

# An Improvement of Retrieval Techniques for Ionospheric Radio Occultations

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**Summary.** The availability of occultation measurements from GPS receivers on-board several Low Earth Orbiters (GPS/MET, CHAMP, SAC-C. . .) is opening new possibilities to the ionospheric sounding. In this context we will briefly describe several approaches developed in the last years to estimate the electron density distribution. In particular a modified technique that increases the electron density retrieval accuracy is detailed. It consists of generalizing the Abel transform of the slant TEC, taking into account the horizontal gradient of the electron content, and the topside electron content. The improvement obtained in different scenarios, at mid and low latitudes, will be shown using both synthetic and real data from the GPS/MET and CHAMP satellites. Comparison with Ionosonde measurements will also be given.

**Key words:** GPS, LEO, Electron density, Occultations, Abel transform, Separability hypothesis

## 1 Introduction

Several approaches can be used to obtain a 3D description of the ionospheric electron content, but the main consideration to take into account is that a realistic estimation can only be obtained using complementary information, that is, to mix information of both the horizontal and vertical distribution of electron density.

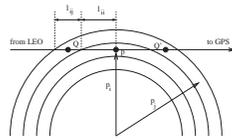
In [7] it was shown how an assimilation scheme based on combining ground and LEO GPS data using a 3D voxel model proved successful, at a global scale, through a comparison with vertical profiles given by inverted Ionosonde data. An alternative scheme considered is to use Ionosonde data or models based on Ionosonde instead of LEO GPS data, as the source of information of vertical distribution of electron density ([8], [4] and [3]). This assimilation scheme gives successful results predicting Slant Total Electron Content (STEC) seen by a Low Earth Orbiter (LEO).

As it is known, Abel inversion techniques are used to obtain high vertical resolution profiles ([5],[6],[10]), with a low computational load. This paper proposes two improvements on the classical approach to overcome spherical symmetry using horizontal Vertical Total Electron Content (VTEC or TEC) gradients and to tackle the assumption of zero electron density above the LEO orbit, which is an unrealistic hypothesis for satellites such as CHAMP (orbiting at 450km).

The classical approach of Abel inversion assumes spherical symmetry (i.e. electron density only dependent on height). Considering that each ray of a Radio occultation defines an onion-layer as shown in Figure 1, an equivalent and discrete expression of the integral formulation of Abel transform is shown in Eq. 1. Therefore the electron density can be computed in a recursive way starting from the outer ray (the one with greatest distance between ray and earth surface).

$$STEC(p_i) = 2 \cdot l_{ii} \cdot N_e(p_i) + \sum_{j=1}^{j=i-1} 2 \cdot l_{ij} \cdot N_e(p_j) \quad (1)$$

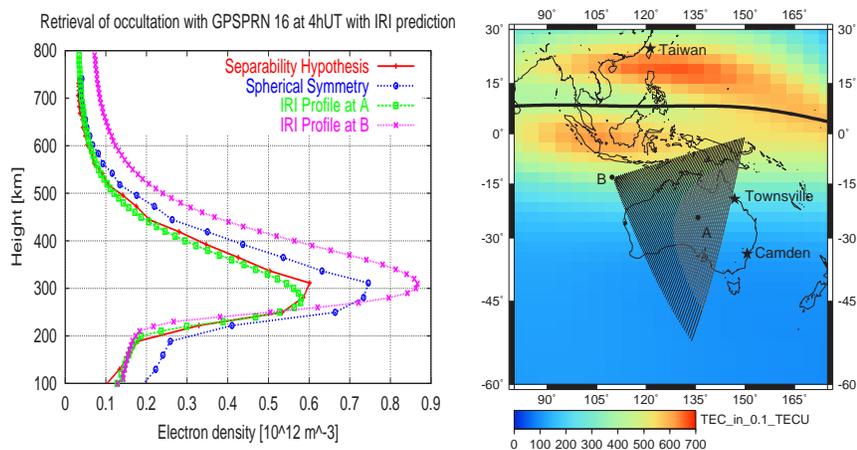
The main features of Abel inversion and the 3D voxel models are compared in Table 1. In the following sections it will be shown how the combination of both techniques may help to improve the results obtained with the classical approach of Abel inversion described above. In particular, the assumption of spherical symmetry is not realistic in general, and in particular for occultations where high VTEC gradients take place. Secondly, for LEOs orbiting at very low altitudes (for instance the CHAMP satellite), assuming zero values for the Electron density above its orbit leads to incorrect vertical profiles.



