

## Ionospheric Tomography Helps Resolve GPS Ambiguities On The Fly At distances Of Hundreds Of Kilometers During Increased Geomagnetic Activity

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*Abstract:* We describe a procedure for resolving ambiguities in real time with the help of very precise ionospheric corrections calculated tomographically using data from several reference stations that are part of a local area automatic control network.

We report successful results during a period of high geomagnetic activity ( $K_p = 6$ ) in the region of the Pacific Northwest (in Washington and British Columbia), using a set of continuously operating IGS stations with separations of between 400 and 1000 km. While the data were collected before the test, our calculations carefully emulated those made in a real-time situation. All GPS receivers were dual-frequency.

This is a proof-of-concept for the use of such a technique in the operation of large regional networks of automatic stations, capable of supporting sub-decimeter surveying and navigation in real time anywhere within their area.

### INTRODUCTION

The free electrons distributed in the ionosphere (between one hundred and thousands km in height) produce a frequency-dependent effect on the Global Positioning System (GPS) signals: a delay in the pseudorange and an advance in the carrier phase. Their spatial-temporal distribution is correlated with the position of the main source of ionization: the Sun. It is also dependent on the solar cycle, on events such as travelling ionospheric disturbances (TID) and, in general, on geomagnetic and "space weather" conditions.

Therefore, the distribution of free electrons in the ionosphere affects precise navigation with GPS, and must be taken into account depending on the distance scale:

- Local Area Differential GPS (LADGPS): when the roving receiver is less than few tens of km from a reference GPS station [1]. The assumption that the ionospheric delays are identical for both receivers and can be ignored in order to fix the double-differenced integer ambiguities in  $L_1$  and  $L_2$ , strongly depends on geomagnetic activity and possible existence of highly localized ionospheric perturbations such strong scintillation and TID's [2].

- Regional Area Differential GPS (RADGPS): Its has been shown in [3] that, even when a dual-frequency rover receiver is at distances of few hundred km from the reference network, the double-difference of the integer phase ambiguities can be fixed, by estimating the ionospheric delay without an ionospheric model (the proof of concept was done close to minimum Solar cycle conditions).

- Wide Area Differential GPS (WADGPS): using the code it is possible to get positioning errors at the meter level, when modeling the ionospheric delay (and other errors, like the tropospheric delay) for the main observable: the  $L_1$  pseudorange smoothed with the carrier phase. This is possible with networks of stations separated typically between 500 and 1000 km [4].

The dual frequency GPS signals, gathered from a set of reference receivers, can be used to compute a real-time ionospheric model in the RADGPS and WADGPS scales, to provide enough precise ionospheric correction to the GPS navigator. Some examples can be found in [5] for WADGPS, using as the main observable the smoothed pseudorange for meter-level positioning, and in [6] for RADGPS, close to the Solar maximum and using the carrier phases for sub-decimeter level positioning.

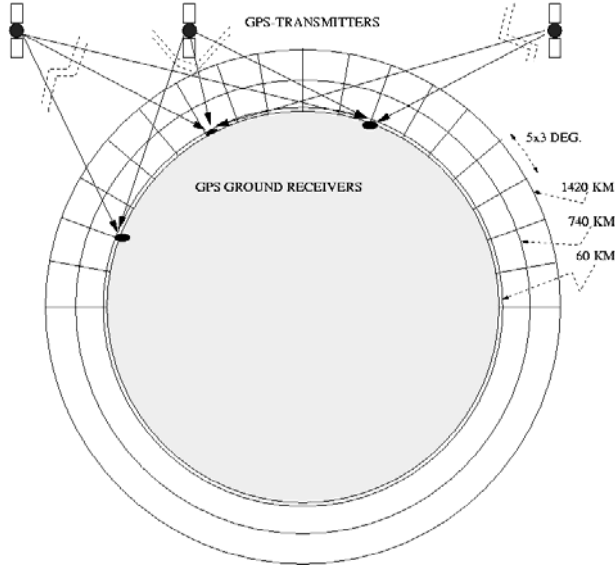
### REAL-TIME TOMOGRAPHIC MODEL

The free electron density can be described as a random walk process in time that can best be estimated in a Sun fixed reference frame where it is relatively stationary (variation of  $\pm 5\%$  during one day with low geomagnetic activity<sup>(1)</sup>). The tomographic model adopted is spatially formed by a set of cells or volume elements (voxels), especially suitable to detect local features, that cover all the ionosphere sampled by the GPS satellites/receivers rays. In these voxels the electron density is considered uniform at any given time. Despite other possibilities to choose the voxel distribution (for instance adapted to the data density

