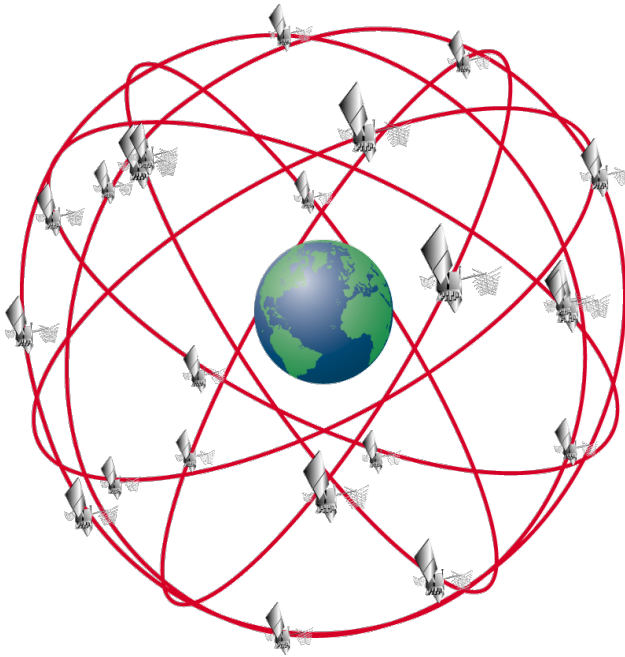


The GPS navigation message provides pseudo-Keplerian elements to compute satellite coordinates



$$(X, Y, Z, V_x, V_y, V_z) \rightarrow (a, e, i, \Omega, \omega, V)$$

6 values are needed (x, y, z, v_x, v_y, v_z) to provide the position and velocity of a body. They can be mapped into the **six Keplerian elements** $(a, e, i, \Omega, \omega, V)$, which provides the "natural" representation of the orbit!

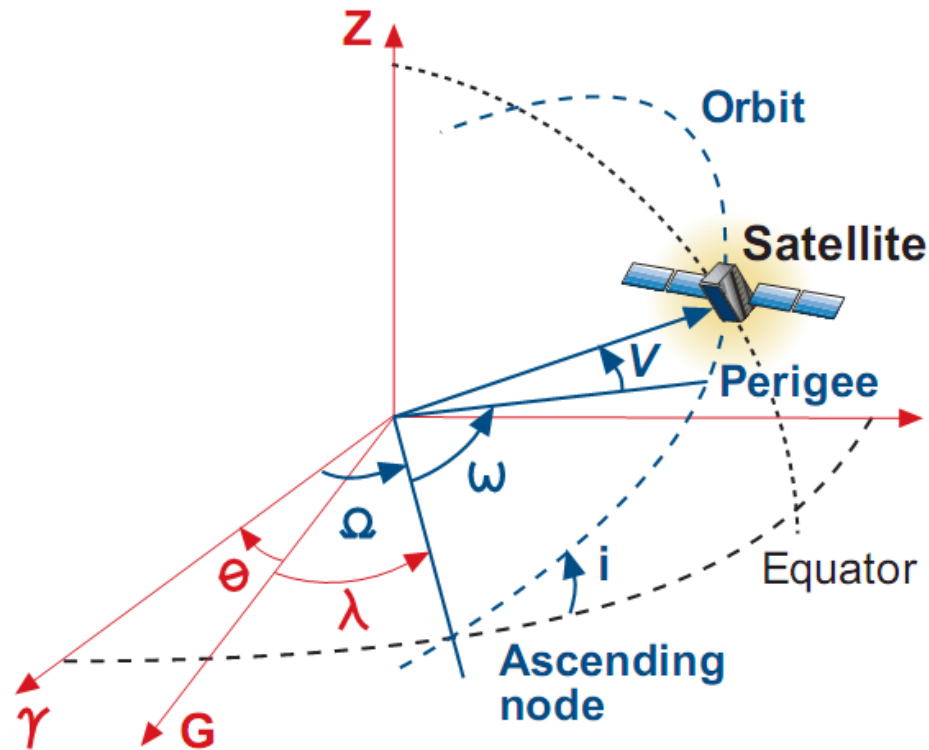
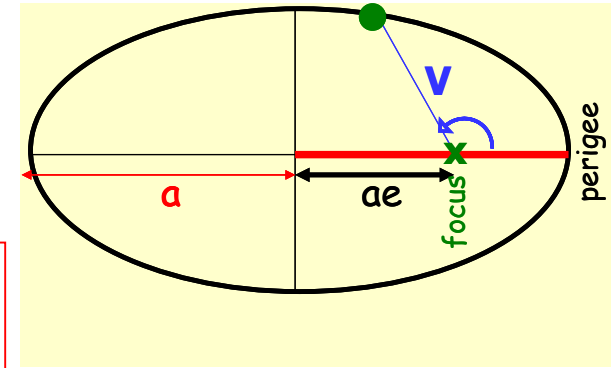


$(a, e, i, \Omega, \omega, V)$

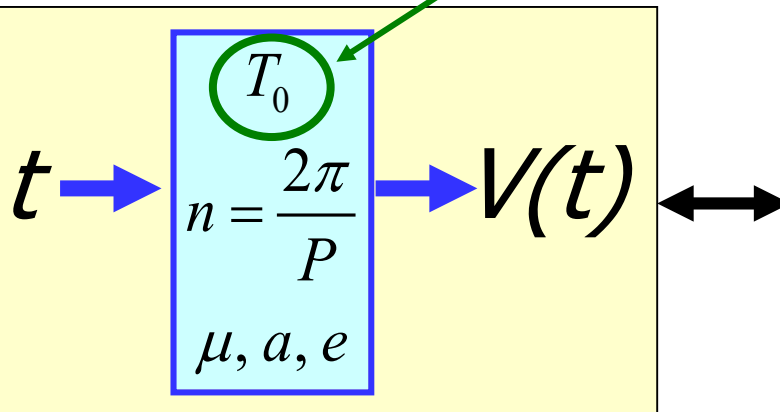
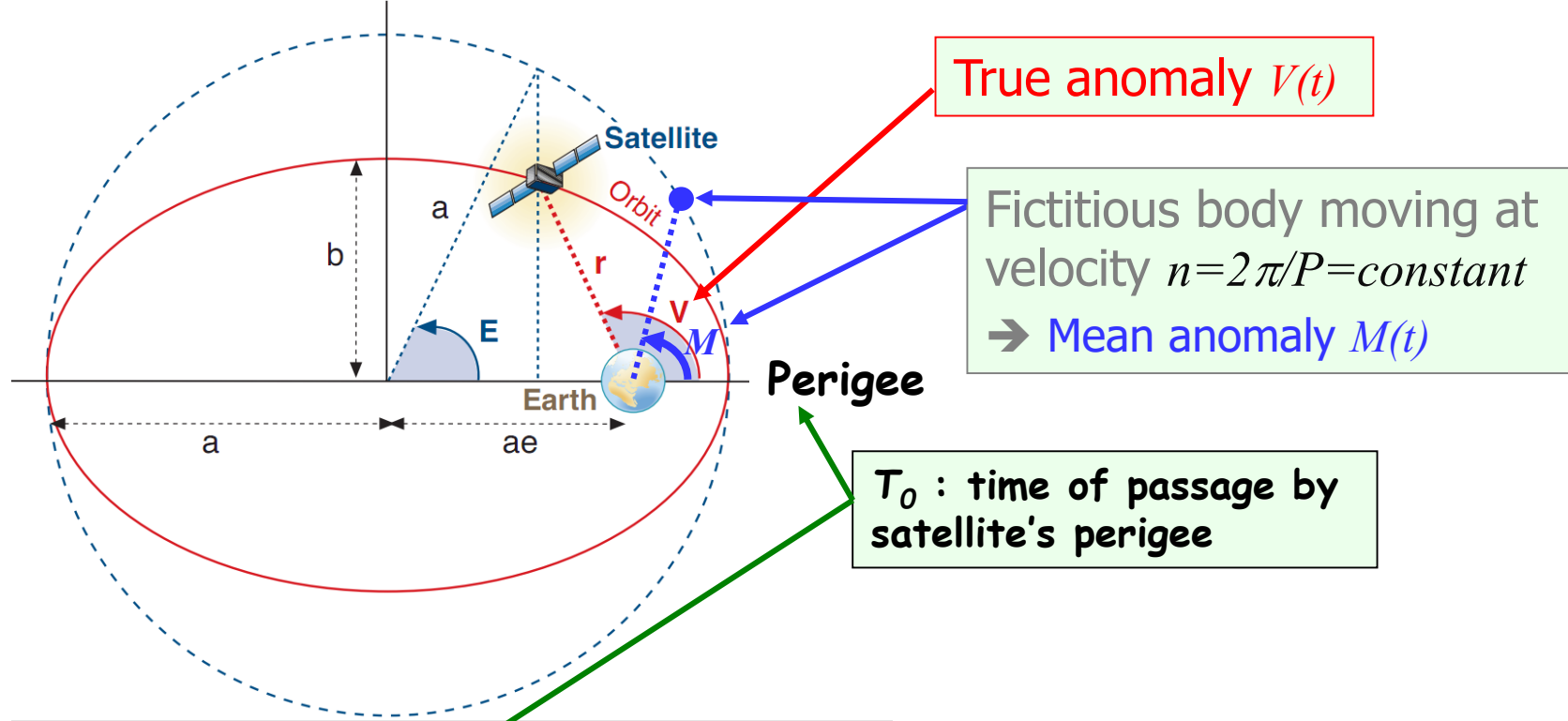
orbit
shape

orbit
orientation

position in
the orbit



- i inclination
- ω argument of perigee
- Ω arg. ascending node (Aries)
- λ arg. ascending node (Greenwich)
- V true anomaly
- θ sidereal time
- γ vernal equinox
- G Greenwich meridian



$$M(t) = n(t - T_0) \quad ; \quad n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

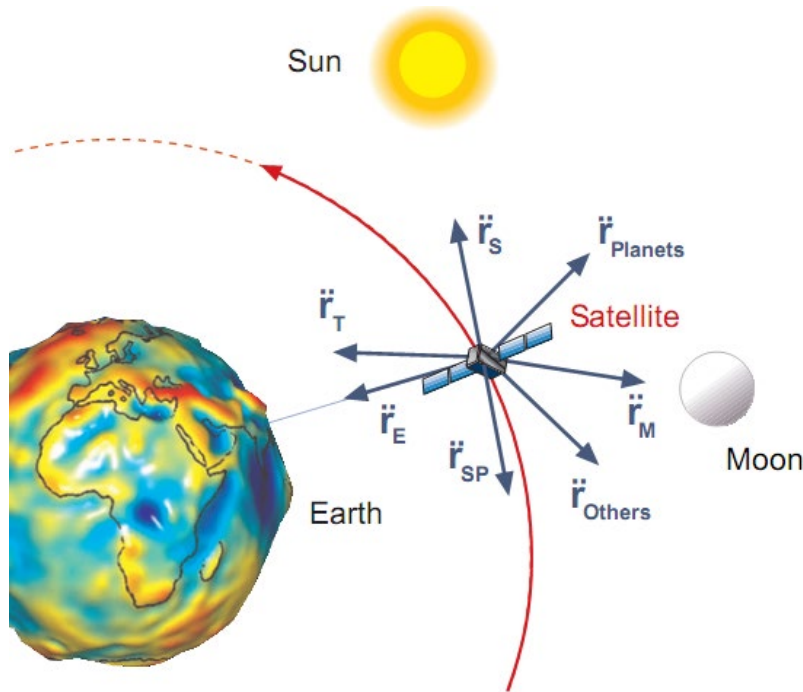
$$E(t) = M(t) + e \sin E(t)$$

$$V(t) = 2 \arctan \left[\sqrt{\frac{1+e}{1-e}} \tan \frac{E(t)}{2} \right]$$

Contents

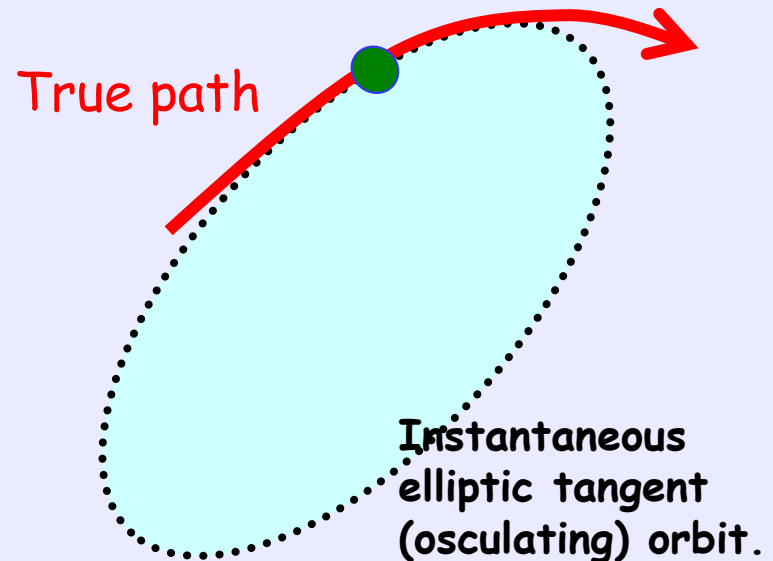
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Due to the non-spherical nature of gravitational potential, the attraction of the Sun and Moon, the solar radiation pressure, etc., **the true satellite path deviates from the elliptic orbit.**



At any time an elliptical orbit tangent to the true path can be defined. This is the “osculating orbit”, whose Keplerian elements vary with time “t”:

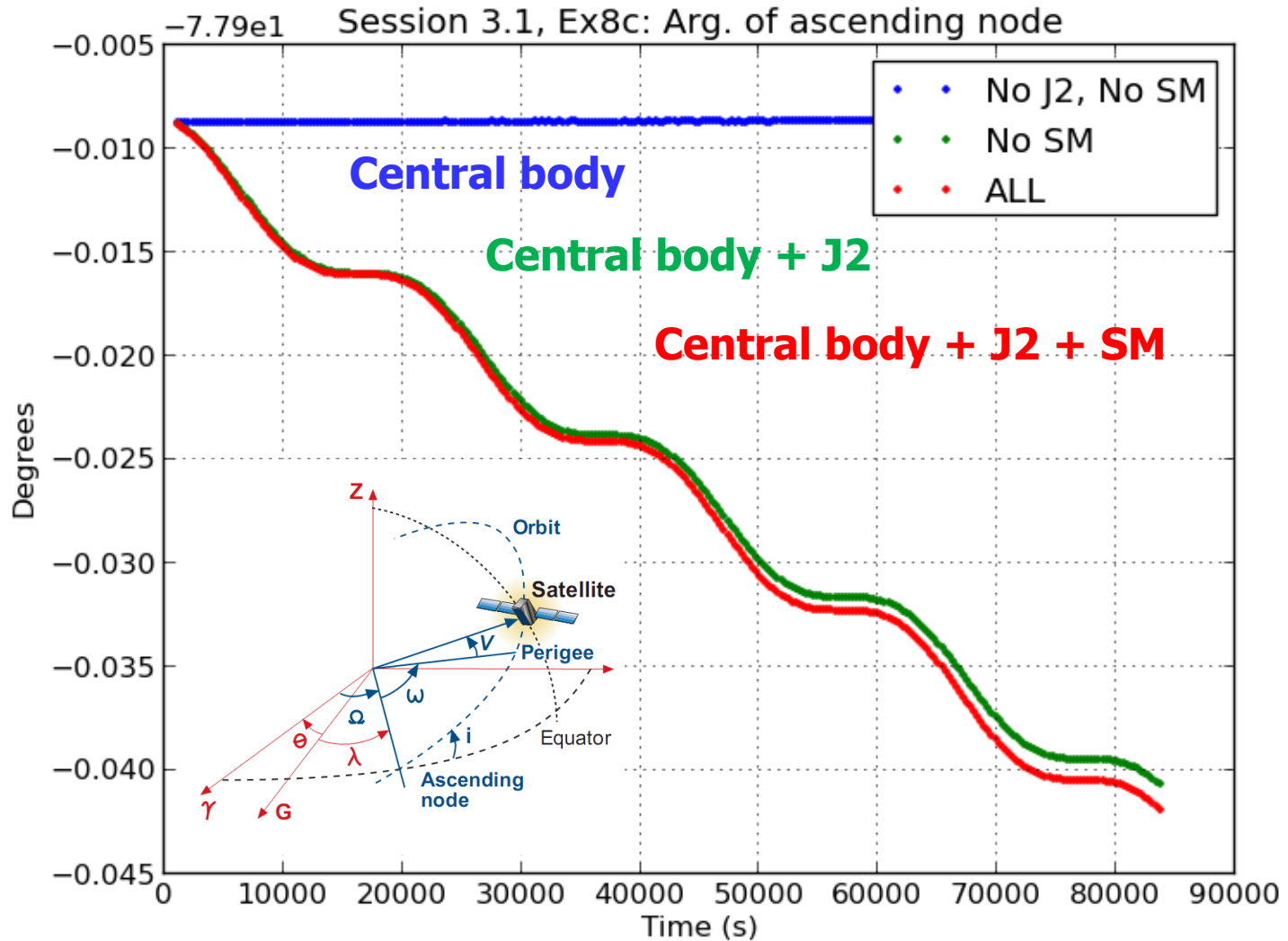
$$a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$$

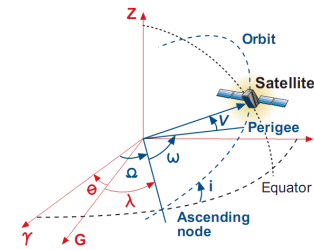
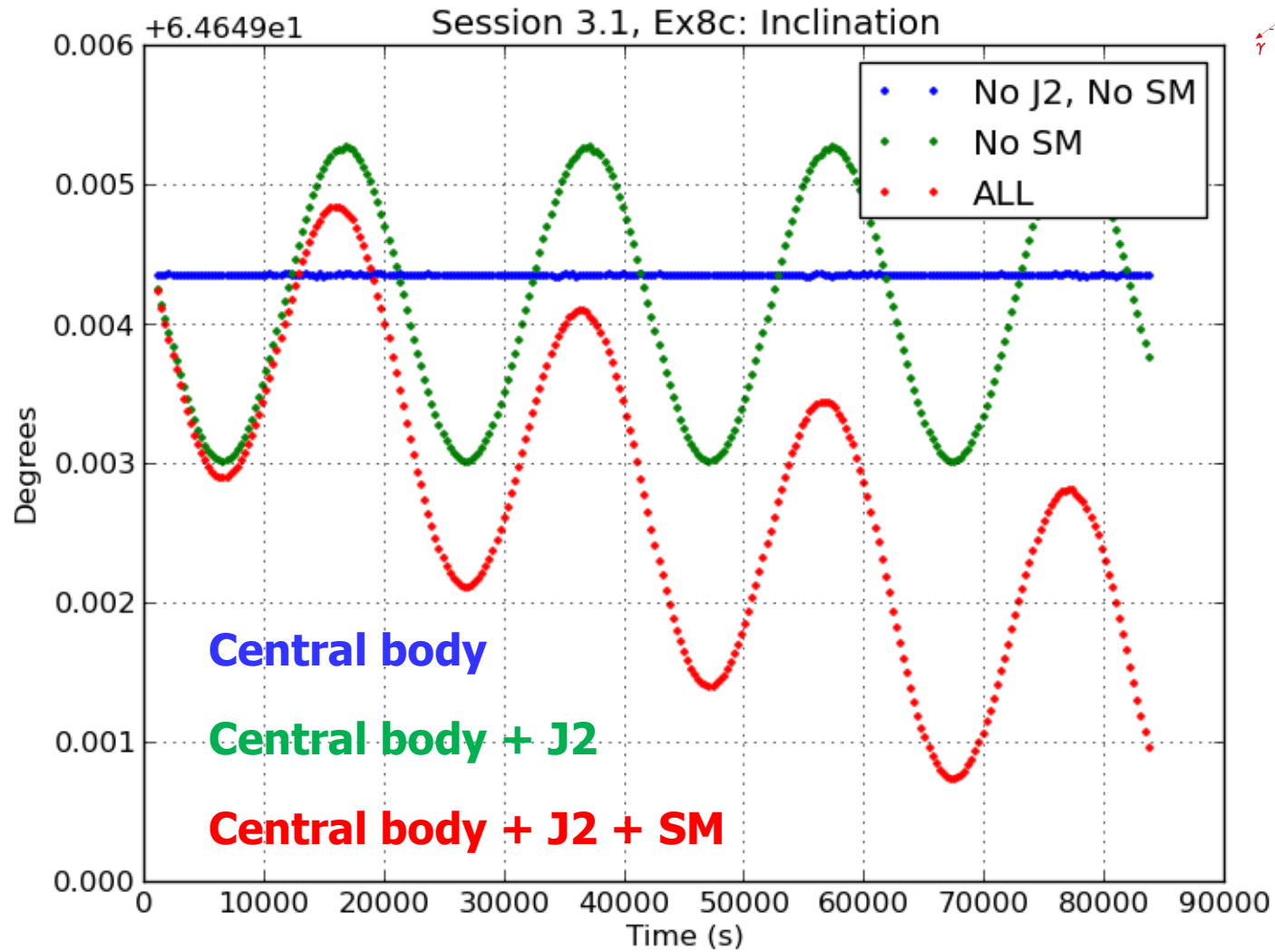


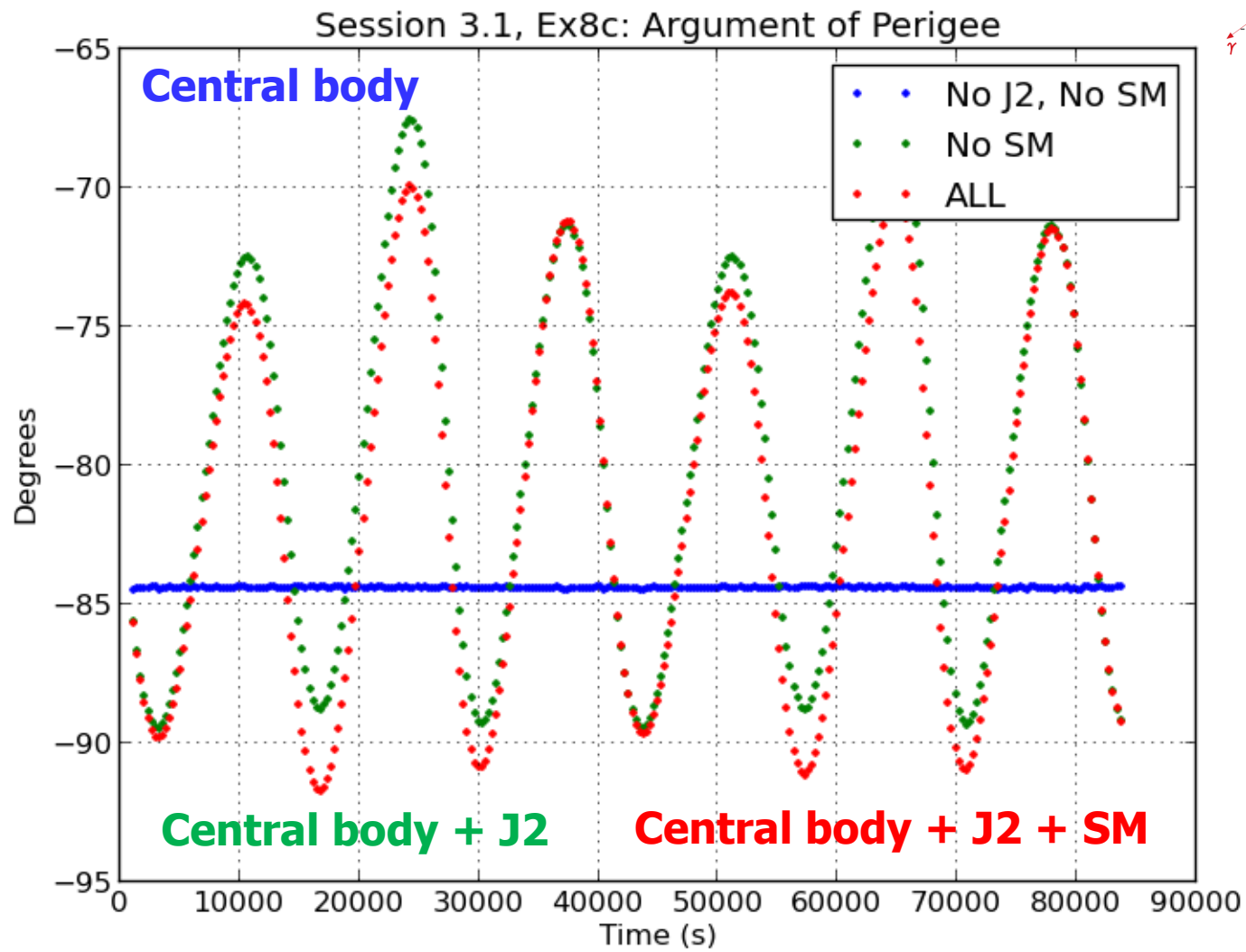
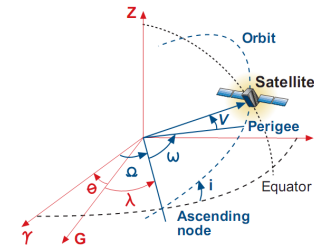
Different magnitudes of perturbation and their effects on GPS orbits

Perturbation	Acceleration (m/s ²)	Orbital effect	
		in 3 hours	in 3 days
Central force (as a reference)	0.56		
J_2	$5 \cdot 10^{-5}$	2 km	14 km
Rest of the harmonics	$3 \cdot 10^{-7}$	50–80 m	100–1500 m
Solar + Moon grav.	$5 \cdot 10^{-6}$	5–150 m	1000–3000 m
Tidal effects	$1 \cdot 10^{-9}$	–	0.5–1.0 m
Solar rad. pressure	$1 \cdot 10^{-7}$	5–10 m	100–800 m

GLONASS Broadcast orbit integration terms







Calculation of osculating orbital elements from position and velocity (**rv2osc.f**)

$$(x, y, z, v_x, v_y, v_z) \Rightarrow (a, e, i, \Omega, \omega, M)$$

$$\vec{c} = \vec{r} \times \vec{v} \Rightarrow p = \frac{c^2}{\mu} \Rightarrow p$$

$$v^2 = \mu(2/r - 1/a) \Rightarrow a$$

$$p = a(1 - e^2) \Rightarrow e$$

$$\vec{c} = c\vec{S} \Rightarrow \Omega = \arctan(-c_x/c_y); \quad i = \arccos(c_z/c) \Rightarrow \Omega, i$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r \cos(V) \\ r \sin(V) \\ 0 \end{pmatrix} = r \begin{pmatrix} \cos \Omega \cos(\omega + V) - \sin \Omega \sin(\omega + V) \cos i \\ \sin \Omega \cos(\omega + V) + \cos \Omega \sin(\omega + V) \cos i \\ \sin(\omega + V) \sin i \end{pmatrix} \Rightarrow \omega + V$$

$$r = \frac{p}{1 + e \cos(V)} \Rightarrow \omega, V$$

$$\tan(E/2) = \left(\frac{1-e}{1+e}\right)^{1/2} \tan(V/2) \quad ; \quad M = E - e \sin E \Rightarrow M$$

Calculation of position and velocity from osculating orbital elements (**osc2rv.f**)

$$(a, e, i, \Omega, \omega, \underbrace{T; t}_V) \Rightarrow (x, y, z, v_x, v_y, v_z)$$

$$t \xRightarrow{M = n(t - T)} M \xRightarrow{M = E - e \sin E} E \xRightarrow{r = a(1 - e \cos E)} (r, V)$$

$$\tan(V/2) = \left(\frac{1+e}{1-e}\right)^{1/2} \tan(E/2)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r \cos(V) \\ r \sin(V) \\ 0 \end{pmatrix} ; \quad \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{na^2}{r} \{ \vec{Q}(1 - e^2)^{1/2} \cos E - \vec{P} \sin E \}$$

Where:

$$\begin{aligned} R &= R_3(-\Omega) R_1(-i) R_3(-\omega) = \\ &= \begin{pmatrix} \cos \Omega & -\sin \Omega & 0 \\ \sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos \omega & -\sin \omega & 0 \\ \sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} P_x & Q_x & S_x \\ P_y & Q_y & S_y \\ P_z & Q_z & S_z \end{pmatrix} = [\vec{P} \ \vec{Q} \ \vec{S}] \end{aligned}$$