**Fig. 1.** Abel inversion is solved in a recursive way starting from the outer ray of the radio occultation

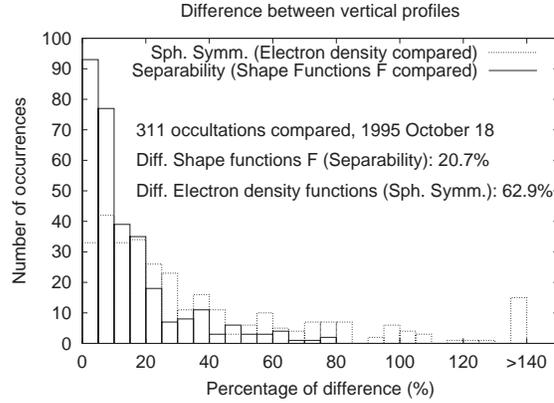
**Table 1.** Summary of Pros (in bold) and Cons of two tomographic techniques

	Abel transform	3D Voxels
Vertical resolution	$\approx 1$ km	10 ~ 100 km
Computational load	<b>Low</b> ( $\approx 10^2$ unk.)	High ( $\geq 10^3$ unk.)
Sph. Sym. assum.	Yes	<b>No</b>
Topside mismod.	Yes	<b>No</b>
Assimilation capab.	No	<b>Yes</b>

**Fig. 2.** Effect of Horizontal Gradients of VTEC in an occultation

## 2 Taking into account horizontal gradients in the Abel Inversion

A first improvement is to consider horizontal gradients of vertical TEC. These gradients can be obtained by means of a climatological model such as the IRI [1], or a data driven model, for instance those provided by the IGS in IONEX format [2]. In the right map of Figure 2 is depicted a typical occultation footprint of the LEO GPS/MET (orbiting at 750km). It can be noticed that this footprint may cover wide geographic areas, in this case  $50^\circ$  in latitude and longitude, besides it takes place in the southern Appleton anomaly. High gradients of VTEC take place in this location, the corresponding IRI profiles for points A (southern mid-latitudes) and B (near the Appleton anomaly) are quite different, these differences may reach values close to 40%.

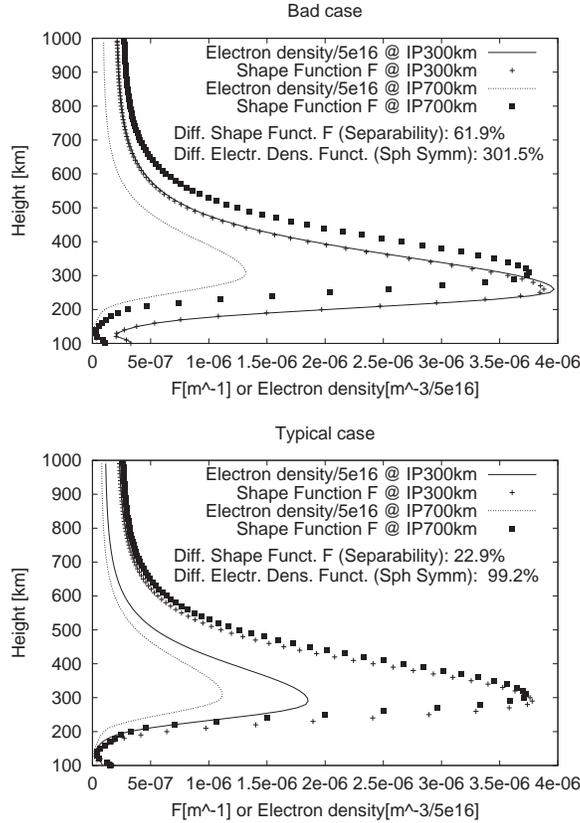


**Fig. 3.** Histogram with the percentage differences between the profiles at the geographic locations of impact parameters (IP) of 700km and 300km. In case of spherical symmetry Electron density profiles are directly compared. In the case of separability, Shape Functions are compared instead. Bottom plots depict two particular examples: high difference and typical case.