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<sup>1</sup> This has been deduced from the International Reference Ionosphere (IRI), at the geomagnetic equator -Fortaleza, Brazil-, at noon during the last solar maximum, 1 January '99.

as in [7]) the uniform distribution is adequate for describing a region sampled from an approximately homogeneously distributed network of reference stations. Voxels of  $3 \times 5$  degrees in latitude and solar longitude, forming two layers with boundaries at 60-740-1420 km, have been adopted (see figure 1). This resolution is adequate to get precise ionospheric estimates from ground GPS data (e.g., [8]).



**Figure 1.** Layout of the tomographic model of the ionosphere

The resolution of the model, by means of the Kalman filter, initialized with data from the previous day, is performed using the geometry-free combination of phases,  $L_J$ , of the transmitter  $T$  measured from the receiver  $R$ :

$L_I = L_1 - L_2 = \sum_{i,j,k} (N_e)_{i,j,k} \Delta s_{i,j,k} + b$  being  $L_1$  and  $L_2$  the carrier phases (in meters) at frequencies  $f_1 = 154 f_0$  and  $f_2 = 120 f_0$  ( $f_0 = 10.23$  MHz); where  $i, j, k$  are the indices for each cell corresponding to solar longitude, geodetic latitude and height;  $(N_e)_{i,j,k}$  is the free electron density; and  $\Delta s_{i,j,k}$  is the length of the ray path crossing the "illuminated cells"; and  $b$  is the alignment term (constant in a given transmitter-receiver arch of continuous phase) that includes the  $L_1, L_2$  integer ambiguities and instrumental delays. This approach extends the model described in [9] by Hernández-Pajares et al., to real-time applications.

## RESOLVING THE AMBIGUOUS STEC IN REAL TIME, FOR THE REFERENCE SITES

In [6] we, with Julia Talaya, studied the resolution of the carrier phase ambiguities for networks of 200-300 km,

showing the value of using an accurate tomographic ionospheric model to solve in real-time the ambiguities in the wide lane combination first, and then those in  $L_1$  and  $L_2$ . The purpose of this paper is to assess, for larger networks and higher geomagnetic activity, the same technique for solving the double difference of integer ambiguities (in the reference stations and in the rover).

### (a) Resolving the wide lane

If we consider the wide lane combination for the reference stations  $L_\delta = (f_1 \cdot L_1 - f_2 \cdot L_2) / (f_1 - f_2)$  then its double difference, first between a satellite and the reference satellite ( $D$ ), and then between station and reference station ( $\Delta$ ), can be written as

$$D\Delta L_\delta = D\Delta\rho + D\Delta T + D\Delta I_\delta + \lambda_\delta D\Delta N_\delta \quad (1)$$

Where:  $\rho$  the distance satellite-receiver,  $T$  the tropospheric delay,  $I_\delta$  the wide lane ionospheric correction and  $N_\delta$  the integer wide lane ambiguity (centimetric terms like the phase multipath have been neglected).

To fix in real time  $D\Delta N_\delta$  to the right integer value from equation (1), it is necessary to find the other three terms with a maximum total error less than  $\lambda_\delta/2 = 40$  cm, i.e. with an error standard deviation less than 20 cm to guarantee the 95% of successful determination, assuming the error is normally distributed. An error of few centimeters can be expected for the satellite-receiver distance term  $D\Delta\rho$ , if the satellite positions are obtained from extrapolated precise ephemeris. If the broadcast ephemeris are used instead, this error term is typically less than 10 cm at distances of 500 km, and can be made negligible if the orbit error is estimated in real-time using the stations GPS data [10]. Regarding the double-differenced tropospheric correction  $D\Delta T$ , and for stations at distances of few tens of km, the maximum error using the models for the hydrostatic and wet components is typically lower than 10 cm for elevations greater than 20 degrees (this corresponds to a worst case in the study of [2] by Coster et al., figure 5d). But this error can diminish to a few centimeters if the tropospheric refraction is estimated in real time, in particular in the reference stations where the coordinates can be accurately known. In our case, using precise orbits and modeled tropospheric corrections, a final maximum error of 30-40 cm in  $\Delta I_\delta$  is allowed. This means a standard deviation of 20 cm for the wide lane ionospheric term to guarantee the 95% successful determination of  $\Delta N_\delta$  (i.e.  $1 \text{ TECU} = 10^{16} \text{ electrons/m}^2$  or 10 cm in  $L_1 - L_2$  delay units, approximately).