Exercise: Orbital elements variation:

File 1995-10-18.eci contains the precise position and velocities of GPS satellites every 5 minutes for October 18th, 1995 in a Earth-Centred Inertial system (ECI)

[from JPL/NASA server:

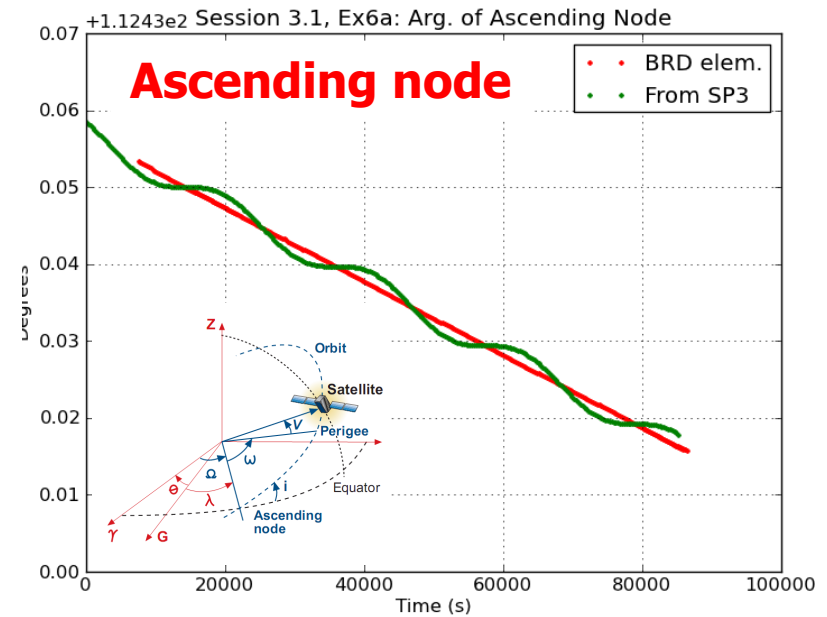
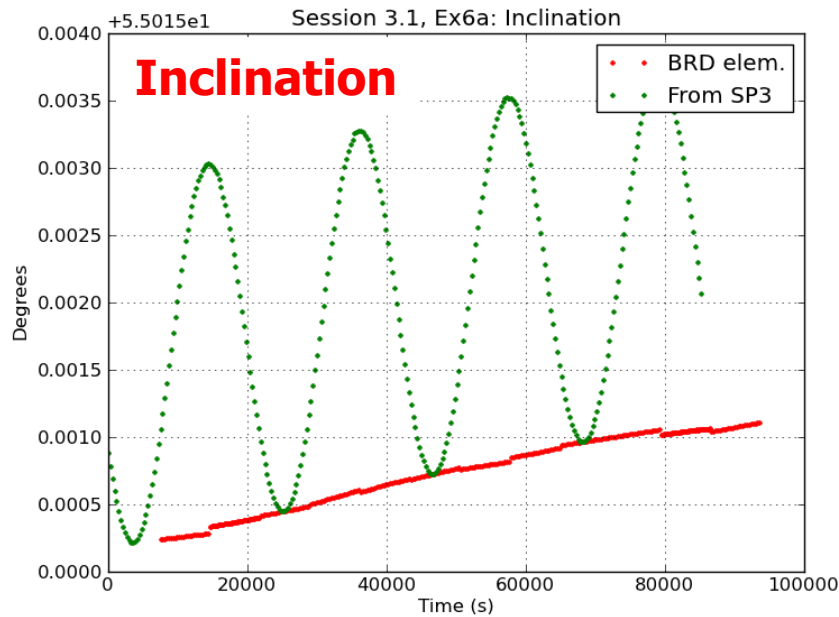
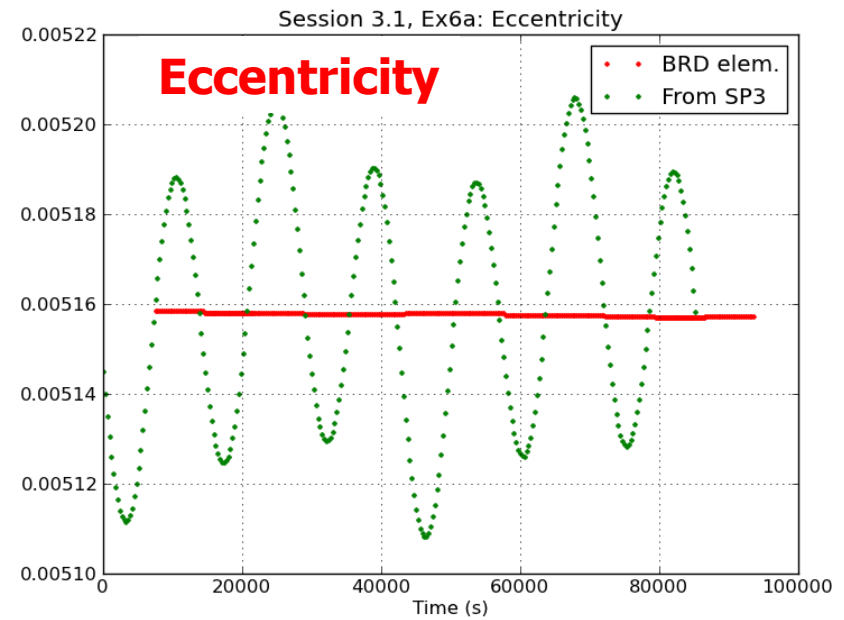
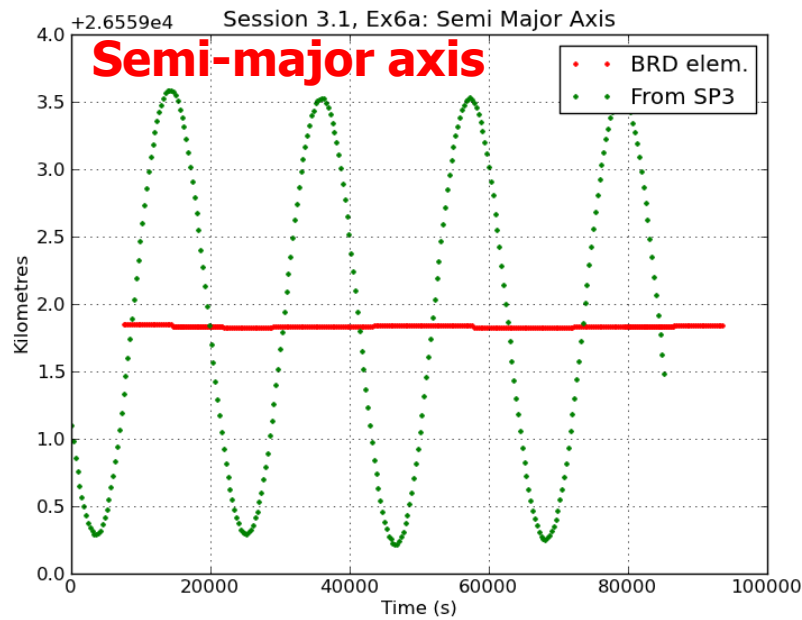
ftp://sideshow.jpl.nasa.gov/pub/gipsy_products]

- Use program "**rv2osc**" to compute the instantaneous orbital elements $(X, Y, Z, V_x, V_y, V_z) \rightarrow (a, e, i, \Omega, \omega, V)$
- Plot the orbital elements in function of time to show their variation: $a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$
- Compare with the broadcast orbital elements

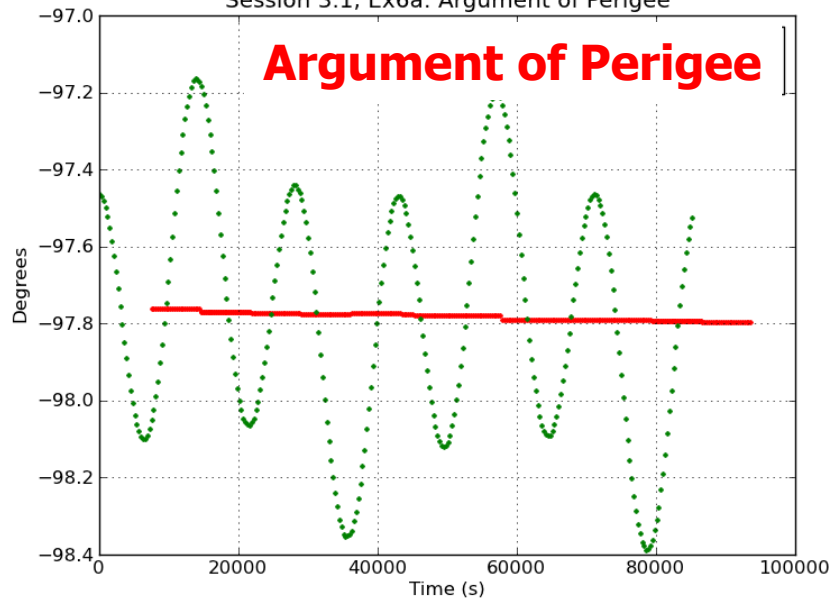
Solution:

a) `cat 1995-10-18.eci|rv2osc> orb.dat`

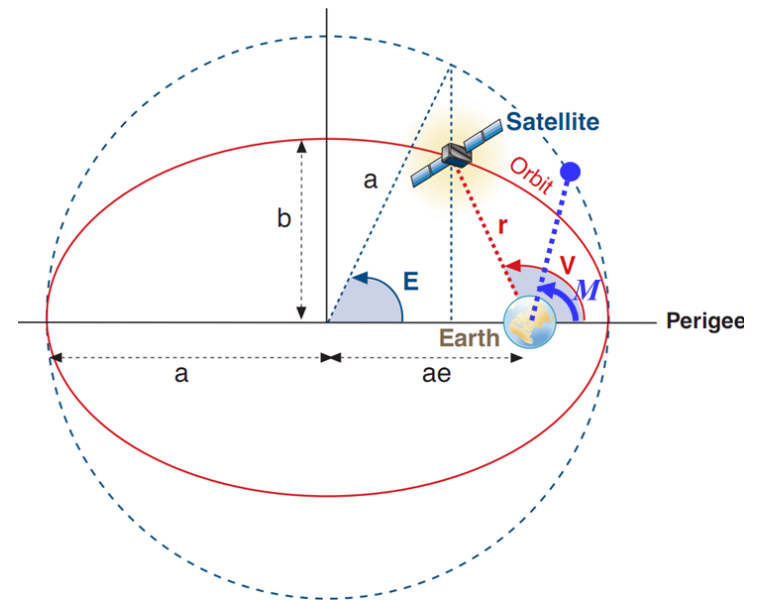
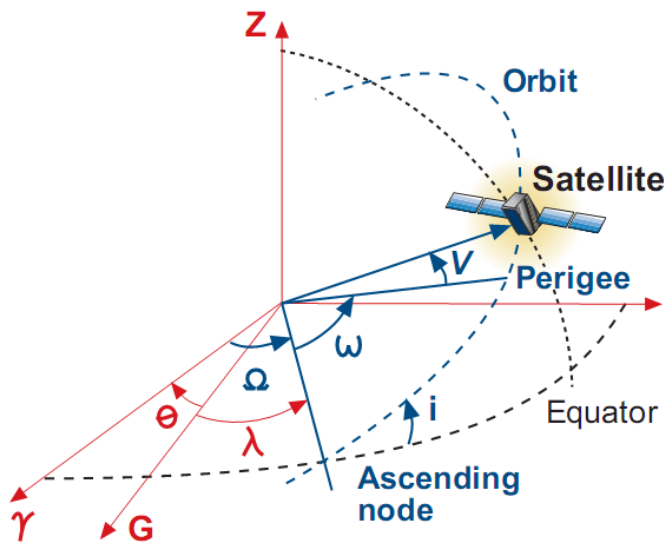
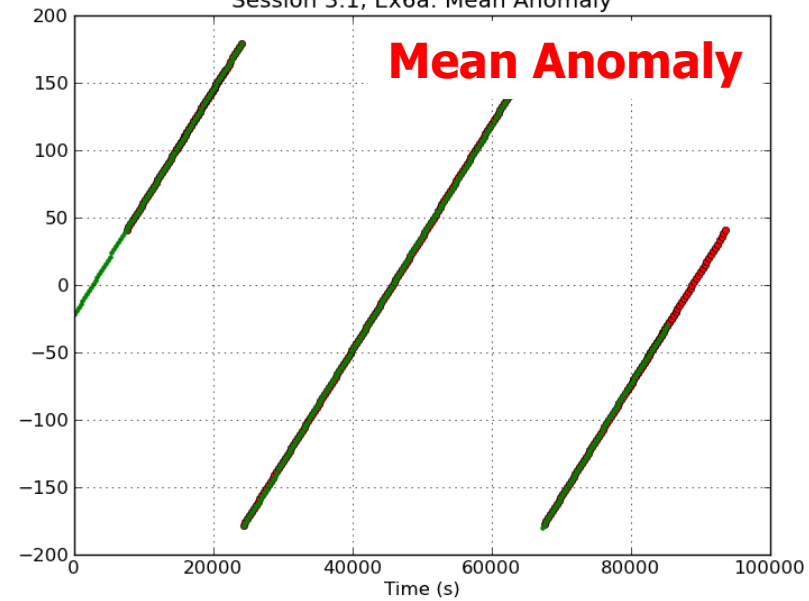
b) See the following plots