To take into account these variations, an alternative formulation was introduced in [9] where it was stated a *separability hypothesis* based in the fact that a vertical profile can be approximated by Eq. 2.

$$N_e(LT, LAT, H) = VTEC(LT, LAT) \cdot F(H) \quad (2)$$

That is, the vertical profile can be regarded as the product between a Vertical TEC function depending on the geographic location and time and a Shape Function  $F$  (i.e. Normalised Electron density function with height dependency), the unknown to be determined through the iterative process of Abel inversion. Although there is a geographic dependency on the function  $F$ , this is smaller than assuming Spherical Symmetry. In fact, the comparison of this  $F$  function at the geographic locations corresponding to Impact Parameters at 700km and 300km for a single occultation reveal that the average differences between the corresponding normalised profiles are about 20% (on the other hand spherical symmetry assumption has to deal with typical differences between profiles of electron density of more than 60%). This study has been made simulating the vertical profiles with the IRI at the corresponding geographic locations of the mentioned Impact Parameters for all occultations of day 1995 October 18 (311 occultations studied). Figure 3 shows a statistics and Figure 4 two particular examples of the horizontal variation of the Shape Function  $F$  with respect to the Electron density function.



**Fig. 4.** Electron densities and Shape Functions: Two particular examples with high differences between profiles (top) and a typical case (bottom).

A reformulation of Eq. 1 is shown in Eq. 3, where it expresses the Slant TEC seen by the LEO considering Vertical TEC information.

$$\begin{aligned}
 STEC(p_i) = & 2l_{ii}VTEC(LT_{ii}, LAT_{ii})F(p_i) + \\
 & \sum_{j=1}^{j=i-1} l_{ij}[VTEC(LT_{ij}, LAT_{ij}) + VTEC(LT'_{ij}, LAT'_{ij})] F(p_j) \quad (3)
 \end{aligned}$$

This formula shows how different Vertical TECs are considered in the different parts of the rays, allowing the horizontal gradients of TEC to be taken into account. General results obtained with either spherical symmetry and this separability hypothesis can be seen in [9], where estimations of  $f_oF2$  and  $f_oE$  using these two methods and the values provided by a Ionosonde were compared. It was shown that spherical symmetry gives poorer results than

the separability assumption. Figure 2 shows the difference between vertical profiles at different locations (A and B). The result of inverting that particular occultation with spherical symmetry gives an averaged profile between the two IRI profiles. If the assumption of separability is applied and the obtained normalised electron density function is multiplied by the corresponding TEC at the location, one is able to obtain the profile provided by IRI. In Figure 2 the Shape Function  $F$  has been multiplied by the TEC at the point A, but the IRI profile at location B would be obtained if the TEC at point B was used instead. Note that the IRI profile and the one of separability match closely. In [9] the inversion of synthetic data obtained with IRI using separability and spherical symmetry were compared with the vertical profiles obtained with IRI. The results obtained with the separability assumption were closer to the values of height and electron density provided by IRI.

Two additional examples with real data taking place near the Appleton anomaly can be seen in Figure 5, in this location the VTEC gradients are higher and this leads to significant differences between the two Abel inversion approaches. There is more agreement with the NmF2 value indicated by the Ionosonde when Horizontal TEC information is used, for these epochs and Ionosondes the hmF2 data was not available. Besides, the profiles obtained with the separability hypothesis have lower values in the lower boundary of the Ionosphere compared with spherical symmetry, this is more realistic since at that point electron density must be close to 0.

### 3 Taking into account the Topside Electron Content on the Abel Inversion

An additional assumption that is made when vertical profiles of electron density are retrieved from radio occultation is that the electron content above the LEO is negligible. This may be realistic to a certain extent for LEOs orbiting at high altitudes such as the GPS/MET or SAC-C (both with orbits above 700km), but this assumption clearly fails for LEOs with low altitude orbit such as the CHAMP (at about 400 km).

In [9] it was assumed an exponential decreasing function for the topside ionosphere, then one may compute the whole vertical profile iteratively in such a way that this assumption helps to converge the normalised electron density profile to the condition formulated in Eq. 4 when TEC based a real data are used.

$$\int F(h) \cdot dh = 1 \quad (4)$$

Figure 6 shows the effect of considering this iterative process with an exponential decreasing topside for two different satellites (GPSMET and