(b) Resolving the L1 and L2 ambiguities

Once  $\Delta N_{\delta}$  is fixed, it becomes possible to fix the  $L_1$  and  $L_2$  double-differenced integer ambiguities,  $N_1$  and  $N_2$  at the reference stations, by using an accurate enough determination of the double-differenced bias  $\Delta B_c$  of the ionospheric-free combination  $L_c$ ,

$L_c = (f_1^2 L_1 - f_2^2 L_2) / (f_1^2 - f_2^2)$ . The following relationships illustrate these steps:

$$D\Delta B_c = 0.5[\lambda_{\delta} D\Delta N_{\delta} + \lambda_n D\Delta(N_1 + N_2)] , \quad (2)$$

$$D\Delta(N_1 + N_2) = NI[(2D\Delta B_c - \lambda_{\delta} D\Delta N_{\delta}) / \lambda_n] , \quad (3)$$

$$D\Delta N_1 = 0.5[D\Delta N_{\delta} + D\Delta(N_1 + N_2)] , \quad (4)$$

and, finally,

$$D\Delta N_2 = D\Delta N_1 - D\Delta N_{\delta} \quad (5)$$

being  $\lambda_n = c / (f_1 + f_2) = 10.7$  cm, and  $NI$  the nearest integer.

Hence, from  $D\Delta N_1$  and  $D\Delta N_2$ , the unambiguous double-differenced ionospheric Slant Total Electron Content (STEC) can be computed for the reference stations:  $D\Delta STEC = D\Delta(L_1 - L_2) - (\lambda_1 D\Delta N_1 - \lambda_2 D\Delta N_2)$ . Finally, and following the procedure described in [6], one can try to solve the carrier phase ambiguities for a roving receiver, correcting them with the unambiguous double-differenced ionospheric STEC interpolated from the reference stations to the rover. In this work we have applied the linear interpolation of unambiguous  $D\Delta STEC$  proposed in [3].

## RESOLVING AMBIGUITIES ON THE FLY WITH A ROVER

Two main steps are required to resolve on the fly the L1 and L2 ambiguities in the double differences between HOLB and the "rover" (ALBH), as part of a precise kinematic solution:

(a) Resolving the wide lane

The double-differenced wide lane is first corrected for the ionosphere using the modeled STEC interpolated to the rover. Then the ionosphere-free combination  $L_c$  is subtracted from the corrected wide lane. Finally, an estimate of the  $L_c$  bias is subtracted as well. This is repeated for every double difference for which there might be a reliable ionospheric correction (satellites above 25 degrees in elevation).

The result is a noisy estimate of the wide lane ambiguity (in meters), with an error that is a combination of the carrier phase noise, the error in the ionospheric correction, and the error in the estimated  $L_c$  bias.

The wide lane ambiguity can be found by rounding off each of these estimates to the nearest integer number of wide lane wavelengths. Errors in computed tropospheric refraction, reference station coordinates, and satellite ephemeris, cancel out when subtracting  $L_c$  from the wide lane. Assuming the ionospheric correction is sufficiently accurate to resolve the L1 and L2 ambiguities ( $< 2.7$  cm error in  $L_1$ ), the main uncertainty is that in the  $L_c$  bias. Assuming, further, that the main sources of uncertainty are normally distributed, then the error in the  $L_c$  bias should be less than 1/4 of a wide lane ( $< 21.5$  cm) for the procedure to work well at least 95% of the time.

The  $L_c$  bias must be estimated simultaneously with the position of the rover, which, in general, cannot be considered sufficiently well known beforehand.