Session 3.1, Ex6a: Argument of Perigee



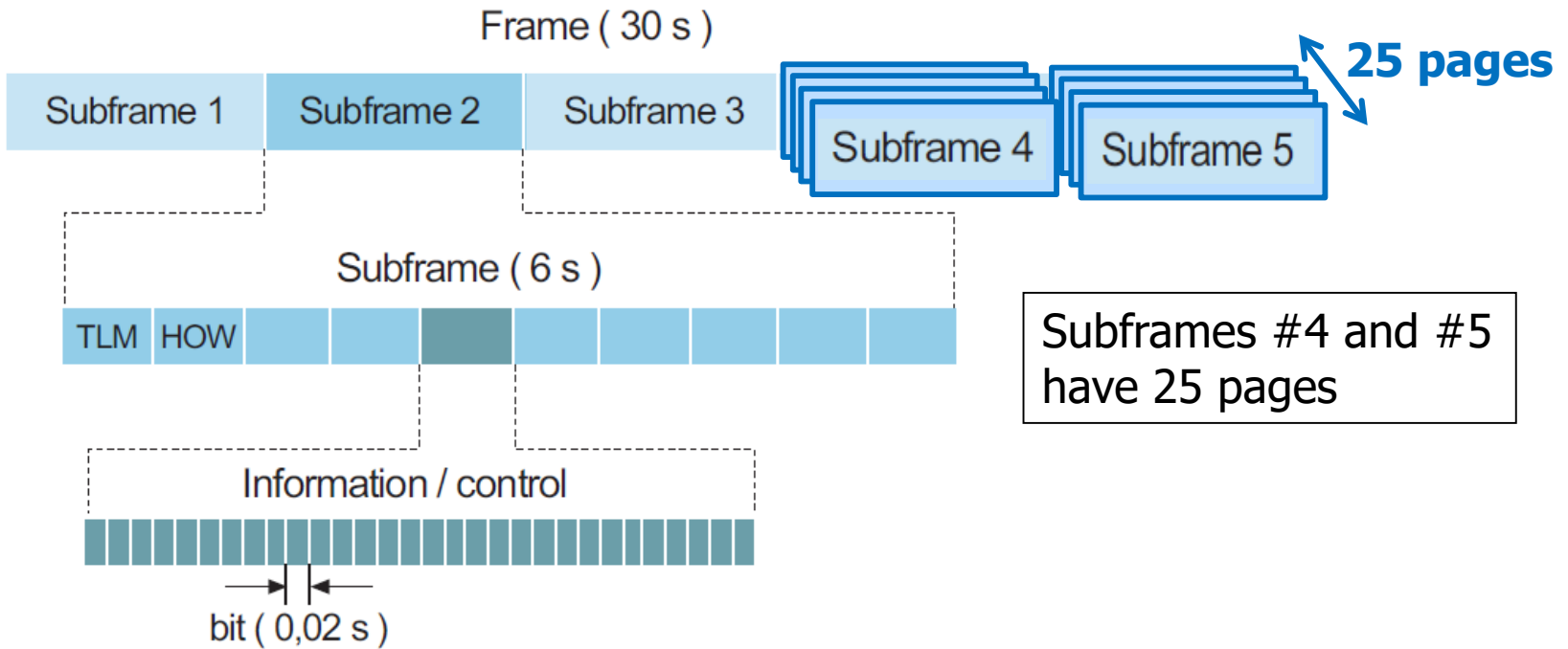
Session 3.1, Ex6a: Mean Anomaly



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GPS navigation message



One Master Frame includes All 25 pages of Subframes #4 and #5 → $25 \times 30s = \mathbf{12.5 \text{ min}}$

Subframe 1 contains information about the parameters to be applied to **satellite clock** status for its correction. These values are polynomial coefficients that allow time onboard to be converted to GPS time. The subframe also contains information on satellite health condition.

Subframes 2 and 3 contain **satellite ephemerides**.

Subframe 4 provides **ionospheric model** parameters (in order to adjust for ionospheric refraction), UTC information, part of the **almanac**, and indications whether the A/S is activated or not (which transforms the P code into encrypted Y code).

Subframe 5 contains data from the **almanac** and on constellation status. It allows rapid identification of the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.

Ephemeris in navigation message

Parameter	Explanation
$IODE$	Series number of ephemerides data
t_{oe}	Ephemerides reference epoch
\sqrt{a}	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω	Ascending node's right ascension
Δn	Mean motion difference
\dot{i}	rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
c_{uc}, c_{us}	Latitude argument correction
c_{rc}, c_{rs}	Orbital radius correction
c_{ic}, c_{is}	Inclination correction

In order to calculate WGS84 satellite coordinates, you should apply the following algorithm [GPS/SPS-SS, table 2-15] (see in the book FORTRAN subroutine orbit.f)

RINEX ephemeris file

```

      2          NAVIGATION DATA      GPS      RINEX VERSION/ TYPE
XPRINT v1.1          gAGE      00/08/17 09:31:37  PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE      COMMENT
+1.7695E-08 +2.2352E-08 -1.1921E-07 -1.1921E-07      ION ALPHA
+1.1878E+05 +1.4746E+05 -1.3107E+05 -3.2768E+05      ION BETA
+1.955777406693E-08+1.598721155460E-14  405504      1064 DELTA.UTC: A0,A1,T,W
13      LEAP SECONDS
      END OF HEADER

```

```

03 00 5 30 10 0 0.0+7.855705916882E-06+3.524291969370E-12+0.000000000000E+00

```

```

+1.010000000000E+02+6.500000000000E+01+5.456298524109E-09+5.530285585107E-01

```

Mo

```

+3.475695848465E-06+1.308503560722E-03+2.641230821609E-06+5.153678266525E+03

```

e, \sqrt{a}

```

+2.088000000000E+05+1.117587089539E-08+7.472176136643E-01-1.862645149231E-09

```

TOE, Ω

```

+9.412719852649E-01+3.163750000000E+02+1.125448382894E+00-8.826796182859E-09

```

io, ω

```

+1.239337382719E-10+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00

```

```

+4.000000000000E+00+0.000000000000E+00-4.190951585770E-09+6.130000000000E+02

```

TGD

```

+2.044980000000E+05+0.000000000000E+00+0.000000000000E+00+0.000000000000E+00

```

```

06 00 5 30 10 0 0.0+1.636799424887E-06+0.000000000000E+00+0.000000000000E+00

```

```

+6.000000000000E+01+5.100000000000E+01+5.198073527168E-09-5.601816471398E-01

```

```

+2.635642886162E-06+6.763593177311E-03+2.468004822731E-06+5.153726325989E+03

```

```

+2.088000000000E+05+1.862645149231E-08+7.894129138508E-01+8.195638656616E-08

```

```

+9.487675576456E-01+3.229687500000E+02-2.409256713064E+00-8.734292400447E-09

```

```

+4.714481929846E-11+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00

```

3.1. Computation of satellite coordinates from navigation message (**orbit.f**)

- Computation of t_k time since ephemerids reference epoch t_{oe} (t and t_{oe} are given in GPS seconds of week):

$$t_k = t - t_{oe}$$

- Computation of mean anomaly M_k for t_k :

$$M_k = M_0 + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n \right) t_k$$

- Iterative resolution of Kepler's equation in order to compute eccentric anomaly E_k :

$$M_k = E_k - e \sin E_k$$

- Calculation of true anomaly v_k :

$$v_k = \arctan \left(\frac{\sqrt{1-e^2} \sin E_k}{\cos E_k - e} \right)$$

- Computation of latitude argument u_k from perigee argument W , true anomaly v_k and corrections c_{uc} and c_{us} :

$$u_k = \omega + v_k + c_{uc} \cos 2(\omega + v_k) + c_{us} \sin 2(\omega + v_k)$$

- Computation of radial distance r_k taking into consideration corrections c_{rc} and c_{rs} :

$$r_k = a(1 - 2 \cos E_k) + c_{rc} \cos 2(\omega + v_k) + c_{rs} \sin 2(\omega + v_k)$$

- Calculation of orbital plane inclination i_k from inclination i_0 at reference epoch t_{oe} and corrections c_{ic} and c_{is} :

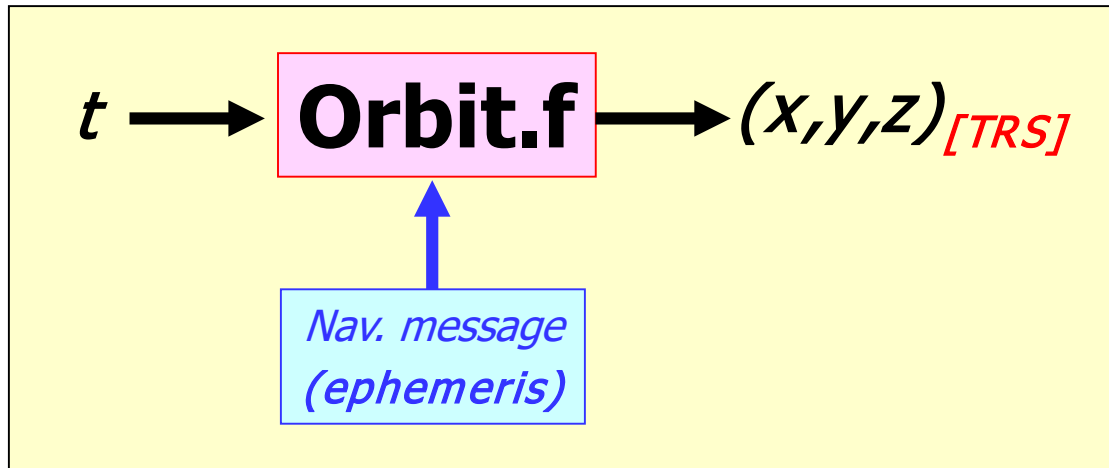
$$i_k = i_0 + it_k + c_{ic} \cos 2(\omega + v_k) + c_{is} \sin 2(\omega + v_k)$$

- Computation of ascending node longitude Ω_k (Greenwich), from longitude Ω_0 at start of GPS week, corrected from apparent variation of sidereal time at Greenwich between start of week and reference time $t_k = t - t_{oe}$ and also corrected from change of ascending node longitude since reference epoch t_{oe} .

$$\Omega_k = \Omega_0 + (\Omega - \omega_E)t_k - \omega_E t_{oe}$$

- Calculation of coordinates in CTS system, applying three rotations (around u_k , i_k , Ω_k):

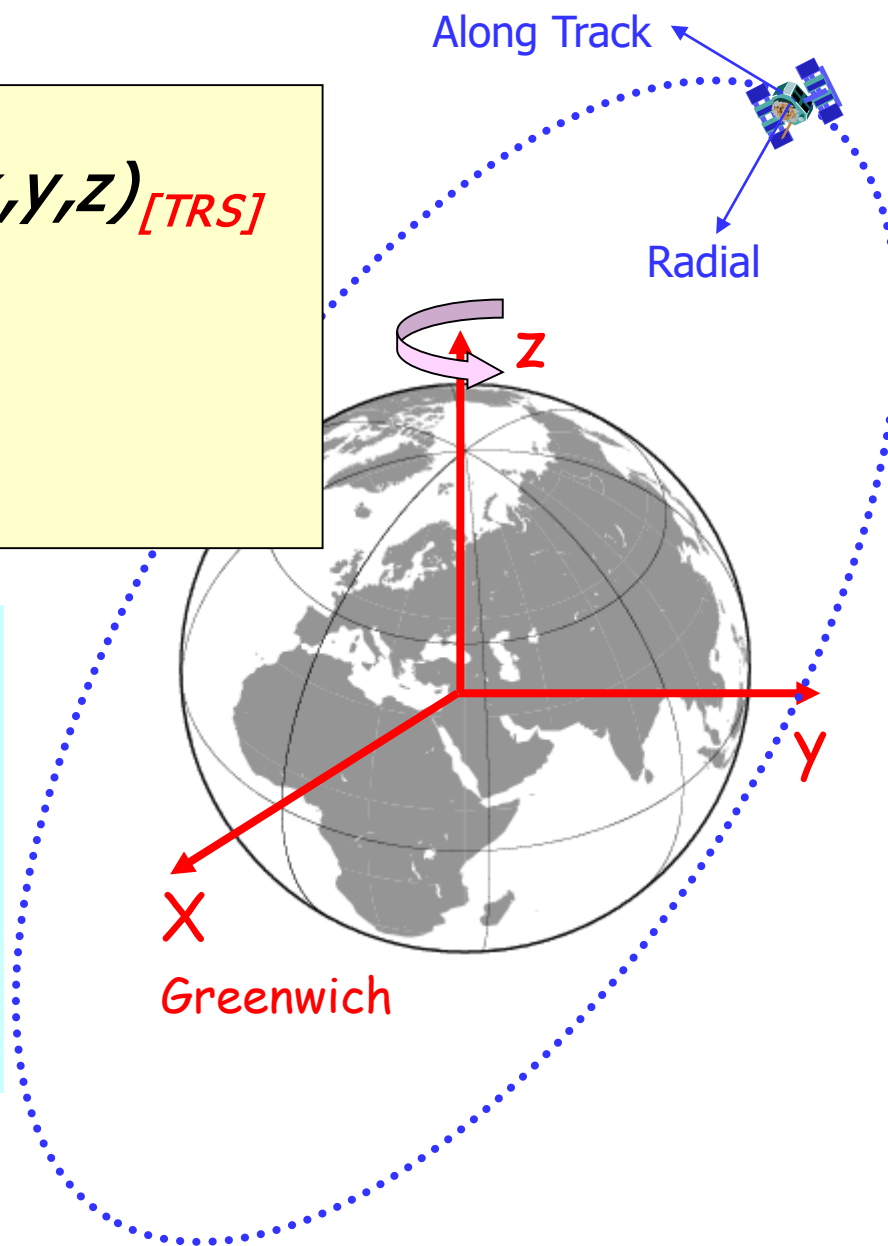
$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = \mathbf{R}_3(-\Omega_k) \mathbf{R}_1(-i_k) \mathbf{R}_3(-u_k) \begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$