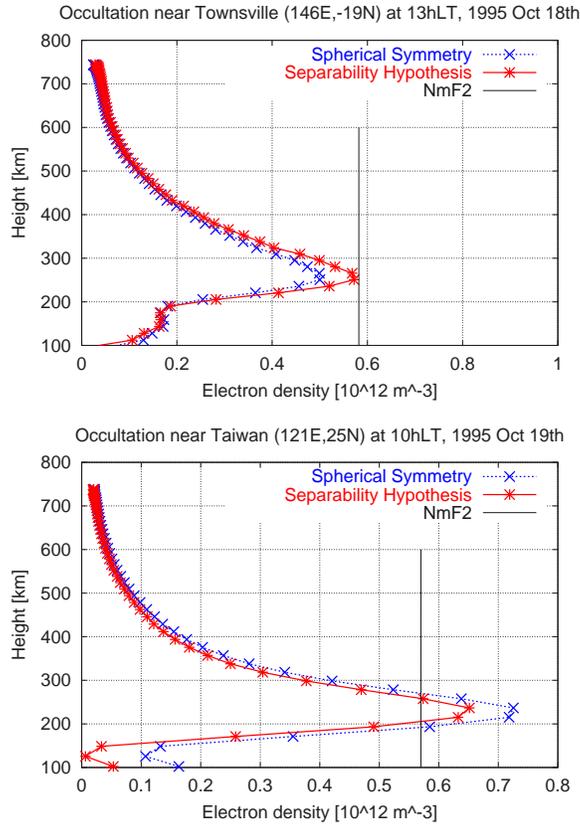
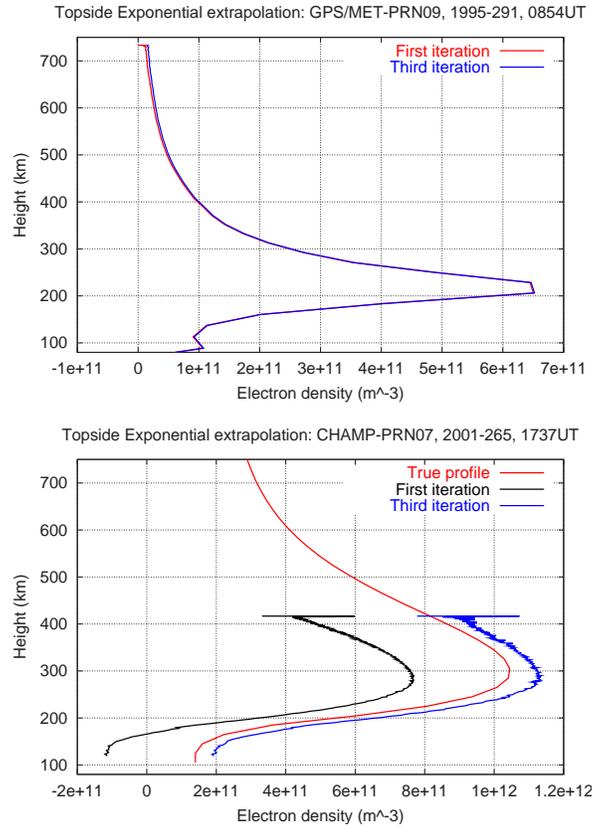


Fig. 5. Examples of retrieved occultations in a low latitude scenario

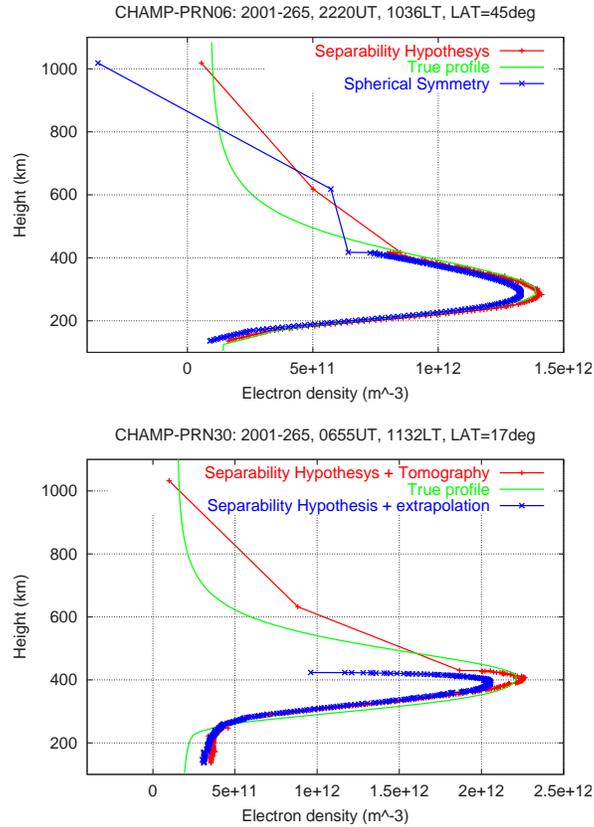
CHAMP). The left-hand side graphic shows two different iterations for a profile corresponding to a retrieved occultation of the GPS/MET with both actual geometry and data. In this case both iterations give similar results, indicating the fact that for this satellite the topside does not contribute in a significant way in computing the vertical profile of electron profile. This is not the case of the satellite CHAMP (is depicted in the right plot), where different iterations and the truth are shown (actual geometry and STEC values provided by the IRI are used). For this satellite it can be seen that the differences between iterations are greater. This indicates that neglecting the topside ionosphere must lead to incorrect results.