Other unknowns are errors in tropospheric correction and in broadcast ephemeris. As in real-time, a Kalman filter is used to obtain a joint solution for the present position and all the other unknowns (in post-processing one would go further and use a smoother). The data are dual-frequency carrier phase plus, whenever possible, ionosphere-corrected L1 pseudo-ranges

This joint solution has been obtained using software developed by the first author for precise, long-range kinematic and static positioning [11], [12], and modified to use the ionospheric corrections. This software has been employed repeatedly to calculate static and kinematic position with sub-decimeter precision, over distances of more than 1000 km. All the unknowns required for the present work are already part of the observation equations. In particular, the long-range technique involves "floating" the ambiguities, that is to say, using the ion-free combination  $L_c$  as the main data type, and solving for its biases as "real numbers". The calculation also gives the precision of each estimated  $L_c$  bias, to decide when this estimate is precise enough to use safely.

(b) Resolving the L1 and L2 ambiguities

Once the wide lane integer ambiguity is known, one can proceed to determine the L1 and L2 phase ambiguities as explained in the previous section, expressions (1 - 5), for the fixed reference sites.

Having found those ambiguities, the exact  $L_c$  bias can be calculated and assimilated in the Kalman filter, as an additional (pseudo-)observation. Since the filter in question is of the usual covariance type, some "noise" uncertainty must be assigned to every observation. In this case, a noise with  $\sigma = 3$  cm has been found to be adequate.

The procedure outlined above does not require an integer search, as long as the various uncertainties are smaller than the specified bounds. In particular, the effect of data noise has been reduced by using data averages. This has been

easy to do, because this long-range technique already involves using data compression (averaging) to economize time and hard-disk scratch space. By trial and error, a data-averaging interval of 2 minutes has been chosen. Generally speaking, it takes between two and six consecutive filter updates with 2-minute carrier phase averages, to make the Lc bias estimates with better than the 20 cm precision needed to resolve the wide lane. (For faster resolution, an integer search could do better, if taking properly into account the various uncertainties, including that of the ionospheric correction.)

No ambiguity is considered resolved unless the absolute value of the corrected, averaged, and presumably unbiased  $LI$  is less than 1.5 cm. No resolved Lc biases are accepted unless they pass a null-hypothesis error test before being assimilated in the Kalman filter. And, while many Lc biases might be satisfactorily resolved every two minutes, only those known with less than 3 cm-precision are updated with their resolved values. So, while the values of the integer ambiguities may be estimated many times during a run, those values will be used only when strictly necessary.

### THE PACIFIC NORTHWEST TEST

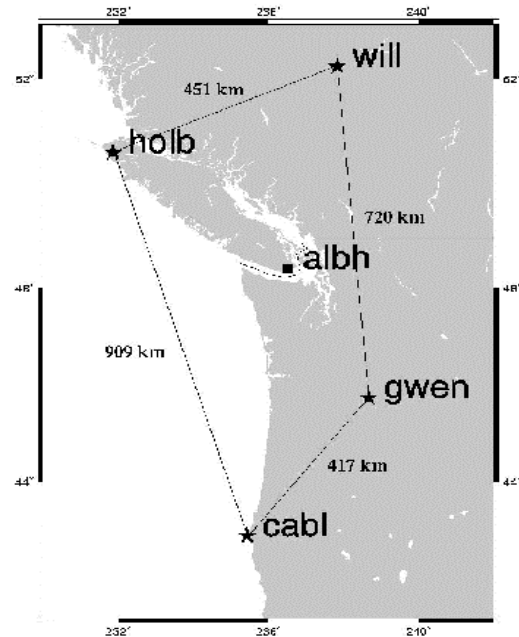
The data set (Kp=6, May 3th, 20-23 UT) came from five North-American reference stations belonging to the International GPS Service (IGS). The baselines ranged from 400 to 1000 km in length. All observations were dual-frequency carrier-phase and pseudorange, collected at the typical IGS rate of 30 seconds. Data sets from four of them (CABL, GWEN, HOLB, WILL), were used to create the ionospheric model. Data from all five sites (those mentioned, plus ALBH, near Victoria) were used in the tests described below. While all data had been collected before we carried out our calculations, we took care to process them as they would be processed in a truly real-time application.

## RESULTS

### *Checking the ionospheric corrections against the known STEC at ALBH*

The difference between the true and the interpolated STEC at ALBH show an RMS of 9 cm (0.9 TECU approximately) in  $L_1-L_2$  delay units, which is below the critical value of 1 TECU discussed in the previous section, allowing us to successfully solve most of the wide lane ambiguities.