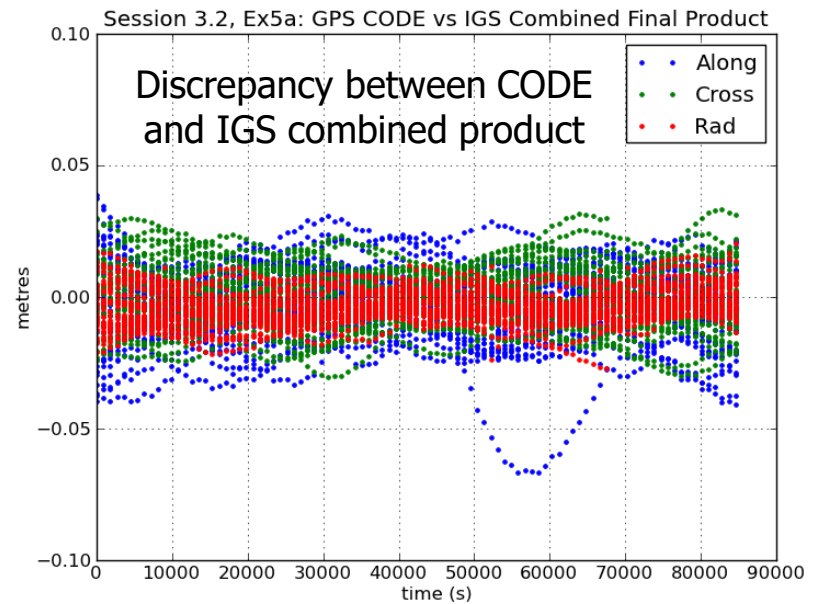
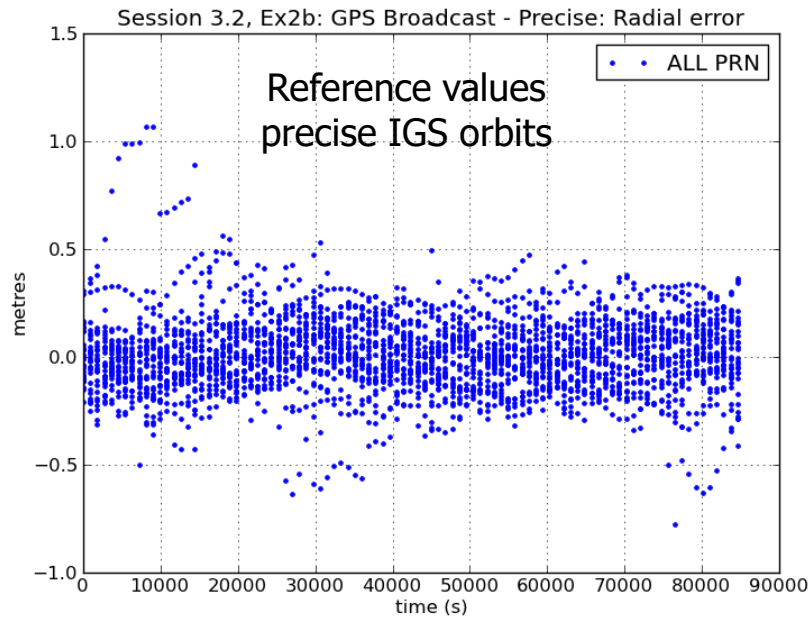
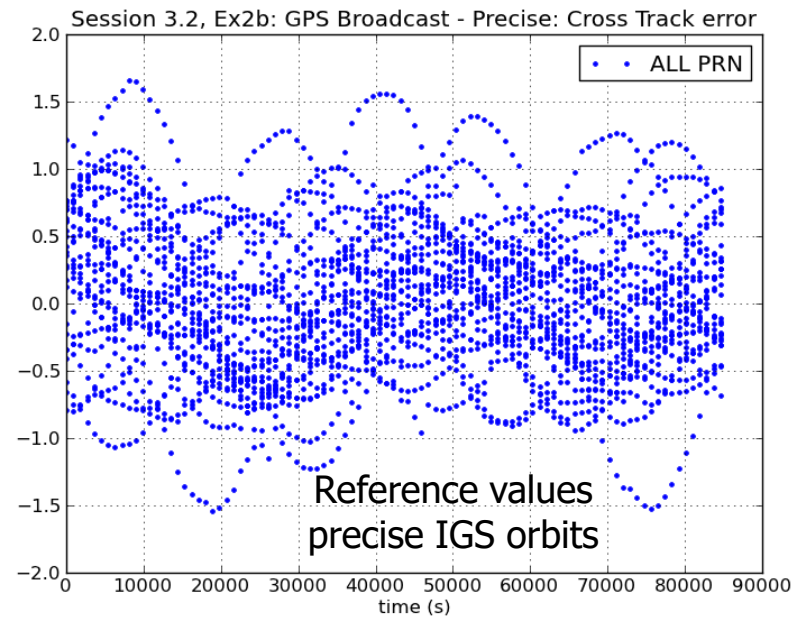
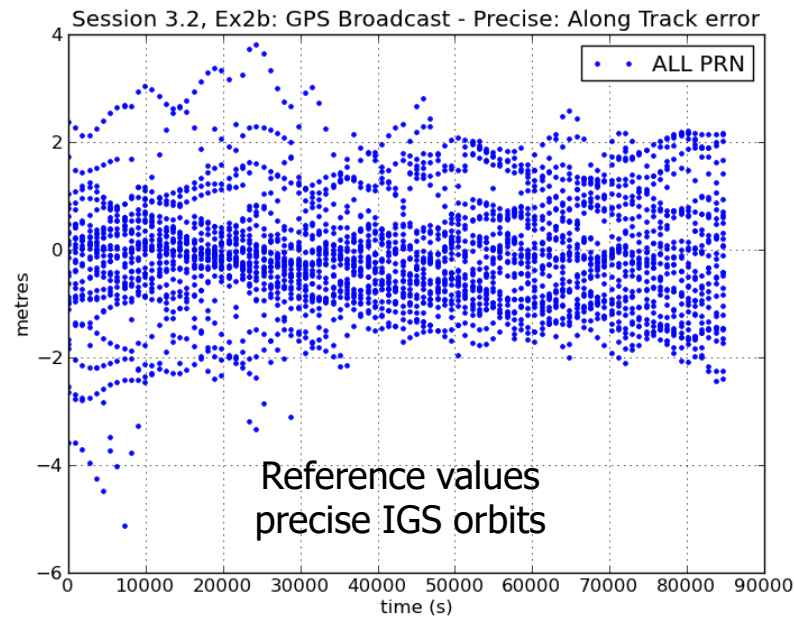


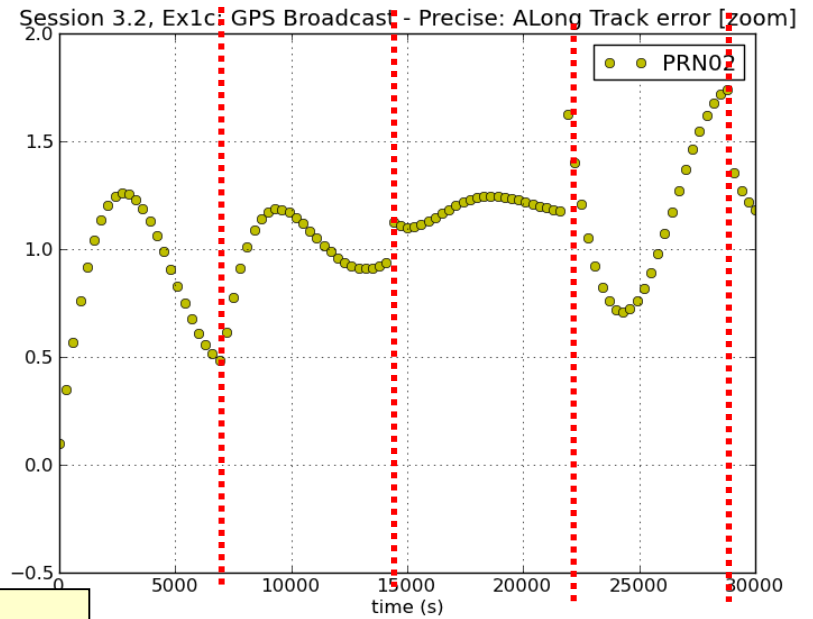
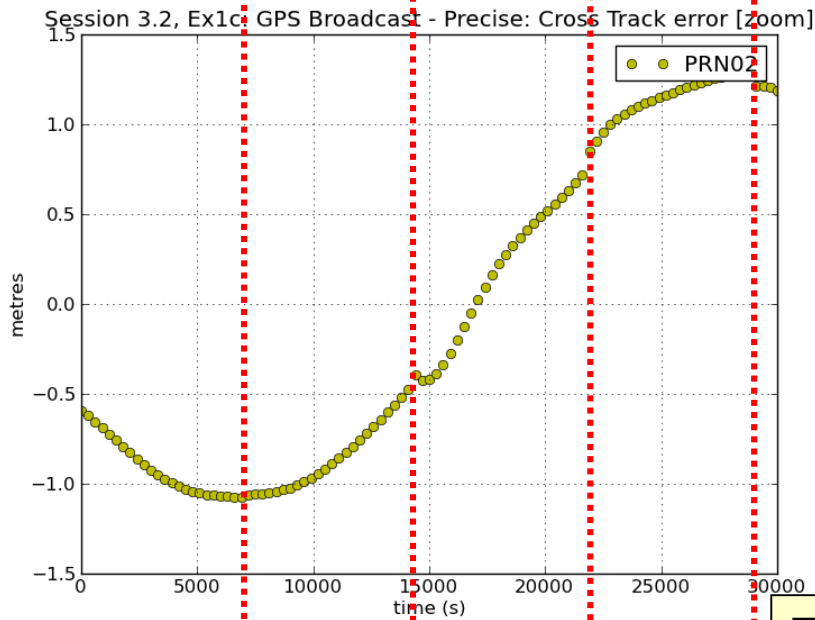
Conventional Terrestrial Reference System (**TRS**):

Earth Centered, Earth-Fixed (ECEF) →

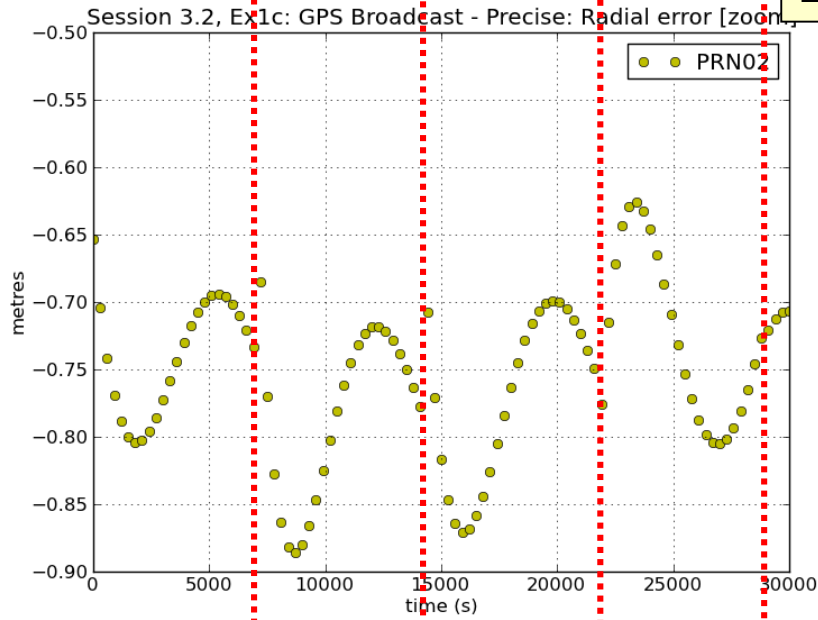
the reference system rotates with Earth.







Zoom



Broadcast Orbit Updates

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3.2 Computation of satellite coordinates from precise products.

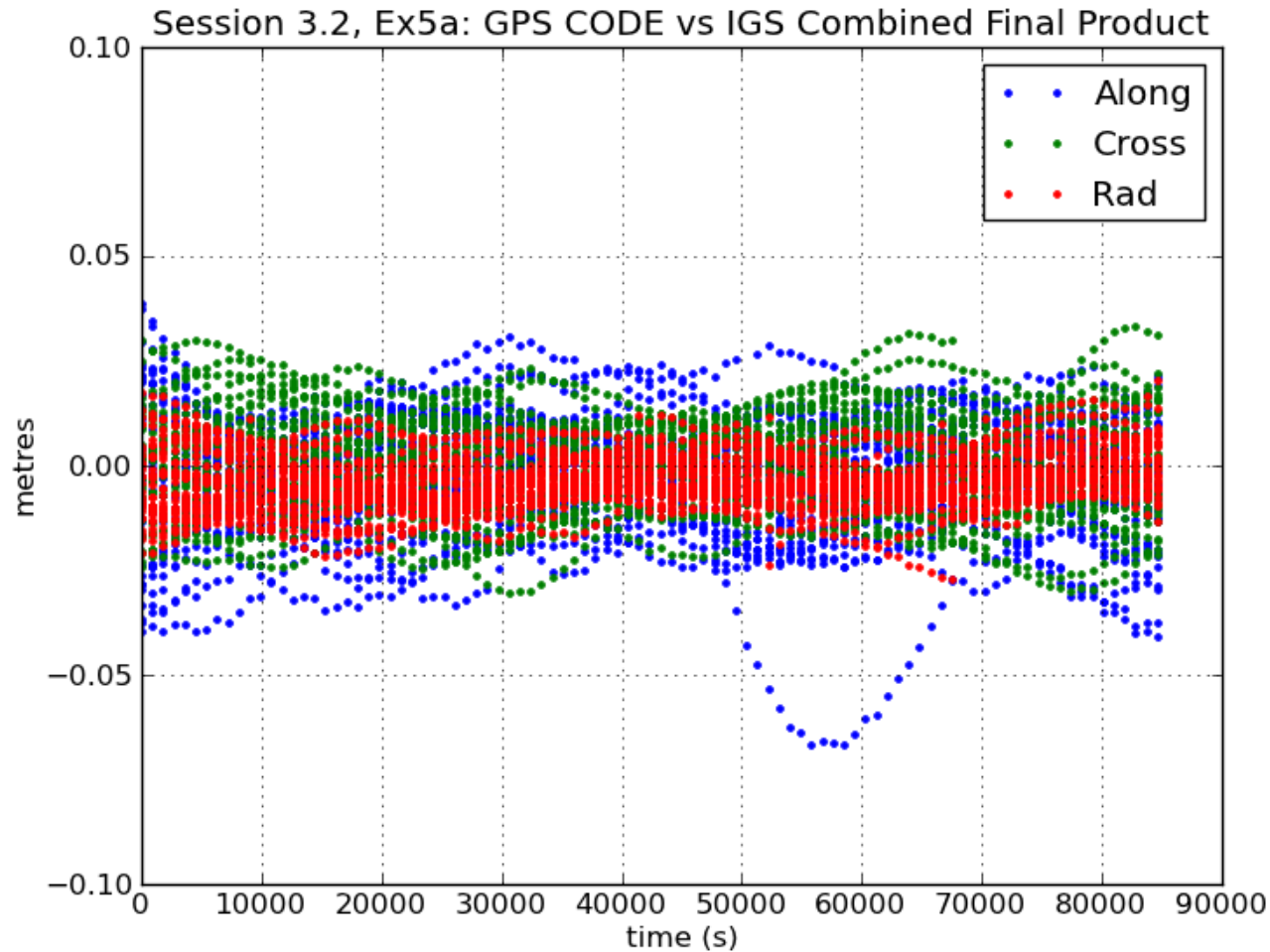
Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server <http://igscb.jpl.nasa.gov>

Orbits are given by (x,y,z) coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

$$\begin{aligned}
 P_n(x) &= \sum_{i=1}^n y_i \frac{\prod_{j \neq i} (x - x_j)}{\prod_{j \neq i} (x_i - x_j)} \\
 &= y_1 \frac{x - x_2}{x_1 - x_2} \cdots \frac{x - x_n}{x_1 - x_n} + \cdots \\
 &\quad + y_i \frac{x - x_1}{x_i - x_1} \cdots \frac{x - x_{i-1}}{x_i - x_{i-1}} \frac{x - x_{i+1}}{x_i - x_{i+1}} \cdots \frac{x - x_n}{x_i - x_n} + \cdots \\
 &\quad + y_n \frac{x - x_1}{x_n - x_1} \cdots \frac{x - x_{n-1}}{x_n - x_{n-1}}
 \end{aligned}$$

IGS orbit and clock products (for PPP):

Discrepancy between CODE and IGS combined product.