One way to improve this is by using observations with positive elevations in order to estimate the electron density of several extra layers above the CHAMP (for instance with 2 extra layers the number of unknowns only in-



**Fig. 6.** Effect of considering a negligible topside for the satellites GPS/MET (top, using both actual geometry and delays) and CHAMP (bottom, using actual geometry and synthetic-IRI-delays)

creases in two) and these values are combined with negative elevation data, more realistic profiles than the ones obtained with an exponential extrapolation are obtained. An example of this technique compared using spherical symmetry and separability can be seen in the left plot of Figure 7. The use of separability hypothesis improves the results of spherical symmetry even in conjunction with the proposed technique with tomography. A more difficult scenario happens when the Electron density peak coincides with the orbit height of the CHAMP (right plot of the panel), in this case the use of tomography clearly improves the result provided assuming an exponential decreasing function for the topside.

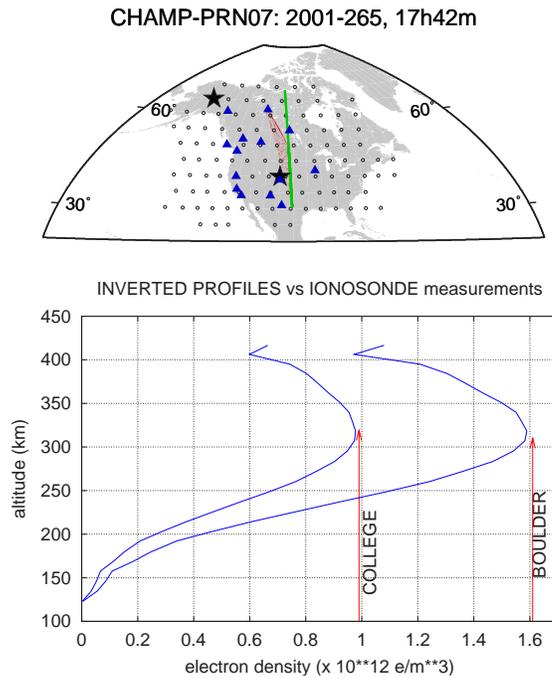


**Fig. 7.** (Top) Comparison of tomographic technique using spherical symmetry and separability. (Bottom) Comparison of separability assumption using tomographic approach or exponential extrapolation

An additional example corresponds to an occultation taking place over North America, see Figure 8. In the right plot of Figure 8 it is shown that the use of separability and topside voxel modeling provides estimation closer to the values of NmF2 and hmF2 given by Ionosonde.

## 4 Summary and Conclusions

This study has been focused on the advantages that the Abel transform provides to obtain vertical profiles of electron density: high resolution and low computational load. To exploit this potential, two weak spots are to be



**Fig. 8.** (Top) Map of occultation taking place over United States. The stars indicated the position of the ionosondes which data will be used for comparison (Boulder, in central North America, and College, in Alaska), while the triangles are the ground GPS stations used to compute the VTEC map that will provide the model with the horizontal VTEC gradients. (Bottom) Comparison of retrieved occultation with ionosonde values, the arrows point to NmF2 and hmF2.

tackled: spherical symmetry and zero value for the topside electron density content. This work proposes two approaches to solve this: horizontal gradients of VTEC have been introduced to the model (separability hypothesis), which can be applied not only on the LI observable but to the bending angle as well) and the topside electron content is estimated simultaneously. This is important to obtain realistic profiles either at mid and low latitudes. Besides topside contribution is computed by means of 2 extra layers above the LEO orbit using positive elevations observations. This is specially important for LEOs with very low orbits such as CHAMP. An overall improvement of up to 40% is obtained in the studied occultations.

## 5 Acknowledgements

IRI model has been provided by Dr. Bilitza. We are grateful to the IGS, UCAR and the GeoForschungsZentrum for providing with the ground GPS data, the GPS/MET data and the CHAMP observations respectively. The maps have been generated with the software package GMT. This work has been partially supported by the 'Generalitat de Catalunya' under fellowship number 2000FI-00395 and the Spanish projects TIC-2000-0104-P4-03 and TIC2001-2356-C02-02.

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