The ALBH data have been reserved for testing. The actual double-differenced STEC is to be compared to the tomographic predictions (see map in figure 2). The  $L_1, L_2$  ambiguities of the carrier phase, double-differenced relative to HOLB, are to be resolved on the fly, as part of a kinematic solution in which the station is treated as the "rover" (with HOLB as base station).

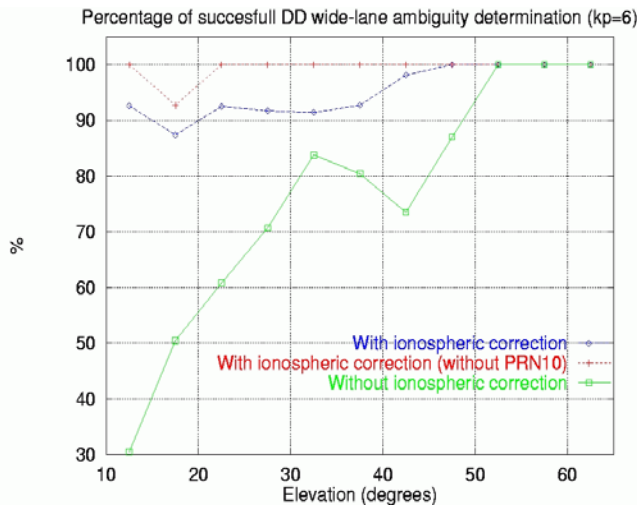


**Figure 2.** IGS sites used in the Pacific Northwest test.

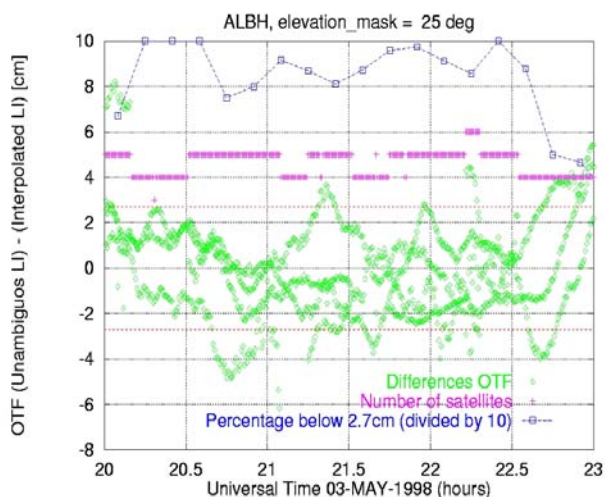
As expected, more than the 90% of successful wide lane fixing is obtained for elevations greater than 20 degrees (100% without the satellite PRN10)<sup>2</sup>, in contrast to the minimum elevation of 50 degrees when the double difference ionospheric correction is neglected (figure 3).

By interpolating the unambiguous  $D\Delta STEC$  values at the reference stations, it is possible to solve the full set of ambiguities, when this value is more accurate than 2.7 cm.

<sup>2</sup>The signals from this satellite, most of them at low elevation, illuminate a part of the ionosphere, to the South, not sounded with other satellites. The consequence, poor determination of the ion content there, could be avoided by extending the area with more reference stations.



**Figure 3.** Percentage of successful wide lane integer ambiguity determination as a function of the lowest satellite elevation.



**Figure 4.** Percentage of successful determination of the double-differenced STEC (i.e., correction within 2.7 cm of the true refraction value), for "rover" site ALBH. Also: number of satellites at more than 25° elevation, and possible ionospheric correction errors (in cm of STEC delay).

This happens 80-100% of the time at the test station ALBH. Notice that the common reference satellite used in forming the double differences (PRN30) is at a low elevation in the last part of the period, which coincides with the worst results, between 22.5 and 23 UT (figure 4).

### Resolving the L1 and L2 ambiguities on the fly for the "Rover" (ALBH)

The instantaneous position of ALBH relative to HOLB, 420 km away and on the opposite end of Vancouver Island, was calculated kinematically and then compared to the known position of ALBH. Solutions were made using: (a) precise IGS ephemeris (from SP3 orbit files); (b) broadcast ephemeris (from the navigation message). In both cases the L1 and L2 ambiguities were found on the fly, using the ionospheric corrections and on-the-fly technique discussed in previous sections. The actual position of ALBH was known with centimeter-level precision from an independent IGS solution, after taking into account the solid-earth tidal displacement. This was not used to help the kinematic solutions, but was used to test their accuracy. When using the broadcast ephemeris, as in true a real-time case, their errors were estimated, along with tropospheric refraction correction errors, floated Lc biases, and the position of the vehicle, with the technique described in [12]. The a priori uncertainties for the vehicle were  $\sigma = 100$  m per coordinate. These coordinates were treated as zero-memory, or white noise, error states. So no dynamics were assumed for the vehicle (kinematic solution).