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GPS Satellite Clock computation: Broadcast message

$$dt^{\text{sat}} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2$$

PRN		t0 YY MM DD H M S		a0	a1	a2
2	NAVIGATION DATA	GPS		RINEX VERSION / TYPE		
sr1/v1.8.1.4	BAI	95/10/19 03:18:35		PGM / RUN BY / DATE		
CASA				COMMENT		
-2444431.2031	-4428688.6270	3875750.1442		COMMENT		
END OF HEADER						
14	95 10 18 00 51 44.0	1.129414886236D-05		1.136868377216D-13		0.000000000000D+00
1.730000000000D+02-5.175000000000D+01 4.375182243902D-09-5.836427291652D-01						
-2.712011337280D-06 2.427505562082D-03 8.568167686462D-06 5.153718931198D+03						
2.623040000000D+05 4.470348358154D-08 1.698435481558D+00 1.676380634308D-08						
9.636381916043D-01 2.153437500000D+02 3.056960010495D+00-8.030691653399D-09						
-5.178787145843D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00						
3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.730000000000D+02						
2.592180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00						

Computation of satellite clocks from precise products

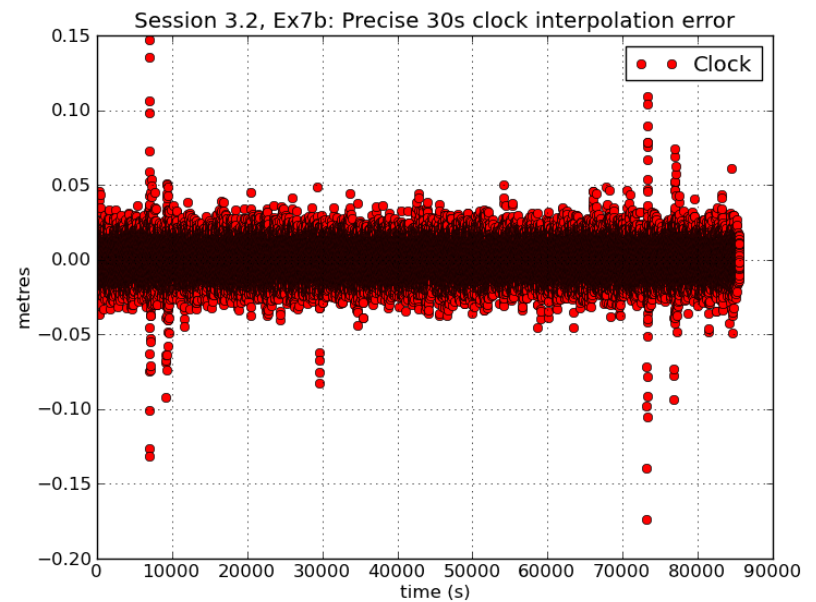
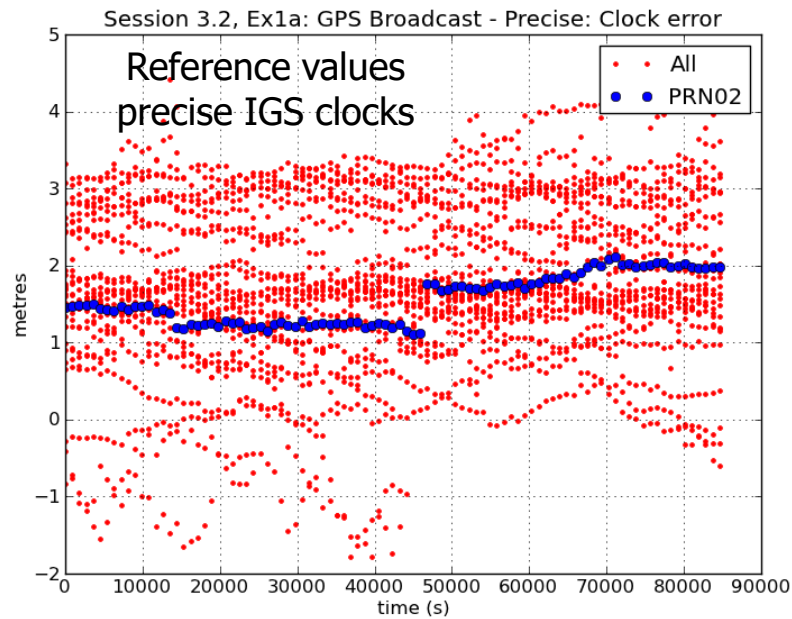
Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server <http://igs.cb.jpl.nasa.gov>

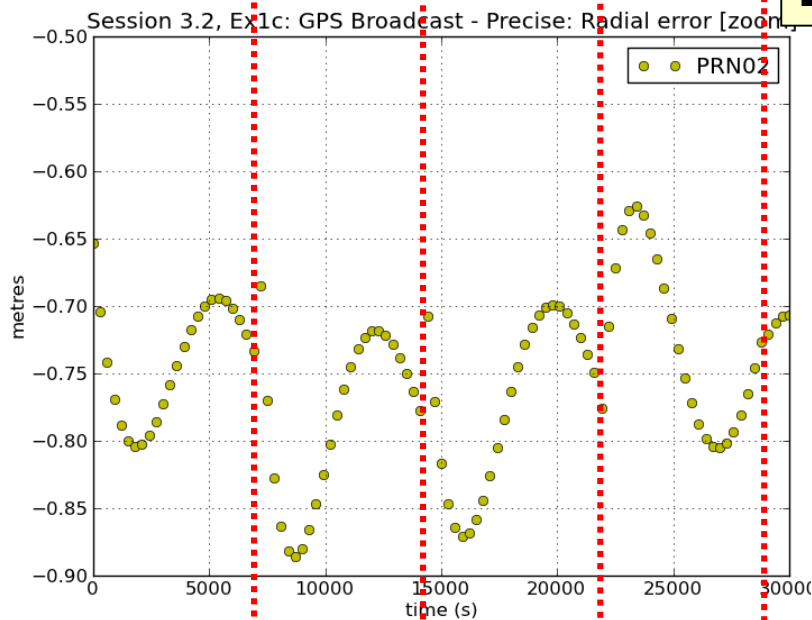
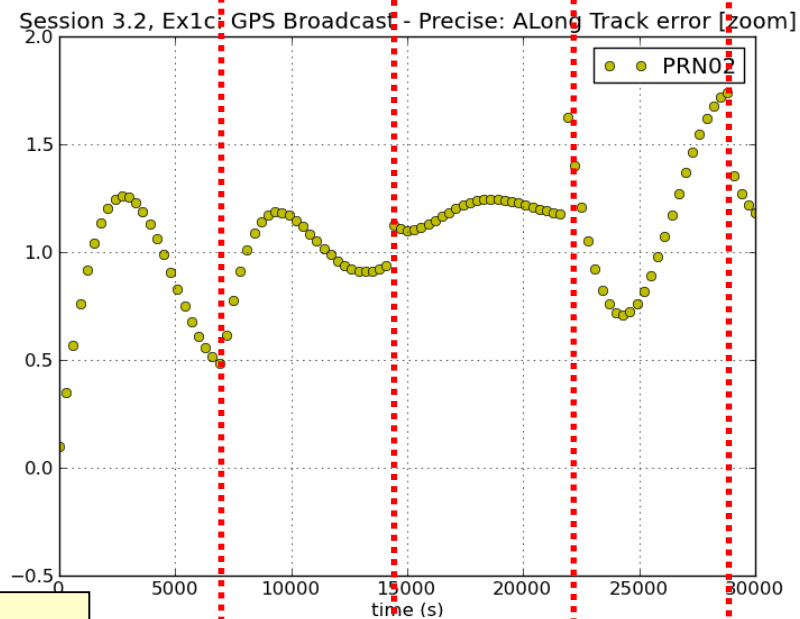
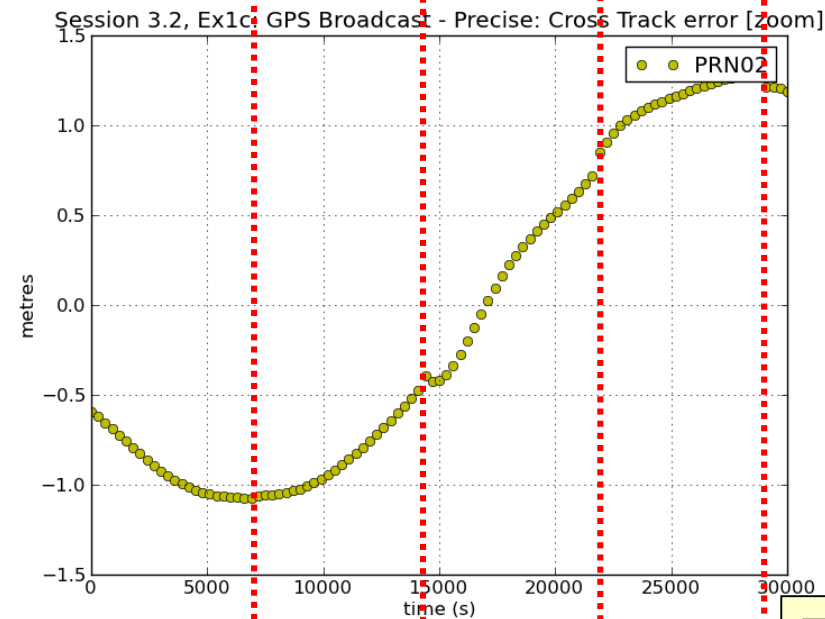
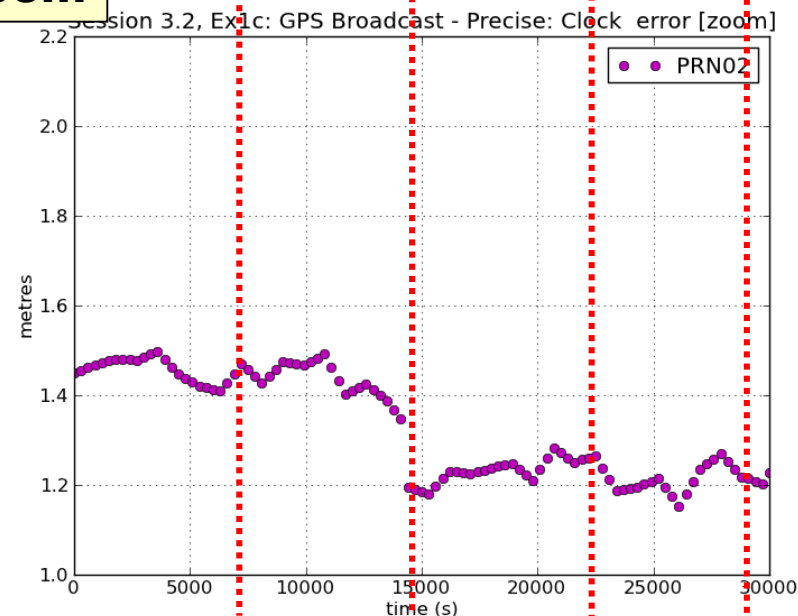
They are providing precise orbits and clock files with a sampling rate of 15 min (SP3 files), as well as precise clock files with a sample rate of 5 min and 30 s (CLK files).

Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.

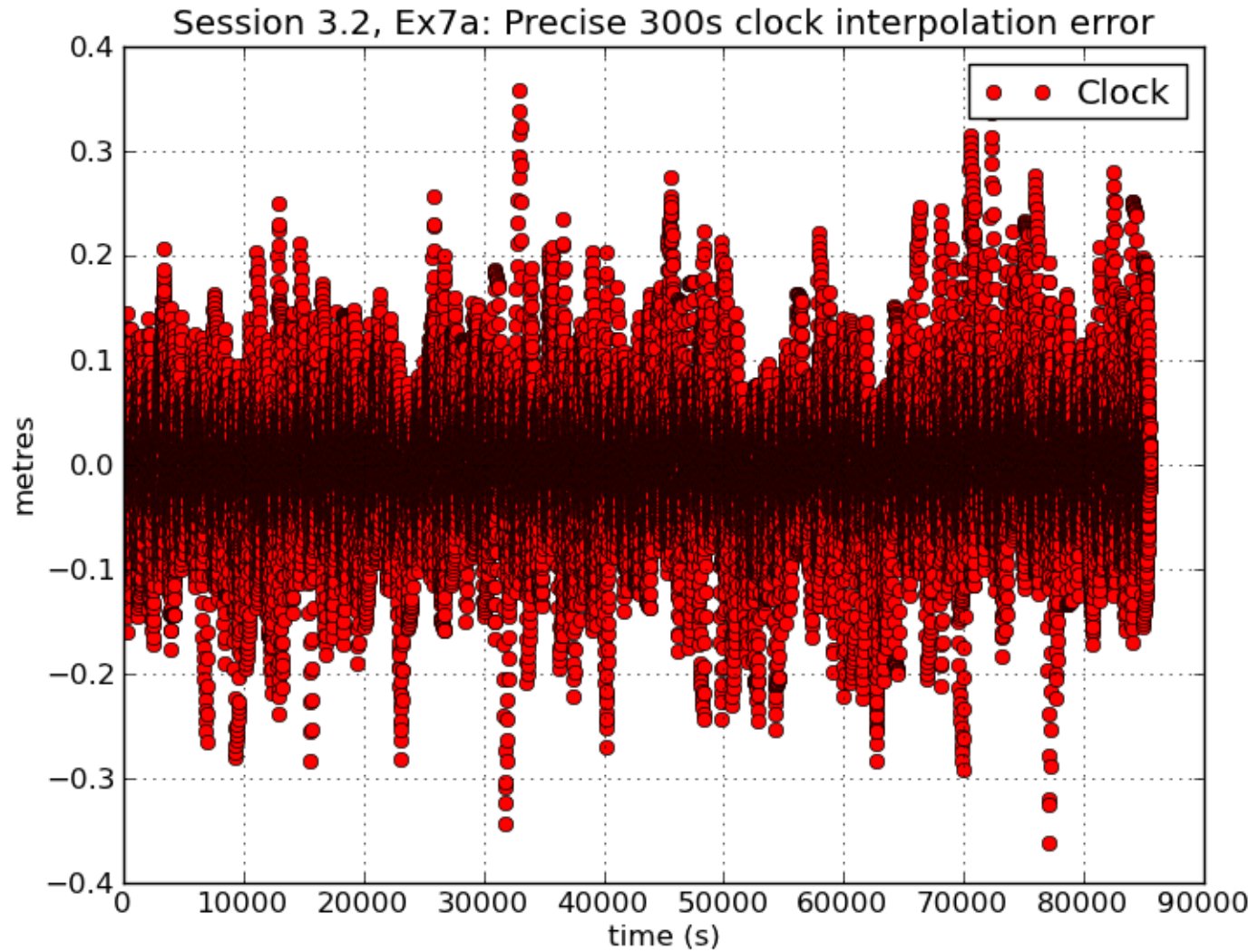
Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.