Each on-the-fly solution has been tested in two ways:

(a) Observing, in lieu of position errors, the discrepancies between known and kinematic position of ALBH. Since the resolved ambiguities were used solely when needed, this first test only covers attempts to resolve ambiguities obtained soon after gaining, or regaining, lock (mostly at the start of the run).

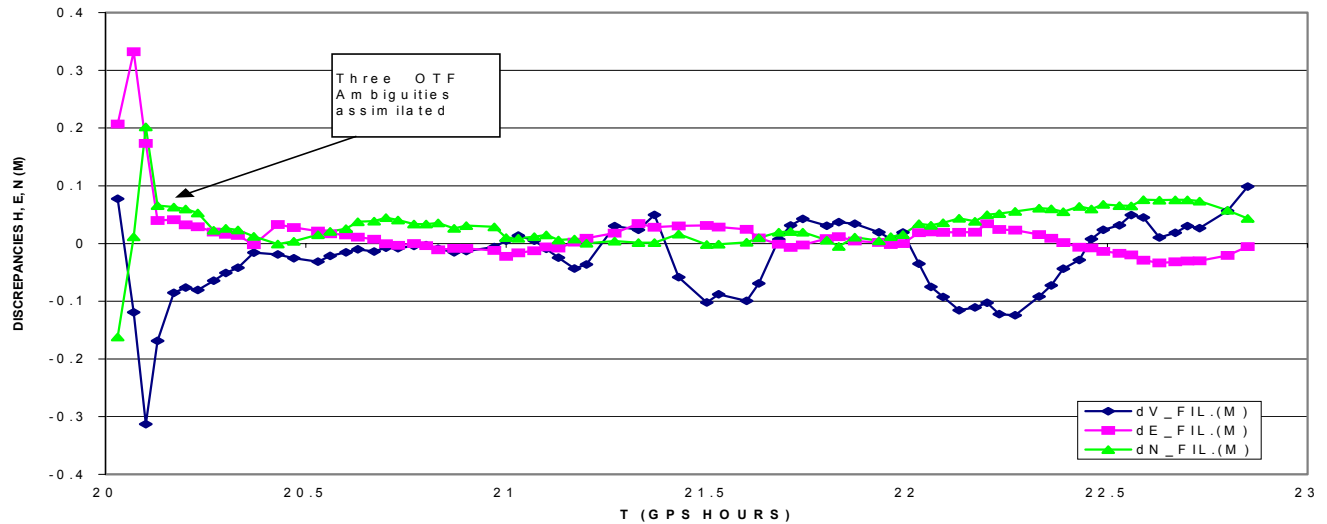
The resulting Vertical (Height), East, and North discrepancies are shown in figures 5 and 6. At the start of the run, it takes ten minutes, if using the (adjusted) broadcast orbits, for three double differences to be fully resolved and all three position discrepancies to fall below 10 cm. That time is down to eight minutes when using the (fixed) SP3 orbits. With the broadcast orbits, there are two small excursions outside the +/-10 cm bounds, later in the run. This happens while the Kalman filter states associated with the orbit errors of newly risen satellites are in the process of converging. There are no signs of incorrectly resolved ambiguities, such as clearly discernible linear trends in the discrepancies. The small excursion with the SP3 orbits (figure 6) is not related to resolved ambiguities. Since these better orbits were not adjusted, the Lc bias converged so fast that their resolved values only had to be used in the first half an hour of the run.

(b) Comparing the Lc biases Bc calculated from the resolved N1 and N2 integers (whether used or not) to their precise post-processed estimates Bc\*. These were obtained while fixing the well-known positions of ALBH and HOLB, and the precise SP3 orbits from the IGS.