SA=off

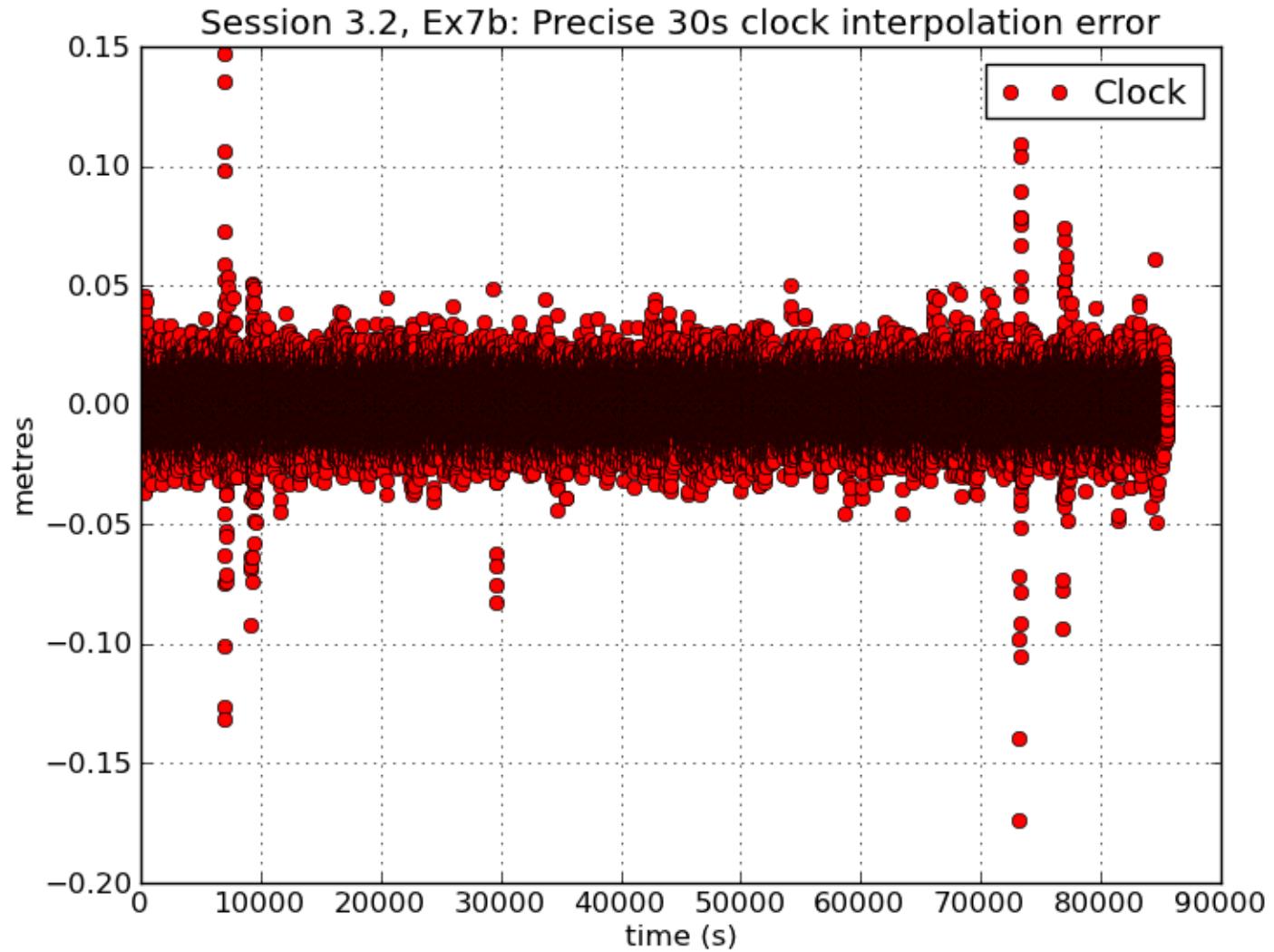


**Zoom**

Precise Clock Interpolation: 300s samples

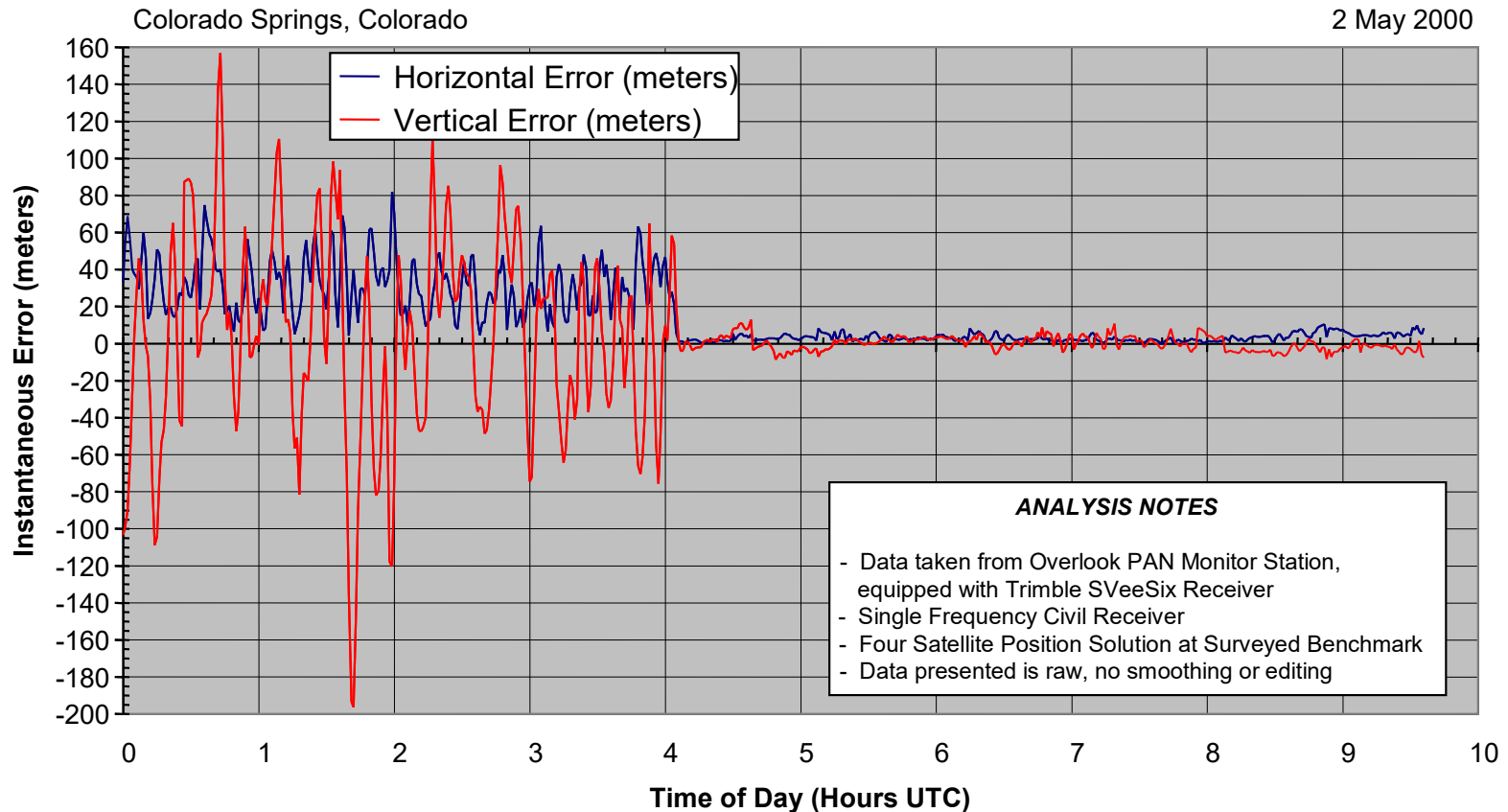


Precise Clock Interpolation: 30s samples

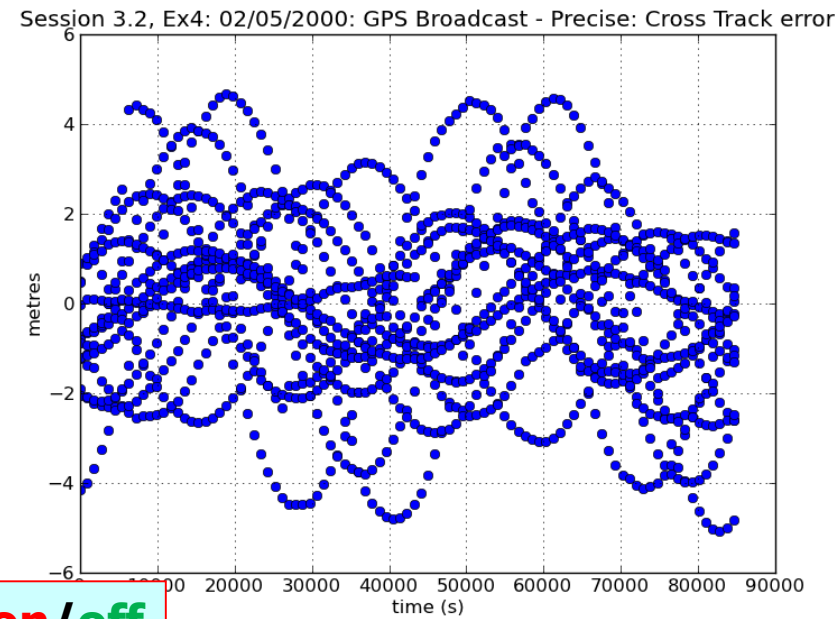
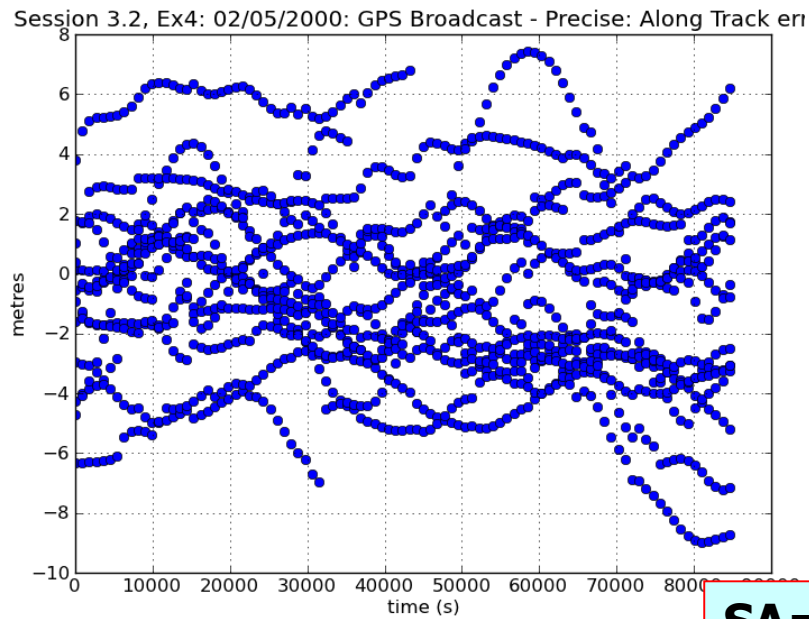


Selective Availability (S/A): Intentional degradation of satellite clocks and broadcast ephemeris. (from 25 March, 1990)

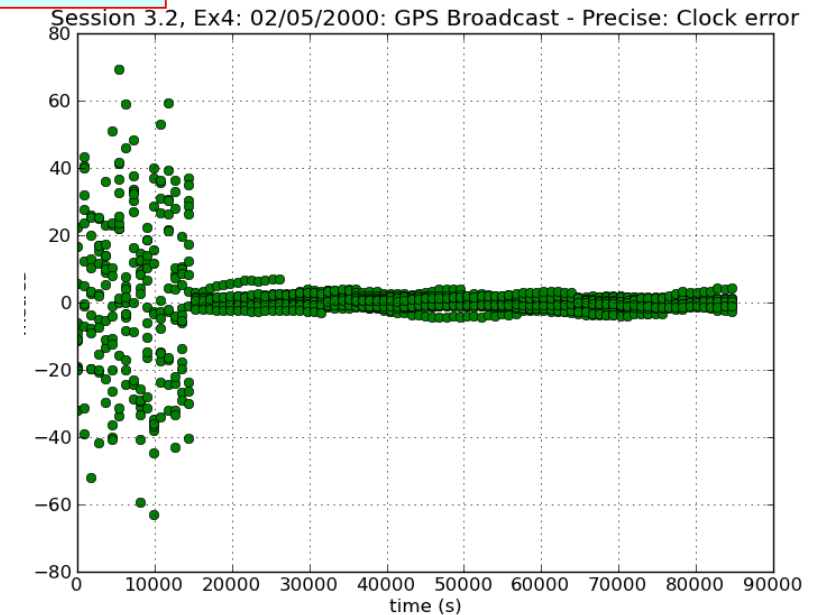
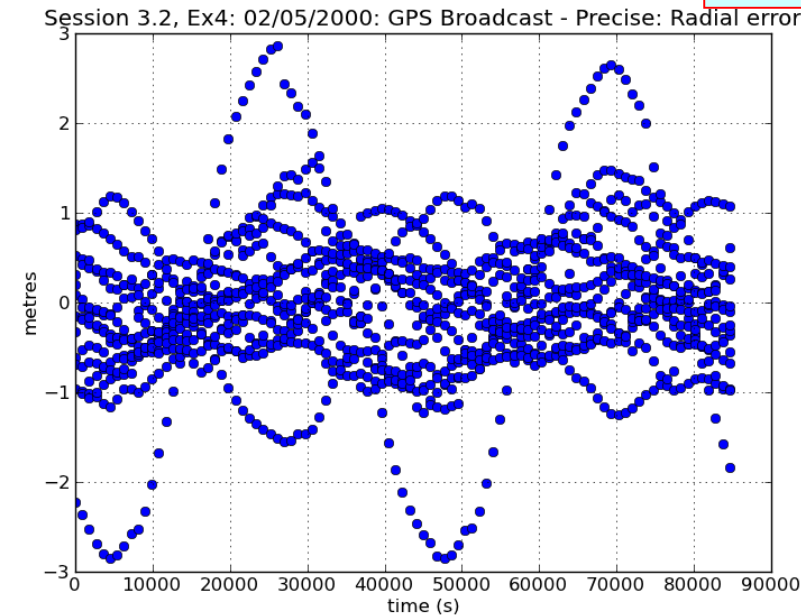
GPS Before and After S/A was switched off

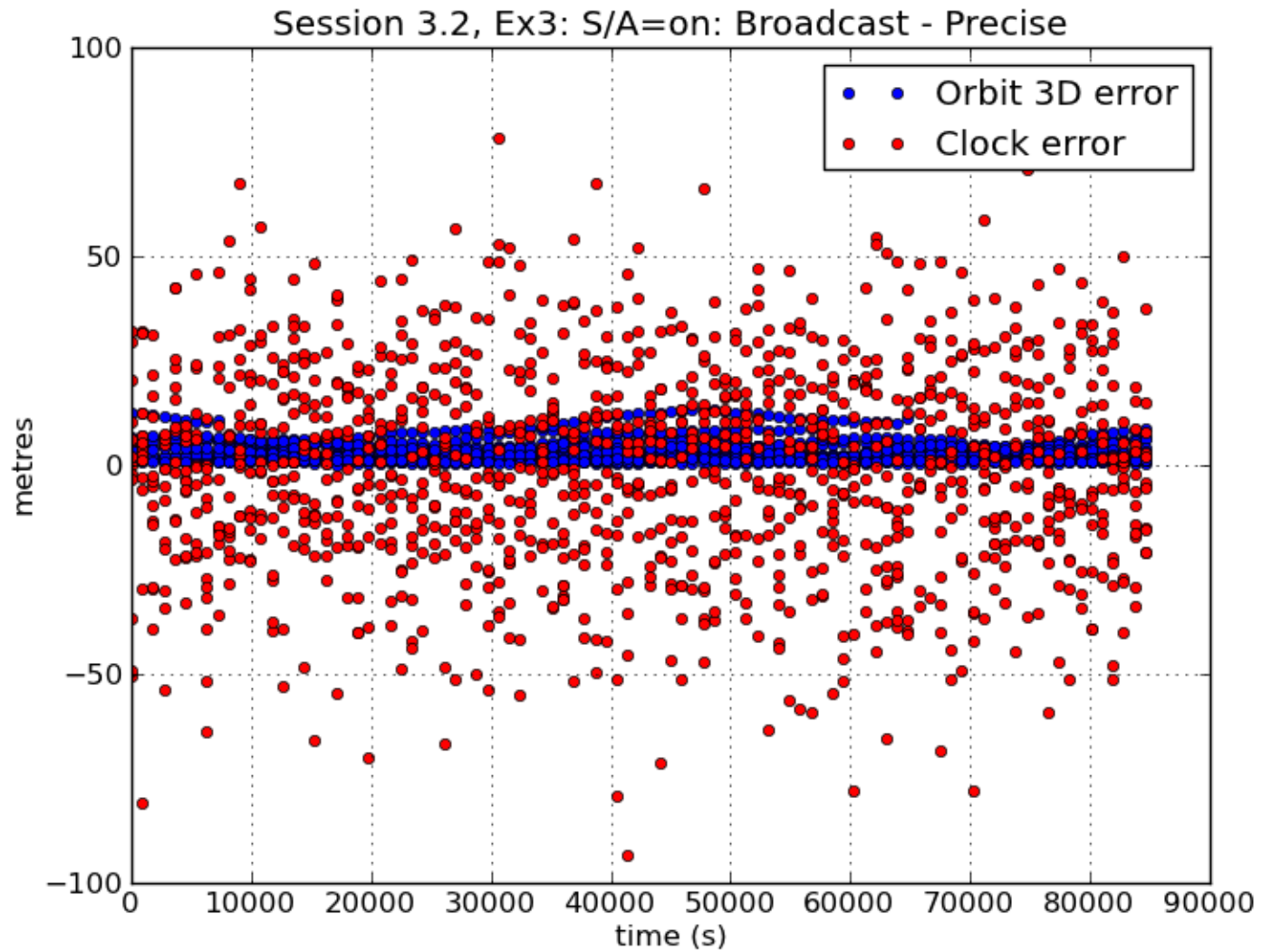


S/A was switched off at 2nd May 2000 and **Permanently removed in 2008**

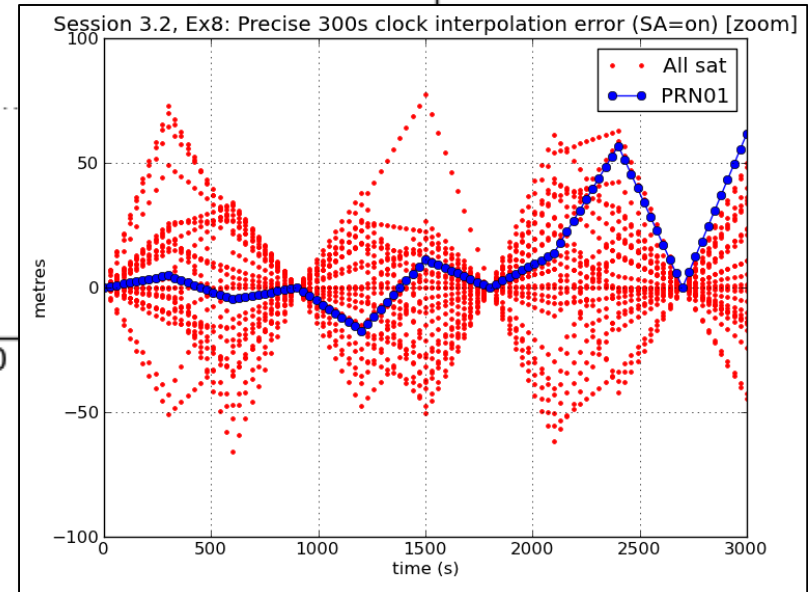
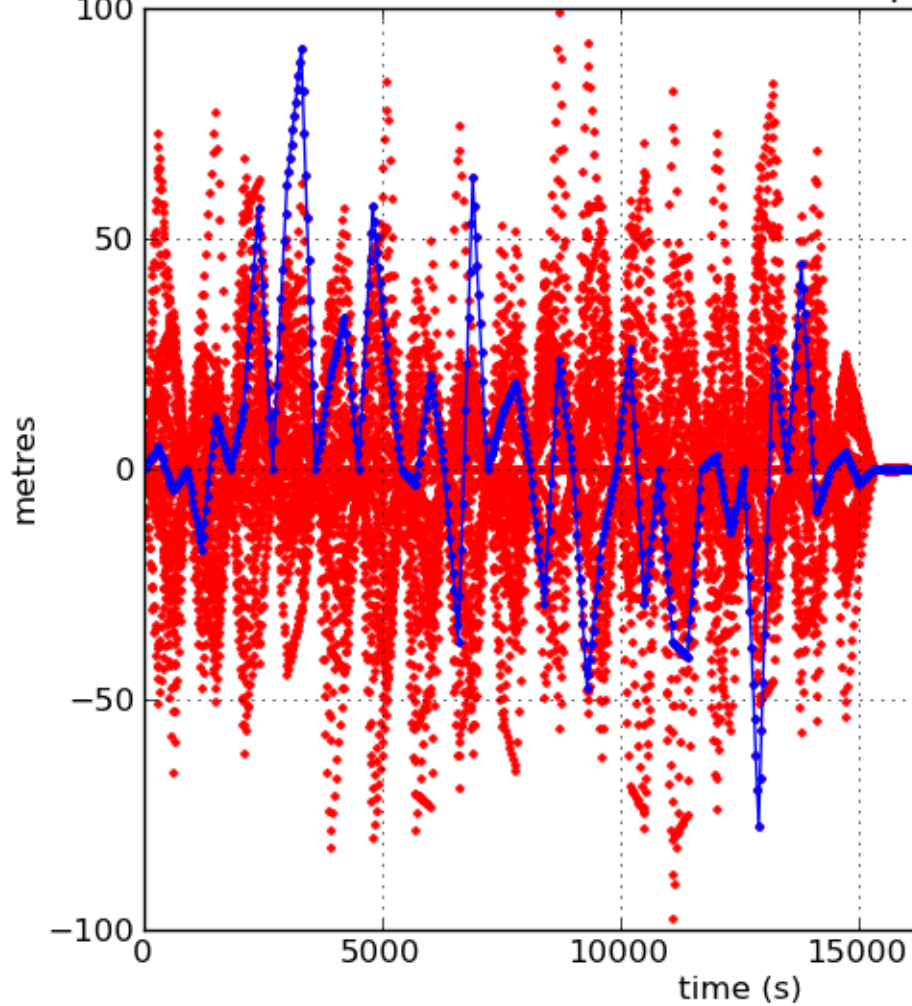


SA=on/off





Session 3.2, Ex8: Precise 300s clock interpolation error (SA=on/off)



References

- [RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.
- [RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.
- [RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.
- [RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

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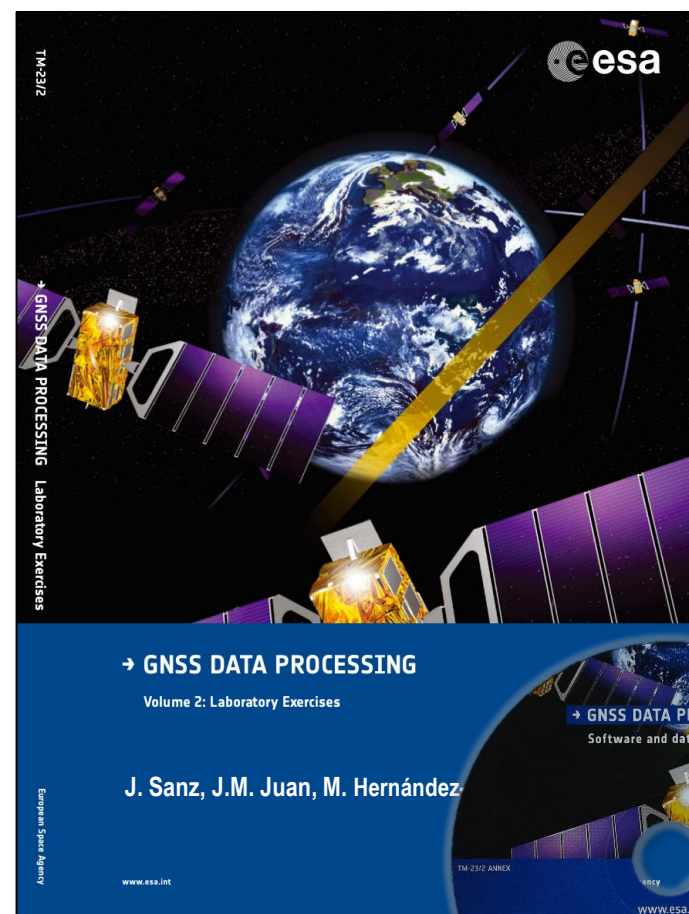
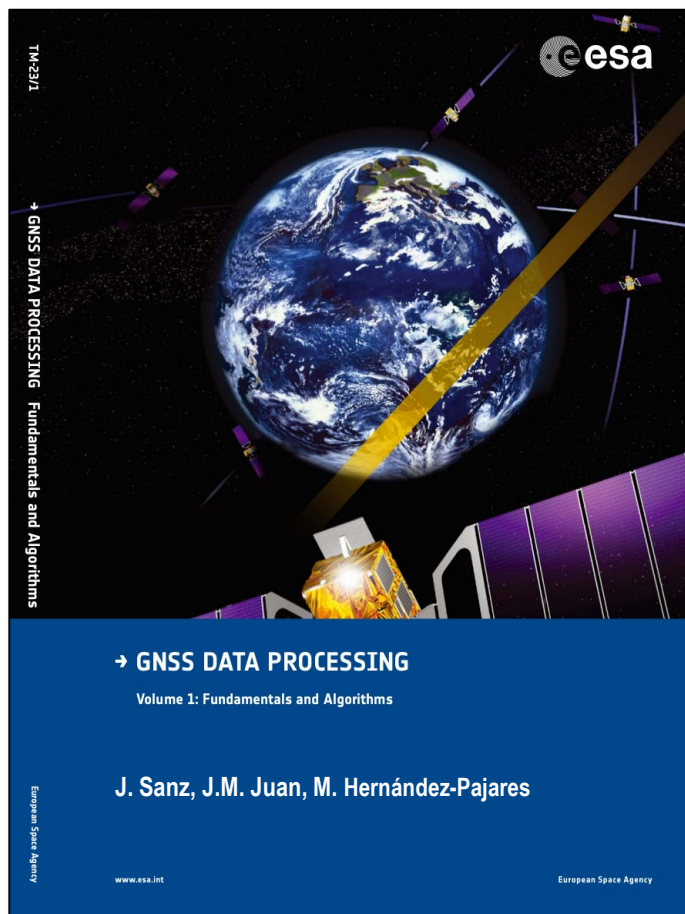
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