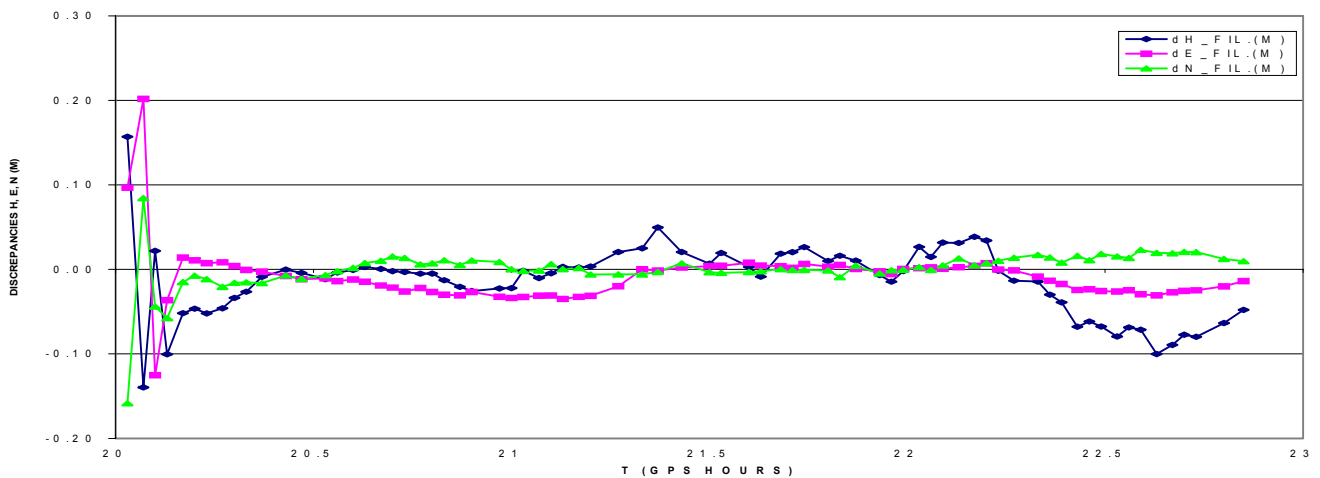
The L1, L2 ambiguities can be regarded as successfully resolved if for the Lc bias is  $|Bc - Bc_{\text{ref}}| < 2$  cm.

There were a total of 180 attempts to resolve the ambiguities in the 3-hour period (for all double differences with satellites above  $25^\circ$ ). Criteria (b) was satisfied 71% of the time with both the broadcast and the SP3 orbits. The

possible errors did not appear to exceed  $\pm 1$  cycle in both L1 and L2 (the wide lane seemed always correctly resolved). If so, the error in the Lc biases was  $\pm 11$  cm at most. This error can be accommodated assigning a standard deviation of 6 cm to the resolved biases.



**Figure 5.** Vertical, East, and North discrepancies (in meters) between ALBH kinematically-determined position relative to HOLB, 420 km away, and its precise static position. Broadcast orbits used (and simultaneously adjusted) in the kinematic solution, as they would be in real time. All receivers used were dual-frequency. After ten minutes, three double differences are fully resolved, and the solution error falls and stays mostly below 10 cm



**Figure 6.** Same as in Figure 5, but using precise SP3 orbits (from the IGS) instead of the broadcast ephemerides. The SP3 orbits were not adjusted in this run. Accuracy reaches 10 cm in eight minutes.

## CONCLUSIONS

We conclude that the tomographic modelling of the ionosphere, if implemented in real time with dual-

frequency receiver data, as proposed here, should give enough precision (better than 1 TECU, or 10 cm in  $L_1-L_2$  delay units) in the double differences of the slant TEC, for the successful resolution on the fly of the wide lane and

STEC ambiguities, even with high geomagnetic activity ( $K_p=6$ ), and with distances of many hundreds of kilometers between receivers.

Inside an area surrounded by a few GPS reference sites, the estimated unambiguous STEC, interpolated to the location of a roving receiver, can be used there to correct the ionospheric delay well enough to resolve most of the L1 and L2 ambiguities, and calculate the Lc biases exactly, or with only small errors.

The testing of such a procedure clearly suggests that better than 10 cm accuracy can be achieved within 10 minutes of starting a run with all ambiguities unknown, and that this accuracy can be maintained most of the time thereafter.

All calculations needed to implement and test the techniques described here were made using ordinary personal computers. (The kinematic solutions were done in the same laptop in which this paper was written.)

Our results indicate that local area control networks have the potential for supporting very precise GPS navigation and surveying work, both in real time and in post-processing, over areas hundreds of kilometers across, and even in conditions of high ionospheric activity.

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